Written Examination Special Relativity F8066

Academic Year 2004–2005: 5 September 2005, 2.30-4.30 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer, and full marks for two complete answers.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

1a. In a high energy accelerator the energy available to create new particles is that of the centre of mass frame. Calculate the available energy in the centre of mass frame when

(i) a proton with energy $E = 200\ Gev$ is incident on a target proton at rest (the proton rest energy is 938 Mev). **Ans.** ${E^2}_{CM} = 2mc^2(E+mc^2), E_{CM} \approx 20Gev$.

(ii) a proton with the same energy E heads east and collides with another proton with the same energy E heading west. **Ans.** $E_{CM} = 2E = 400Gev$.

1b. What energy would be needed in a fixed target experiment to obtain the energy of (ii)? (This explains why most large accelerators are now colliders). **Ans.** $\frac{{E^2}_{CM}}{2mc^2} \approx 80Tev$.

2a. Given the components of a tensor M^{ab} as the matrix

$$M^{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & -2 & 0 \end{pmatrix}$$

find

(i) the components of its symmetric part $M^{(ab)}$
(ii) the components of its antisymmetric part $M^{[ab]}$
(iii) the components of ${M^a}_b$
(iv) the components of ${M_a}^b$
(iv) the components of M_{ab}

2b. For the tensor whose components are ${M^a}_b$, does it make sense to speak of its symmetric and antisymmetric parts? If so, define them. If not, say why.

3. i) Write out the Lorentz matrix condition and give two explicit examples of Lorentz matrices.
ii) Derive at least 3 properties of Lorentz matrices from the Lorentz matrix condition.

4. A source of blue light moves in a circle of radius R around an observer, who sees it as red. What is the angular velocity of the source of radiation, assuming that the wavelength of red light is twice that of blue light? **Ans.** $\gamma = 2, v = \frac{c\sqrt{3}}{2}, \omega = \frac{v}{R}$.