# Jets in heavy-ion collisions at RHIC and LHC

Carlos A. Salgado
Universidade de Santiago de Compostela

International School on Quark-Gluon Plasma and Heavy Ion Collisions: past, present, future

carlos.salgado@usc.es http://cern.ch/csalgado

# Contents

# Summary of Lecture 1

- QCD corrections to naive parton model given by parton splitting
  - Evolution of parton distribution functions PDF
  - Jet structures
- Hadronic cross sections present a factorization between long and short distance contributions
  - PDFs and FF are universal
- Hard processes are excellent probes of the medium formed in heavy ion collisions
  - Computable in pQCD
  - Framework to compute medium-effects (jet quenching)
  - New regimes at the LHC

# LHC physics program

Fundamental Interactions
Searches – Higgs, SUSY, extra-dimensions...

→

Increase energy density

simple systems

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Collective properties
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simple systems

Collective properties
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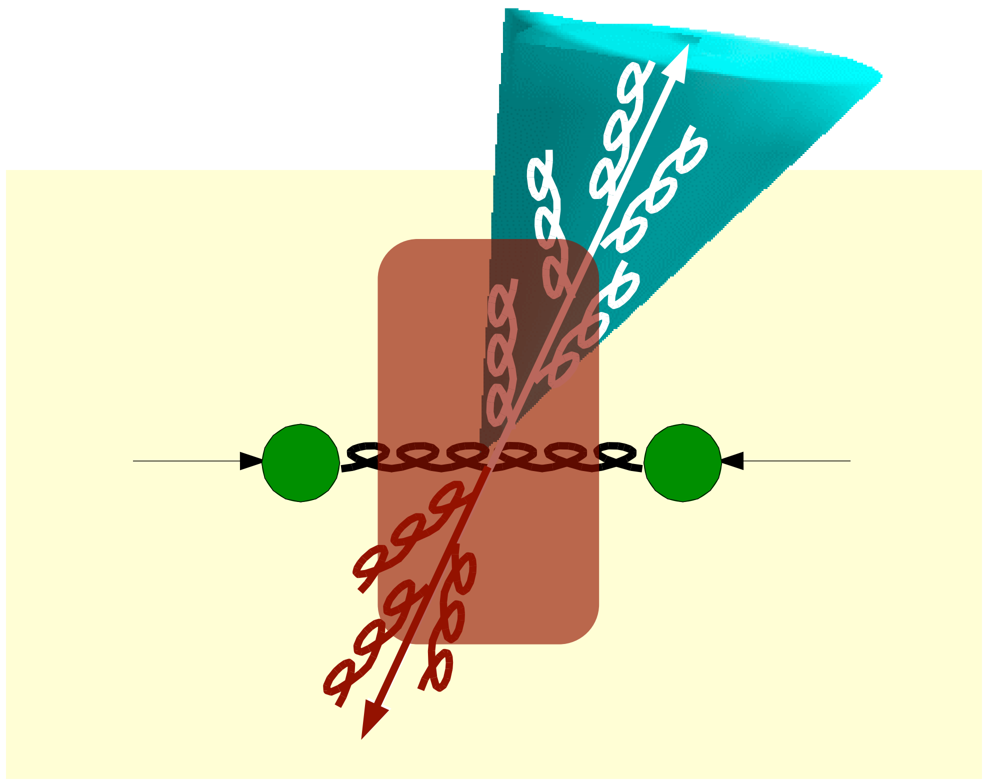
→ **Increase extended energy density**
"less simple" systems

**How?:** Symmetries and breaking of the symmetries

- Chiral symmetry/confinement (ALICE, ATLAS, CMS)
- EW symmetry breaking: Higgs (CMS, ATLAS)
- CP violation (LHCb)

# Jet quenching



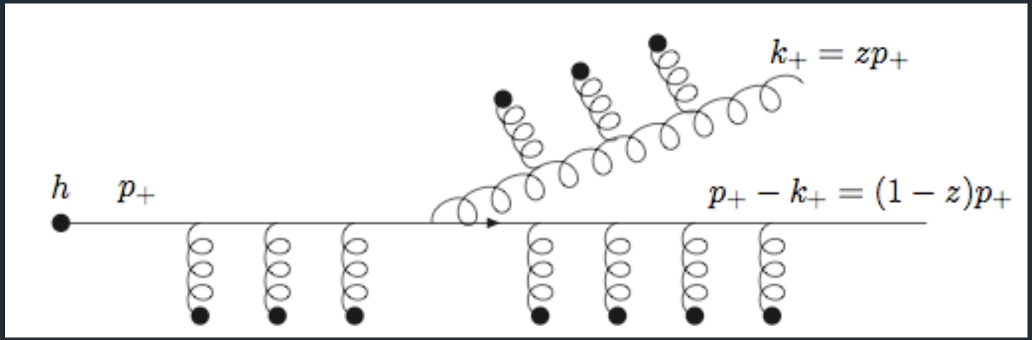
A jet is an **extended object**:

How does the extension of the medium modify the structure of the jet?

Is there a modification of the evolution equations?

# *Medium-induced gluon radiation:*

*We need the modification of the splitting probability:*




## Particle propagation in matter

- *Notice that we normally compute Feynman diagrams in momentum space*
- *This is ok for the vacuum where the space-time picture is not important*
- *For a finite-length medium we need to work in* ***configuration space***

# High-energy variables

⇛ Light-cone variables

$$x^{\pm} = x_0 \pm x_3 \qquad p^{\pm} = p_0 \pm p_3$$

⇛ So that, the scalar product

$$p \cdot x = \frac{1}{2}(p^{+}x^{-} + x^{-}x^{+}) - \mathbf{p}_{\perp} \cdot \mathbf{x}_{\perp}$$

⇛ Rapidity

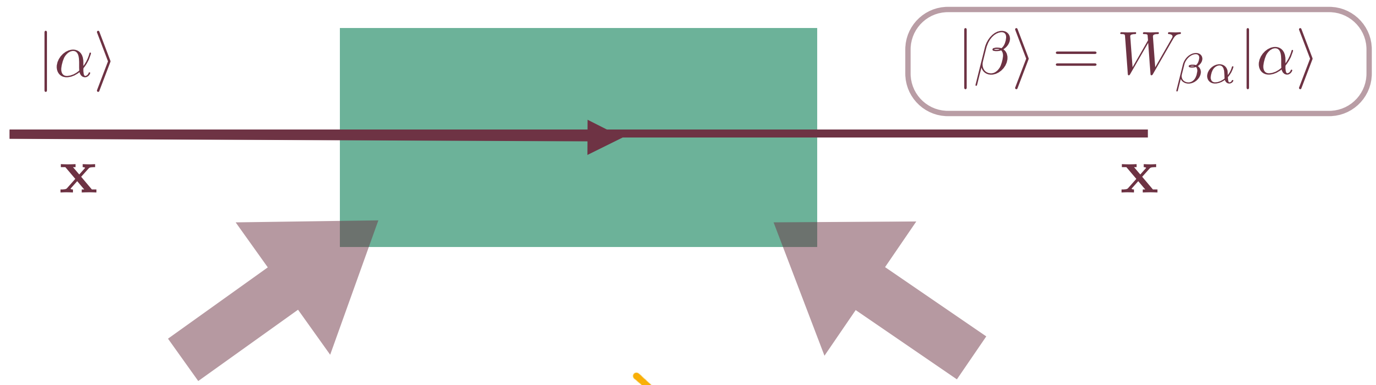
$$y = \frac{1}{2} \ln \left[ \frac{p_0 + p_3}{p_0 - p_3} \right] = \frac{1}{2} \ln \left[ \frac{p^{+}}{p^{-}} \right]$$

⇛ Boost is just adding a factor → *additive velocity*

$$y' = y + y_{\beta} \qquad \Longrightarrow \qquad y_{\beta} = \frac{1}{2} \ln \left[ \frac{1+\beta}{1-\beta} \right]$$

# Particle propagation in matter: Eikonal limit

⇛ At high energies → Eikonal approximation $E \gg k_\perp$




⇛ Particle does not change its direction of propagation

⇛ The medium rotates the color of the probe

$$W(\mathbf{x}) = \mathcal{P} \exp\left[ig \int dx_+ A_-(x_+, \mathbf{x})\right]$$

Wilson line

⇛ Recoil is neglected → medium is a background field

Note: We will follow the derivations in the lectures [see for more details]
J. Casalderrey-Solana and C.A. Salgado, arxiv:0712.3443

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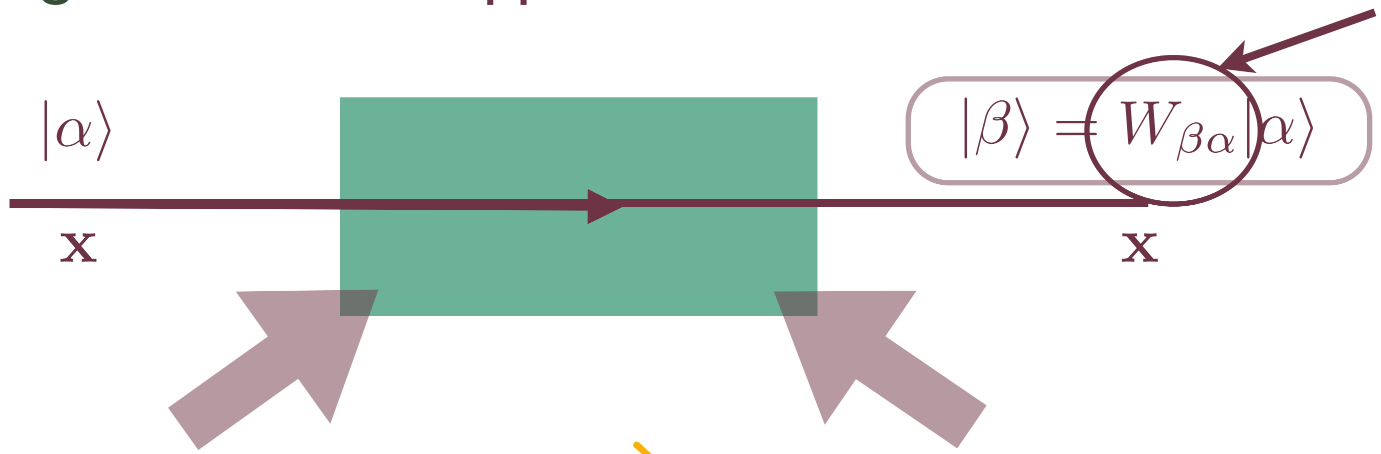


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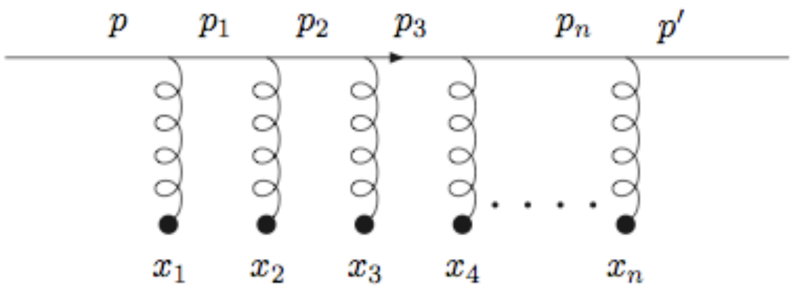
# Intuitive derivation from multiple scattering

- Consider a medium as a collection of **static scattering centers**
  - Equivalently: discretize the space for a recoil-less medium
  - For one scattering center...




$$S_1(p',p) = \int d^4x \, \mathrm{e}^{i(p'-p)\cdot x} \, \bar{u}(p') \, igA^a_\mu(x)T^a\gamma^\mu \, u(p)$$

  - In eikonal approximation $p \simeq p'$

$$\frac{1}{2}\sum_\lambda \bar{u}^\lambda(p)\gamma^\mu u^\lambda(p) = 2p^\mu \qquad p^\mu A^a_\mu \simeq 2p_+ A^a_-$$

$$S_1(p',p) \simeq 2\pi\delta(p'_+ - p_+)2p_+ \int d\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} \left[ ig \int dx_+ A_-(x_+, \mathbf{x}_\perp) \right],$$

  - Where we have used that the fields do not depend on $x_-$

[Equivalent to the eikonal approximation]

# Intuitive derivation from multiple scattering

- For two scattering centers:

$$S_2(p',p) = \int \frac{d^4p_1}{(2\pi)^4} d^4x_1 d^4x_2 \; e^{i(p_1-p)\cdot x_1} e^{i(p'-p_1)\cdot x_2} \bar{u}(p') \; igA^{a_1}_{\mu_1}(x_1)T^{a_1}\gamma^{\mu_1} \times$$
$$\times \quad i\frac{p_{1,\nu}\gamma^\nu}{p_1^2+i\epsilon} igA^{a_2}_{\mu_2}(x_2)T^{a_2}\gamma^{\mu_2} \; u(p)$$

- Applying the Dirac equation $\bar{u}(p)\gamma_{\mu_1}p_{1,\nu}\gamma^\nu\gamma_{\mu_2}u(p') \simeq (2p_+)^2 g_{\mu_1 -}g_{\mu_2 -}$

$$S_2(p',p) = -ig^2(2p_+)^2 \int \frac{d^4p_1}{(2\pi)^4} d^4x_1 d^4x_2 \; \frac{e^{i(p_1-p)\cdot x_1 + i(p'-p_1)\cdot x_2}}{p_1^2+i\epsilon} A_-(x_1)A_-(x_2),$$

- The integrals in $p_+$, $p_{1,\perp}$ give delta-functions, the remaining one

$$\int dp_{1-} \frac{e^{i(x_{1+}-x_{2+})p_{1-}}}{2p_{1+}p_{1-}+i\epsilon} = -\Theta(x_{2+}-x_{1+})\frac{2\pi i}{2p_+}$$

- Giving

$$S_2(p',p) \simeq 2\pi\delta(p'_+-p_+)2p_+ \int d\mathbf{x}_\perp e^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp-\mathbf{p}_\perp)} \frac{1}{2}\mathcal{P}\left[ig\int dx_+ A_-(x_+,\mathbf{x}_\perp)\right]^2 ;$$

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# Intuitive derivation from multiple scattering

⇛ So, summing all the contributions from n-scattering centers

$$S(p',p) = \sum_{n=0}^{\infty} S_n(p',p) \simeq 2\pi\delta(p'_+ - p_+)2p_+ \int d\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} W(\mathbf{x}_\perp);$$

⇛ Interpretation

- The Wilson line gives the S-matrix (equiv. scattering amplitude)
- It describes the propagation of an eikonal particle
- The particle propagates in a straight-line, no change in direction
- The only effect of the medium is to induce color rotation

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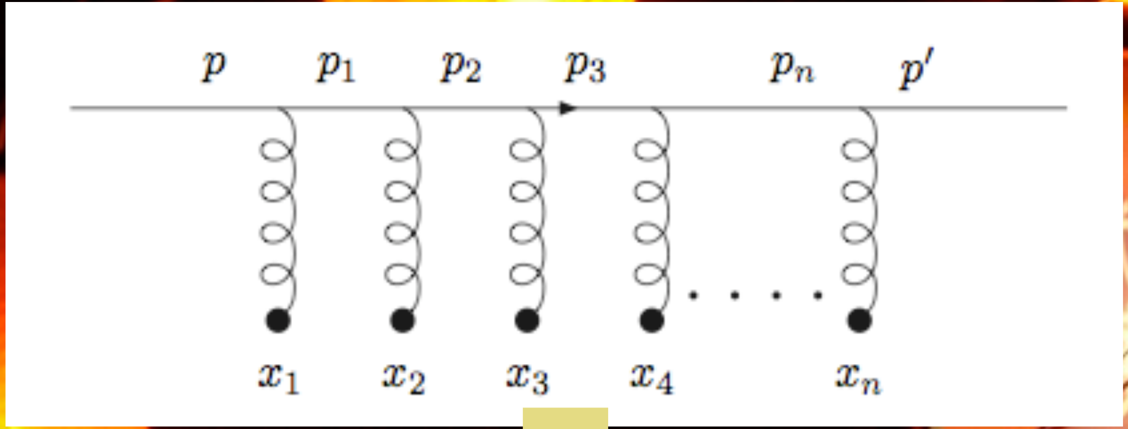


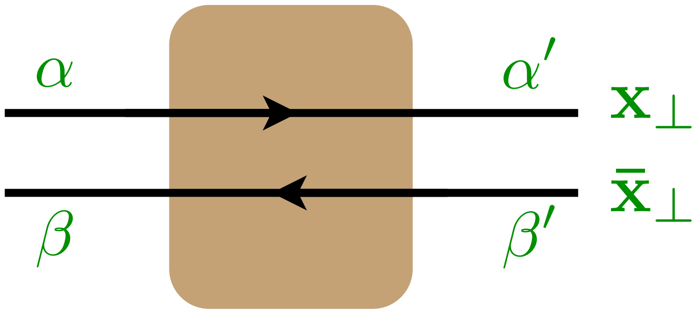

$$W(\mathbf{x}) = \mathcal{P} \exp \left[ ig \int dx_+ A_-(x_+, \mathbf{x}) \right]$$

**Wilson line**

# First example: the dipole scattering

⇛ Each propagation is a Wilson line at the relevant (fixed) transverse position

$$W(\mathbf{x}) = \mathcal{P} \exp \left[ ig \int dx_+ A_-(x_+, \mathbf{x}) \right]$$




⇛ So, the S-matrix

$$|\alpha'; \beta'\rangle \equiv S_{\alpha'\beta'\alpha\beta} |\alpha; \beta\rangle = W_{\alpha'\alpha}(\mathbf{x}_\perp) W^\dagger_{\beta'\beta}(\bar{\mathbf{x}}_\perp) |\alpha; \beta\rangle$$

⇛ Total probability of interaction (cross-section w/ needed factors)

$$P^{q\bar{q}}_{\rm tot} = \left\langle 2 - \frac{2}{N_C} {\rm Tr} \left[ W(\mathbf{x}_\perp) W^\dagger(\bar{\mathbf{x}}_\perp) \right] \right\rangle$$

[Ex. check these formulas, use e.g. the optical theorem]

# <u>*Medium averages*</u>

*The colorless object*

$$\frac{1}{N_c}\mathrm{Tr}W^\dagger(\mathbf{x}_\perp)W(\mathbf{y}_\perp)$$

*provides the scattering probability for a **<u>given configuration</u>** of the fields in the medium*

*To compute an observable, we need to **<u>average</u>** over all the possible medium configurations*

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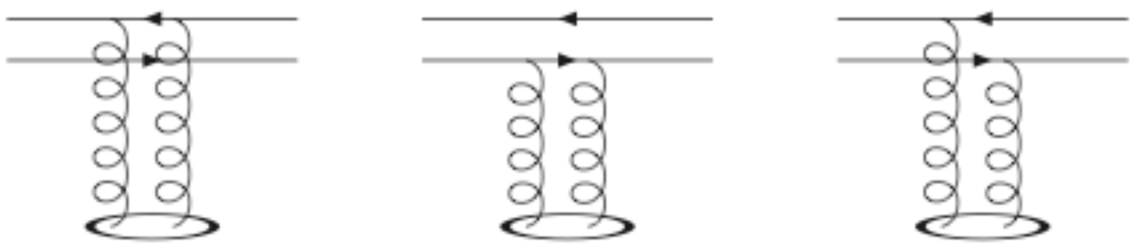
# Medium averages I

⇛ The Wilson lines appear always in colorless combinations as

$$\frac{1}{N}\text{Tr}\langle W^\dagger(\mathbf{x}_\perp)W(\mathbf{y}_\perp)\rangle = \frac{1}{N}\text{Tr}\Big\langle \exp\{-ig\int dx_+ A_-^\dagger(x_+,\mathbf{x}_\perp)\}\exp\{ig\int dx_+ A_-(x_+,\mathbf{y}_\perp)\}\Big\rangle$$

⇛ Expanding the exponents, the leading contribution is quadratic



⇛ Dipole cross section

$$\sigma(\mathbf{y}_\perp-\mathbf{x}_\perp) = 2\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,|a(\mathbf{q})|^2\left[1-\text{e}^{i(\mathbf{y}_\perp-\mathbf{x}_\perp)\mathbf{q}}\right]$$

⇛ For a Yukawa screened potential

$$|a(\mathbf{q})|^2 = \frac{\mu^2}{\pi(\mathbf{q^2}+\mu^\mathbf{2})^2} \qquad \sigma(r) = \frac{1}{(2\pi)^2\mu^2}\left[1-\mu r K_1(\mu r)\right] \simeq \frac{1}{(2\pi)^2}\,r^2\log\left[\frac{2}{\mu r}\right]$$

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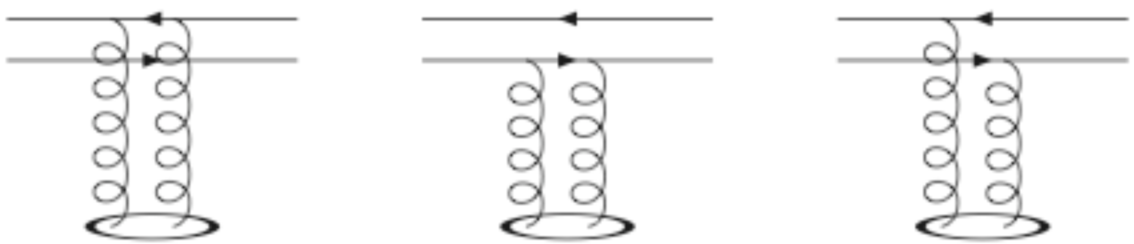

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# Two main averages used in jet quenching

- Single-hard approximation (GLV)

$$\frac{1}{N_c^2-1}\mathrm{Tr}\left\langle W_A^\dagger(\mathbf{x}_\perp)W_A(\mathbf{y}_\perp)\right\rangle \simeq 1-\frac{N_c}{2}\int d\xi\, n(\xi)\sigma(x_\perp-y_\perp)$$

- Multiple soft scattering approximation (BDMPS-Z/AWS...)

$$\frac{1}{N_c^2-1}\mathrm{Tr}\left\langle W_A^\dagger(\mathbf{x}_\perp)W_A(\mathbf{y}_\perp)\right\rangle \simeq \exp\left\{-\frac{N_c}{4}\int d\xi\, \hat{q}(\xi)(x_\perp-y_\perp)^2\right\}$$

- So, the transport coefficient is given by the density times the factor of the quadratic term in the cross section (neglect logs)

$$\hat{q}(\xi)\equiv 2n(\xi)\,C \qquad \text{with} \qquad \sigma(r)\simeq C\,r^2$$

- Relation with the Color Glass Condensate: $Q^2_{\rm sat}\ \to\ \hat{q}\,L$

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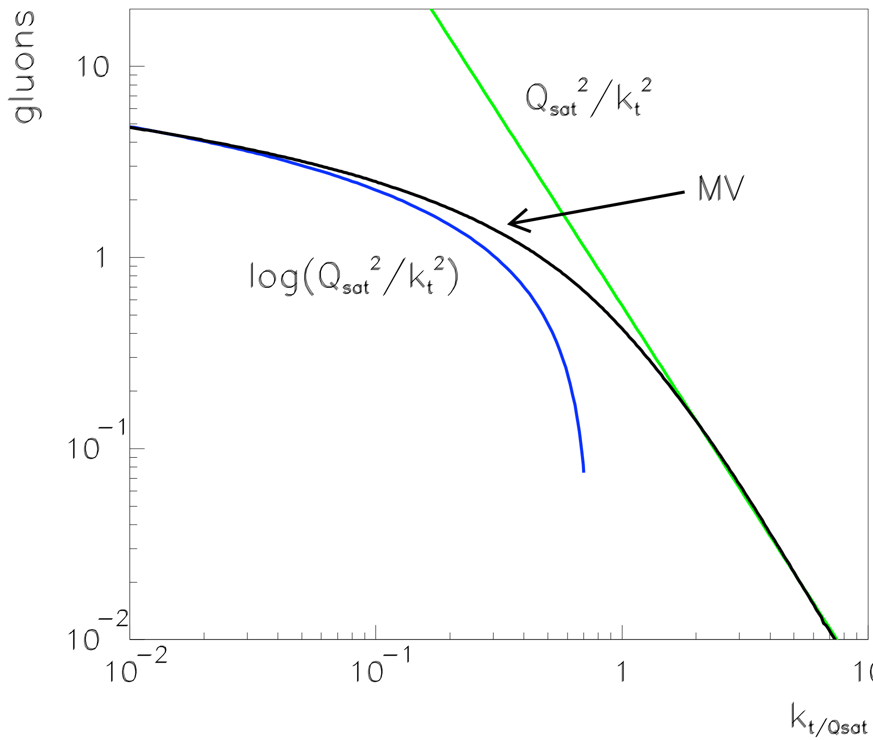
# So, coming back to the dipole

⇛ The dipole 'counts' the number of gluons, of a given size $r$, in the nucleus, so the (unintegrated) gluon distribution:

$$N(r) = 1 - \exp\left[-\frac{1}{8} Q_{\rm sat}^2 r^2\right] \quad \Longrightarrow \quad \phi(k) = \int \frac{d^2 r}{2\pi r^2} e^{i\mathbf{r}.\mathbf{k}} N(r)$$

[up to logs: McLerran, Venugopalan 1994]




Two important consequences:

- Saturation scale cuts-off the small momentum region
- Geometric scaling:

$$\phi = \phi(k^2/Q_{\rm sat}^2)$$

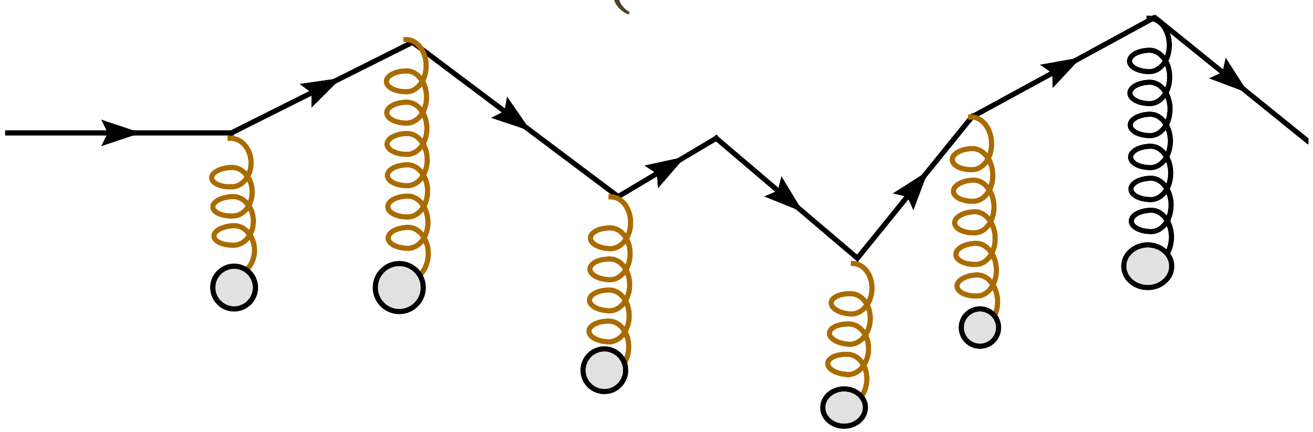
# Non-eikonal terms

⇛ To compute the medium-induced gluon radiation, we will take into account small departure from a straight line for the gluon

$$\int dp_- \frac{\mathrm{e}^{ip_-(x_{i+}-x_{(i+1)+})}}{2p_+p_- - p_\perp^2 + i\epsilon} = -i\frac{2\pi}{2p_+}\Theta(x_{(i+1)+} - x_{i+})\mathrm{e}^{i\frac{p_\perp^2}{2p_+}((x_{i+}-x_{(i+1)+})}$$

⇛ In this case, instead of the Wilson line we obtain a path integral

$$G(a,b) = \int \mathcal{D}\mathbf{r}(x_+)\exp\left\{i\frac{p_+}{2}\int dx_+\left[\frac{d\mathbf{r}}{dx^+}\right]^2 + ig\int dx_+ A_-(x_+, r(x_+))\right\}$$



Brownian motion in transverse plane

$$\hat{q} \sim \frac{\mu^2}{\lambda}$$

⇛ Notice that for $p_+ \to \infty$ the Wilson line is recovered

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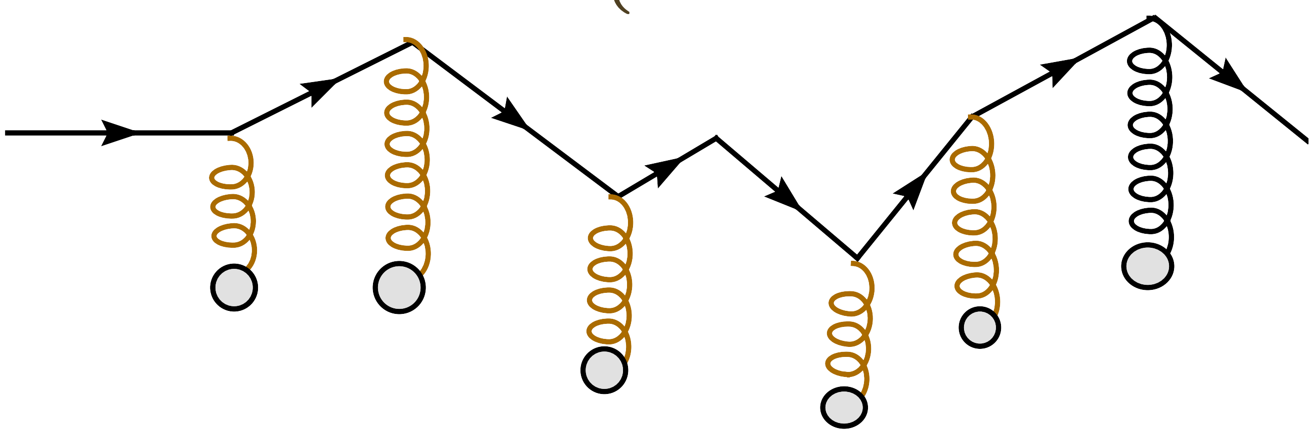

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# Medium-induced radiation (sketch of calculation)

⇛ We work in the approximation of a very highly energetic quark which radiates a soft gluon

$$E_q \gg \omega \gg k_\perp$$

- Eikonal propagators for quarks
- Non-eikonal corrections for gluons

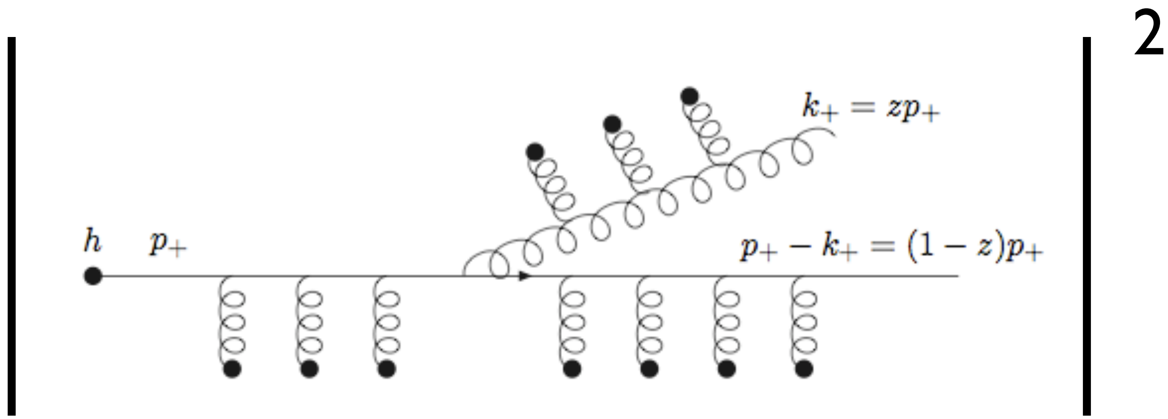
⇛ "Recipe": write

- Quark propagation $W(\mathbf{x}_\perp, x_+, y_+)$
- Gluon propagation $G(\mathbf{x}_\perp, x_+; \mathbf{y}_\perp, y_+)$
- Quark-gluon hard vertex

$$\frac{i}{k_+}\epsilon_\perp \cdot \frac{\partial}{\partial \mathbf{y}_\perp}$$

- Then include Fourier transforms, integrals, color traces, factors....

# The medium-induced gluon radiation




$$\langle|\mathcal{M}_{a\to bc}|^2\rangle = \frac{g^2}{N^2-1} 2\mathrm{Re}\Bigg[\frac{1}{k_+^2}\int_{x_{0+}}^{L_+} dx_+ \int_{x_+}^{L_+} d\bar{x}_+ \int d\mathbf{x} d\bar{\mathbf{x}}\; e^{i\mathbf{k}_\perp(\mathbf{x}-\bar{\mathbf{x}})} \times$$

$$\Big\langle W_{aa_1}(\mathbf{0}, x_{0+}, x_+) T^{c_1}_{a_1 b_1} \frac{\partial}{\partial \mathbf{y}} G_{c_1 c}(\mathbf{y}=0, x_+; \mathbf{x}, L_+) W_{b_1 b}(\mathbf{0}, x_+, L_+) \times$$

$$W^\dagger_{b\bar{b}_1}(\mathbf{0}, \bar{x}_+, L_+) \frac{\partial}{\partial \bar{\mathbf{y}}} G_{c\bar{c}_1}(\bar{\mathbf{x}}, L_+; \bar{\mathbf{y}}=0, \bar{x}_+) T^{\bar{c}_1}_{\bar{b}_1 \bar{a}_1} W^\dagger_{\bar{a}_1 a}(\mathbf{0}, x_{0+}, \bar{x}_+)\Big\rangle -$$

$$-\frac{2}{k_+}\frac{\mathbf{k}_\perp}{k_\perp^2}\int_{x_{0+}}^{L_+} dx_+ \int d\mathbf{x} e^{i\mathbf{k}_\perp \mathbf{x}} \Big\langle W_{aa_1}(\mathbf{0}, x_{0+}, x_+) T^{c_1}_{a_1 b_1} \frac{\partial}{\partial \mathbf{y}} G_{c_1 c}(\mathbf{y}=0, x_+; \mathbf{x}, L_+) \times$$

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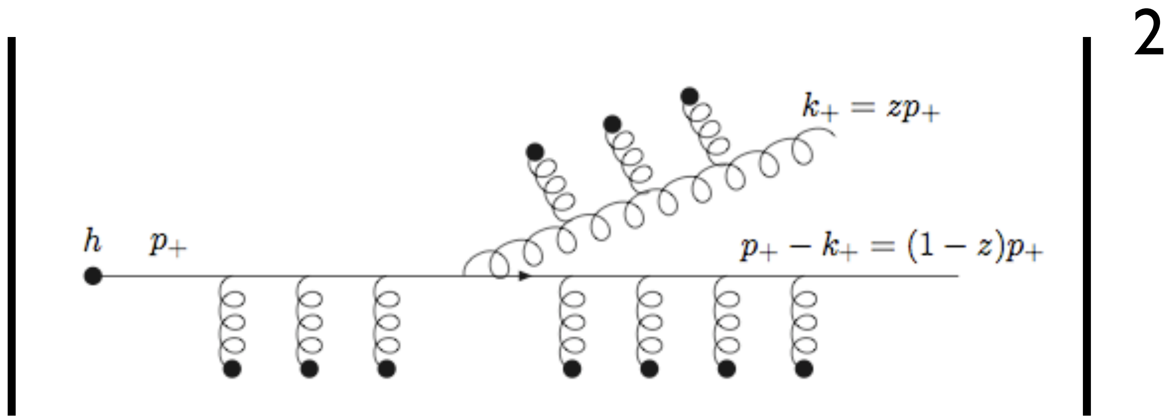
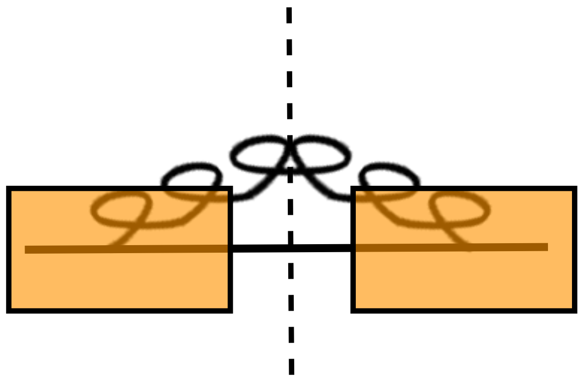
# The medium-induced gluon radiation



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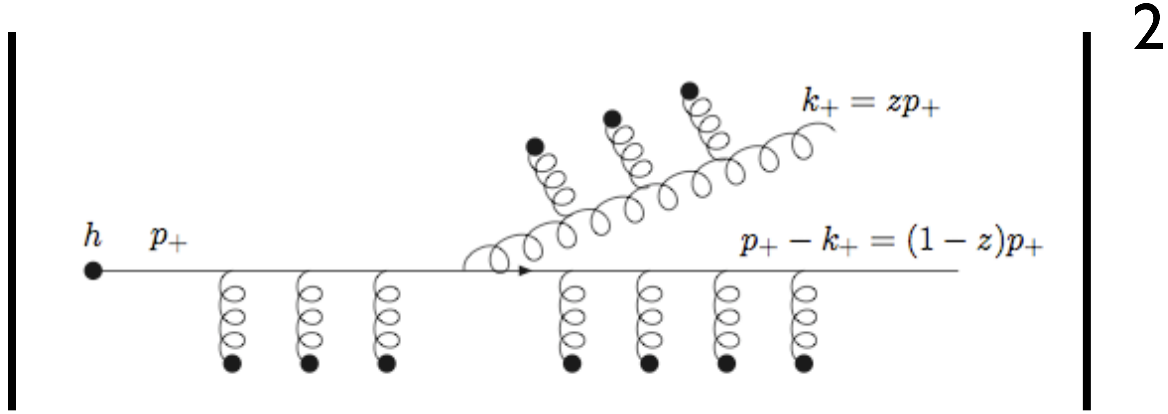
$$\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)W_{b_1b}(\mathbf{0},x_+,L_+)\times$$

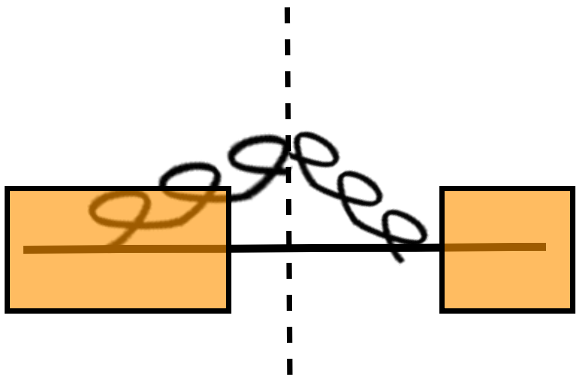
$$W^\dagger_{b\bar{b}_1}(\mathbf{0},\bar{x}_+,L_+)\frac{\partial}{\partial\bar{\mathbf{y}}}G_{c\bar{c}_1}(\bar{\mathbf{x}},L_+;\bar{\mathbf{y}}=0,\bar{x}_+)T^{\bar{c}_1}_{\bar{b}_1\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},\bar{x}_+)\Big\rangle -$$

$$-\frac{2}{k_+}\frac{\mathbf{k}_\perp}{k_\perp^2}\int_{x_{0+}}^{L_+} dx_+\int d\mathbf{x}e^{i\mathbf{k}_\perp\mathbf{x}}\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)\times$$

$$W_{b_1b}(\mathbf{0},x_+,L_+)T^{c_1}_{b\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},L_+)\Big\rangle\Bigg] + \frac{4g^2C_R}{k_\perp^2}$$

# The medium-induced gluon radiation



$$\langle|\mathcal{M}_{a\to bc}|^2\rangle = \frac{g^2}{N^2-1} 2\mathrm{Re}\Bigg[\frac{1}{k_+^2}\int_{x_{0+}}^{L_+} dx_+ \int_{x_+}^{L_+} d\bar{x}_+ \int d\mathbf{x}d\bar{\mathbf{x}}\; e^{i\mathbf{k}_\perp(\mathbf{x}-\bar{\mathbf{x}})}\times$$

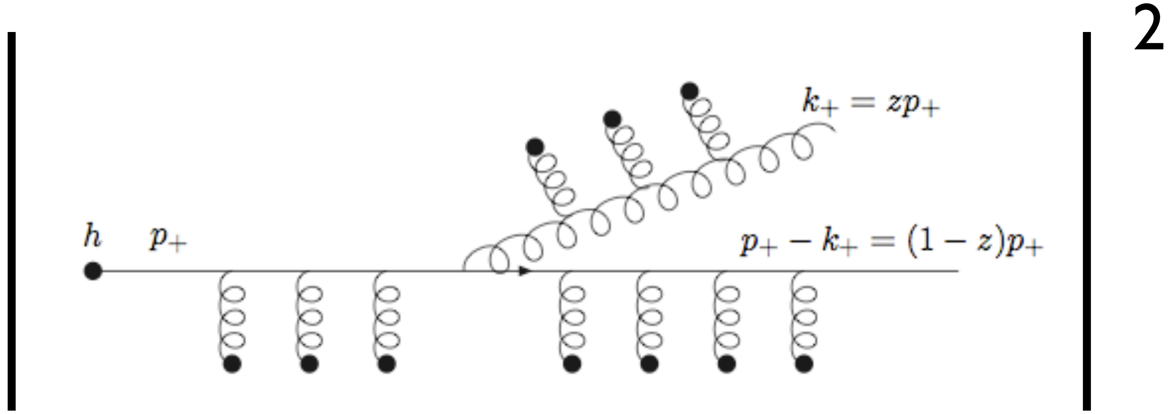
$$\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)W_{b_1b}(\mathbf{0},x_+,L_+)\times$$

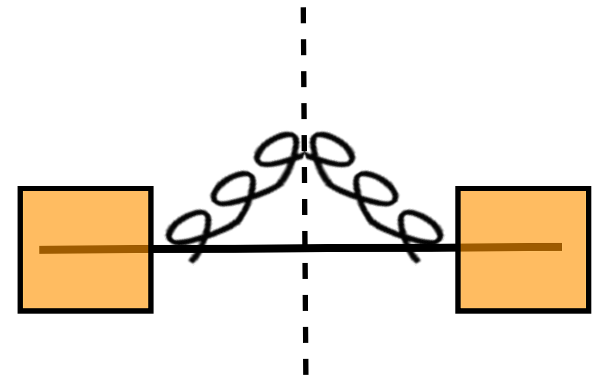
$$W^\dagger_{b\bar{b}_1}(\mathbf{0},\bar{x}_+,L_+)\frac{\partial}{\partial\bar{\mathbf{y}}}G_{c\bar{c}_1}(\bar{\mathbf{x}},L_+;\bar{\mathbf{y}}=0,\bar{x}_+)T^{\bar{c}_1}_{\bar{b}_1\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},\bar{x}_+)\Big\rangle -$$

$$-\frac{2}{k_+}\frac{\mathbf{k}_\perp}{k_\perp^2}\int_{x_{0+}}^{L_+} dx_+ \int d\mathbf{x}e^{i\mathbf{k}_\perp\mathbf{x}}\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)\times$$

$$W_{b_1b}(\mathbf{0},x_+,L_+)T^{c_1}_{b\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},L_+)\Big\rangle\Bigg] + \frac{4g^2C_R}{k_\perp^2}$$

# The medium-induced gluon radiation



$$\langle|\mathcal{M}_{a\to bc}|^2\rangle = \frac{g^2}{N^2-1} 2\mathrm{Re}\Bigg[\frac{1}{k_+^2}\int_{x_{0+}}^{L_+} dx_+ \int_{x_+}^{L_+} d\bar{x}_+ \int d\mathbf{x}d\bar{\mathbf{x}}\; e^{i\mathbf{k}_\perp(\mathbf{x}-\bar{\mathbf{x}})}\times$$

$$\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)W_{b_1b}(\mathbf{0},x_+,L_+)\times$$

$$W^\dagger_{b\bar{b}_1}(\mathbf{0},\bar{x}_+,L_+)\frac{\partial}{\partial\bar{\mathbf{y}}}G_{c\bar{c}_1}(\bar{\mathbf{x}},L_+;\bar{\mathbf{y}}=0,\bar{x}_+)T^{\bar{c}_1}_{\bar{b}_1\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},\bar{x}_+)\Big\rangle -$$

$$-\frac{2}{k_+}\frac{\mathbf{k}_\perp}{k_\perp^2}\int_{x_{0+}}^{L_+} dx_+ \int d\mathbf{x}e^{i\mathbf{k}_\perp\mathbf{x}}\Big\langle W_{aa_1}(\mathbf{0},x_{0+},x_+)T^{c_1}_{a_1b_1}\frac{\partial}{\partial\mathbf{y}}G_{c_1c}(\mathbf{y}=0,x_+;\mathbf{x},L_+)\times$$

$$W_{b_1b}(\mathbf{0},x_+,L_+)T^{c_1}_{b\bar{a}_1}W^\dagger_{\bar{a}_1a}(\mathbf{0},x_{0+},L_+)\Big\rangle\Bigg] + \boxed{\frac{4g^2C_R}{k_\perp^2}}$$

***So, now all the problem reduces to compute all the medium averages***

## *So, now all the problem reduces to compute all the medium averages*

*This, in fact, takes a while.....*

# Summary

- High-energy limit: Eikonal approximation
  - Particle propagates in a **straight line** without energy loss
  - Described by Wilson lines
- Non-eikonal corrections
  - Allow for changes in the transverse position
  - **Brownian motion** in transverse plane
- Medium-induced gluon radiation
  - Take parent as completely eikonal, and apply corrections to gluon
  - Energy loss by radiation

# Title