Written Examination Special Relativity F8066

Academic Year 2009–2010: 29 June 2010 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. What is the velocity of the centre of mass for a system consisting of a photon of energy E and a stationary atom of rest mass M?
i) Would this velocity change (and if so, how) if instead of a photon, there was a particle of rest mass m and the same energy E?
ii) What is the ratio between the photon frequencies in the centre-of-mass and laboratory frames?
Ans: $v = \frac{Ec}{E+Mc^2}$, i) Yes, $\frac{\sqrt{E^2-m^2c^4}}{E+Mc^2}$, ii) $\sqrt{\frac{Mc^2}{Mc^2+2E}}$

2a. Given the components of a tensor M^{ab} as the matrix

$$M^{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & -2 & 0 \end{pmatrix}$$

find
(i) the components of its symmetric part $M^{(ab)}$
(ii) the components of its antisymmetric part $M^{[ab]}$
(iii) the components of $M^a{}_b$
(iv) the components of $M_a{}^b$
(iv) the components of M_{ab}

2b. For the tensor whose components are $M^a{}_b$, does it make sense to speak of its symmetric and antisymmetric parts? If so, define them. If not, say why.

3. i) Write out the Lorentz matrix condition and give two explicit examples of Lorentz matrices.
ii) Derive at least 3 properties of Lorentz matrices from the Lorentz matrix condition.