Written Examination Special Relativity F8066

Academic Year 2008–2009: 29 June 2009, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. Find the matrix for the Lorentz transformation consisting of a boost $\beta_{\mathbf{1}}$ in the x–direction followed by a boost $\beta_{\mathbf{2}}$ in the y–direction. Show that the boosts performed in the reverse order would give a different transformation. Discuss how to construct a generic Lorentz transformation.

2. Two particles of rest masses m_1 and m_2, and collinear velocities $\vec{u}_1$ and $\vec{u}_2$, coalesce to form a single particle. Show that the mass m and speed u of the resulting particle are given by

$$m^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma_1 \gamma_2 \left(1 - \frac{u_1 u_2}{c^2}\right)$$

$$u = \frac{m_1 \gamma_1 u_1 + m_2 \gamma_2 u_2}{m_1 \gamma_1 + m_2 \gamma_2}$$

,
where ${\gamma_1}^{-2} = 1 - \frac{u_1^2}{c^2}, {\gamma_2}^{-2} = 1 - \frac{u_2^2}{c^2}$.

Ans. Conservation of four–momentum.

3. An airplane sets out to fly at $800 km/h$. Show that it would have to fly for more than a thousand years in order to make a difference of $\frac{1}{100} sec$ between the times recorded by a clock in the airplane and a clock on the ground.

Ans. $\Delta t - \Delta\tau = (\gamma - 1)\Delta\tau = 10^{-2}s, \frac{v}{c} = 0.8.10^{-6}$ so $\Delta\tau \approx 3.6.10^{10}s > 10^3$ years.