Written Examination Special Relativity F8066

Academic Year 2004–2005: 30 June 2005, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. In an inertial frame S two photons of frequencies ν_1 and ν_2 travel in the positive and negative x directions respectively. Find the velocity of the CM (centre of mass) frame of these photons relative to S.
Ans. $\left(\frac{\nu_1-\nu_2}{\nu_1+\nu_2}\right)$

2. Show that the sequence of n consecutive parallel Lorentz boosts, each with velocity $u = c\ \tanh\theta$, is equivalent to a single Lorentz boost in the same direction with velocity $c\left(\frac{z^n-1}{z^n+1}\right)$, where $z = e^{2\theta}$.
Ans. velocity is $c\tanh n\theta = c\left(\frac{e^{n\theta}-e^{-n\theta}}{e^{n\theta}+e^{-n\theta}}\right) = ...$

3. Show, by considering the complex vector $\vec{\Phi} = \vec{E} + i\vec{B}$, ($\vec{E}$ and $\vec{B}$ are the electric and magnetic fields) that the only two independent Lorentz invariants of the electromagnetic field are the quantities $\vec{E}.\vec{B}$ and $\vec{E}^2 - \vec{B}^2$. Express these invariants in terms of the electromagnetic tensor $F_{\mu\nu}$.

4. In an inertial frame two particles are shot out simultaneously from a given point, with equal speeds v, in orthogonal directions. What is the speed of each particle relative to the other?
Ans.$\gamma(u) = \gamma^2(v), \quad u = v\sqrt{2 - \frac{v^2}{c^2}}$