# Written Examination Special Relativity F8066

## Academic Year 2006–2007: 25 June 2007, 2.30-4.30 PM

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**Please read the following INSTRUCTIONS**

**A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.**

**B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.**

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**1.** In an inertial frame of reference $S$ event $A$ occurs before event $B$. These two events are separated by a time interval $\Delta t_{AB}$ and a distance $\Delta x_{AB}$.

(a) What are the conditions on $\Delta t_{AB}$ and $\Delta x_{AB}$ such that event $A$ will **always** be observed to occur before event $B$ in all inertial frames? **Ans.** causally connected i.e. $|\Delta x_{AB}| \leq c|\Delta t_{AB}|$.

(b) Suppose that $A$ and $B$ both occur on the $x$–axis (i.e. $y = z = 0$) in frame $S$. Another inertial frame $S'$ moves with velocity $\vec{v} = v\hat{x}$ relative to $S$. Under what conditions would the two events be *simultaneous* in $S'$? **Ans.** $\Delta t'_{AB} = 0$ when $\beta = \frac{c\Delta t_{AB}}{\Delta x_{AB}}$

(c) Again suppose that $A$ and $B$ both occur on the $x$–axis (i.e. $y = z = 0$) in frame $S$, and that $S'$ moves with velocity $\vec{v} = v\hat{x}$ relative to $S$. Under what conditions would the two events occur at the same point in space in $S'$? **Ans.** $\Delta x'_{AB} = 0$ when $v = \frac{\Delta x_{AB}}{\Delta t_{AB}}$

**2.** A $\mu$–meson with an average lifetime of $2 \times 10^{-6}$sec is created in the upper atmosphere at an altitude of 6000 m. When it is created it has a velocity of $0.998c$ in a direction towards the earth.

(a) What is the average distance that it will travel before decaying, as determined by an observer on earth?**Ans.** $v\gamma(v)\tau = 9473m$

(b) Consider an observer at rest with respect to the $\mu$–meson. What is the distance this observer measures from the point of creation of the $\mu$–meson to the earth? **Ans.** $\gamma(v) = 15.82$, so $\frac{6000m}{\gamma(v)} = 379.27$m

(c) Comment on the physical significance of these results. **Ans.** Relativistic effects allow observance of otherwise unobservable phenomena.

**3.** A particle of rest mass $m_1$ moving along the $x$–axis collides elastically with a stationary particle of rest mass $m_2$. As a result $m_1$ and $m_2$ are deflected through angles $\alpha$ and $\beta$ respectively. If $E$ and $E'$ are the total energies of the particle $m_1$ before and after the collision, show that

$$\cos\alpha = \frac{(E + m_2c^2)E' - m_2c^2E - m_1^2c^4}{\sqrt{(E^2 - m_1^2c^4)(E'^2 - m_1^2c^4)}}$$

**Ans.** Conservation of total 4–momentum, eliminate $\beta$.