Written Examination Special Relativity F8066

Academic Year 2003–2004: 28 June 2004, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. Which of the following statements is false? Justify your answer.

1. The four–velocity of a massive particle is time–like.
2. The four–acceleration of a massive particle is space–like.
3. If the magnitude of the force on a particle vanishes in one inertial frame, it vanishes in all inertial frames.
4. One of the above statements is false.

Ans. Number 3 is false since $f^\alpha f_\alpha = (f^0)^2 - (\vec{f})^2$ is invariant but $(\vec{f})^2$ is not.

2. An observer receives light from a source receding with velocity $v = 0.6c$. At what angle would the observer see a red shift? And a blue shift? And if the source were approaching the observer?
Ans. for a red shift $\cos\phi < \frac{\gamma-1}{\gamma\beta} = \frac{1}{3}$, so $|\phi| > 70°$, otherwise blue. If β is negative, for a red shift, $\cos\phi < -\frac{1}{3}$, $|\phi| > 110°$, otherwise blue.

3. Consider a system of two identical particles of rest mass m. In the laboratory frame, one of them is at rest and the other has energy E. In the centre–of–mass frame each particle has energy E'.

i) Express E' in terms of E and m. **Ans.** $(E')^2 = \frac{mc^2(E+mc^2)}{2}$

ii) What is the velocity of the centre–of–mass frame? **Ans.** $\beta^2 = \frac{E-mc^2}{E+mc^2}$

4.
i) Write the Lorentz force law on a particle of mass m and charge e

$$\frac{d\vec{p}}{dt} = e\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)$$

as the spatial part of a tensor equation. **Ans.** $\frac{dp^\alpha}{d\tau} = \frac{e}{c}F^\alpha{}_\beta u^\beta$

ii) Write out the zero (temporal) component of this tensor equation, and give a physical interpretation. **Ans.** $\frac{d}{dt}(m\gamma c^2) = e\vec{E}.\vec{v}$

iii) Show that the particle's acceleration is

$$\frac{d\vec{v}}{dt} = \frac{e}{m}\sqrt{1 - \frac{\vec{v}.\vec{v}}{c^2}}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} - \frac{\vec{v}(\vec{v}.\vec{E})}{c^2}\right)$$

Ans. $\frac{d\vec{p}}{dt} = \frac{d(m\gamma\vec{v})}{dt} = ...$