Written Examination Special Relativity MFN 1313

Academic Year 2013–2014: 25 June 2014, 2.00-4.00 PM

Please read the following INSTRUCTIONS

Answer at most TWO questions. You may answer in english or in italian. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

1.a) What is the velocity of the centre of mass for a system consisting of a photon of energy E and a stationary atom of rest mass M? **Ans:** $v = \frac{Ec}{E+Mc^2}$

b) Would this velocity change (and if so, how) if instead of a photon, there was a particle of rest mass m and the same energy E? **Ans:** Yes, $\frac{\sqrt{E^2-m^2c^4}}{E+Mc^2}$,

2. Show that the sequence of n consecutive parallel Lorentz boosts, each with velocity $u = c\ \tanh\theta$, is equivalent to a single Lorentz boost in the same direction with velocity $c\left(\frac{z^n-1}{z^n+1}\right)$, where $z = e^{2\theta}$.**Ans:** $\tanh\theta = \frac{u}{c}$.

3. In an inertial frame of reference S event A occurs before event B. These two events are separated by a time interval Δt_{AB} and a distance Δx_{AB}.

a) What are the conditions on Δt_{AB} and Δx_{AB} such that event A will **always** be observed to occur before event B in all inertial frames? **Ans.** causally connected i.e. $|\Delta x_{AB}| \leq c|\Delta t_{AB}|$.

b) Suppose that A and B both occur on the x–axis (i.e. $y = z = 0$) in frame S. Another inertial frame S' moves with velocity $\vec{v} = v\hat{x}$ relative to S. Under what conditions would the two events be *simultaneous* in S'? **Ans.** $\Delta t'_{AB} = 0$ when $\beta = \frac{v}{c} = \frac{c\Delta t_{AB}}{\Delta x_{AB}}$

c) Again suppose that A and B both occur on the x–axis (i.e. $y = z = 0$) in frame S, and that S' moves with velocity $\vec{v} = v\hat{x}$ relative to S. Under what conditions would the two events occur at the same point in space in S'? **Ans.** $\Delta x'_{AB} = 0$ when $\beta = \frac{v}{c} = \frac{\Delta x_{AB}}{c\Delta t_{AB}}$