Written Examination Special Relativity MFN 1313

Academic Year 2013–2014: 17 September 2014, 2.00-4.00 PM

Please read the following INSTRUCTIONS

A. Answer at most TWO questions. You may answer in english or in italian. A pass is obtained for one complete answer.

B. You may not use notes or textbooks, but the course notes are available for consultation at the front desk.

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1. In an inertial frame S two photons of frequencies ν_1 and ν_2 travel in the positive and negative x directions respectively.

a) Find the velocity of the CM (centre of mass) frame relative to S.
Ans. $\left(\frac{\nu_1-\nu_2}{\nu_1+\nu_2}\right)$

b) Calculate the photon frequencies in the CM frame
Ans.. $\sqrt{\nu_1\nu_2}$

2. A stick of proper length l sits at rest in frame S, lying in the $x-y$ plane at an angle $\theta = \arctan(3/4)$ with the x axis. Another frame S' moves with velocity v along the positive x axis of S. In S' the stick is angled at 45° with respect to the x' axis.
(a) What is v? **Ans.** $\gamma = \frac{\tan\theta'}{\tan\theta} = \frac{4}{3}$ so $v = \frac{\sqrt{7}}{4}c$
(b) What is the length l' of the stick as measured is S'? **Ans.** $l' = \frac{3\sqrt{2}}{5}l$.

3. Derive the Einstein formula $E = \gamma mc^2$ from the variation of the action $S = \int -mc^2 d\tau$ for a free particle of mass m, where τ is its proper time. Show that for velocities $v << c$ it differs from the non-relativistic kinetic energy by a constant.