Written Examination Special Relativity F8066

Academic Year 2005–2006: 11 July 2006, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. A radioactive nucleus moves with a constant speed $0.6c$ relative to the laboratory. The nucleus decays and emits an electron with a speed $0.8c$ in a direction perpendicular to the direction of (the laboratory's) motion as determined by an observer at rest with respect to the nucleus. Find the velocity and the direction of motion of the electron as measured by an observer in the laboratory frame.
Ans. from the velocity transformations $\vec{v}' = (u, \frac{v}{\gamma(u)}, 0)$ and with $u = 0.6c$, $v = 0.8c$ $\vec{v}' = (\frac{3}{5}, \frac{16}{25}, 0)\ c$.

2. The equation for a spherical pulse of light starting from the origin at $t = t' = 0$ is

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

Show from the Lorentz transformations that an observer O' will also measure this same pulse to be spherical, in accord with Einstein's second postulate stating that the velocity of light is the same for all observers.

3. In some frame, the four-vectors U^α and D^α have components (in units where c=1):

$$U^\alpha = (1 + t^2, t^2, \sqrt{2}\ t, 0), \quad D^\alpha = (x, 5tx, \sqrt{2}\ t, 0)$$

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(a) Find $U^\alpha U_\alpha, U^\alpha D_\alpha, D^\alpha D_\alpha$. Is U^α suitable as a four-velocity? Why? Is D^α?
Ans. $U^\alpha U_\alpha = 1$ so yes, D^α no.
(b) Find the spatial velocity of a particle whose four-velocity is U^α, for arbitrary t. What happens to it in the limits $t \to 0, t \to \infty$?
Ans. $(t^2, \sqrt{2}\ t, 0)/(1 + t^2)$, limits $(0, 0, 0)$ and $(1, 0, 0)$
(c) Find $U^\alpha{}_{,\beta}$ for all α, β. **Ans.** $U^\alpha{}_{,i} = 0, U^\alpha{}_{,0} = (2t, 2t, \sqrt{2}, 0)$
(d) Show that $U_\alpha U^\alpha{}_{,\beta} = 0$ for all β.

4. A stick of proper length l sits at rest in frame S, lying in the $x - y$ plane at an angle $\theta = \arctan(3/4)$ with the x axis. Another frame S' moves with velocity v along the positive x axis of S. In S' the stick is angled at 45° with respect to the x' axis.
(a) What is v? **Ans.** $\gamma = \frac{\tan\theta'}{\tan\theta} = \frac{4}{3}$ so $v = \frac{\sqrt{7}}{4}c$
(b) What is the length l' of the stick as measured is S'? **Ans.** $l' = \frac{3\sqrt{2}}{5}l$.