

Electroweak physics and the LHC

an introduction to the Standard Model

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Outline

- ❑ Prologue on weak interactions
- ❑ Express review of gauge theories
- ❑ SM gauge sector
- ❑ Hidden symmetries
- ❑ SM Higgs sector
- ❑ Precision tests of the SM
- ❑ anomalous magnetic moments
- ❑ Computing G_F
- ❑ Global fit and the Higgs mass
- ❑ Electroweak physics at LHC

A few references

books:

- Peskin & Schoeder, *Quantum Field Theory*
- Donoghue, Golowich, Holstein, *Dynamics of the SM*
- Cheng & Li, *Gauge theory of elementary particle physics*
- Becchi & Ridolfi, *Introduction to relativistic processes and the SM*

lectures sets:

- Altarelli, *hep-ph/0011078*
- Ridolfi, see <http://www.ge.infn.it/~ridolfi/>

Prologue

Weak forces are weak

$$\mathcal{L} = -\frac{G^{(\beta)}}{\sqrt{2}} \bar{p} \gamma^\alpha (1 - a\gamma_5) n \bar{e} \gamma_\alpha (1 - \gamma_5) \nu_e - \frac{G^{(\mu)}}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha (1 - \gamma_5) \mu \bar{e} \gamma_\alpha (1 - \gamma_5) \nu_e$$

Fermi Lagrangian describes beta (semileptonic) decays $a=1.269 \pm 0.003$

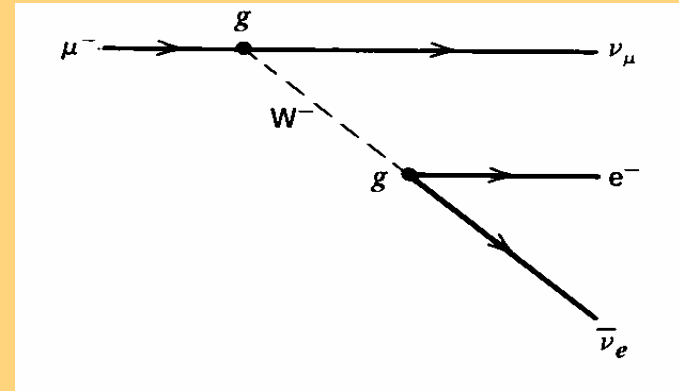
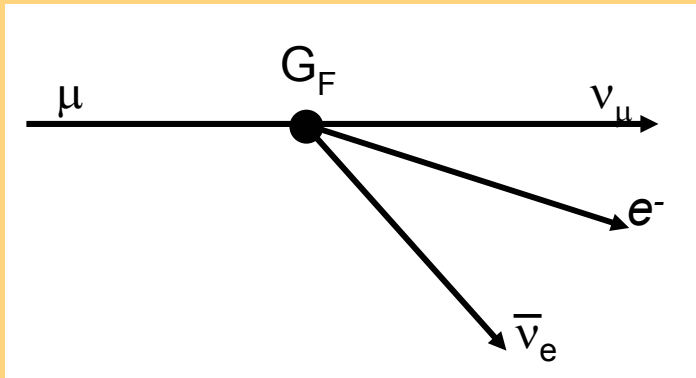
$$\mathcal{L} = - \sum_i \frac{G_F^{(i)}}{\sqrt{2}} J_i^\mu J_\mu^e$$

$$G^{(\mu)} \simeq 1.16639 \times 10^{-5} \text{ GeV}^{-2}; \quad G^{(\beta)} \simeq G^{(\mu)} = G_F$$

$$\text{typical process} \sim G_F E^2; \quad n \text{ decay } G_F m_p^2 \sim 10^{-5} \ll \alpha = 1/137$$

- ❑ growth with energy incompatible with unitarity: only valid up to $\Lambda \sim 100 \text{ GeV}$
- ❑ non-renormalizable: gives good predictions, but they cannot be consistently improved
- ❑ short range interaction $r_W \sim 1 \times 10^{-3} \text{ fm}$
- ❑ vector currents: Intermediate Vector Boson hypothesis

Intermediate Vector Boson



$$\frac{g^2}{p^2 - M_W^2} \approx -\frac{g^2}{M_W^2} \propto -G_F$$

Further lessons from $\mathcal{L}^{\text{Fermi}}$

$$\mathcal{L} = -\frac{G^{(\beta)}}{\sqrt{2}}\bar{p}\gamma^\alpha(1 - a\gamma_5)n\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e - \frac{G^{(\mu)}}{\sqrt{2}}\bar{\nu}_\mu\gamma^\alpha(1 - \gamma_5)\mu\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e$$

$$G^{(\mu)} \simeq 1.16639 \times 10^{-5} \text{ GeV}^{-2};$$

Improved fundamental theory should moreover include:

- **chiral** structure, P and C violation
- **universality** of weak coupling $G^{(\beta)} \simeq G^{(\mu)} = G_F$
- **flavor** violation (K_{l3}) but no flavor changing neutral currents (FCNC)
- common vector interactions hint at possible **electroweak unification** (Schwinger 1957, Glashow...)

All these points have a natural solution in
the framework of **Gauge theories**

Express review of gauge theories

Continuous symmetries

- **Classical Mechanics**: invariance of a system under cont. transf.
→ constants of motion $E, \vec{p}, \vec{J} \dots$
- **Quantum Mechanics**: O_i conserved if $[O_i, H]=0$, O_i generator of unitary transf. that leaves system unchanged
- **Field Theory** T: $\phi_i(x) \rightarrow U_{ij}\phi_j(x)$ leaves EOM or $S = \int d^4x \mathcal{L}$ unchanged:
it is a *symmetry*.

Noether Theorem: the current $j^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta \phi$
is conserved: $\partial_\mu j^\mu(x) = 0$.

$Q = \int d^3x j_0(x)$ is constant and generates the transformation

At the quantum level the symmetry generates Ward identities
between Green's functions

Noether currents \leftrightarrow physical weak currents
?

Examples of global symmetries

- $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 |\phi|^2 - \lambda |\phi|^4$

$\phi(x) \rightarrow e^{i\alpha} \phi(x)$ change of phase

charge current $J_\mu = i [\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi]$ is conserved

- $\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda/2 (\phi^\dagger \phi)^2$ $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

$SU(2)$ invariant under $\phi \rightarrow \phi + \frac{1}{2} i \varepsilon_j \tau_j \phi$ τ_j Pauli matrices

3 conserved currents, 3 charges satisfy Lie algebra $[Q_a, Q_b] = i \varepsilon_{abc} Q_c$

- $\mathcal{L} = \bar{N}(i\not{\partial} - M)N + \frac{1}{2}[\partial_\mu \vec{\pi} \partial^\mu \vec{\pi} - m_\pi^2 \vec{\pi}^2] + ig \bar{N} \vec{\tau} \cdot \vec{\pi} \gamma_5 N - \frac{\lambda}{4} \vec{\pi}^4$

with $N = \begin{pmatrix} p \\ n \end{pmatrix}$, $\vec{\pi} = \pi_i$ $i = 1, 2, 3$ $m_\pi \approx 140 \text{ MeV}$, $M \approx 940 \text{ MeV}$

is $SU(2)$ invariant wrt $N \rightarrow e^{-i \frac{\vec{\tau} \cdot \vec{\pi}}{2}} N = U N$ and $\vec{\tau} \cdot \vec{\pi} \rightarrow U \vec{\tau} \cdot \vec{\pi} U^\dagger$

Degenerate multiplets, universal couplings, conserved currents

$V_\mu^i = \bar{N} \gamma_\mu \frac{\tau_i}{2} N + \varepsilon^{ijk} \pi^j \partial_\mu \pi^k \simeq J_\mu^W + \dots$ CVC hypothesis $\rightarrow \pi \ell_3$ decay

Gauge theories: local abelian symmetry

Dirac free $\mathcal{L} = \bar{\psi}(i \not{\partial} - m) \psi$ invariant under

$$\psi \rightarrow e^{i e \alpha} \psi, \quad \bar{\psi} \rightarrow e^{-i e \alpha} \bar{\psi}$$

If $\alpha = \alpha(x)$ *local invariance* requires

$\partial^\mu \rightarrow \mathbf{D}^\mu \equiv \partial^\mu - i e \mathbf{A}^\mu$, covariant derivative [*minimal coupling*]

$\mathbf{A}^\mu \rightarrow \mathbf{A}^\mu + \partial^\mu \alpha(x)$, \mathbf{A}^μ real vector field [*E&M gauge invariance*]

$$\begin{aligned} \mathcal{L}^{QED} &= \bar{\psi} (i \mathbf{D} - m) \psi - \frac{1}{4} \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} \\ &= \bar{\psi} (i \not{\partial} - m) \psi + e \mathbf{A}^\mu \mathbf{J}_\mu - \frac{1}{4} \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} \end{aligned}$$

with $\mathbf{F}^{\mu\nu} \psi = i/e [\mathbf{D}^\mu, \mathbf{D}^\nu] \psi$ gauge invariant

No photon gauge invariant mass term. Ward identities $k_\mu M^\mu(k) = 0$

Non-abelian local symmetry

SU(N): N^2-1 generators t^a in representation R,

$$[t^a, t^b] = i f^{abc} t^c; \quad f^{abc} \text{ antisymmetric}$$

Generic element $U = e^{ig\alpha^a t^a}$ identifies a gauge t. of ψ **Yang-Mills (1954)**

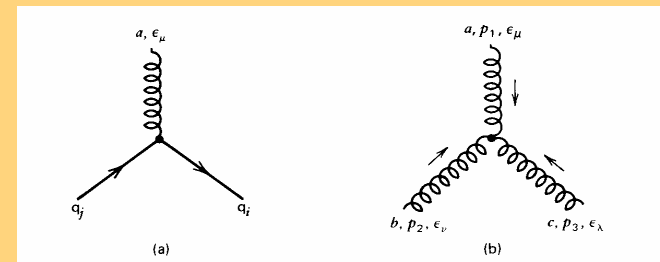
Covariant derivative $D^\mu = \partial^\mu - i g A^\mu_a t^a$, $D^\mu \rightarrow U D^\mu U^{-1}$ (D^μ transf. like ψ)

$$F_a^{\mu\nu} t^a \psi = i/g [D^\mu, D^\nu] \psi$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

kinetic term $-\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$ is gauge invariant, unlike $F_a^{\mu\nu}$

Gauge field self-interaction imposed by gauge invariance. Yang-Mills theories are non-trivial even without matter fields. Gauge fields carry charge: cons. currents include a pure gauge term



Gauge theories: symmetry dictates dynamics

- By promoting global to local **symmetry** [*local gauge principle*] gauge theories allow for vector bosons
- **Symmetry** makes some d.o.f. redundant: $A^0, \nabla \cdot \mathbf{A}$ are *c-numbers*. Gauge fixing necessary to quantize theory.
- **Symmetry** dictates form of allowed interactions. Gauge fields self-interact, hence single universal coupling for each group $-g J_\mu^a A^\mu_a$
- **Symmetry** permits the renormalization of gauge theory
- **Symmetry** forbids mass terms for the vector bosons. QCD, based on SU(3) is the most beautiful realization [*asymptotic freedom*]

The gauge sector of the SM

Which gauge symmetry?

We assume that currents in $\mathcal{L}^{\text{Fermi}}$ are Noether currents

$$J_\mu^\ell = \bar{\nu} \gamma_\mu \frac{1}{2} (1 - \gamma_5) e = \bar{\nu}_L \gamma_\mu e_L$$

Let's group e_L, ν_L in a doublet $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

$$\tau^+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad J_\mu = \bar{L} \gamma_\mu \tau^+ L, \quad J_\mu^\dagger = \bar{L} \gamma_\mu \tau^- L$$

$$J_\mu^3 = \bar{L} \gamma_\mu \tau^3 L = \bar{\nu}_L \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L$$

SU(2) group in fundamental representation, simplest choice ν_R, e_R singlets
Neutral current $J_\mu^3 \neq J_\mu^\gamma$ (because of chirality & neutrinos) what is it?

Now promote SU(2) to local symmetry: $D^\mu = \partial^\mu - i g W_\mu^\pm T_\pm$; $W_\mu^\pm = (W_\mu^1 \mp i W_\mu^2) / \sqrt{2}$

$$\mathcal{L} = i \bar{L} \not{D} L + i \bar{\nu}_R \not{\partial} \nu_R + i \bar{e}_R \not{\partial} e_R = \text{kin.} + \frac{g}{\sqrt{2}} J_\mu W^\mu + \text{h.c.} + \frac{g}{2} J_\mu^3 W_3^\mu$$

Explains G_F in terms of gauge couplings $G_F = \sqrt{2} g^2 / 8 M_W^2$

Neutral currents: electroweak unification

One way to solve the NC problem is to extend the gauge group $SU(2) \rightarrow SU(2) \times U(1)$. Extra abelian **hypercharge** Y differs for L,R fields. $D^\mu = \partial^\mu - i g W_i^\mu T_i - \frac{1}{2} i g' Y B^\mu$

$$\mathcal{L}^{NC} = \sum_\psi \left[g \bar{\psi} \gamma_\mu T_3 \psi W_3^\mu + g' \bar{\psi} \gamma_\mu \frac{Y}{2} \psi B^\mu \right] \quad T_3 = \pm \frac{1}{2}, 0$$

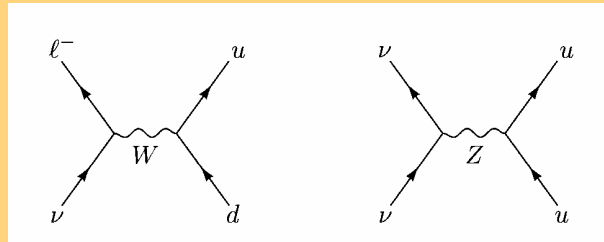
$$B^\mu = A^\mu \cos \theta_W - Z^\mu \sin \theta_W; \quad W_3^\mu = A^\mu \sin \theta_W + Z^\mu \cos \theta_W$$

In order to have J_μ^γ coupled to A^μ : $T_3 g \sin \theta_W + \frac{1}{2} Y g' \cos \theta_W = eQ$

The choice $Y(L)=-1$ $Y(e_R)=-2$ $Y(\nu_R)=0$ implies **$g \sin \theta_W = g' \cos \theta_W = e$**

The Z neutral current has charge $Q_Z = (T_3 - Q \tan^2 \theta_W) / \cos \theta_W$, $\tan \theta_W = \sin \theta_W / \cos \theta_W$

A definite prediction: weak NCs have been first observed in 1973!



Hadronic currents

Using **QUARK L doublets and R singlets**, it's like for leptons **but flavor change** has long been observed in charged currents (CC).

On the other hand, **FCNCs strongly suppressed** (higher order effects):

$$\text{ex.: } \frac{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)}{\Gamma(K^+ \rightarrow \pi^0 e^+ \nu)} \simeq \left| \frac{\bar{s} \rightarrow \bar{d} e^+ e^-}{\bar{s} \rightarrow \bar{u} e^+ \nu} \right|^2 \approx 10^{-5}$$

Solution (Glashow, Iliopoulos, Maiani 1970): quark doublets

$$Q = \begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \text{ where } \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

in this way NC conserve flavor (at lowest perturbative order):

$$J_3^\mu \propto \bar{d}'_L \gamma^\mu d'_L = \bar{d}_L \gamma^\mu V_{CKM}^\dagger V_{CKM} d_L = \bar{d}_L \gamma^\mu d_L$$

The CKM matrix → Ciuchini's lectures

describes Flavor Violation (mixing between generations of quarks) in the SM

$$\mathcal{L}_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_{i,L} \gamma^\mu W_\mu^+ d_{j,L} + \text{h.c.}$$

Wolfenstein parameterization

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9740 \pm 0.0010 & 0.2196 \pm 0.0023 & 0.0040^{+0.0006}_{-0.0007} \\ 0.224 \pm 0.016 & \mathbf{0.91 \pm 0.16} & \mathbf{0.0402 \pm 0.0019} \\ < 0.010 & \simeq 0.0400 & 0.99 \pm 0.29 \end{pmatrix}$$

3 angles and 1 phase with strong hierarchy:

$\lambda \sim 0.22$ sine of Cabibbo angle, $A, \rho, \eta = O(1)$

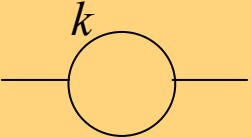
The CKM phase is the only source of CP violation in the SM

Summary of matter fields

	SU(2)	U(1) (Y)	SU(3) _{QCD}
L	2	-1	1
e _R	1	-2	1
Q	2	1/3	3
u _R	1	4/3	3
d _R	1	2/3	3
ν _R	1	0	1

NB SU(2)xU(1) is semisimple: Y is arbitrary → no charge quantization in SM

Renormalizability

- * Loops integrations  generally divergent in the UV

$$\int d^4k \frac{1}{(k^2 - m^2)^2} \propto \ln \frac{\Lambda^2}{m^2} + \dots \quad \Lambda \text{ cutoff}$$

- * A theory is **renormalizable** if all divergences can be reabsorbed at each pert. order in a redefinition of the parameters of \mathcal{L} .
- * systematics of renormalization: $\dim \leq 4$ terms in \mathcal{L} are *generally* renormalizable. Cutoff dependence is **power-suppressed**.
- * Yang-Mills gauge theories are renormalizable, like QED
- * Renormalizability **guiding principle** in SM evolution: weak coupling renormalizable th. are predictive, have small pert. corrections
- * Massive vector bosons: $\mathcal{L} = -\frac{1}{4}(\partial^\mu W^\nu - \partial^\nu W^\mu)(\partial_\mu W_\nu - \partial_\nu W_\mu) + M_W^2 W_\mu W^\mu / 2$

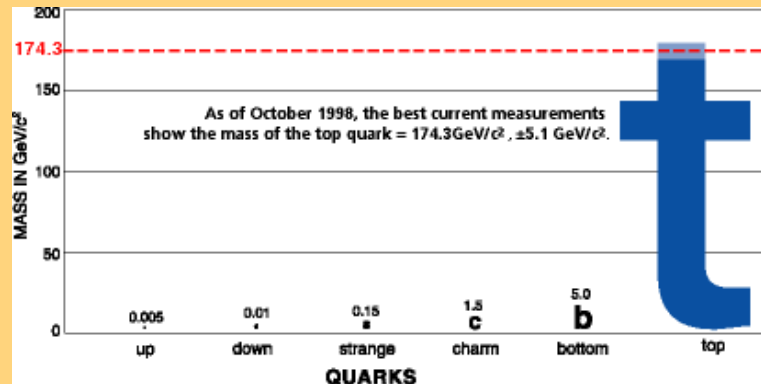
$$\Delta^{\mu\nu} = \frac{i}{k^2 - M_W^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{M_W^2} \right) \rightarrow \text{const for } k \rightarrow \infty$$

A massive problem

Also fermion masses break SU(2) symmetry:

$$m_e \bar{e}e = m_e [\bar{e}_L e_R + \bar{e}_R e_L]$$

because e_R and e_L belong to different multiplets.



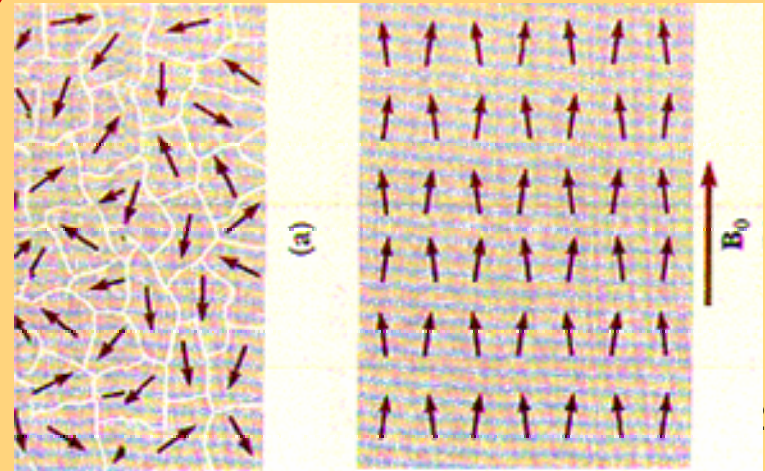
Exp: Currents are conserved to high accuracy!
SU(2)xU(1) works beautifully

BUT HOW DO WE GET THE MASSES?

On the fate of symmetries

- Symmetries can be **exact**: $U(1)_{em}$, $SU(3)_{QCD}$, B-L
- Or they can be **explicitly broken** by (small) terms: $SU(2)$ isospin is broken by $m_{u,d}$ and by QED Still useful.
- They can be broken by quantum corrections, have **anomalies**: eg scale invariance in massless QFT is anomalous, a new scale appears.
- They can be **Spontaneously broken**: the ground state is NOT symmetric, although the interactions respect the symmetry. Two possibilities: a scalar field acquires a vev, or dynamical breaking (chiral $SU(2)_R \times SU(2)_L$ of strong int)

It is quite common in nature that the lowest energy state is not symmetric: ex **ferromagnet** below the Curie temperature



A miraculous cancellation

Axial anomaly: impossible to regularize a field th in a way that preserves both axial and vector conservation
Ward id. (gauge invariance) spoiled by loops (triangle)

$$J_A^\mu = \psi \gamma^\mu \gamma^5 \psi; \quad \partial_\mu J_A^\mu = \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}/4\pi^2 + \text{mass terms}$$

Non-abelian case: **anomaly** $\propto \text{Tr}(\{T^a, T^b\} T^c)$ $T^i = \tau^i, Y$
SU(2) not anomalous, yet

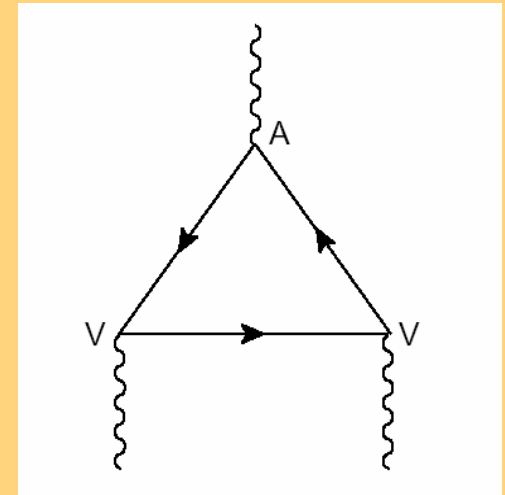
$$\text{ex } \text{Tr}(\{\tau^a, \tau^b\} Y) = 2\delta^{ab} \text{Tr } Y_L =$$

$$2\delta^{ab} [n_q \times 3 \times 2 \times 1/3 + n_l \times 2 \times (-1)] \propto n_q - n_l$$

and similarly for all other gauge currents of SM.

Why? hint of GUTs?

SM has also **accidental** global symmetries, rephasing invariance that is a consequence of the assumed gauge symmetry and renormalizability: B, L_e, L_μ, L_τ .
B and L are anomalous, B-L is not. $B(\mu \rightarrow 3e) < 10^{-12}$ but cosmological consequences



Hidden symmetry

we need a mechanism of Spontaneous Symmetry Breaking (SSB)

$$m \neq 0 \iff \langle 0 | X | 0 \rangle \neq 0 \quad (\text{v.e.v.})$$

X could be a scalar field or condensate, should be SU(2) doublet, $v_{ev} \sim 250 \text{ GeV}$
Langrangian symmetric, currents conserved, spectrum and vacuum not invariant.

Goldstone theorem: as many massless bosons as the broken continuous symmetries: if $Q_i |0\rangle \neq 0$ but $[H, Q_i] = 0$, $Q_i |0\rangle$ is degenerate with $|0\rangle$.

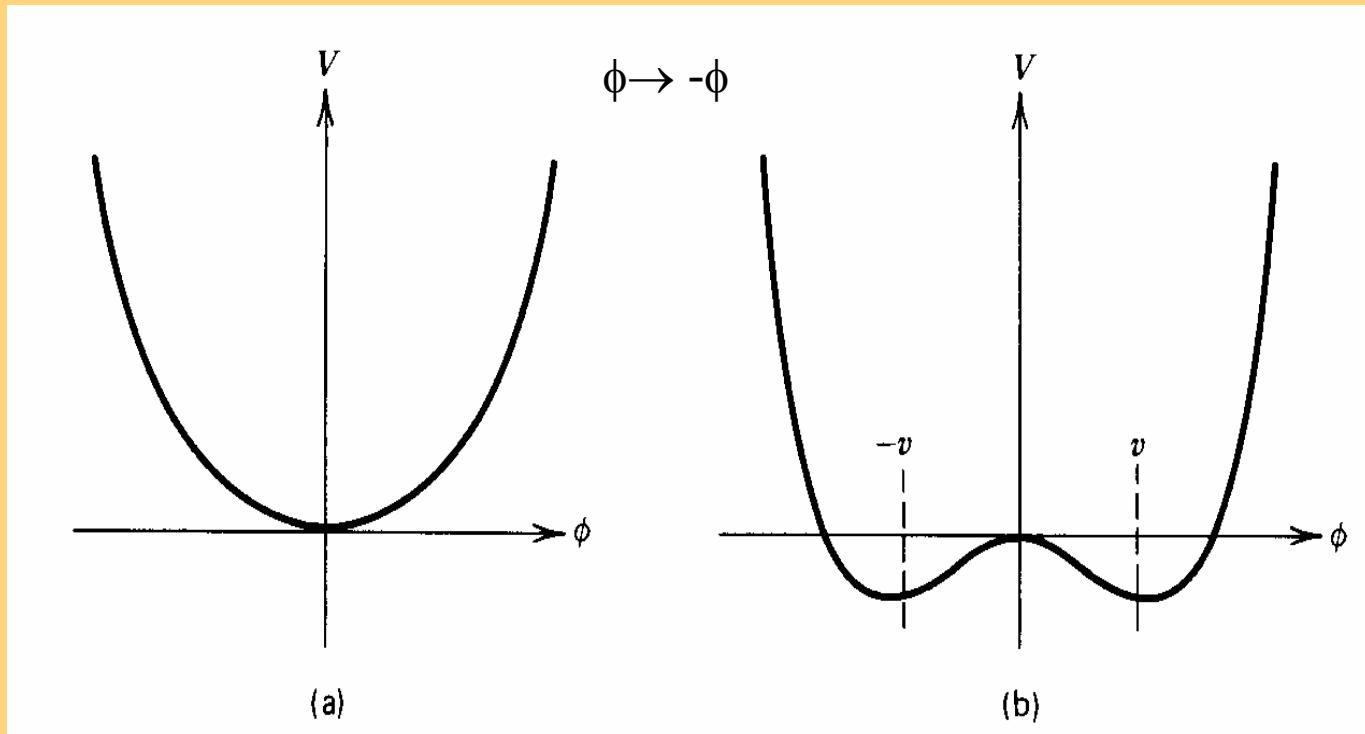
Goldstone th is EVADED in gauge theories due to longitudinal vector bosons.

SSB does not spoil renormalizability (soft breaking)

The Higgs mechanism realizes SSB in SM in the most economical way:
X is single complex doublet of fundamental scalars, predicting the existence of a new particle, the **HIGGS BOSON**.

At the same time massive vector bosons are quantized without spoiling renormalizability and unitarity.

Hidden (discrete) symmetry



$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \rightarrow \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4$$

Potential minimized by $\phi=0 \rightarrow \phi = \pm \mu/\lambda^{1/2} = v \quad \langle 0|\phi|0 \rangle = v$

excitations around vacuum: $\phi = v + \phi'$ symmetry is no longer manifest

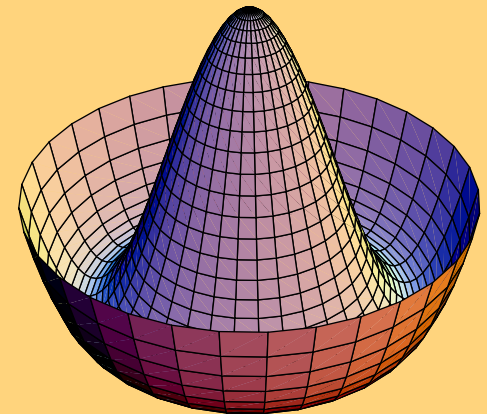
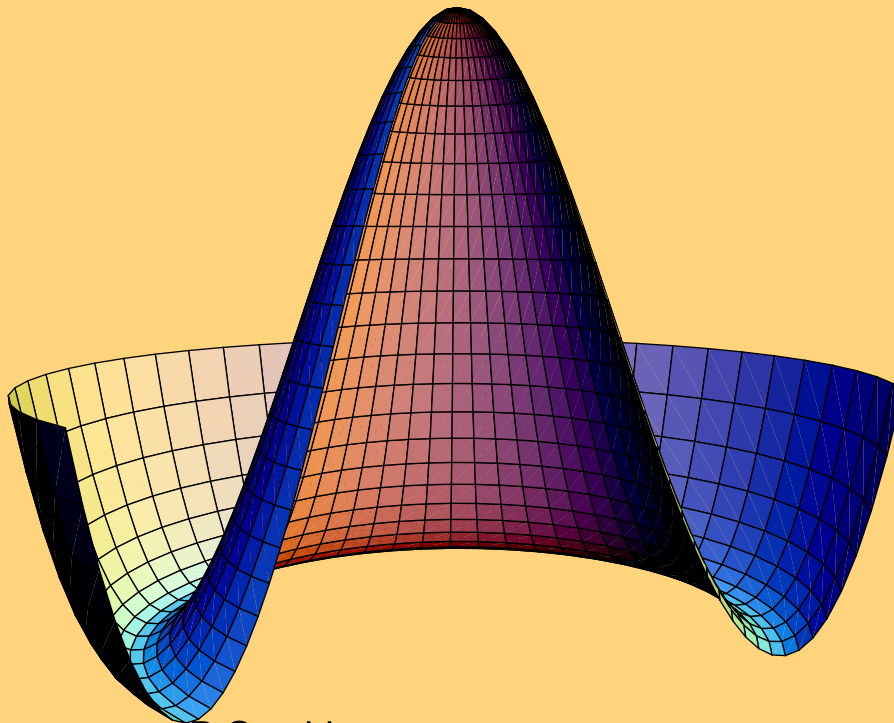
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi')^2 - m^2 \phi'^2 - \lambda v \phi'^3 - \frac{1}{4} \lambda \phi'^4$$

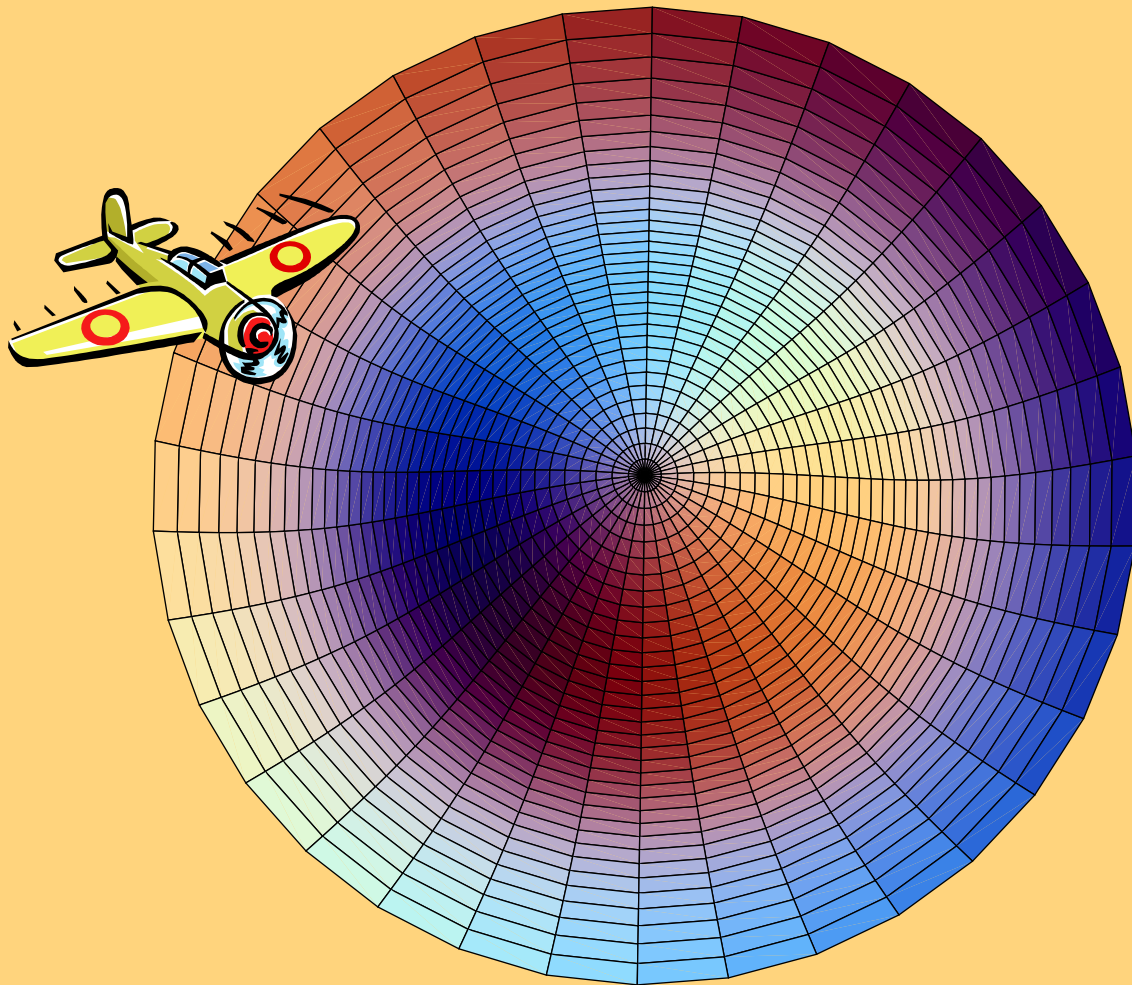
Abelian Higgs mechanism

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$ U(1) invariant

with $D_\mu = \partial_\mu - i e A_\mu$ and $V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$ (most general renorm.)

If $m^2 < 0$, $\lambda > 0$, we have an infinite number of degenerate vacua for $|\phi|^2 = -m^2/2\lambda \equiv \frac{1}{2} v^2$ connected by gauge transf.





Symmetry becomes apparent only at high energies

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Abelian Higgs mechanism (II)

Decompose $\phi(x) = [v + H(x) + i G(x)]/\sqrt{2}$ H, G real

In L the terms $\propto H, G^2$ vanish ie G is a Goldstone boson.

$(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2} e^2 v^2 A_\mu A^\mu + \dots$ **VB has acquired a mass $M = e v$**

After proper gauge fixing $-(\partial^\mu A_\mu + e v \xi G)^2/2\xi$

the A propagator becomes

$$\Delta^{\mu\nu} = \frac{i}{k^2 - M_A^2} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - M_A^2 \xi} \right]$$

good UV behaviour. In the limit $\xi \rightarrow \infty$ (unitary gauge)

the Goldstone boson decouples and one recovers the usual propagator

The number of dof is constant: we had a complex ϕ , now we have H and the longitudinal polarization of A

Higgs mechanism in the SM

we want $\langle 0|\phi|0\rangle=v\neq 0$ ϕ in red. repr. of $SU(2)\times U(1)$ but must preserve $U(1)_{em}$: SSB concerns 3 generators

$$V(\phi)=m^2|\phi|^2+\lambda|\phi|^4 \quad \text{minimized by } |\phi_0|^2=-m^2/2\lambda\equiv v^2/2$$

Simplest solution: ϕ doublet $\rightarrow U(1)_{em}$ inv. imposes $Y=1$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \text{ in unitary gauge } \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$V(H) = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4 \quad M_H = \sqrt{2\lambda}v$$

from covariant derivatives

$$M_W^2 = \frac{1}{4}g^2v^2 \quad M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2 \quad m_\gamma = 0$$

$$v = \sqrt{1}G_F\sqrt{2} \simeq 246 \text{ GeV}$$

$$\text{natural relation } \rho_0 = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{g^2}{(g^2 + g'^2) \cos^2 \theta_W} = 1$$

$$\text{with more } \phi \text{ doublets, triplets... } \rho_0 = \frac{\sum_i [(t_i)^2 + t_i - (t_i^3)^2] v_i^2}{\sum_i 2(t_i^3)^2 v_i^2} \neq 1$$

Yukawa couplings

ϕ can couple to matter fields as well. Most general gauge inv and renorm form

$$\mathcal{L}_Y^{hadr} = -\bar{Q}' \phi h'_d d'_R - \bar{d}'_R \phi^\dagger h'^{\dagger}_d Q' - \bar{Q}' \phi_c h'_u u'_R - \bar{u}'_R \phi_c^\dagger h'^{\dagger}_u Q'$$

$$\text{where } \phi_c = \epsilon \phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$$

The Yukawa matrices are diagonalized by biunitary transf

$$h_{u,d} \equiv V_L^{u,d\dagger} h'_{u,d} V_R^{u,d}$$
$$u'_{L,R} = V_{L,R}^u u_{L,R} \quad d'_{L,R} = V_{L,R}^d d_{L,R}$$

$$\text{Unitary gauge: } \mathcal{L}_Y^{hadr} = -\frac{1}{\sqrt{2}}(v + H) \sum_f (h_d^f \bar{d}^f d^f + h_u^f \bar{u}^f u^f)$$

SM accomodates flavor: there is no theory of flavor!
masses \propto Yukawas only for 1 doublet

Leptons (no ν mass): same but no LFV, $\text{BR}(\mu \rightarrow e\gamma) < 10^{-11}$