Written Examination Special Relativity F8066

Academic Year 2002–2003: 23 June 2003, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

C. On your answer paper, please rewrite and sign the pledge "I swear on my honour that I have neither given nor received help during this examination."

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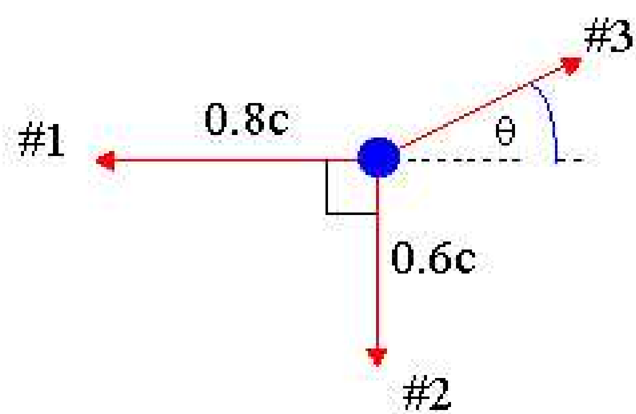
1. A particle of rest mass M is at rest in the laboratory when it decays into three identical particles of rest mass m. Two of the particles have velocities and directions as shown.



Find the speed and direction of the third. What is the ratio M/m?
Ans: $v = 0.84c, \tan\theta = 9/16, M/m = 4.75$

2. What is the velocity of the centre of mass for a system consisting of a photon of energy E and a stationary atom of rest mass M?
i) Would this velocity change (and if so, how) if instead of a photon, there was a particle of rest mass m and the same energy E?
ii) What is the ratio between the photon frequencies in the centre-of-mass and laboratory frames? **(continued)**

Ans: $v = \frac{Ec}{E+Mc^2}, i) Yes, \frac{\sqrt{E^2-m^2c^4}}{E+Mc^2}, ii)\sqrt{\frac{Mc^2}{Mc^2+2E}}$

3. What is the four–acceleration a^α of a particle with three-velocity u?

i) What are the components of a^α in the rest frame ($u = 0$) of the particle?
ii) What is the norm of a^α? i.e. calculate $a^\alpha a_\alpha$.
iii) A particle moves on the circle $x^2 + y^2 = r^2, z = 0$ with constant velocity u. Calculate the components of its four–acceleration when it crosses the negative y–axis, i.e. when $x = 0$, and $y = -r$.

Ans: $a^\alpha = \frac{du^\alpha}{d\tau} = \gamma(u)\frac{du^\alpha}{dt}$
i) $\vec{u} = 0, \gamma = 1, a^\alpha = (0, \frac{d\vec{u}}{dt})$
ii) $a^\alpha a_\alpha = -\frac{d\vec{u}}{dt}^2$ (Lorentz invariant)
iii) $(0, 0, \frac{u^2\gamma^2}{r}, 0)$

4. Prove that if a two–tensor is symmetric (antisymmetric) in one frame, then it is symmetric (antisymmetric) in all frames. Give two examples of each type of tensor (four in all).