Written Examination Special Relativity MFN 1313

Academic Year 2015–2016: 4 July 2016, 2-4 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

1.

i) What is the relativistic Lagrangian for a free particle of mass m and speed v? Show that for velocities $v << c$ it differs from the non–relativistic Lagrangian by a constant.

ii) Calculate the relativistic 3-momentum and relativistic energy for the particle of i), and show how they are related.

2. A rocket propels itself rectilinearly through empty space by emitting pure radiation (photons) in the direction opposite to its motion. If V is its final velocity relative to its initial rest - frame, show that the ratio of the initial to the final rest mass of the rocket is given by

$$\frac{M_i}{M_f} = \left(\frac{c+V}{c-V}\right)^{\frac{1}{2}}$$

HINT: Choose a suitable frame, and use conservation of 4-momentum.

3. Find the 4×4 matrix for the Lorentz transformation consisting of a boost $\beta_{\mathbf{1}}$ in the x–direction followed by a boost $\beta_{\mathbf{2}}$ in the y–direction. Show that the boosts performed in the reverse order would give a different transformation. Would this change if $\beta_{\mathbf{1}} = \beta_{\mathbf{2}}$? Discuss how to construct a generic Lorentz transformation.

4. Consider the components of a 4-vector V^α as the matrix (with $i^2 = -1$)

$$\mathcal{V} = \begin{pmatrix} V^0 + V^3 & V^1 + iV^2 \\ V^1 - iV^2 & V^0 - V^3 \end{pmatrix}$$

i) Show that V^α satisfies $V^\alpha V_\alpha = Det\ \mathcal{V}$

ii) Show that the transformation $\mathcal{V} \to \mathcal{V}' = \mathcal{A}\mathcal{V}\mathcal{A}$ where

$$\mathcal{A} = \begin{pmatrix} \omega^{-\frac{1}{2}} & 0 \\ 0 & \omega^{\frac{1}{2}} \end{pmatrix}$$

and ω is a constant, corresponds to a Lorentz boost along the z axis. What is ω in terms of the boost velocity v, and the rapidity parameter θ?