Written Examination Special Relativity MFN 1313

Academic Year 2014–2015: 26 June 2015, 3.00-5.00 PM

Please read the following INSTRUCTIONS

Answer at most TWO questions. You may answer in english or in italian. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

1. Derive the Einstein formula $E = \gamma mc^2$ from the variation of the action $S = \int -mc^2 d\tau$ for a free particle of mass m, where τ is its proper time. Show that for velocities $v << c$ it differs from the non-relativistic kinetic energy by a constant.

2. Two snowballs of equal rest mass $m = 60\ gm$ are thrown together head-on, and fuse to form one snowball. If the velocity of each snowball was $v = 0.8c$, calculate

i) the velocity of the resultant snowball

ii) the rest mass of the resultant snowball
Answeri) zero ii) $M = 2\gamma m, \gamma = \frac{5}{3}, M = 200gm$

3. Consider the components of a four–vector V^α as the matrix (with $i^2 = -1$)

$$\mathcal{V} = \begin{pmatrix} V^0 + V^3 & V^1 + iV^2 \\ V^1 - iV^2 & V^0 - V^3 \end{pmatrix}$$

i) Show that V^α satisfies $V^\alpha V_\alpha = Det\ \mathcal{V}$

ii) Show that the transformation $\mathcal{V} \to \mathcal{V}' = \mathcal{A}\mathcal{V}\mathcal{A}$ with

$$\mathcal{A} = \begin{pmatrix} \omega^{-\frac{1}{2}} & 0 \\ 0 & \omega^{\frac{1}{2}} \end{pmatrix}$$

and ω constant, corresponds to a Lorentz boost along the z axis. What is ω in terms of the rapidity parameter θ and the boost velocity v?
Ans. $\omega = \exp\theta = \sqrt{\frac{c+v}{c-v}}$.

4.a) What is the velocity of the centre of mass for a system consisting of a photon of energy E and a stationary atom of rest mass M?
Ans: $v = \frac{Ec}{E+Mc^2}$

b) What is the ratio between the photon frequencies in the centre-of-mass and laboratory frames?
Ans: $\sqrt{\frac{Mc^2}{Mc^2+2E}}$