Written Examination Special Relativity MFN 1313
Academic Year 2011–2012: 4 September 2012, 2.30-4.30 PM

Please read the following INSTRUCTIONS

**A. Answer at most TWO questions. You may answer in english
or in italian. A pass is obtained for one complete answer.**

**B. You may not use notes or textbooks, but the course notes are
available for consultation at the front desk.**

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1. Show that the sequence of n consecutive parallel Lorentz boosts, each with
velocity $u = c \tanh \theta$, is equivalent to a single Lorentz boost in the same direction with velocity $c\left(\frac{z^n - 1}{z^n + 1}\right)$, where $z = e^{2\theta}$.

Ans. velocity is $c \tanh n\theta = c\left(\frac{e^{n\theta} - e^{-n\theta}}{e^{n\theta} + e^{-n\theta}}\right) = \ldots..$

2. The equation for a spherical pulse of light starting from the origin at $t = t' = 0$ is

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

Show from the Lorentz transformations that an observer O' will also measure
this same pulse to be spherical, in accord with Einstein's second postulate stating that the velocity of light is the same for all observers.

3. In an inertial frame S two photons of frequencies ν_1 and ν_2 travel in the
positive and negative x directions respectively. Find the velocity of the CM
(centre of mass) frame of these photons relative to S.

Ans. $\left(\frac{\nu_1 - \nu_2}{\nu_1 + \nu_2}\right)$