

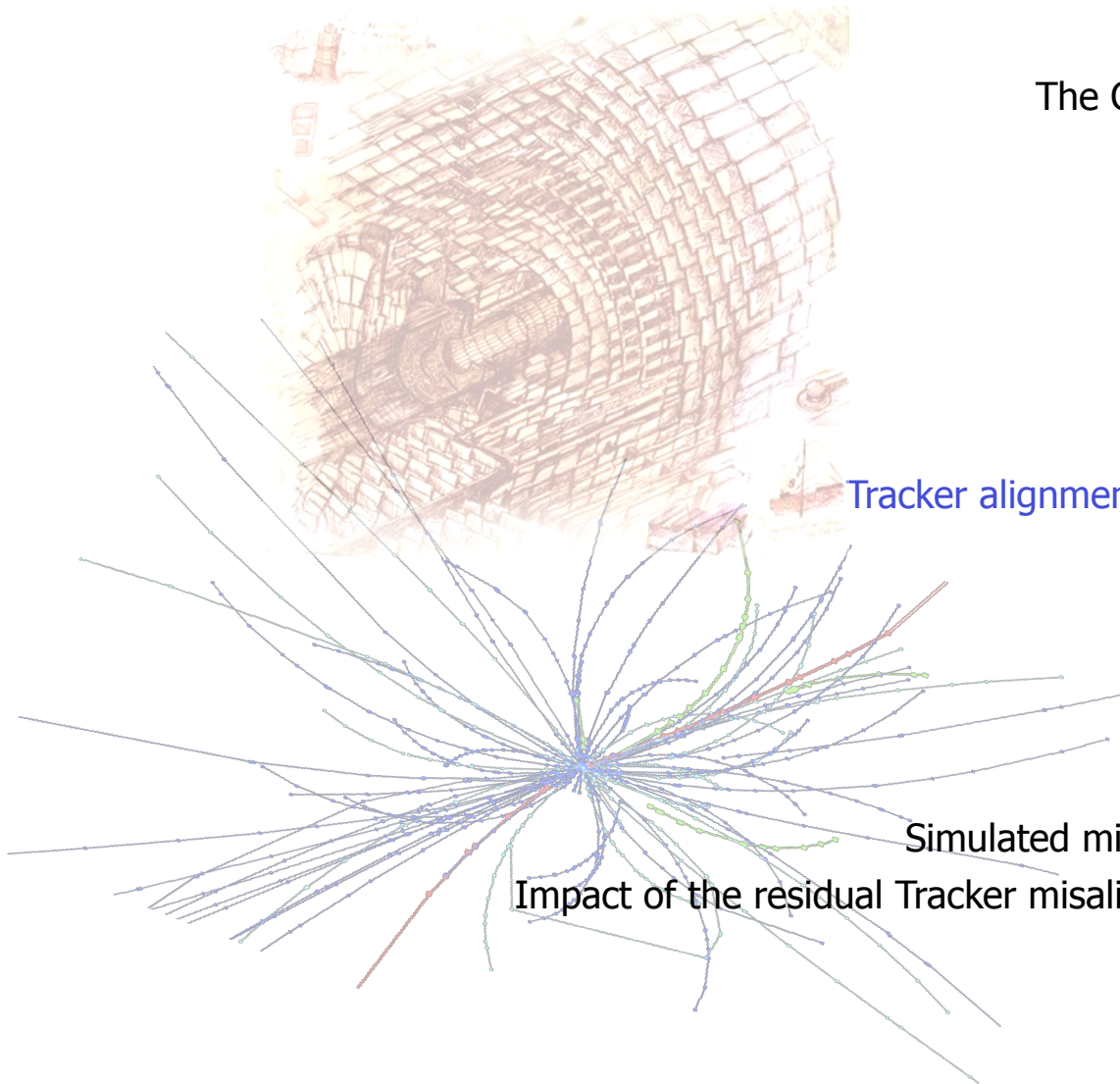


Scuola di dottorato in Scienza ed Alta Tecnologia, Indirizzo Fisica ed Astrofisica
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The alignment of the CMS silicon Tracker and its influence on early physics performance

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Introduction

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Alignment

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Tracker alignment with real data (cosmics muon tracks)

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Simulated misalignment scenario for physics studies

Impact of the residual Tracker misalignment on the muon momentum scale

Summary

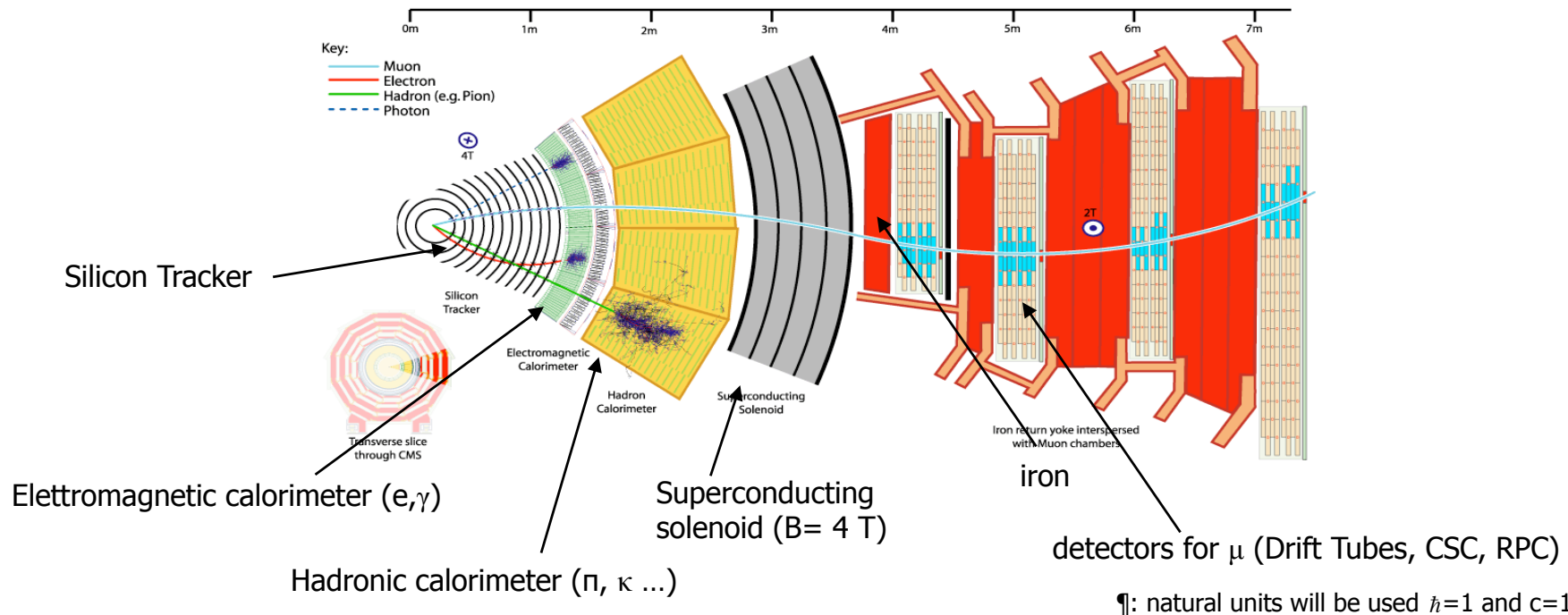
Introduction



The Compact Muon Solenoid



- One of the four Large Hadron Collider experiments (with Alice, Atlas and LHCb)
- Multi-purpose experiment (search for Higgs, Supersymmetry,...)
- pp collisions at $\sqrt{s}=14$ TeV (design): parton scattering $\sqrt{\hat{s}}=1-2$ TeV
- A system to identify muons and to measure their momentum up to the TeV scale
- A *CMS muon* is defined as a charged particle capable to produce a signal (*hit*) in the μ -chambers (*trigger*)
- Minimal p_T value to reach the μ -chambers is about 3 GeV^{\natural} (using solenoidal $B=4\text{T}$)

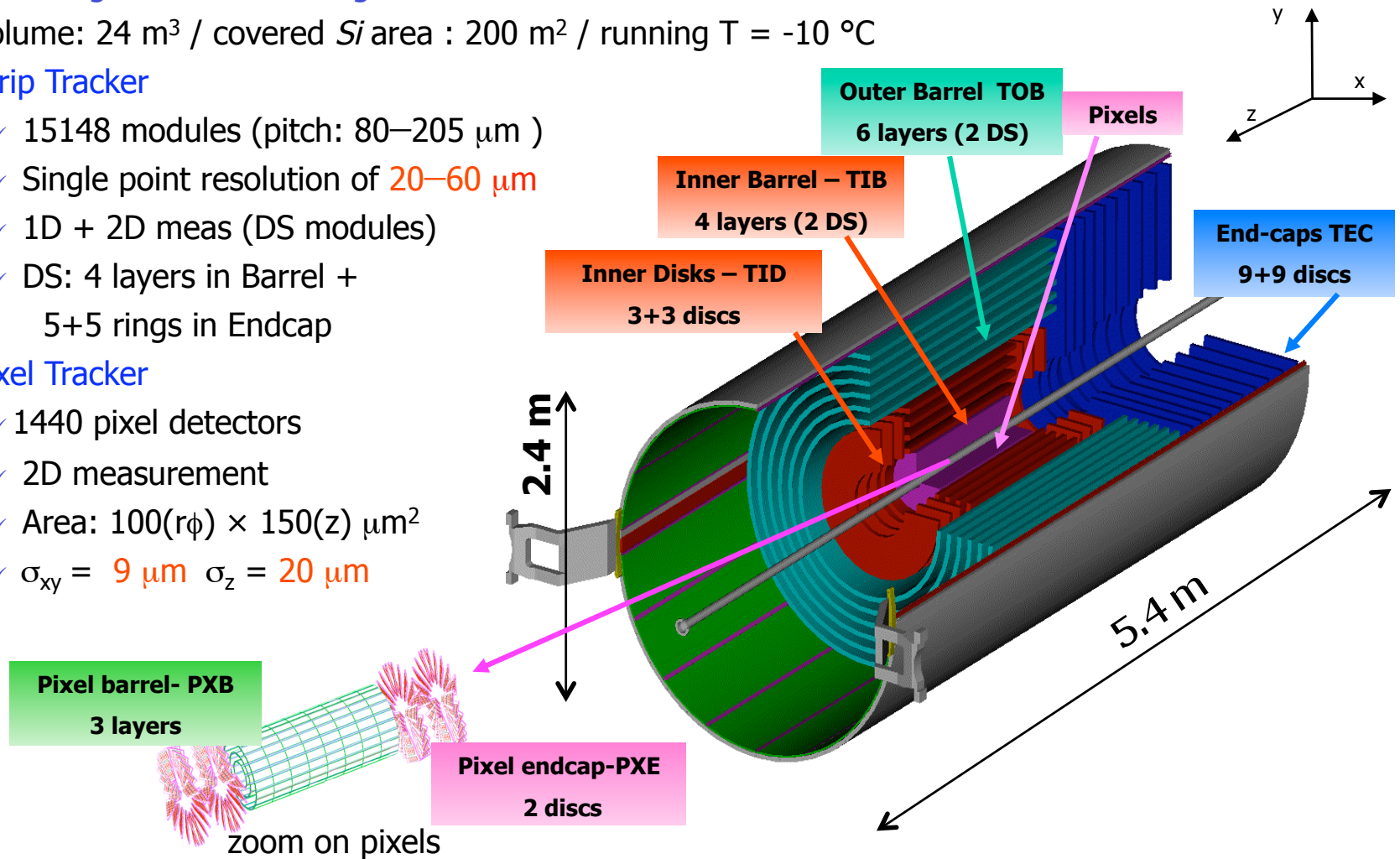


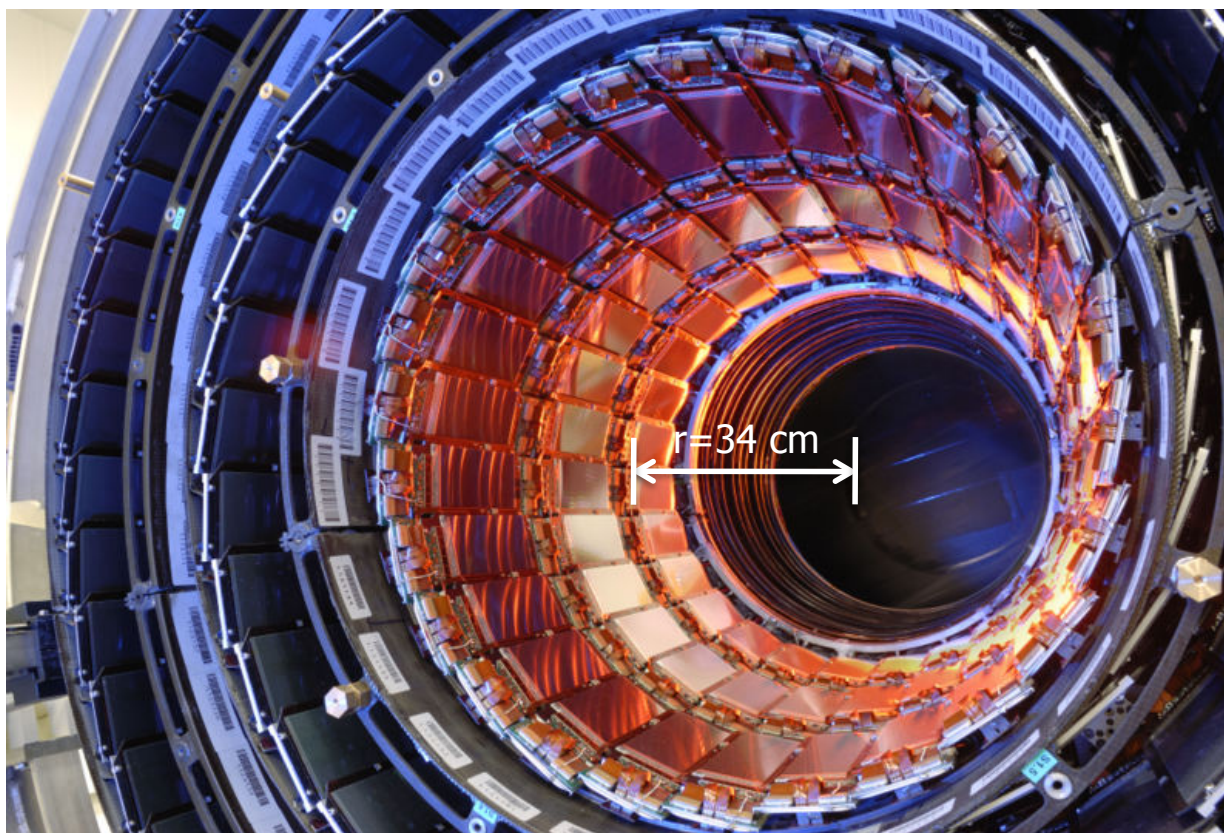


The CMS silicon Tracker

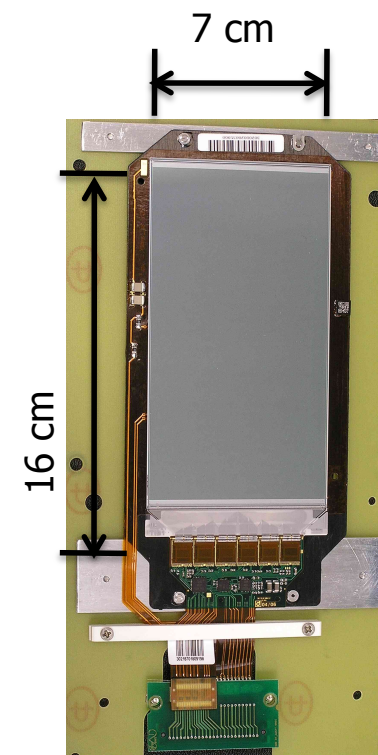


- World's largest silicon tracking detector
- Volume: 24 m³ / covered Si area : 200 m² / running T = -10 °C
- Strip Tracker
 - ✓ 15148 modules (pitch: 80–205 μm)
 - ✓ Single point resolution of 20–60 μm
 - ✓ 1D + 2D meas (DS modules)
 - ✓ DS: 4 layers in Barrel + 5+5 rings in Endcap
- Pixel Tracker
 - ✓ 1440 pixel detectors
 - ✓ 2D measurement
 - ✓ Area: 100(rφ) × 150(z) μm²
 - ✓ $\sigma_{xy} = 9 \mu\text{m}$ $\sigma_z = 20 \mu\text{m}$





A layer of the Tracker Inner Barrel (TIB)

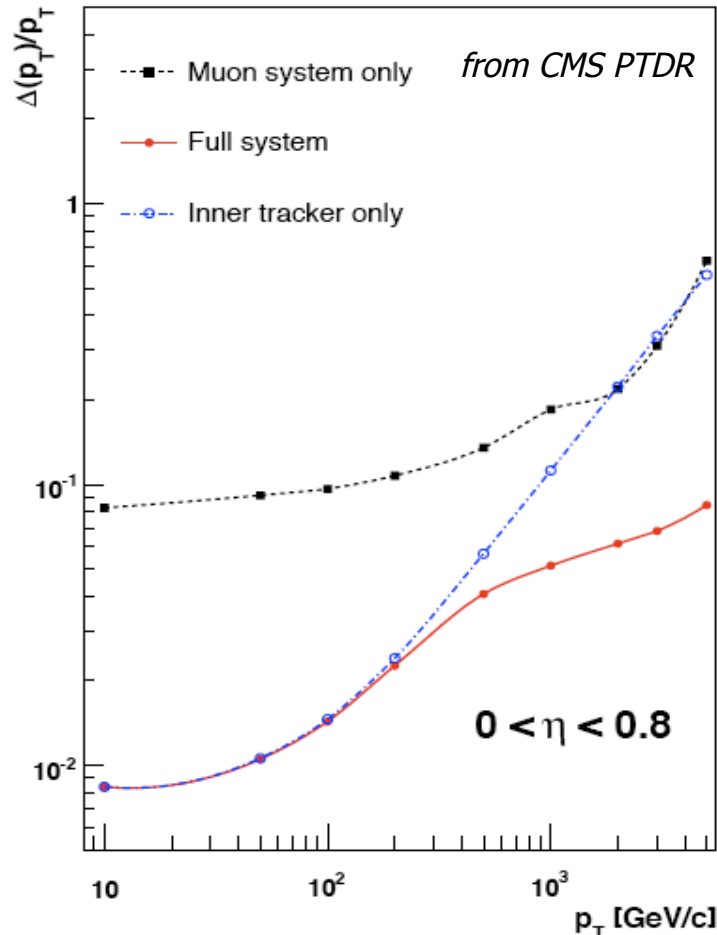


A TIB module

Alignment

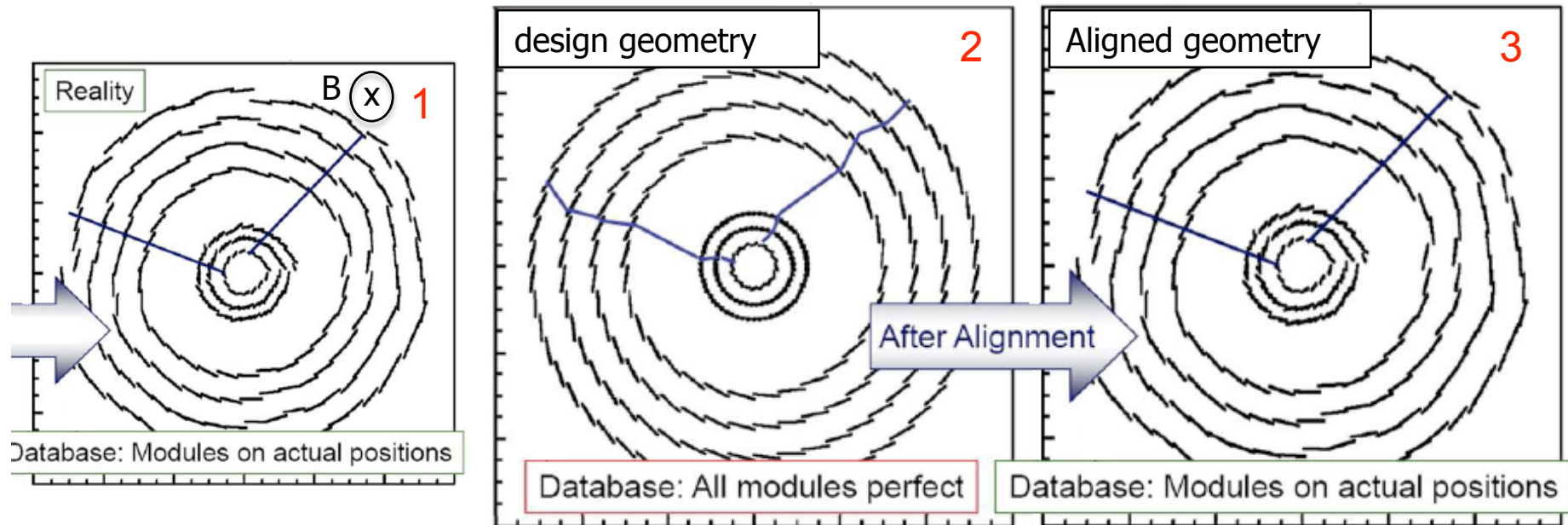


Why alignment is needed?



$\Delta p_T/p_T$ in the central region

- Tracker is essential to measure the momentum of the particles
- $\Delta p_T/p_T = C_1 p_T + C_2$
- C_1 : proportional to single point resolution σ_x
- C_2 : depending on the Multiple Coulomb Scattering (material)
- For $p < 20$ GeV the $\Delta p_T/p_T$ is dominated by C_2
- For high momentum muons, systematic effects of misaligned detectors become relevant (C_1)
- This contribution is minimised by alignment procedures



- ❑ In the reality the detector is misaligned: a particle of high momentum (e.g. $p=1$ TeV) is a 'straight line' assuming real geometry (fig.1)
- ❑ Using the design geometry the track reconstruction could assign a curvature and consequently give a wrong momentum estimate (fig.2)
- ❑ After **alignment** the track is re-fitted with the new geometry (near to the real one) and a correct measurement of the momentum is performed (fig.3)
- ❑ Same for the other parameters of a track



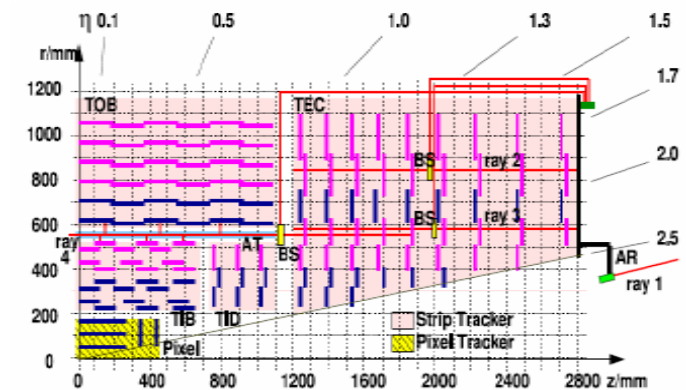
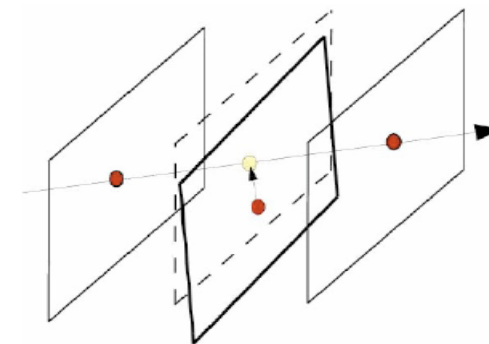
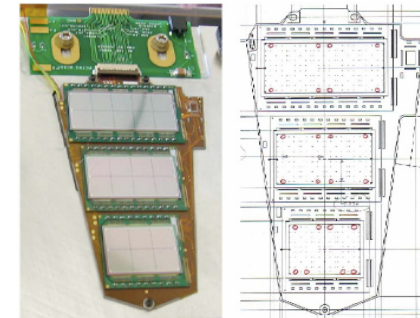
Inputs to alignment



- Survey measurements:
 - ✓ during assembly of the Tracker using Coordinates Measure Machine (CMM): precision of the sensor on carbon fiber $10\ \mu\text{m}$
 - ✓ Photogrammetry: precision of $100\ \mu\text{m}$

- Track-based alignment
 - ✓ different kind of tracks (cosmic ray μ , μ from Z and W decay, etc..)
 - ✓ final expected precision on the module position of less than $10\ \mu\text{m}$ along their sensitive coordinate

- Laser Alignment System (LAS):
 - ✓ continuous position measurements of large scale structures using laser beams
 - ✓ TEC disks position with spatial precision of $100\ \mu\text{m}$ and $100\ \mu\text{rad}$
 - ✓ relative alignment of TIB, TOB vs. TEC

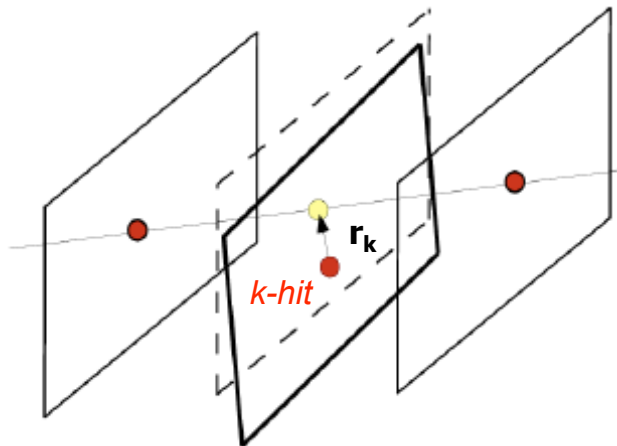




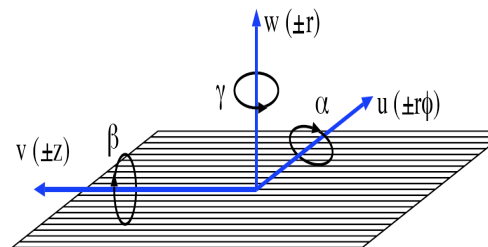
Track-based alignment



- The issue: recorded measurement of the k -hit (u_k^{hit}) and position extrapolated from the reconstructed trajectory (u_k^{fit}) are systematically shifted
- one-dimensional *residual* is defined: $\vec{r}_k = \vec{u}_k^{hit} - \vec{u}_k^{fit}$
- u_k^{fit} prediction depends on track q and alignment parameters p

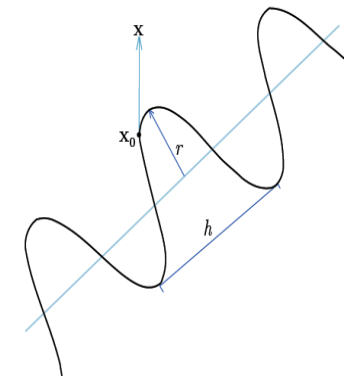


local modules coordinates



$$p = (\Delta u, \Delta v, \Delta w, \Delta \alpha, \Delta \beta, \Delta \gamma)$$

track parameters



- Final aim of track-based alignment is to minimize the track-residuals
- 6 d.o.f. x 15k modules = $O(100k)$ unknowns



Track-based alignment algorithms



- A track-based alignment algorithm is aimed at minimizing a global χ^2 function, determining the *alignment parameters* :

$$\chi^2 = \sum_k^{hits} r_k^T(p, q) V_k^{-1} r_k(p, q)$$

V = covariance matrix from fit

p = alignment parameters

q = track parameters

$$r_k = u_k^{hit} - u_k^{fit}(p, q)$$

r_k = residual depending from p and q

- A complex system of equations to solve
- Three alignment algorithms available in CMS software:
 - ✓ **Local method (HIP algorithm)** - Iterative procedure: local analytical χ^2 equation solution for p only.
 - ✓ **Global method (Millepede II algorithm)** - Global solution of the χ^2 equation for p and q : all correlations considered.
 - ✓ **Sequential method (Kalman algorithm)**- Sequential method updating alignment parameters after every track.
- My work consisted in performing track-based alignment with **global method** algorithm

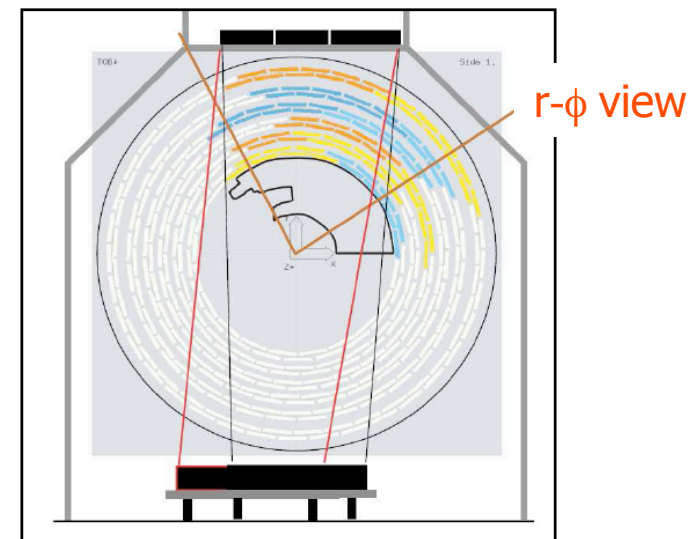
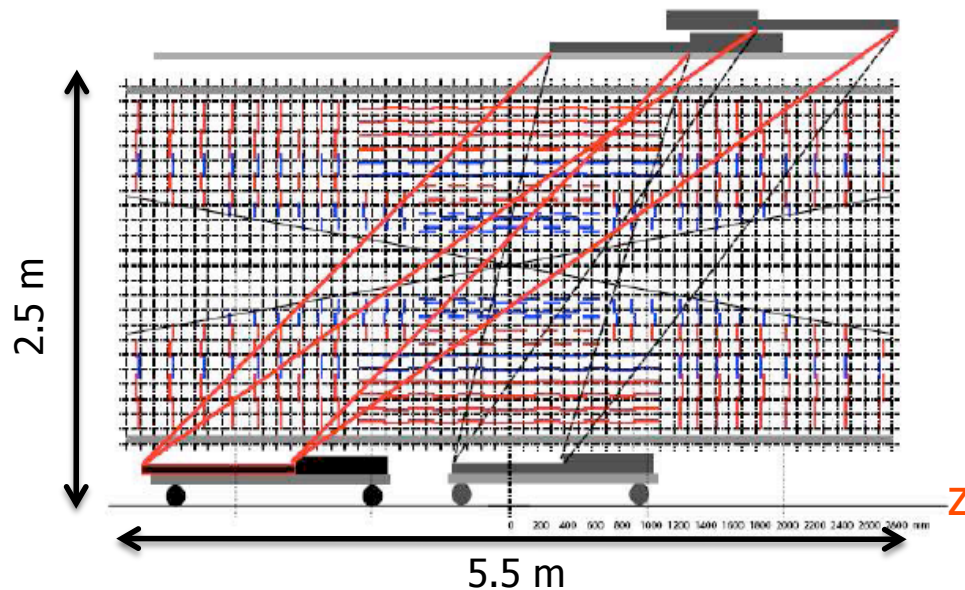
Tracker alignment with real data using cosmic rays



The alignment at Integration Facility (TIF)



- ❑ First attempt of (partial) CMS Tracker alignment with real data (2007)
- ❑ 15 % of the total strip tracker was read out
- ❑ Tracker not yet in the final position inside CMS
- ❑ Detector operating in different thermal conditions and in different mechanical conditions
- ❑ 4 different sets of scintillators to trigger cosmic events
- ❑ 5 M of collected events
- ❑ No p measurement (since no B field)



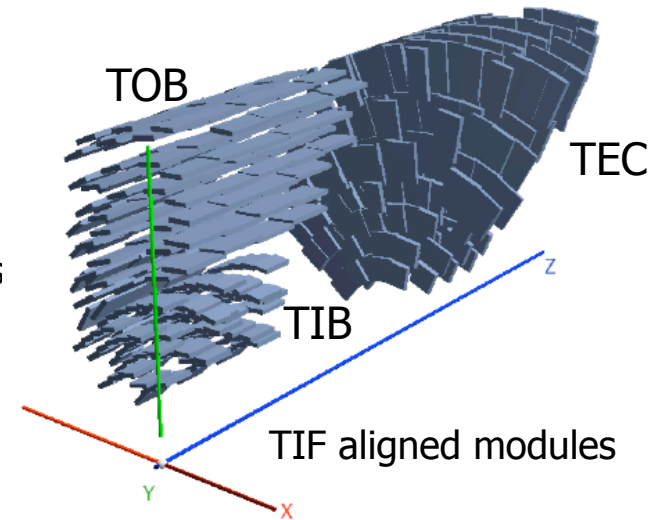


TIF: strategy for the analysis of collected data



- ❑ Dataset: 900k events taken at -10° C
- ❑ One single track per event
- ❑ Requirements for selecting a track on:
 - ✓ fiducial scintillator geometrical region
 - ✓ minimum number of hits and hits on DS modules
 - ✓ $\chi^2_{\text{track}}/\text{ndf}$
- ❑ Requirements for associating a hit to a track:
 - ✓ minimum cluster charge
 - ✓ isolation (no other hit within 8 mm)
 - ✓ outlier rejection ($\chi^2_{\text{hit}} < 5$)

- ❑ Selection of alignment parameters is different for each sub-detector at TIF:
 - ✓ TIB/TOB: high statistics, aligned at level of single module
 - ✓ TIB $u, (v), w, \gamma$ (for DS) | TOB $u, (v), \gamma$ (for DS)
 - ✓ TEC: low statistics, aligned at the level of disks
 - ✓ TID: very low statistics, not aligned
- ❑ Track based alignment with all 3 algorithms (global, local and sequential method)

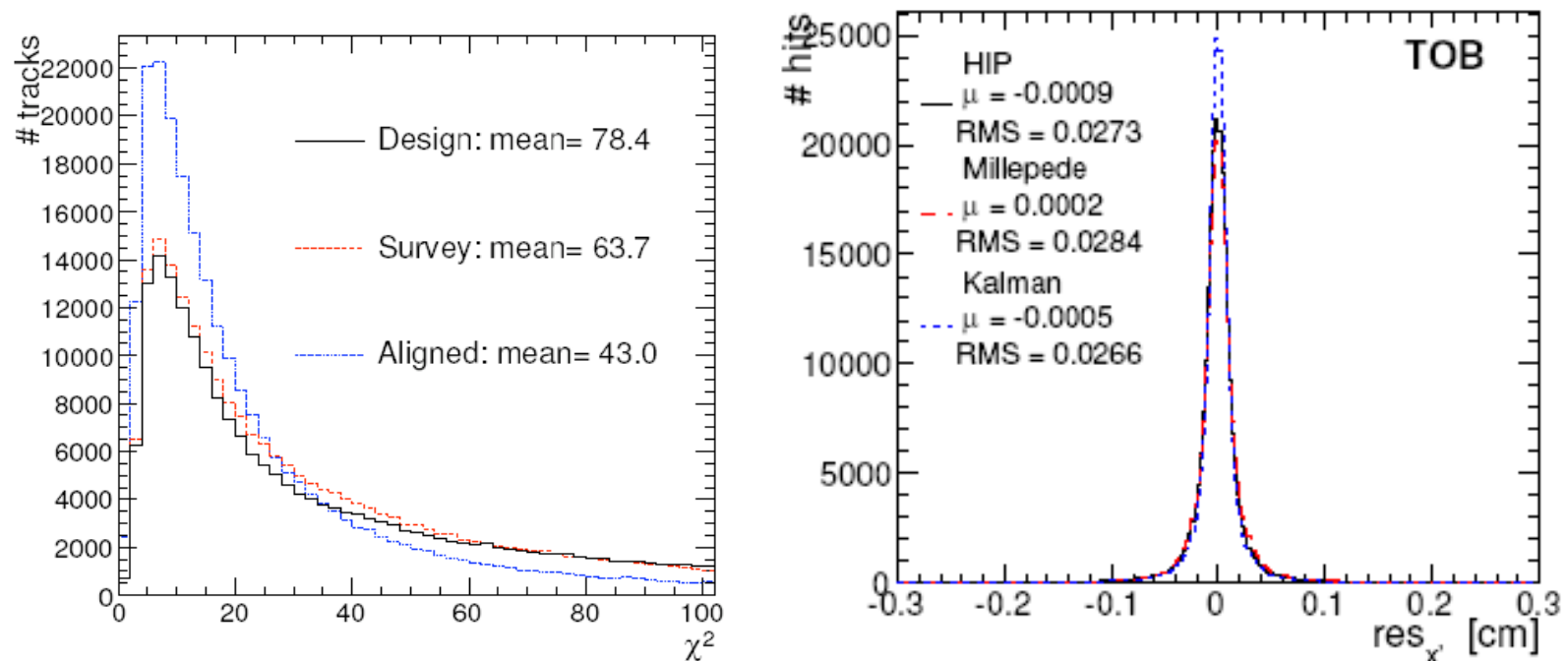




TIF: validation of survey and track based alignment



- Goodness of alignment given by track χ^2 distribution: overall improvement
 - ✓ Design = alignment constants not applied
 - ✓ Survey = survey measurements applied (TIB: module positions within layer only, TOB: layer level survey)
 - ✓ Aligned = alignment constants from track based alignment applied



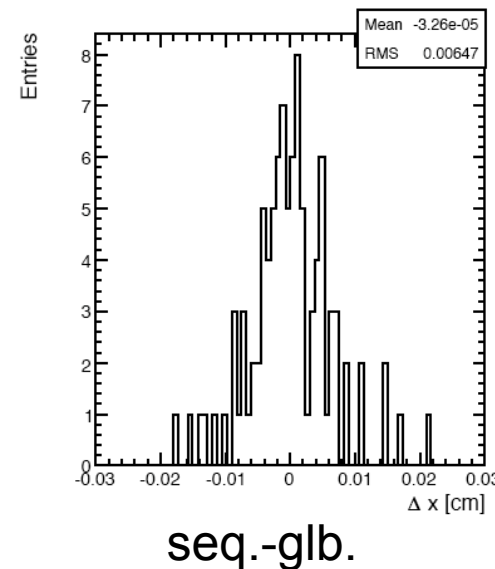
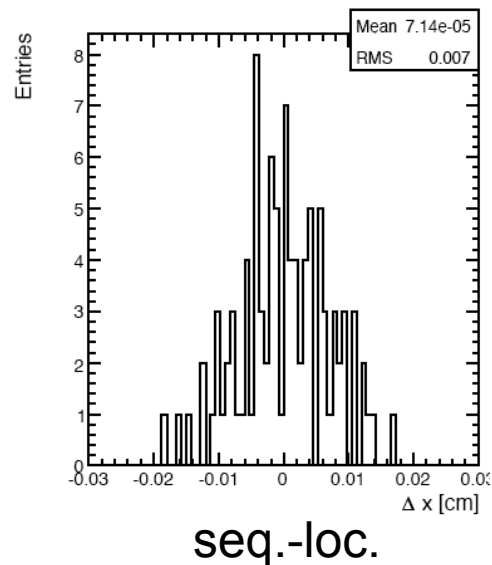
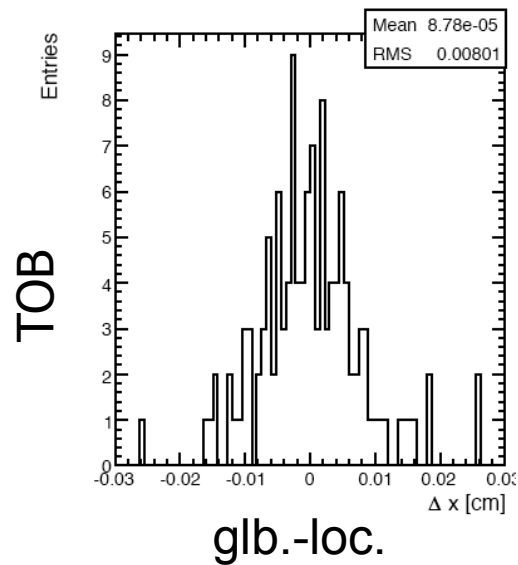
- Same algorithm performances on the residuals distribution for global/local/sequential method (e.g. TOB ss modules, pitch= 120-180 μm)



TIF: comparison of geometries from the different methods



- Differences between modules in local x:
 - ✓ $\Delta x_{\text{RMS}} < 80 \mu\text{m}$ (TOB) and $\Delta x_{\text{RMS}} < 150 \mu\text{m}$ (TIB)



Further studies at TIF:

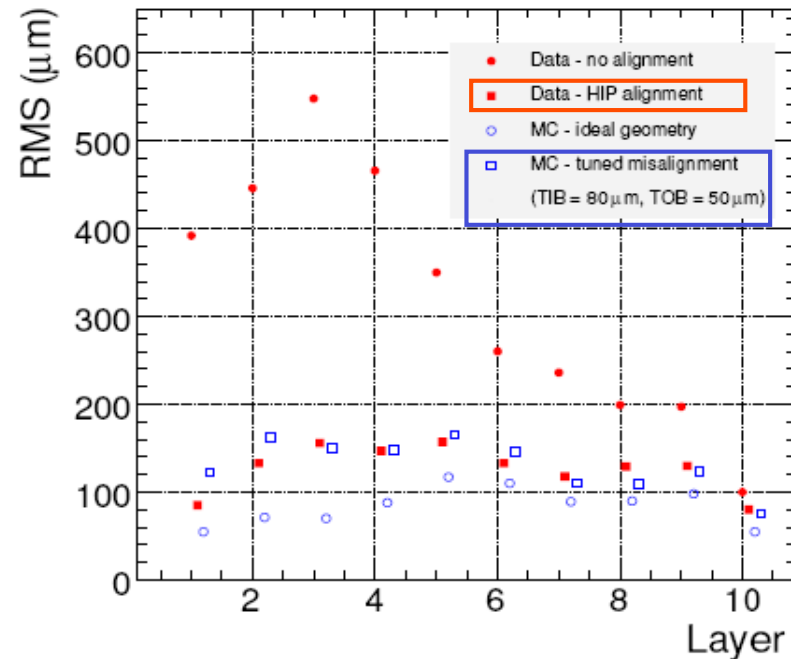
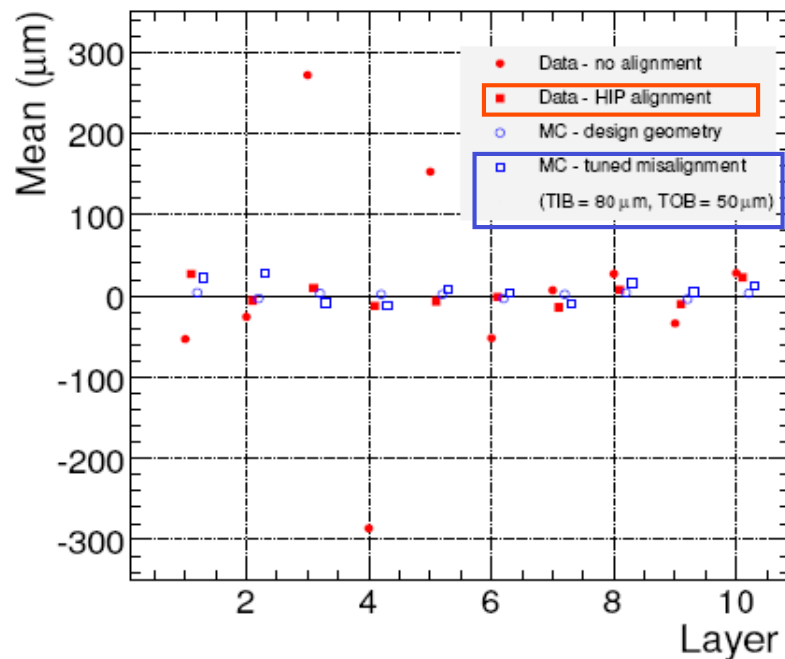
- Stability of the alignment **within 100 μm** in different:
 - ✓ mechanical conditions : before and after TEC at $z < 0$ insertion
 - ✓ thermal conditions: temperature ranging from -15°C to 15°C
- Comparison of LAS alignment and track based alignment in TEC discs: good agreement



TIF: estimation of achieved alignment precision



- Using Monte Carlo cosmic sample (500k evts)
- Track reconstruction performed with ideal tracker geometry with increasing misalignment applied to detectors
- Procedure repeated until reconstructed quantities (residuals, χ^2) match those observed in data after applying alignment constants
- The value of the misalignment introduced in MC is taken as an estimate of the maximum remaining misalignment on data: $TIB = 80 \mu\text{m}$, $TOB = 50 \mu\text{m}$

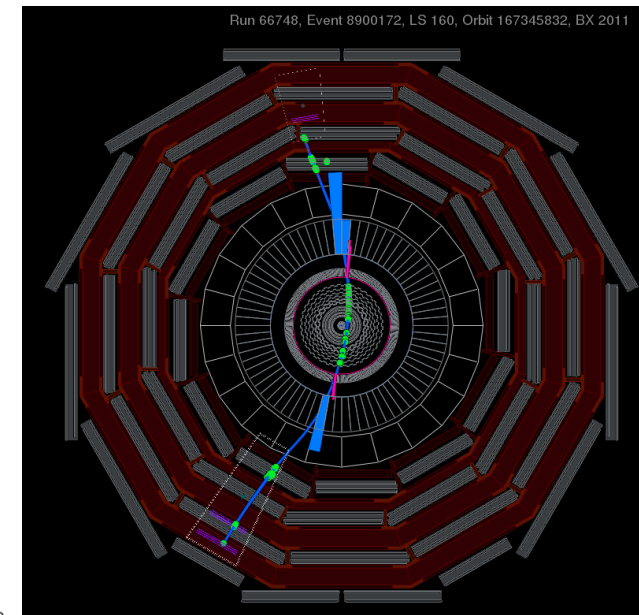
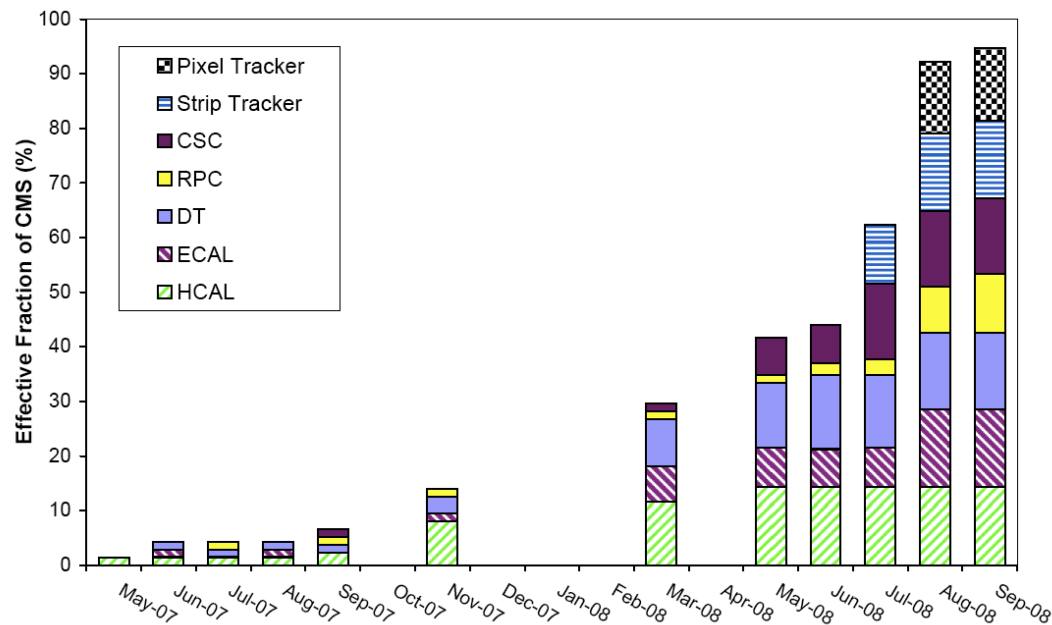




Tracker alignment at Cosmic Run At Four Tesla (CRAFT)



- First attempt of **full** CMS Tracker alignment during the CMS global run
- Tracker operating with all other CMS subdetectors
- TIF experience with cosmic data on a larger set of aligned objects
- **270 M** of cosmics collected with magnetic field switched on (only **~2%** in Tracker)
- **300 Hz** cosmic muon *Level 1* trigger rate (**6 Hz** in the Tracker)
- $\Delta t_{\text{top-bottom}} = 2 \times \text{BC} = 2 \times 25 \text{ ns} = \mathbf{50 \text{ ns}}$ (muon *time of flight*)



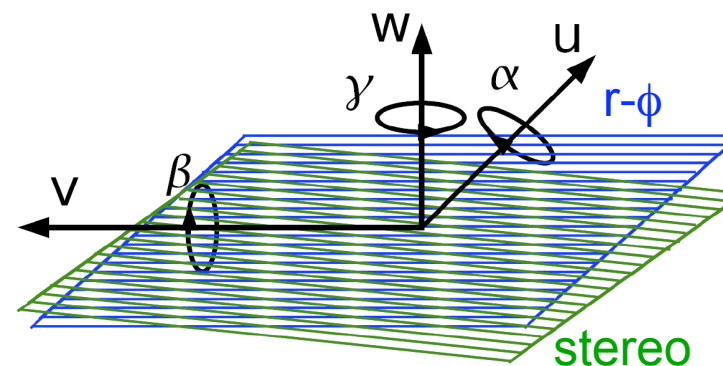


CRAFT: alignment strategy



- ❑ Dedicated alignment stream (AlCaReco) 4.5 M of events
- ❑ 3.2 M of tracks selected for alignment (only ~3.5% have at least 1 hit in Pixel volume)
- ❑ Large statistics available allow for a separate alignment of stereo and r-phi components of the DS modules (module unit)
- ❑ DS modules:
 - ✓ 2-D measurement in the combined plane
 - ✓ 100 mrad stereo angle between two components: $\Delta v \sim 10 \times \Delta u$
- ❑ Alignment in v of a DS module found to be not consistent with assembly accuracy
- ❑ Separate alignment of $r-\phi$ and stereo component improve dramatically residuals

Track quality cuts	value
momentum p	$> 4 \text{ GeV}/c$
number of hits	≥ 8
number of 2-d hits (on Pixel or DS modules)	≥ 2
χ^2/ndf of the track fit	< 6.0
Hit quality cuts	value
S/N (Strip modules)	> 12
pixel hit prob. matching template shape in u (v) dir.	> 0.001 (> 0.01)
track angle relative to the local uv plane	$< 20^\circ$
square pull of the hit residual	< 15

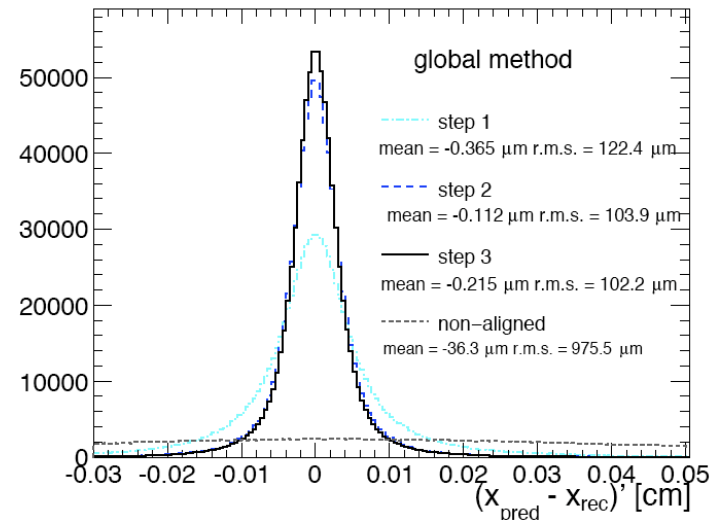
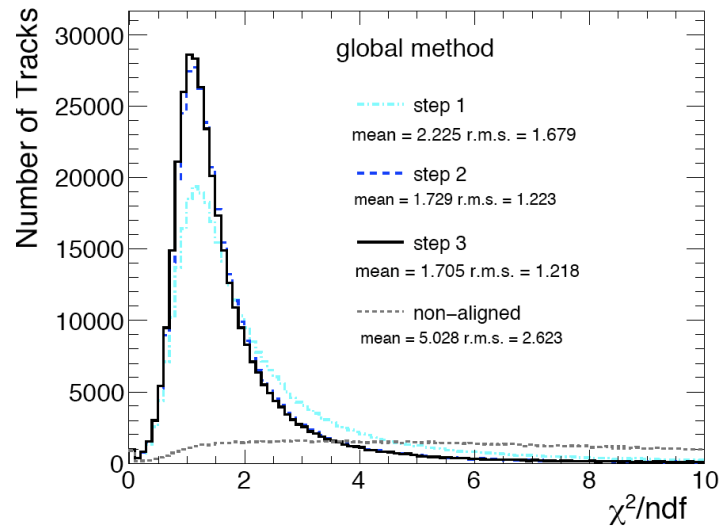




CRAFT: alignment strategy



- *Local* (HIP) and *global* (Millepede) statistical method run independently
- Best results from the combination of the two: *combined* method



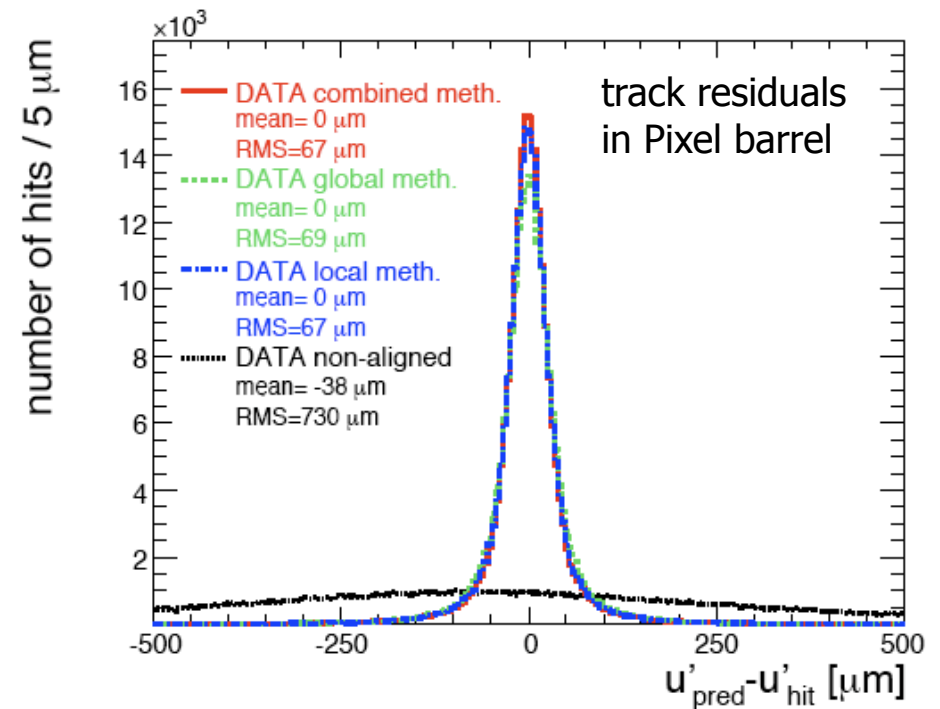
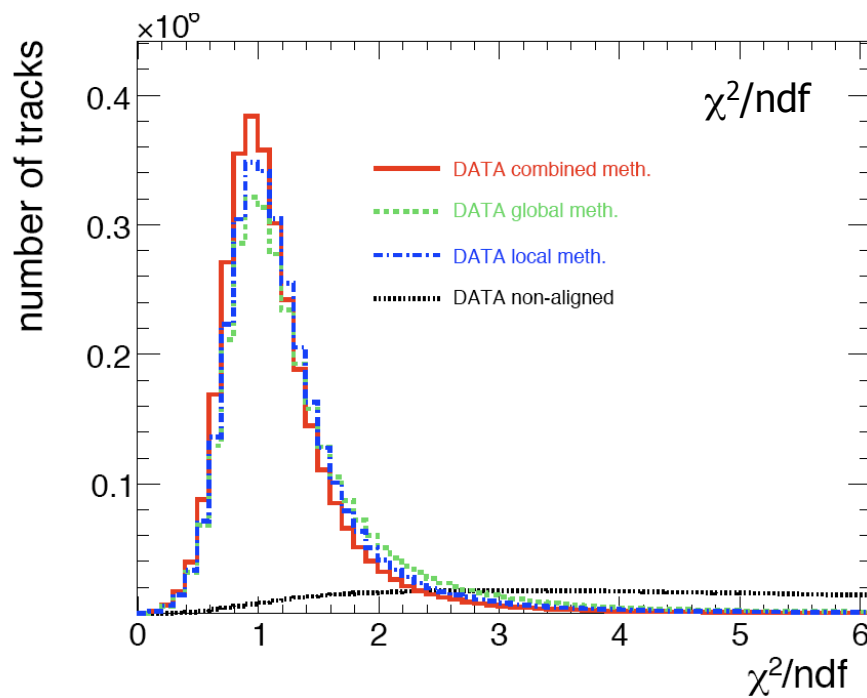
- E.g. the *global* method alignment strategy:
 - ✓ step1: Highest level structures aligned together with module unit (u, w, γ , and v for pixel)
 - ✓ step2: DS and SS modules aligned together ($u, w, \alpha, \beta, \gamma$ in TIB) starting from step1
 - ✓ step3: as step1, starting from step2 to recover lost correlations between first steps
- Strong correlation in step1 between highest-lowest level structures preventing systematic expansion and possibility to align more degrees of freedom in step2



CRAFT: track fit and residuals



- Clear improvement after alignment compared with the non-aligned geometry
- Best results in terms of χ^2 and residuals given by *combined* method
- Local method (best match with the track model in all *d.o.f.s*) run on the result of the global method (solve correlations quickly)
- Track refitted with properly tuned Alignment Position Errors (APE)



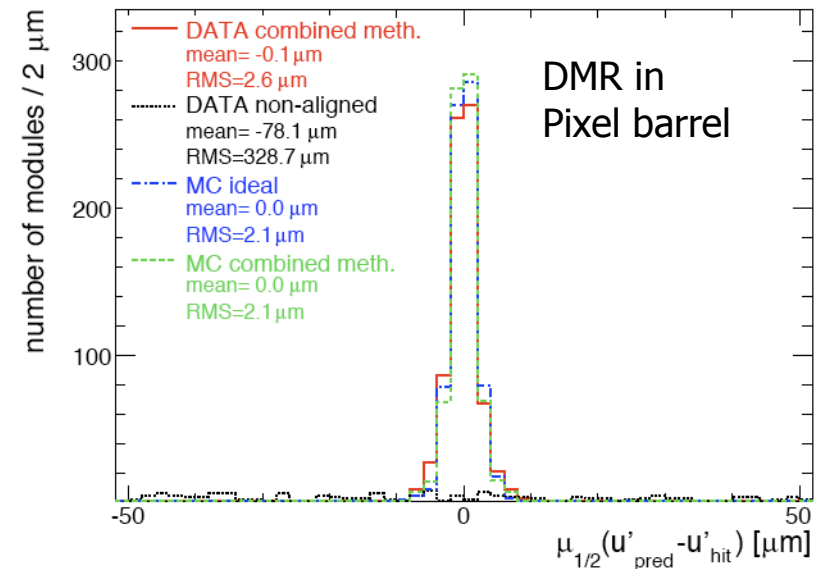


CRAFT: evaluation of remaining misalignment



- Residual width dominated by multiple scattering and hit uncertainties
- Goal: disentangle random effects from systematic ones produced by remaining misalignment
- Median* chosen as estimator of the position of the peak of the residuals for each module
- RMS of the Distribution of the Median of the Residuals (DMR) measure the remaining misalignment in the detector
- module positions w.r.t to cosmic ray trajectory measured with a precision of $3-4 \mu\text{m}$ in the barrel and of $3-14 \mu\text{m}$ in the endcap (along $r\phi$)

	non-aligned [μm]	global [μm]	local [μm]	combined [μm]	combined MC [μm]	ideal MC [μm]
PXB (u')	328.7	7.5	3.0	2.6	2.1	2.1
PXB (v')	274.1	6.9	13.4	4.0	2.5	2.4
PXE (u')	389.0	23.5	26.5	13.1	12.0	9.4
PXE (v')	385.8	20.0	23.9	13.9	11.6	9.3
TIB (u')	712.2	4.9	7.1	2.5	1.2	1.1
TOB (u')	168.6	5.7	3.5	2.6	1.4	1.1
TID (u')	295.0	7.0	6.9	3.3	2.4	1.6
TEC (u')	216.9	25.0	10.4	7.4	4.6	2.5



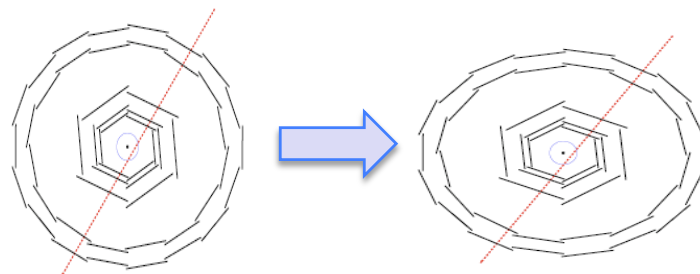
Impact of alignment on early physics



Towards the collision phase



- ❑ Statistical precision reached after CRAFT is not the final step of alignment
- ❑ **Non-trivial transformation** (*weak modes*) can affect Tracker, surviving after track based alignment



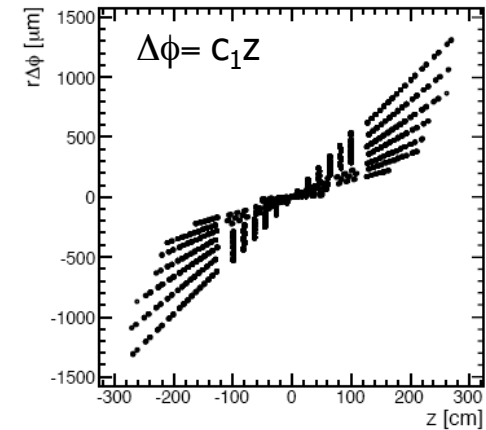
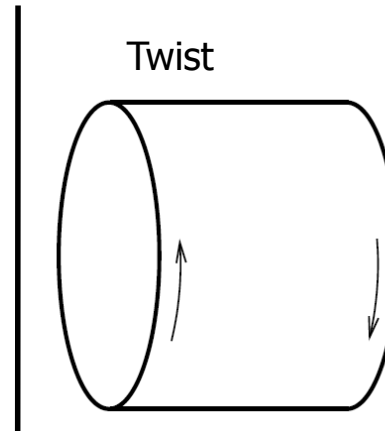
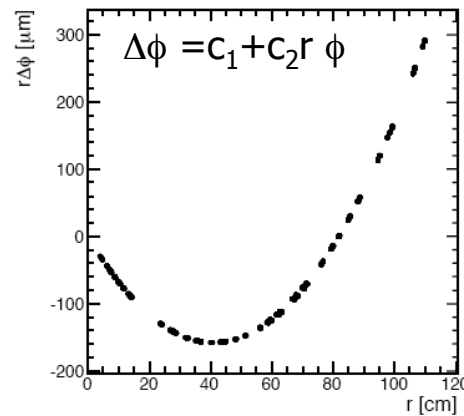
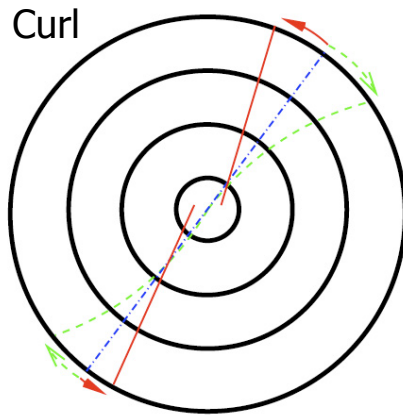
- ❑ **Physics is affected:**
 - ✓ artificial charge-dependent asymmetry can be induced by a not corrected *layer rotations*
 - ✓ expansion on the z-scale can induce distorted lifetime measurement
- ❑ Global transformations and remaining shifts are simulated in *misalignment scenarios* to be used in the study of early physics process
- ❑ Rather than implementing misalignment effects at generation level, they are introduced in the reconstruction level:
 - Generation with misaligned geometry + reconstruction with design geometry = misalignment**
is replaced by (assuming small shifts of the modules)
 - Generation with design geometry + reconstruction with misaligned geometry = misalignment**



Systematic misalignment studies with CRAFT data



- 9 systematic distortions modeled introducing a misalignment for cylindrical geometry (r, ϕ, z)



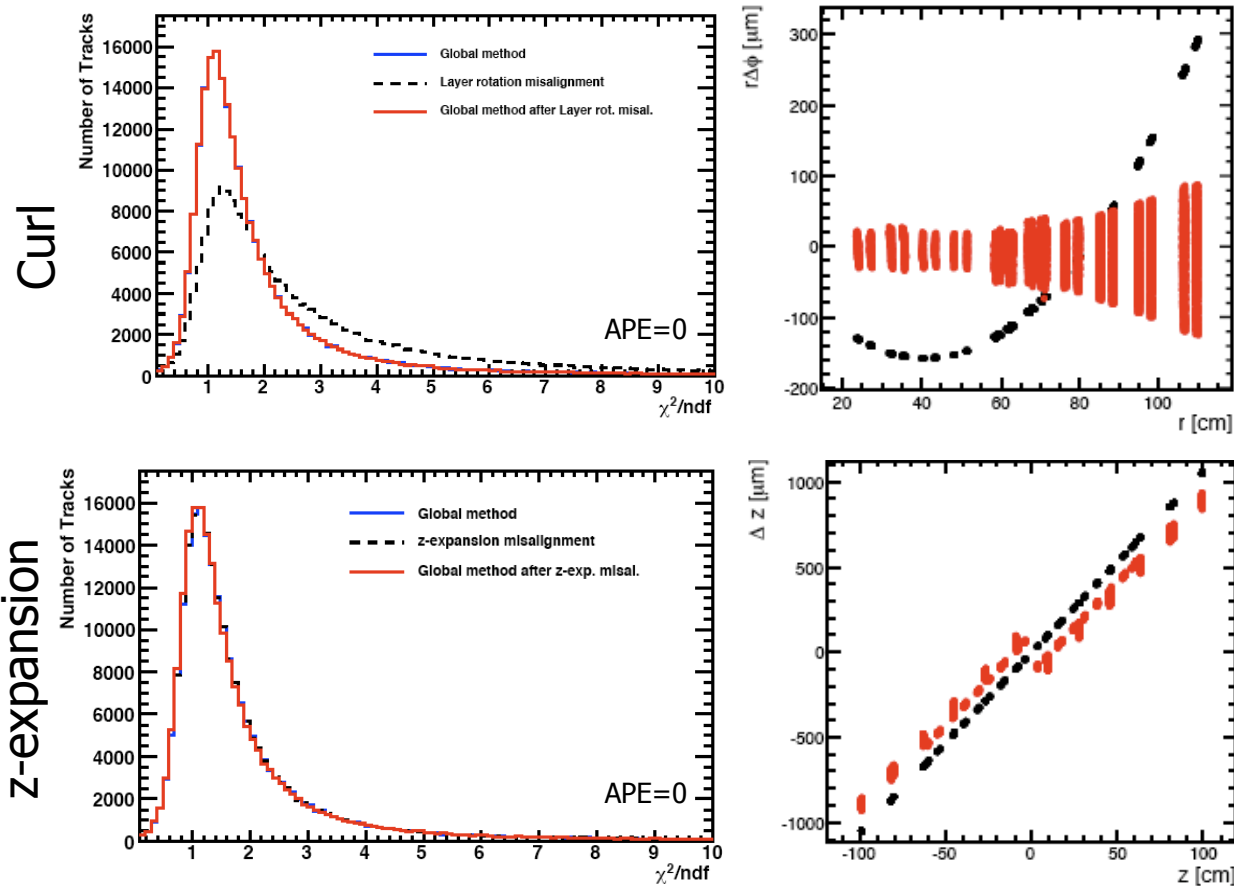
- Sensitivity to deformations checked with CRAFT cosmic rays tracks with following method:
 - ✓ global method geometry after CRAFT (**geom.A**) misaligned according to each modes: geom.B
 - ✓ geom. B used as starting point for alignment with cosmics using same strategy: **geom. C**
 - ✓ **geom. C** compared module by module with **geom. A**
- [**geom. C** - **geom. A**] expected to be zero if the mode is recovered
- χ^2 behavior is an indication of the potential in recovering
- Scale of misalignment tuned as expected in the reality (can influence the recovering)



Results of the analysis with cosmic ray tracks



- Curl ($\Delta\phi = c_1 + c_2 r \phi$): track χ^2 worsens but recovered after: geometric scattering $< 100 \mu\text{m}$
- Zexpansion ($\Delta z = c_1 z$): *weak* mode, χ^2 insensitive to geometrical deformation
- Not all systematics recoverable with cosmics only, better sensitivity from collision tracks (constrain)





Misalignment scenarios for early physics process

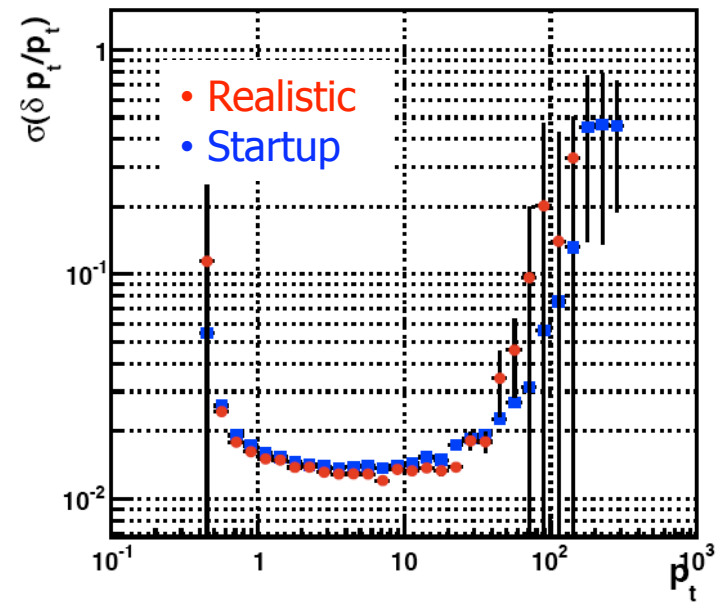
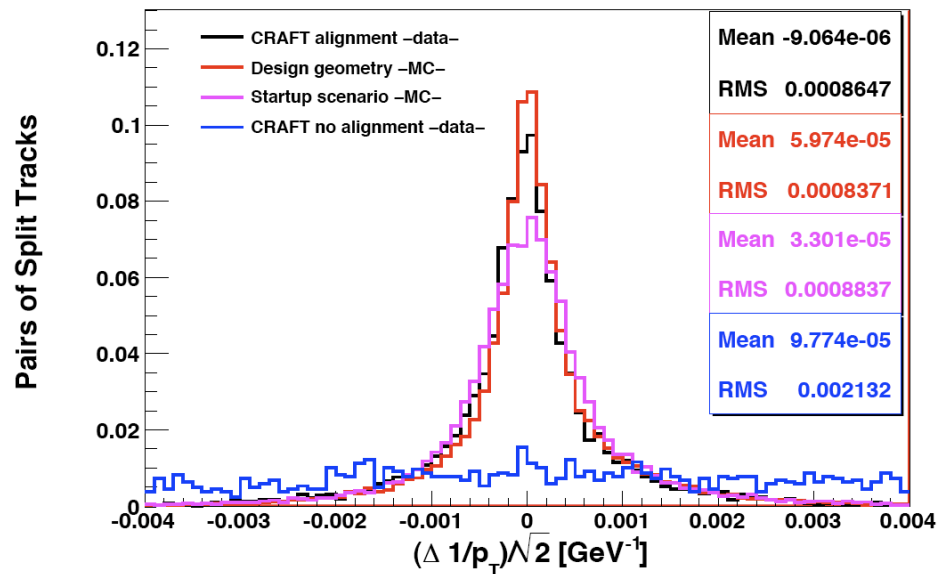


Tracker *Startup* scenario

- ✓ Random misalignment driven by the residual distribution observed in early analysis of CRAFT data
- ✓ precision: $L=100 \text{ pb}^{-1}$ in TIB/TOB, $L=10 \text{ pb}^{-1}$ in TID/TEC/ BPIX, survey in FPIX
- ✓ Alignment Position Errors constant for modules within same subdetectors $100\mu\text{m}/300\mu\text{m}/200\mu\text{m}$ for Barrel/EndCap/Pixel

Tracker *Realistic* scenario

- ✓ No random numbers used: CRAFT geometry+ alignment with MC cosmics = realistic scenario
- ✓ a bit optimistic, could contain some global movements inherited from CRAFT geometry
- ✓ Alignment Position Error as on CRAFT data



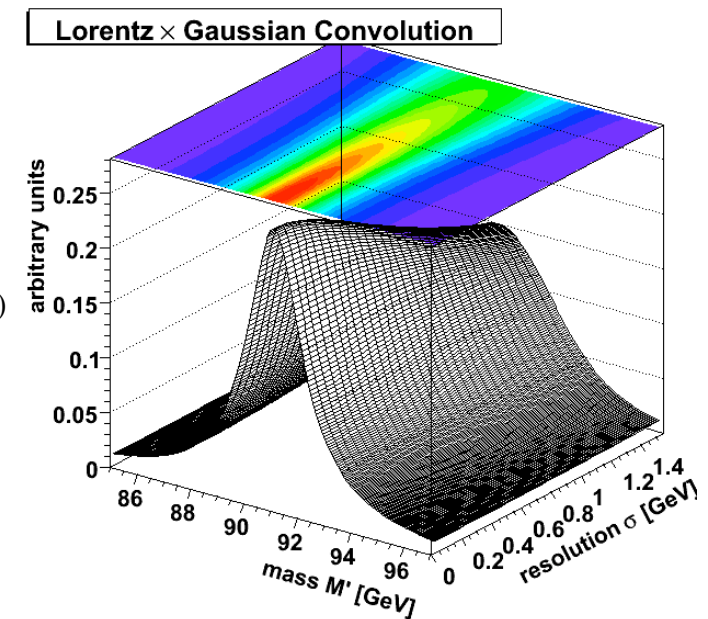


The muon momentum measurement and the *MuscleFit* algorithm



- ❑ Interest in evaluate the effects of the remaining misalignment in the Tracker on the muon momentum measurement
- ❑ Goals:
 - ✓ discover the possible bias by reconstructing the first resonances Z, J/ψ, Y as a function of all the possible muon kinematic variables (η , ϕ , $p_{t\prime}$, charge)
 - ✓ compute the discovered biases with multivariate likelihood approach
 - ✓ Deliver a function of the muon kinematics to correct the muon momentum scale and determine its resolution
- ❑ Calibration method to be applied on the first collected physics events (before the full detector calibration)
- ❑ Available algorithm in CMS: *MuSclFit*
- ❑ Each di-muon resonance (e.g. Z) fitted with:

$$F(c_j; M) = \text{Lorentz}(M_{ref}, \Gamma, M) \times \text{Gaussian}(\mu, \sigma, M - M') + \text{background}(c_j; M')$$
- ❑ Ansatz functions for parametrizing the biases:
 - Scale $p_{t\prime} = F(a_j; p_{t\prime}, \eta, \phi, q) \times p_t \rightarrow M'(p'_{t1}, p'_{t2})$
 - Resolution $\sigma_{\eta}, \sigma_{\phi}, \sigma_{pt} = G_i(b_j; p_t, \eta, \phi)$
- ❑ Minimization process to determine parameters a_j, b_j, c_j



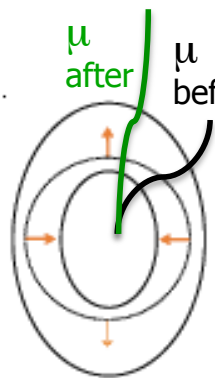


A test of the algorithm performances ~systematic misalignment~



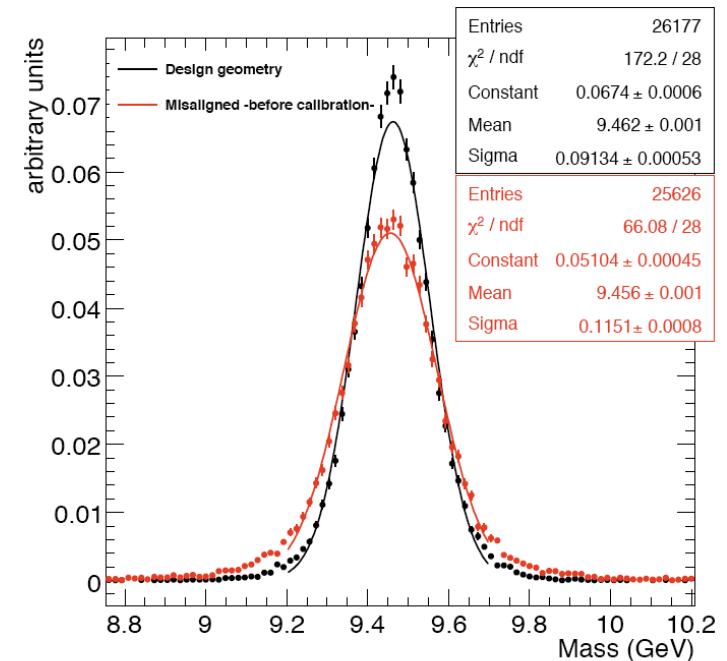
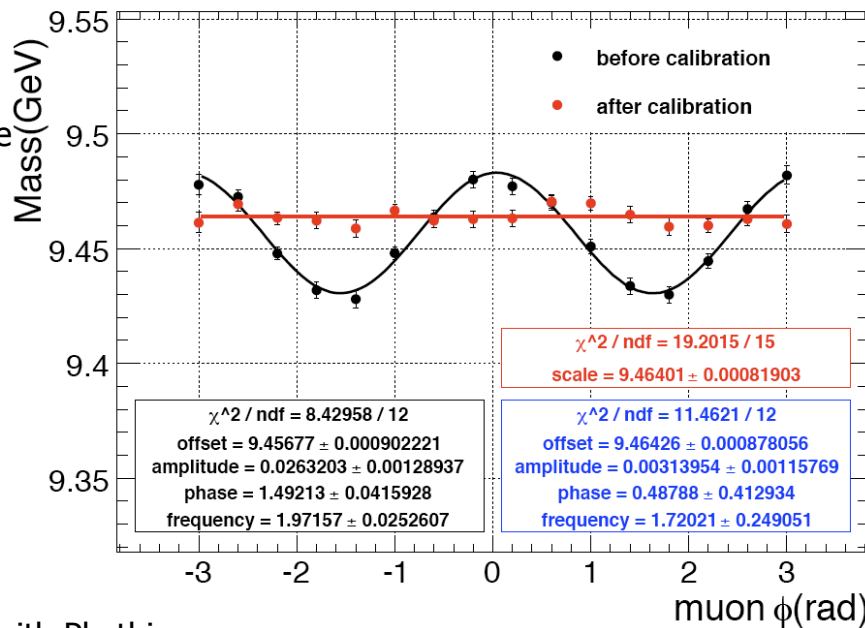
- Tracker systematic elliptical distorsion for testing algorithm capability in recovering it
- For each module: $\Delta r_{\text{global}} = 1 - c_1 \cos 2\phi$ where c_1 enlarged x10 w.r.t. what expected in reality
- 26k $Y \rightarrow \mu\mu^{\S}$ ($L_{\text{int}} \sim 2.6 \text{ pb}^{-1}$) reconstructed with this geometry: $\sigma_{\text{mass}} = 91 \text{ MeV}$ to 115 MeV
- Cosinusoidal dependence in $M_Y(\phi_{\text{muon}})$ properly modeled and recovered

$$p_t' = F(a_j p_t, \eta, \phi_j) \times p_t = (a_0 + a_1 \sin(a_2 \phi + a_3) + a_4 |\eta| + a_5 \eta^2)$$



elliptical distorsion

\S : generated with Pythia





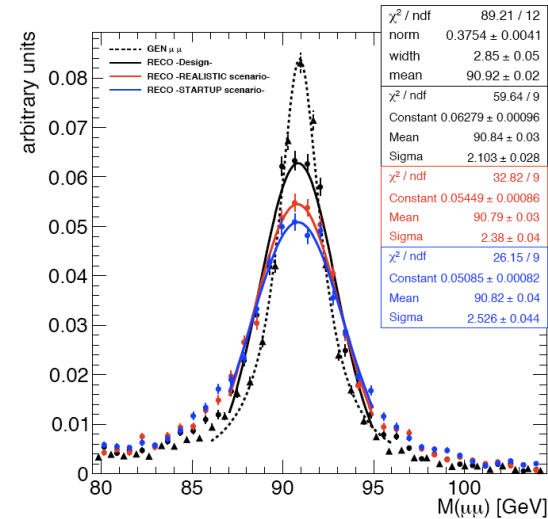
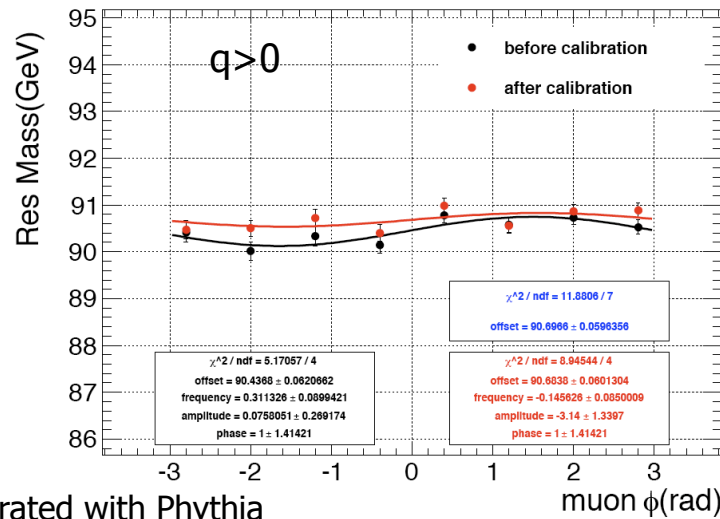
Momentum scale calibration using Z resonance



- $L_{\text{int}}=10 \text{ pb}^{-1}$ conditions: Startup/Realistic misalignment scenario applied on the generated[§] and reconstructed 5000 $Z \rightarrow \mu\mu$
- Impact on the $M(\mu\mu)$: worsening of σ_M (2.10/2.38/2.52 GeV using Design/Realistic/Startup geometry)
- effect of misalignment on the *scale*: ϕ dependence different for pos./neg. charged muons
- *Resolution*: η dependence modelled according regions ($|\eta| < 1.4$, $\eta < -1.4$, $\eta > 1.4$)

$$p_T' = F(a_j p_T, \eta, \phi) \times p_T = (a_0 + a_4|\eta| + a_5\eta^2)p_T + \begin{cases} a_1 \text{sen}(a_2\phi + a_3)p_T(\mu+) \\ a_6 \text{sen}(a_7\phi + a_8)p_T(\mu-) \end{cases} \rightarrow \begin{cases} a_1 = -0.0010 \pm 0.0005 \\ a_6 = -0.0036 \pm 0.0006 \\ a_2 = 0.75 \pm 0.04 \\ a_7 = 1.33 \pm 0.01 \end{cases}$$

- Biases in the reconstructed mass vs η , ϕ , p_T not significant anymore

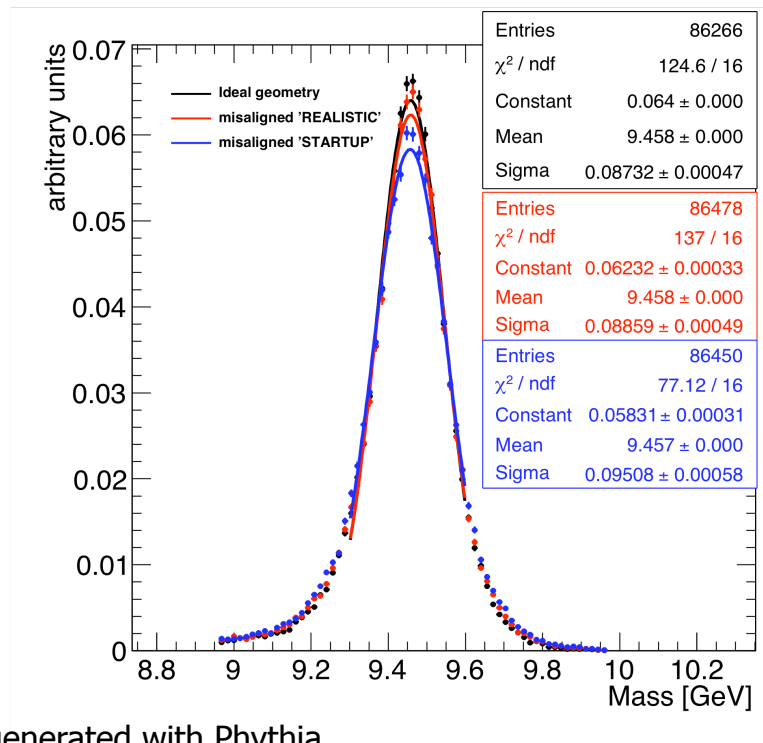




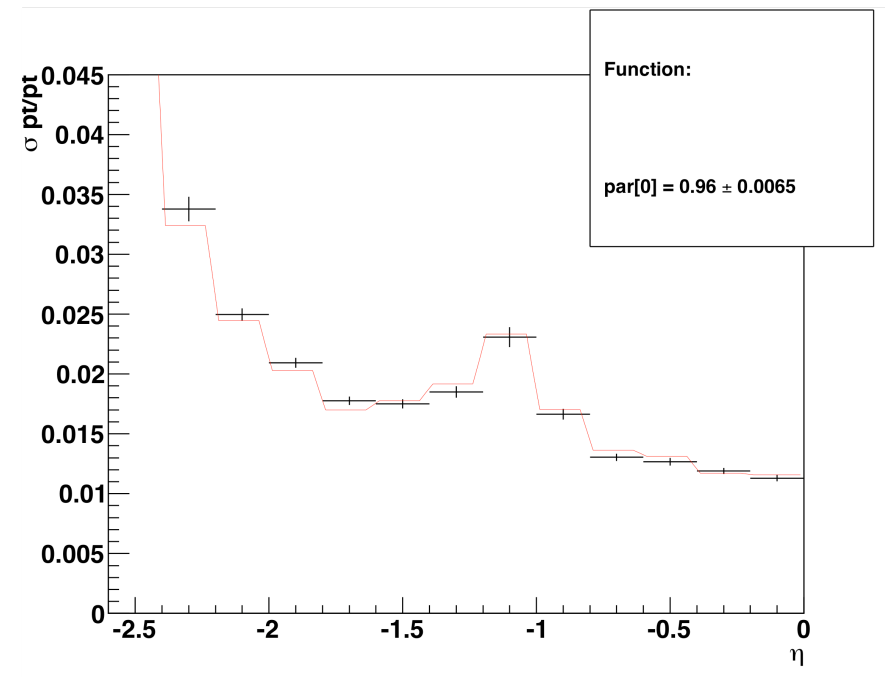
Momentum scale calibration using Υ resonance (I)



- $L_{\text{int}}=10 \text{ pb}^{-1}$ conditions: Startup/Realistic misalignment scenario applied in the reconstruction of 86000 $\Upsilon(1S) \rightarrow \mu\mu^{\pm}$
- Impact on the $M(\mu\mu)$: worsening of σ_M (87/88/95 MeV using Design/Realistic/Startup geometry)
- Low muon momentum tracks, most of the hits in the Tracker: Realistic scenario applied
- *Scale*: again ϕ dependence different for pos./neg. charged muons
- *Resolution*: point-by-point η dependence taken from a single muon generator (*particle gun*)



ξ : generated with Pythia





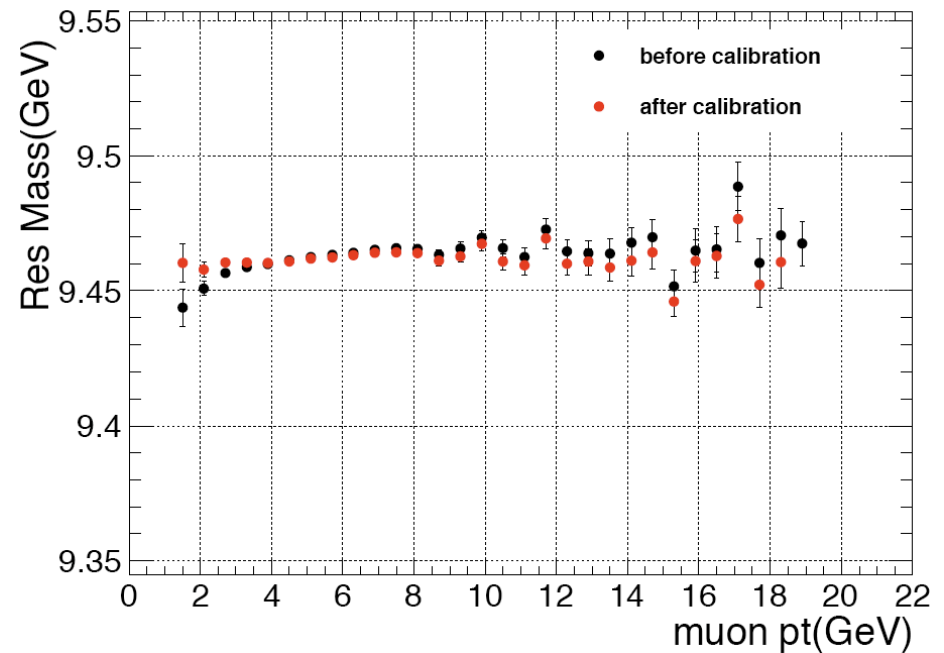
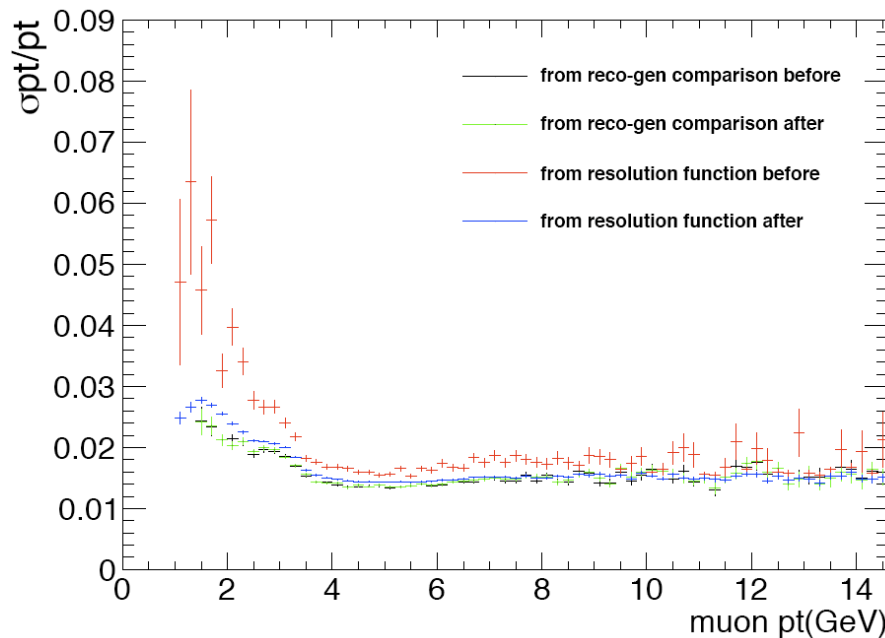
Momentum scale calibration using Υ resonance (II)



- Scale corrected at low p_t and resolution measured for low p_t values (up to ~ 15 GeV)
- Obtained hyperbolic term ($1/p_T$) allows for a better correction in the lowest p_T region

$$p_T' = F(a_j, p_T, \eta, \phi,) \times p_T = (a_0 + a_9/p_T + a_4|\eta| + a_5\eta^2)p_T + \begin{cases} a_1 \text{sen}(a_2\phi + a_3)p_T (\mu+) \\ a_6 \text{sen}(a_7\phi + a_8a)p_T (\mu-) \end{cases}$$

$$\begin{aligned} a_1 &= -0.178 \pm 0.003 \\ a_6 &= -0.0112 \pm 0.0009 \\ a_2 &= 0.00965 \pm 0.00001 \\ a_7 &= -0.036 \pm 0.01 \\ a_9 &= 0.00669 \pm 0.00004 \end{aligned}$$





Summary

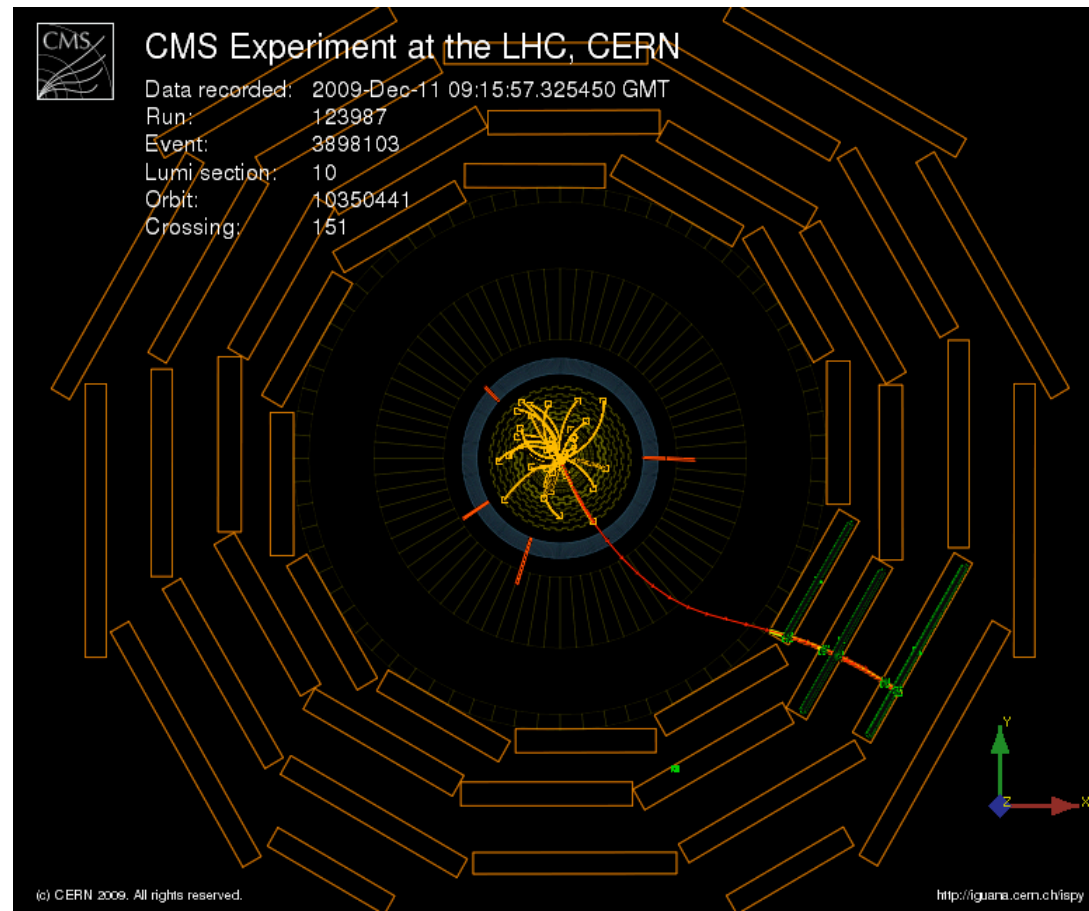


- ❑ Challenging demands of CMS for the momentum measurement led to design a complex inner tracking system.
- ❑ Unknown position of the 15k modules is the main source of systematic error for physics.
- ❑ Stand alone alignment at the TIF led to encouraging results, despite the non availability of the magnetic field: statistical precision in the module position of 50/80 μm in $r\phi$ w.r.t. particle trajectory.
- ❑ Alignment during commissioning with cosmic rays (CRAFT) significantly improved the statistical precision to 3-15 μm .
- ❑ Towards collision phase the study of remaining systematic deformations affecting Tracker using cosmics becomes an important step: collision tracks are needed to better constraint.
- ❑ Remaining biases on the momentum measurement, introduced by misalignment scenario, have been properly evaluated, modelled and corrected using dimuon resonances available after $L_{\text{int}}=10 \text{ pb}^{-1}$.

- ❑ Bibliografy:
 - ❑ W. Adam et al., **Alignment of the CMS silicon strip tracker during stand-alone commissioning**, published on *JINST 4-T07001* (July 2009)
 - ❑ The CMS collaboration, **Alignment of the CMS silicon Tracker during commissioning with cosmic rays**, *arXiv:0910.2505*, accepted by JINST
 - ❑ S. Bolognesi et al., **Calibration of track momentum using di muon resonances in CMS**, CMS note in preparation



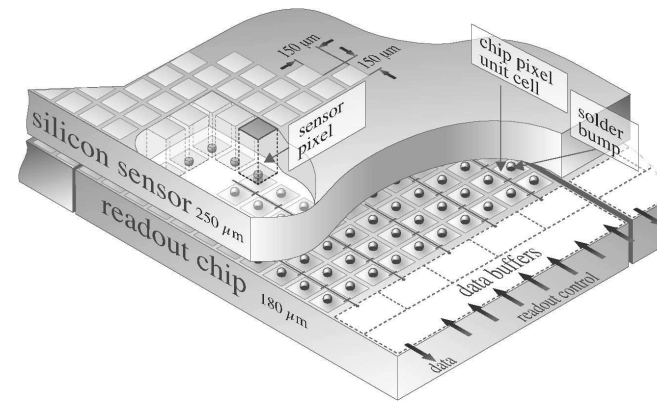
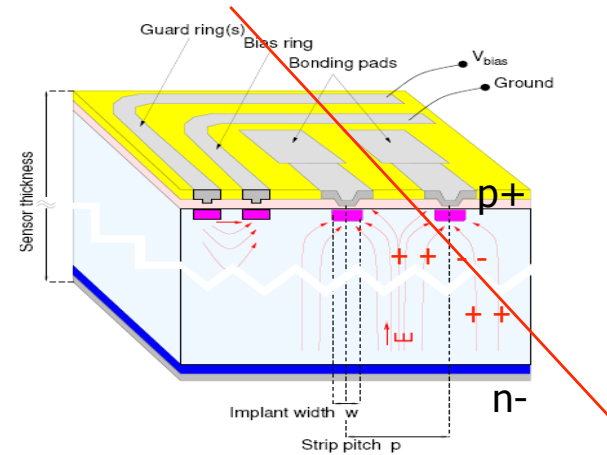
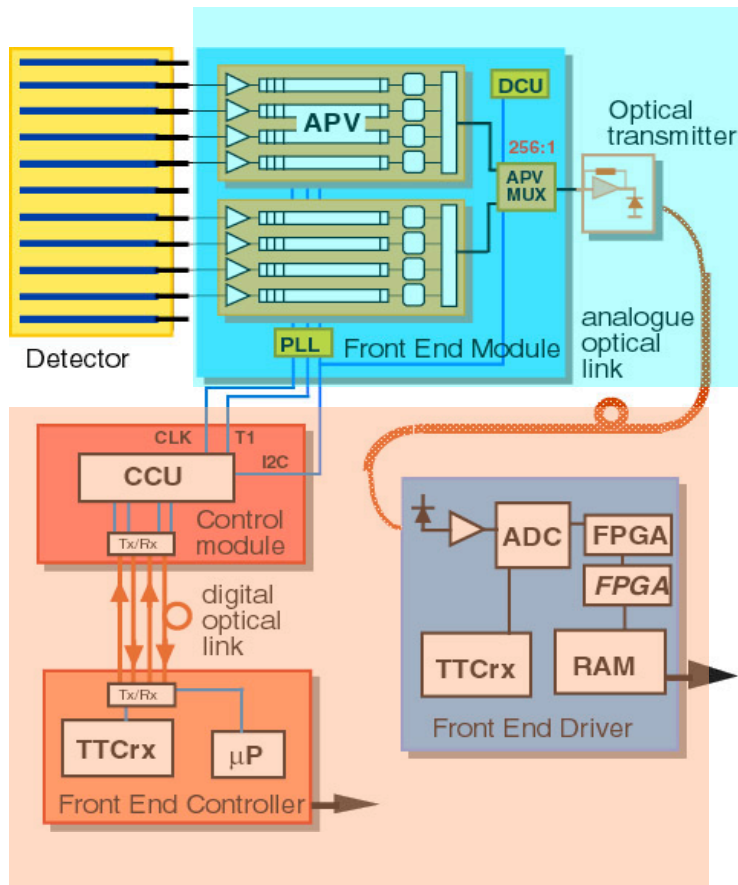
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First muon detected by CMS during pp collision $\sqrt{s}=900$ GeV.
Tracker geometry used in the reconstruction is the one obtained
after CRAFT alignment.

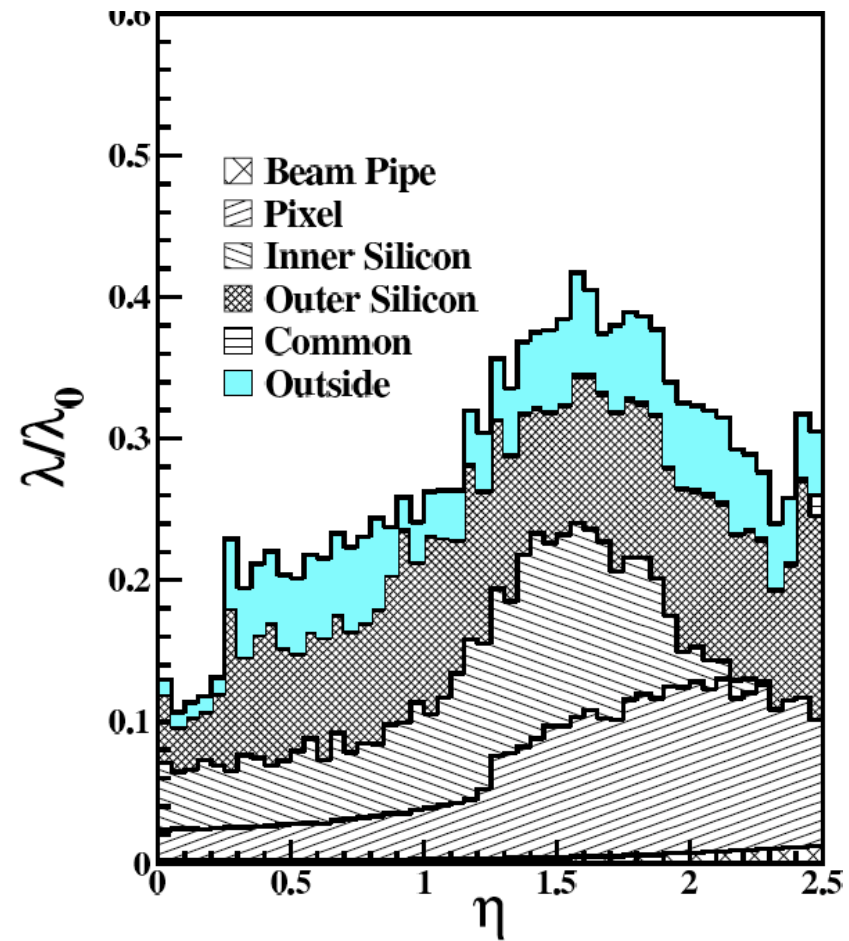
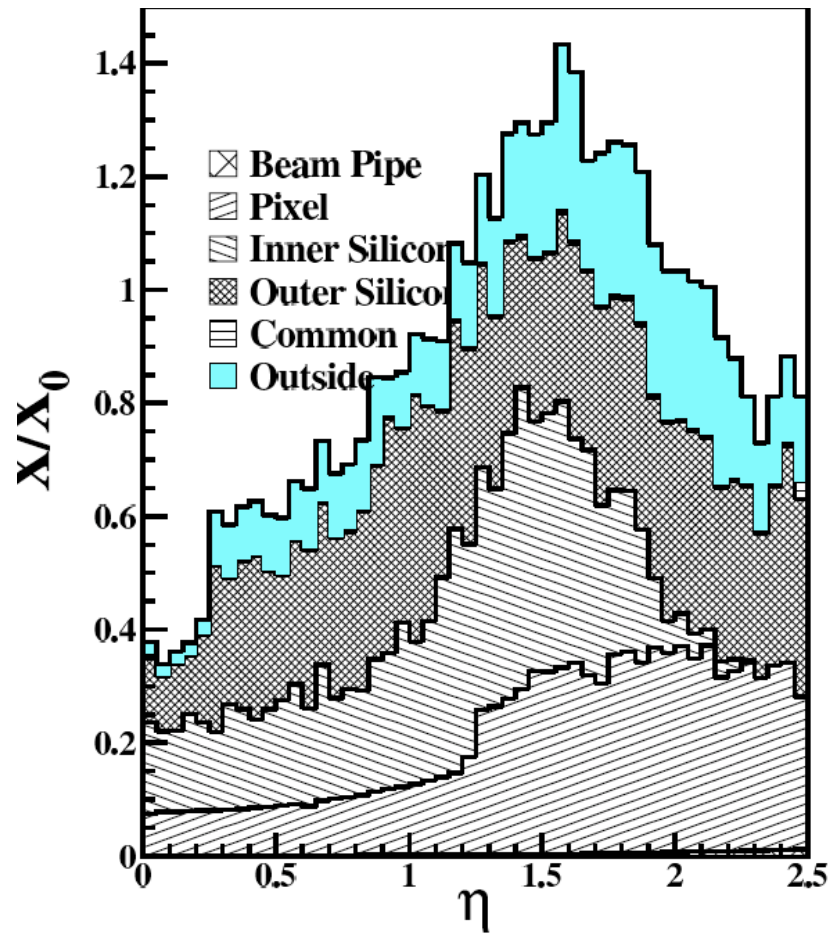
BACKUP slides

□ Solid state detectors (p-n junction)/ Tracker READ OUT





Material in the Tracker





Alignment formalism



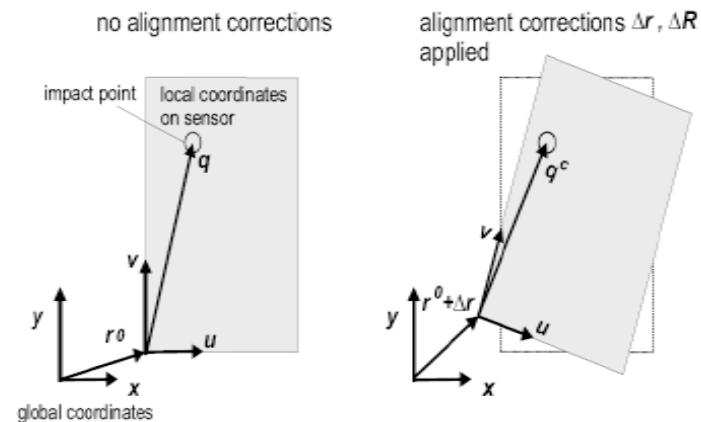
- The track hit position in the global reference frame (r) is expressed in terms of the position in the local frame (q) by the transformation:

$$r = R^T q + r_0$$

- The alignment provides corrections (ΔR , Δq) to the initial position, applied by a roto-translation operator:

$$r = R^T \Delta R (q + \Delta q) + r_0$$

- Final aim of the alignment is to determine the rotation parameters α, β, γ of the ΔR matrix and the translations $\Delta q = (\Delta u, \Delta v, \Delta w)$ to apply to the geometry of each of the Tracker modules





MillePede alignment algorithm



- V. Blobel (University of Hamburg)
- χ^2 function minimisation taking into account *track* and *alignment* parameters
- The global χ^2 function can be expressed as the sum of local contribution

$$\chi^2(p, q) = \sum_j^{\text{tracks}} \chi_j^2(p, q_j)$$

- The local χ_j^2 can be written in terms of residuals between measured hit position (y_i) and the corresponding prediction of the track model, $f_i(p, q_j)$

$$\chi_j^2(p, q_j) = \sum_i^{\text{hits}} \frac{(y_i - f_i(p, q_j))^2}{\sigma_i^2}$$

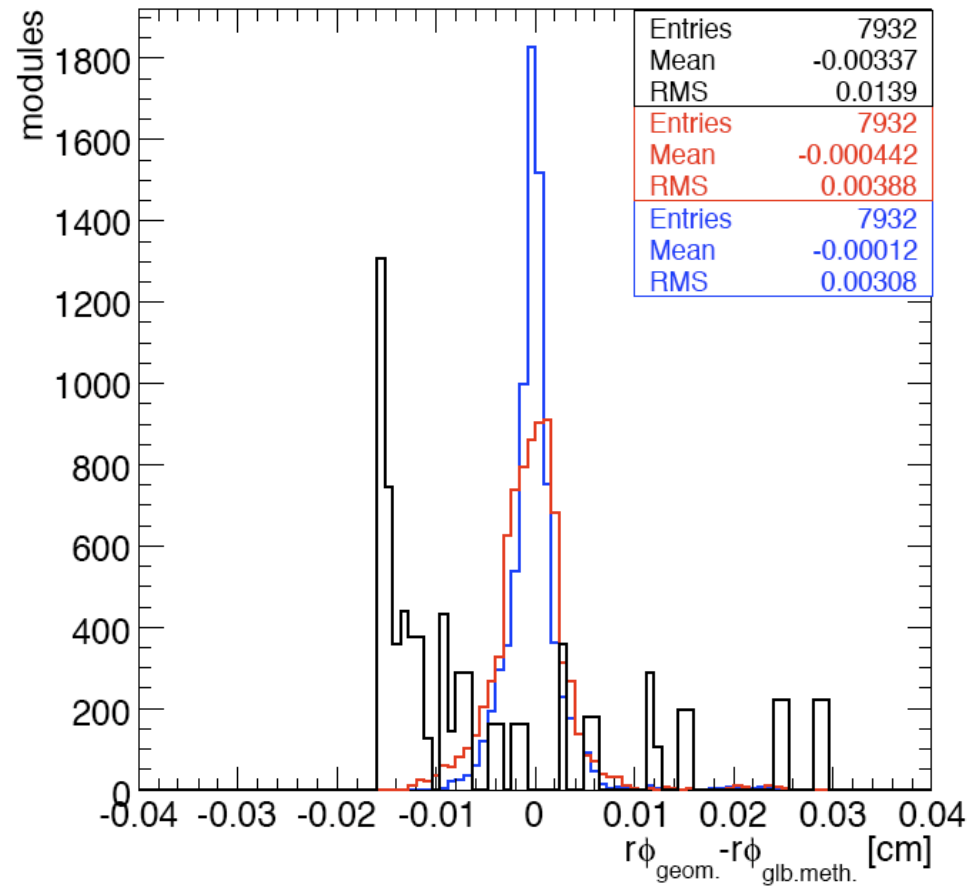
- Given reasonable start values p_0 and q_{j0} as expected in alignment, the track model $f_i(p, q_j)$ can be linearised

$$\chi_j^2(p, q_j) \approx \sum_i^{\text{hits}} \frac{\left(y_i - f_i(p_0, q_{j0}) + \frac{\partial f_i}{\partial p} a + \frac{\partial f_i}{\partial q_j} \Delta q_j \right)^2}{\sigma_i^2}$$

- Minimization leads to the matrix equation $Ca = b$ where C is built from the derivatives and the vector b from derivatives and residuals
- Alignment parameters a are determined



More on systematic misalignment



effect of the subtraction of coherent movements
of the highest level structures: scatter decreases
from **38 μm** to **30 μm**