The deconfined phase of QCD: collective dynamics of the Quark Gluon Plasma

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Quark-Gluon Plasma and Heavy Ion Collisions: past, present, future

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Outline

- The plasma as a dielectric medium:
  - The Maxwell equations in a medium,
  - The dielectric tensor $\epsilon_{ij}(\omega, \boldsymbol{k})$,
  - The propagation of transverse and *longitudinal* excitations;
- Microscopic evaluation of the dielectric tensor in the *Hard (Thermal) Loop* approximation: the result of *classical kinetic theory*;
- Physical applications: study of general medium effects (plasma-oscillations, thermal masses, Debye-screening, damping and energy-loss processes) starting from the dielectric tensor;
- The link with the *Thermal Field Theory* calculation based on Feynman-diagrams;
- Summary

The general setup

We consider a hot-relativistic gauge (e.g. QED, QCD...) plasma

- hot-relativistic: $m \ll T$, $\epsilon \sim T^4$, $n \sim T^3$ (Stephan-Boltzmann law) $\longrightarrow$ massless plasma particles with typical momenta $k \sim T$;
- gauge: coupling through the covariant derivative $\partial_\mu - igA_\mu$
  NB pure thermal fluctuations entail $\langle A^2 \rangle \sim T^2$, hence for a field fluctuation $\delta(x) \sim \sum_k \delta_k e^{-ik\cdot x}$
  - the propagation of *hard modes* ($k \sim T$) is midly modified:

$$\partial_\mu - ig\overline{A}_\mu \sim T - igT$$

  - the propagation of sufficiently long-wavelenght *soft excitations* ($k \sim gT$), *even for a small value of the coupling*, ($g \ll 1$) is strongly affected by thermal fluctuations

$$\partial_\mu - ig\overline{A}_\mu \sim -igT - igT$$

These are the effects we are going to describe!

A plasma as a dielectric medium

- A plasma is a system of *charged particles* (colored in the case of the QGP) free to propagate over macroscopic distances, giving rise to a non-trivial collective dynamics;

- For the purpose of displaying general collective phenomena occurring in a plasma like the QGP it is sufficient to consider an *electromagnetic plasma* of relativistic particles of charge $q = \pm e$.
  A perturbation of the electromagnetic field will produce an *induced current*

$$j^\mu_{\rm ind}(x) = g_s \times e \int \frac{d\boldsymbol{p}}{(2\pi)^3} v^\mu \left[f_+(\boldsymbol{p},x) - f_-(\boldsymbol{p},x)\right] \quad [\text{with} \quad v^\mu = (1, \boldsymbol{v})]$$

  polarizing the medium (spin factor for an $e^\pm$ plasma: $g_s = 2$).

- When necessary a dictionary will be provided to translate the results to the case of a QCD plasma (of $q, \bar{q}, g$).

The Maxwell equations in a medium I

- Studying the propagation of excitations of the e.m. field in a dielectric medium the induced charge and current densities ρ_{ind} and $\boldsymbol{j}_{\text{ind}}$ will act as a further source term. One has:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho_{\text{ext}} + \rho_{\text{ind}} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{j}_{\text{ext}} + \boldsymbol{j}_{\text{ind}} + \frac{\partial \boldsymbol{E}}{\partial t}$$

- It is useful to introduce the field $\boldsymbol{D} \equiv \boldsymbol{E} + \boldsymbol{P}$, which embodies the effect of the medium polarization ($\boldsymbol{P}$).
  Being $\boldsymbol{\nabla} \cdot \boldsymbol{P} = -\rho_{ind}$ and $\partial_t \boldsymbol{P} = \boldsymbol{j}_{\text{ind}}$:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_{\text{ext}} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{j}_{\text{ext}} + \frac{\partial \boldsymbol{D}}{\partial t}$$

The Maxwell equations in a medium II

- It is convenient to consider the Fourier components of the fields e.g.

$$\boldsymbol{E}(t,\boldsymbol{x}) = \boldsymbol{E}(\omega,\boldsymbol{k})e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{x})}$$

- One introduces the *dielectric tensor* $\epsilon^{ij}(\omega,\boldsymbol{k})$

$$D^i(\omega,\boldsymbol{k}) \equiv \epsilon^{ij}(\omega,\boldsymbol{k})E^j(\omega,\boldsymbol{k})$$

which summarizes the medium response.

- The Maxwell equations in Fourier space read:

$$k^i\left[\epsilon^{ij}(\omega,\boldsymbol{k})E^j\right] = -i\rho_{\rm ext} \qquad k^iB^i = 0$$

$$\epsilon^{ijk}k^jE^k = \omega B^i \qquad \epsilon^{ijk}k^jB^k = -ij^i_{\rm ext} - \omega\left[\epsilon^{ij}(\omega,\boldsymbol{k})\right]E^i$$

Combining the 3$^{\rm rd}$ and 4$^{\rm th}$ eqs. and exploiting $\epsilon^{ijk}\epsilon^{klm}=\delta^{il}\delta^{jm}-\delta^{im}\delta^{jl}$...

The Maxwell equations in a medium III

...one gets

$$\underbrace{\left[\epsilon^{ij}(\omega,\boldsymbol{k}) - \frac{k^2}{\omega^2}\left(\delta^{ij} - \frac{k^i k^j}{k^2}\right)\right]}_{\Delta^{ij}(\omega,\boldsymbol{k})} E^j(\omega,\boldsymbol{k}) = -\frac{i}{\omega}\, j^i_{\rm ext}(\omega,\boldsymbol{k})$$

The *normal modes of the e.m. field in the plasma* (i.e. the excitations which propagates even *in the absence of external sources*) are obtained solving

$$\det\left[\Delta(\omega_{\boldsymbol{k}} + i\gamma_{\boldsymbol{k}}, \boldsymbol{k})\right] = 0$$

The solutions are then of the form $\boldsymbol{E}(t,\boldsymbol{x}) = \boldsymbol{E}_{\boldsymbol{k}} e^{\gamma_{\boldsymbol{k}} t} e^{-i(\omega_{\boldsymbol{k}} t - \boldsymbol{k}\cdot\boldsymbol{x})}$:

- $\gamma_{\boldsymbol{k}} < 0$: damped modes;
- $\gamma_{\boldsymbol{k}} = 0$: stable modes;
- $\gamma_{\boldsymbol{k}} > 0$: *plasma instability* (amplitude exponentially growing with t)

More on plasma instabilities

- They represent processes (quite common in a plasma) in which energy is transfered from the *hard* plasma particles to long-wavelength (*soft*) collective excitations (at variance with what we are used looking at waves propagating in an elastic medium dissipating energy);
- They were proposed as a possible mechanism to explain the observed rapid thermalization in heavy-ion collisions[a] ($\tau_0 \lesssim 1$ fm/c);
- They develop, for instance, in the case of an anisotropy in the momentum distribution. In heavy-ion collisions, during the initial free-streaming one has

$$v_z = z/t$$

so that, around $z \approx 0$, one finds only particles with vanishing longitudinal momentum.

[a] S. Mrowczynski and M.H. Thoma, Ann. Rev. Nucl. Part. Sci. 57, 61 (2007) and references therein.

Isotropic plasma

- In the case of an isotropic medium the dielectric tensor can be expressed as

$$\epsilon^{ij}(\omega, \boldsymbol{k}) \equiv \hat{k}^i \hat{k}^j \epsilon_L(\omega, k) + \left(\delta^{ij} - \hat{k}^i \hat{k}^j\right) \epsilon_T(\omega, k)$$

- The Maxwell equations give:

$$\left[\hat{k}^i \hat{k}^j \epsilon_L(\omega, k) + \left(\delta^{ij} - \hat{k}^i \hat{k}^j\right) \left(\epsilon_T(\omega, k) - \frac{k^2}{\omega^2}\right)\right] E^j(\omega, \boldsymbol{k}) = -\frac{i}{\omega} j^i_{\rm ext}(\omega, \boldsymbol{k})$$

  – Transverse modes: $\boldsymbol{E}(\omega, k_z) = E_{x/y}(\omega, k_z) \hat{\boldsymbol{u}}_{x/y}$

$$\epsilon_T(\omega, k) - k^2/\omega^2 = 0$$

  They would be the only excitations propagating in the vacuum (photons/gluons with transverse polarization)

  – Longitudinal mode (only in the plasma!): $\boldsymbol{E}(\omega, k_z) = E_z(\omega, k_z) \hat{\boldsymbol{u}}_z$

$$\epsilon_L(\omega, k) = 0$$

Evaluation **of the dielectric tensor:** **classical kinetic theory**

Calculations in terms of the particle distribution function:

$$f_q(x, \boldsymbol{p})$$

Classical probability of finding a plasma particle of charge $q = \pm e$ with momentum $\boldsymbol{p}$ at the space-time point $x \equiv (t, \boldsymbol{x})$.

The Vlasov equation I

The time-evolution of the distribution function is described by the equation

$$D_t f_q(x, \boldsymbol{p}) = C[f], \quad \text{where}$$

- $D_t \equiv \partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}}$ is the *total derivative* along a trajectory in phase space;
- $C[f]$ is the *collision integral* (gain-loss terms).

For fluctuations of f_q occurring on a very-short time-scale $\delta t \ll \tau_{\text{coll}}$ the collisions cannot modify significantly the particle distribution and one can set the RHS to zero. One gets then the Vlasov equation:

$$\partial_t f_q + \boldsymbol{v} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} f_q + \boldsymbol{F} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f_q = 0 \quad \text{where}$$

$$\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

NB At equilibrium $f_q(x, \boldsymbol{p}) = f^0(\epsilon_p) \equiv 1/[e^{\beta\epsilon_p} \pm 1]$, hence $\boldsymbol{E} = \boldsymbol{B} = \boldsymbol{0}$.

The Vlasov equation II

Let us consider a small perturbation of the q-particle distribution

$$f_q(x, \boldsymbol{p}) = f^0(\epsilon_p) + \delta f_q(x, \boldsymbol{p}).$$

Keeping only the linear terms in the Vlasov equation one has:

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}_{\boldsymbol{x}})\delta f_q(x, \boldsymbol{p}) = -q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\cdot\boldsymbol{v}\,\frac{df^0}{d\epsilon_p} = -q\,\boldsymbol{v}\cdot\boldsymbol{E}\,\frac{df^0}{d\epsilon_p}$$

In order to *solve the equation* it is useful to set

$$f_q(x, \boldsymbol{p}) = f^0\left(\epsilon_p - qW(x, \boldsymbol{v})\right) \approx f^0(\epsilon_p) - qW(x, \boldsymbol{v})\frac{df^0}{d\epsilon_p},$$

with W satisfying the equation

$$v \cdot \partial_x W(x, \boldsymbol{v}) = \boldsymbol{v}\cdot\boldsymbol{E}(x)$$

$$\Longrightarrow W(x, \boldsymbol{v}) = \int_{-\infty}^{t} dt'\boldsymbol{v}\cdot\boldsymbol{E}(t', \boldsymbol{x} - \boldsymbol{v}(t - t'))\,e^{\eta t'}$$

The Vlasov equation III

Hence, for the induced current (spin degeneracy factor $g_s = 2$)

$$j^\mu_{\rm ind}(x) = 2 \times e \int \frac{d\boldsymbol{p}}{(2\pi)^3} v^\mu \left[\delta f_+(\boldsymbol{p}, x) - \delta f_-(\boldsymbol{p}, x)\right],$$

one gets ($\delta f_\pm$ give an equal contribution):

$$j^\mu_{\rm ind}(x) = -\,4e^2 \int \frac{d\boldsymbol{p}}{(2\pi)^3} v^\mu \frac{df^0}{d\epsilon_p} \int_0^\infty d\tau\, \boldsymbol{v}\cdot\boldsymbol{E}(x - v\tau) e^{-\eta\tau}$$

Writing it in Fourier space:

$$j^k_{\rm ind}(\omega, \boldsymbol{k}) = i\, m_D^2 \int \frac{d\Omega_{\boldsymbol{v}}}{4\pi} \frac{v^k v^l}{\omega - \boldsymbol{v}\cdot\boldsymbol{k} + i\eta} E^l(\omega, \boldsymbol{k}),$$

where the *Debye screening mass*

$$m_D^2 = -\frac{2e^2}{\pi^2} \int_0^\infty p^2\, dp\, \frac{df^0}{d\epsilon_p} = \frac{e^2 T^2}{3}$$

was introduced. We will comment more on its role in the following!

Getting the dielectric tensor...

From $\boldsymbol{D} \equiv \boldsymbol{E} + \boldsymbol{P}$, with $\partial_t \boldsymbol{P} = \boldsymbol{j}_{\text{ind}}$, one gets

$$D^i(\omega, \boldsymbol{k}) = \epsilon^{ij}(\omega, \boldsymbol{k}) E^j(\omega, \boldsymbol{k}) = \delta^{ij} E^j(\omega, \boldsymbol{k}) + \frac{i}{\omega} j^i_{\text{ind}}(\omega, \boldsymbol{k})$$

$$\Longrightarrow j^i_{\text{ind}}(\omega, \boldsymbol{k}) = i\omega \left(\delta^{ij} - \epsilon^{ij}(\omega, \boldsymbol{k})\right) E^j(\omega, \boldsymbol{k})$$

From the explicit expression of $\boldsymbol{j}_{\text{ind}}$ one finally obtains:

$$\epsilon^{ij}(\omega, \boldsymbol{k}) = \delta^{ij} - \frac{m_D^2}{\omega} \int \frac{d\Omega_{\boldsymbol{v}}}{4\pi} \frac{v^i v^j}{\omega - \boldsymbol{v} \cdot \boldsymbol{k} + i\eta}$$

It is then possible to evaluate the longitudinal and transverse components:

$$\epsilon_L(\omega, k) = \hat{k}^i \hat{k}^j \epsilon^{ij}(\omega, k)$$

$$\epsilon_T(\omega, k) = \frac{1}{2} \left[\text{Tr}\left(\epsilon^{ij}(\omega, \boldsymbol{k})\right) - \epsilon_L(\omega, k)\right]$$

The dielectric tensor of the plasma

- The transverse/longitudinal dielectric functions

$$\epsilon_T(\omega,k) = 1 - \frac{m_D^2}{2\omega^2}\left[x^2 + \frac{x(1-x^2)}{2}\ln\frac{x+1}{x-1}\right]$$

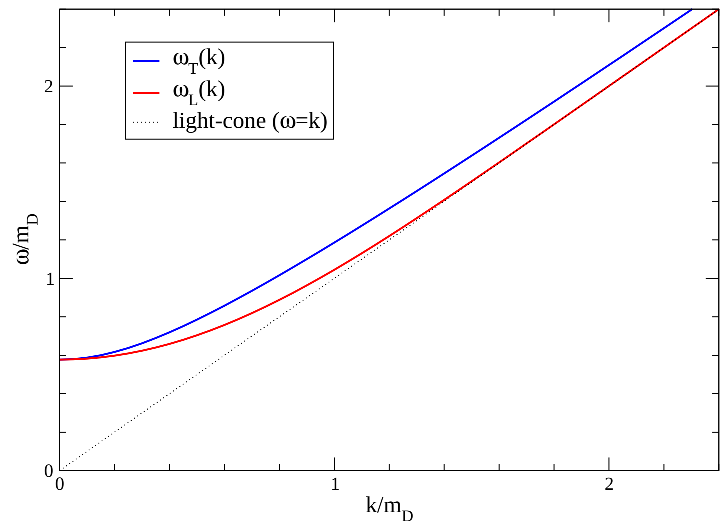
$$\epsilon_L(\omega,k) = 1 + \frac{m_D^2}{k^2}\left[1 - \frac{x}{2}\ln\frac{x+1}{x-1}\right] \quad \text{with} \quad x \equiv \frac{\omega}{k}$$

- Some applications:
  - Study of the excitations propagating in the plasma: appearence of a longitudinal excitation (*plasmon*), thermal mass acquired by the transverse modes (photons/gluons) and possible relevance for the QCD thermodynamics.
  - Screening of electric charges (e.g. dissociation of quarkonia in the QGP: J/ψ suppression);
  - Parton energy-loss: soft-collision contribution.

The normal modes in the plasma

(thermal corrections to the gluon propagator)

The transverse/longitudinal modes



Dispersion relations of the transverse/longitudinal excitations:

$$\epsilon_T(\omega, k) - k^2/\omega^2 = 0; \qquad \epsilon_L(\omega, k) = 0$$

- Both modes at $k=0$ start at $\omega = \omega_{\rm pl} \equiv m_D/\sqrt{3}$ (*plasma frequency*)
- At large momentum ($k \gg m_D$)
  - The longitudinal mode approaches the light-cone;
  - $\omega_T^2(k) \sim k^2 + \frac{m_D^2}{2}$ (*photon/gluon thermal mass* $m_\infty \equiv m_D/\sqrt{2}$!)

The plasma oscillation

- The longitudinale mode (*plasmon* or plasma-wave) is a *collective charge oscillation*
- The *plasma frequency* $\omega_{\rm pl}$ can be obtained expanding the equation $\epsilon_L(\omega, k) = 0$ for $k \ll \omega$:

$$1 - \frac{m_D^2}{k^2}\left[\frac{k^2}{3\omega^2} + \ldots\right] = 0 \quad \Longrightarrow \quad \omega = \omega_{\rm pl} \equiv \frac{m_D}{\sqrt{3}}$$

- Having $\omega_{\rm pl} \sim m_D$, the plasma oscillations occur on a time-scale

$$\Delta t_{\rm pl} \sim 1/\omega_{\rm pl} \sim 1/gT$$

The time required to collision to change significantly the momentum of a particle can be shown to be

$$\Delta t_{\rm coll} \sim 1/g^4 T$$

For weak-coupling $\Delta t_{\rm pl} \ll \Delta t_{\rm coll}$ and *studying the collective modes in the collisionless approximation results a posteriori justified*.

The transverse mode

- For large momenta $k \gg m_D$ the dispersion relation of the transverse excitation is not the one of a massless photon/gluon but rather the one of a massive particle

$$\omega_T^2 \underset{k \gg m_D}{\sim} k^2 + m_\infty^2$$

- The value of the *asymptotic thermal mass* m_∞ of the photon/gluon can be determined substituting $\omega^2 = k^2 + m_\infty^2$ into

$$\epsilon_T(\omega, k) - k^2/\omega^2 = 0$$

  and expanding for $k \gg m_D, m_\infty$. One gets:

$$1 - \frac{m_D^2}{2k^2} - 1 + \frac{m_\infty^2}{k^2} + \cdots = 0 \quad \Longrightarrow \quad m_\infty^2 = \frac{m_D^2}{2}$$

- The fact that the gluon acquires a thermal mass $m_\infty \sim gT$ will be *of relevance for the QCD thermodynamics*!

The relevance for the QCD thermodynamics[a]

- The *entropy-density* of a plasma of non-interacting gluons is given by the Stephan-Boltzmann law

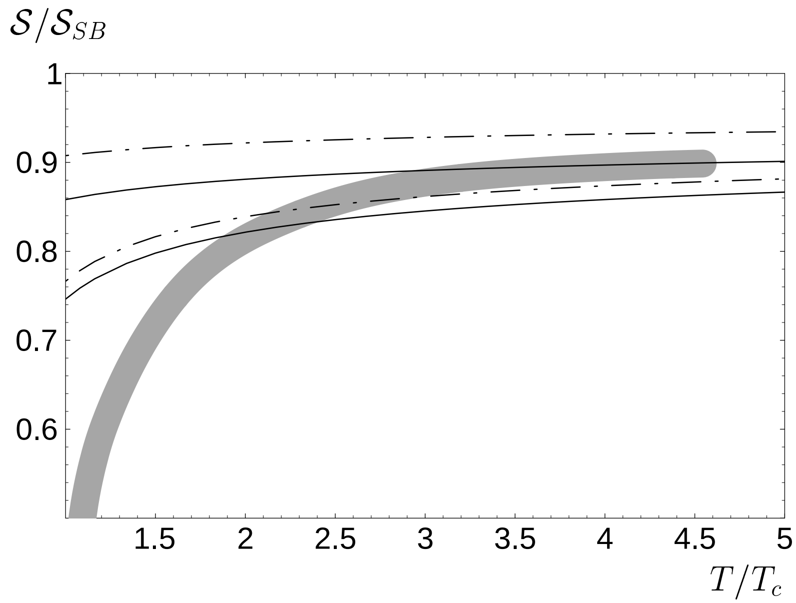
$$\mathcal{S}_{\mathrm{SB}} = \underbrace{2}_{\mathrm{polar.}} \times \underbrace{8}_{colors} \times \int \frac{d\boldsymbol{k}}{(2\pi)^3} \left[(1+N_k)\log(1+N_k) - N_k \log N_k\right]$$

$$\underset{m_g=0!}{=} 2 \times 8 \times \frac{4\pi^2}{30} T^3 \qquad \left(\text{being} \quad N_k \equiv 1/\left[\exp(\beta\epsilon_k) - 1\right]\right)$$

- We have seen that *the main effect of the interaction* is to assign the gluon a thermal mass $m_\infty \sim gT$;
- This should lead to *deviations from the Stephan-Boltzmann result* (referring to a gas of massless particles).

[a] For an overview on the subject see the lectures by C. Ratti

The QCD thermodynamics: numerical results



- Grey band: lattice-QCD data (Boyd *et al.*, Nucl. Phys. B **469**, 419);
- Lines: results of the *Hard Thermal Loop* approximation (Blaizot *et al.*)

The 10-20% deviation from the ideal-gas limit nicely reproduced and mainly due to the thermal mass $m_\infty \sim gT$ acquired by the gluons. Slow approach to $\mathcal{S}_{\rm SB}$ due to the running of the coupling $g(T)$.

The plasma frequency in everyday life

In the case of a medium of non-relativistic particles one can ignore the k-dependence and the dielectric tensor gets a simpler expression:

$$\epsilon^{ij}(\omega) = \delta^{ij}\epsilon(\omega) \qquad \text{with} \quad \epsilon(\omega) \approx 1 - \frac{\omega_{\rm pl}^2}{\omega^2}$$

For a plasma of electrons in a positive background one has $\omega_{\rm pl}^2 \equiv ne^2/m$. The dispersion relation of e.m. waves in the plasma is given by:

$$\epsilon(\omega) = \frac{k^2}{\omega^2} \quad \Longrightarrow \quad \omega^2 = \omega_{\rm pl}^2 + k^2.$$

Only waves with $\omega > \omega_{\rm pl}$ can propagate in the plasma. Waves with $\omega < \omega_{\rm pl}$ are reflected!

- AM ($\nu \sim 1$ MHz) *radio-waves are reflected by the ionosphere* and can reach long distances; FM waves ($\nu \sim 100$ MHz) cross the atmosphere.
- *Visible light is reflected by the surface of metals* (plasma: electrons of the conduction band), which become transparent to UV rays.

Screening and charmonium suppression

Screening of electric charges

Let us consider a charge q moving with velocity $\boldsymbol{v}$ in the plasma:

$$\boldsymbol{\nabla}\cdot\boldsymbol{D}(t,\boldsymbol{x}) = q\,\delta(\boldsymbol{x}-\boldsymbol{v}t) \longrightarrow i\boldsymbol{k}\cdot\boldsymbol{D}(\omega,\boldsymbol{k}) = 2\pi\,q\,\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$$

From $D^i(\omega,\boldsymbol{k}) \equiv \epsilon^{ij}(\omega,\boldsymbol{k})E^j(\omega,\boldsymbol{k})$ one has:

$$ik^j\epsilon_L(\omega,\boldsymbol{k})E^j(\omega,\boldsymbol{k}) = 2\pi\,q\,\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$$

In terms of the gauge potential $A^\mu \equiv (\phi,\boldsymbol{A})$ one has $\boldsymbol{E} \equiv -i\boldsymbol{k}\phi + i\omega\boldsymbol{A}$. In *Coulomb gauge* $\boldsymbol{k}\cdot\boldsymbol{A} = 0$ and one gets:

$$k^2\epsilon_L(\omega,\boldsymbol{k})\phi(\omega,\boldsymbol{k}) = 2\pi\,q\,\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$$

Hence, after FT:

$$\phi(t,\boldsymbol{x}) = q\int\frac{d\boldsymbol{k}}{(2\pi)^3}\,\frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{v}t)}}{k^2\epsilon_L(\omega=\boldsymbol{k}\cdot\boldsymbol{v},k)}$$

The electric field generated by a charge crossing the plasma is entirely described by the *longitudinal dielectric function*!

The static limit: Debye screening

Let us considet the case of a charge *at rest*. In such a case $\boldsymbol{v} = \mathbf{0}$ and only the static limit

$$\epsilon_L(\omega = 0, k) = 1 + \frac{m_D^2}{k^2}$$

of the dielectric function is of relevance. One gets ($r \equiv |\boldsymbol{x}|$):

$$\phi(x) = q \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{x}}}{k^2 + m_D^2} = \frac{q}{4\pi r} e^{-m_D r}$$

The Debye *screening mass* ($m_D \sim eT$) turns the Coulomb interaction into a short range Yukawa-like potential!

The screening results from *the collective effect* of a large number of elementary charges: in a plasma at temperature T the particle density is $n \sim T^3$, hence in a sphere of radius $R = r_D \equiv 1/m_D$ one finds

$$N \sim n r_D^3 \sim \frac{T^3}{e^3 T^3} \sim \frac{1}{e^3} \gg 1 \quad \text{elementary charges}$$

Debye screening in the QGP: J/ψ suppression

In the case of QCD the effective $Q\overline{Q}$ potential in the plasma reads:

$$V_{Q\overline{Q}}(r) = -C_F \alpha_s \frac{e^{-m_D r}}{r} \quad (C_F = 4/3 \text{ color factor})$$

In the QGP both both quarks and gluons carry color charge and contribute to the screeening. One has:

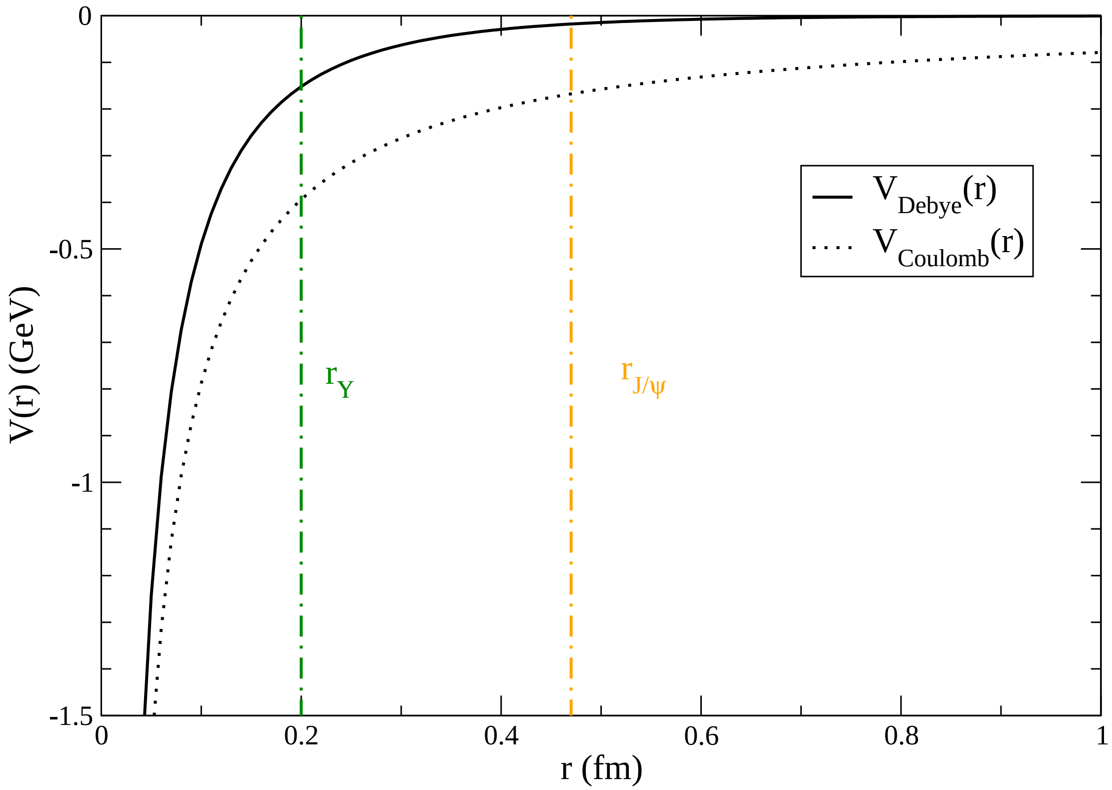
$$m_D = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$$

For $T = 400$ MeV, $N_c = N_f = 3$, $\alpha_s = 0.3$ one has $r_D \equiv 1/m_D \approx 0.21$ fm, to be compared with

- $c\overline{c}$ (1s) ground state: $\overline{r}_{J/\psi} \approx 0.47$ fm ($> r_D$!) $\longrightarrow$ *J/ψ expected to melt in a QGP at a sufficiently high temperature*[a];
- $b\overline{b}$ (1s) ground state: $\overline{r}_{\Upsilon} \approx 0.2$ fm ($< r_D$) $\longrightarrow$ Υ only *midly affected by the QGP* at the experimentally accessible temperatures;

[a]For more details on J/ψ suppression see the lectures by R. Arnaldi.

Debye screening in the QGP: a cartoon



The force $F(r) \equiv -dV/dr$ felt on average by the $c\bar{c}$ pair above the *dissociation temperature* $T_d^{J/\psi}$ is no longer sufficient to ensure the binding.

Around $\overline{r}_{\Upsilon}$ the potential is still steep enough to keep it bound.

Damping and energy-loss processes

The Landau damping: generalities

- The dielectric tensor $\epsilon^{ij}(\omega, k)$ can develop an imaginary part;
- *Physically* such an imaginary part reflects a damping of the field oscillations, leading to an energy transfer to the plasma particles. Remember that, in optics, for the propagation of a plane-wave in a medium with refraction-index $n(\omega)$ one has:

$$e^{-i(\omega t - kx)} = e^{-i\omega t} e^{i\frac{\omega}{c}n(\omega)x} \quad \text{where} \quad n(\omega) \equiv \sqrt{\epsilon(\omega)}.$$

For a complex dielectric function $\epsilon \equiv \epsilon_1 + i\epsilon_2$ one can approximate

$$n(\omega) \approx \sqrt{\epsilon_1(\omega)} + i\frac{\epsilon_2(\omega)}{2\sqrt{\epsilon_1(\omega)}}$$

so that the imaginary part lead to a damping of the oscillations:

$$e^{-\frac{\omega}{c}\frac{\epsilon_2(\omega)}{2\sqrt{\epsilon_1(\omega)}}x}$$

The Landau damping: the HTL result

From the explicit expression of the HTL dielectric tensor:

$$\mathrm{Im}\epsilon^{ij}(\omega,\boldsymbol{k}) = \frac{m_D^2}{2\omega} v^i v^j \int_{-1}^{1} d\cos\theta_{\boldsymbol{v}}\ \underbrace{\delta(\omega - \boldsymbol{v}\cdot\boldsymbol{k})}_{\delta(\omega - k\cos\theta_{\boldsymbol{v}})}$$

one gets:

$$\mathrm{Im}\epsilon_L(\omega,\boldsymbol{k}) = \frac{\pi m_D^2}{2k^2}\frac{\omega}{k}\theta\left(k^2-\omega^2\right) \quad \text{and}$$

$$\mathrm{Im}\epsilon_T(\omega,\boldsymbol{k}) = \frac{\pi m_D^2}{4\omega^2}\frac{\omega}{k}\left(1-\frac{\omega^2}{k^2}\right)\theta\left(k^2-\omega^2\right)$$

- The energy is transfered to particles moving with projection of the velocity $\boldsymbol{v}\cdot\hat{\boldsymbol{k}}$ equal to the *phase velocity* ω/k of the field oscillation;
- It follows that the damping occurs *only for space-like* ($|\omega|<k$) *modes*! NB These are *not* the normal modes of the field. They must be excited by some external current $J_{\rm ext}$, e.g. a fast charge crossing the plasma.

An application: parton energy-loss

- A hard parton ($E \gg T$) crossing the plasma will loose energy (*jet-quenching*)
- Actually so far (at RHIC) the process was studied mainly at the level of single-hadron spectra[a], considering the quantity

$$R_{AA}(p_T) \equiv \frac{dN/dp_T^{AA}}{\langle N_{\rm coll}\rangle dN/dp_T^{pp}}$$

- Various energy-loss mechanisms contribute
  - Radiation of soft gluons ($\omega \ll E$),
  - Collisions involving the exchange of hard ($Q \sim T$ or larger) and soft ($Q \sim m_D$) momenta.

The contribution of soft collisions to the energy-loss can be evaluated starting from $\epsilon_{T/L}(\omega, k)$.

[a] For an overview on jet-reconstruction in heavy-ion see the lectures by E. Bruna

Energy loss: soft-collisions I

- The energy lost (due to soft scatterings) by a charge $q = \pm e$ crossing the plasma is equal to the *work done by the induced electric field* on the charge itself:

$$\left.\frac{dE}{dx}\right|_{\rm soft} = \frac{1}{v}\left.\frac{dE}{dt}\right|_{\rm soft} = \frac{1}{v}\int d\boldsymbol{x}\, \boldsymbol{E}(t,\boldsymbol{x})\cdot\boldsymbol{j}_{\rm ext}(t,\boldsymbol{x})$$

with $\quad \boldsymbol{j}_{\rm ext}(t,\boldsymbol{x}) = q\boldsymbol{v}\delta(\boldsymbol{x}-\boldsymbol{v}t) \longrightarrow \boldsymbol{j}_{\rm ext}(\omega,\boldsymbol{k}) = (2\pi)q\boldsymbol{v}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$.

- One gets:

$$\left.\frac{dE}{dx}\right|_{\rm soft} = \frac{1}{v}q\,\boldsymbol{v}\cdot\boldsymbol{E}(t,\boldsymbol{v}t) = \frac{1}{v}\,q\int\frac{d\omega}{2\pi}\frac{d\boldsymbol{k}}{(2\pi)^3}e^{-i(\omega-\boldsymbol{k}\cdot\boldsymbol{v})t}\boldsymbol{v}\cdot\boldsymbol{E}(\omega,\boldsymbol{k})$$

- The electric field can be obtained from the Maxwell equations:

$$E^i(\omega,\boldsymbol{k}) = \frac{-i}{\omega}\left[\hat{k}^i\hat{k}^j\frac{1}{\epsilon_L(\omega,k)} + \left(\delta^{ij}-\hat{k}^i\hat{k}^j\right)\frac{1}{\epsilon_T(\omega,k)-k^2/\omega^2}\right]j^j_{\rm ext}(\omega,\boldsymbol{k})$$

Energy loss: soft-collisions II

- The final result is then:

$$\left.\frac{dE}{dx}\right|_{\rm soft} = \frac{(-i)e^2}{v}\int \frac{d\boldsymbol{k}}{(2\pi)^3}\left[\frac{\omega}{k^2\epsilon_L(\omega,k)} + \frac{v^2-\omega^2/k^2}{\omega[\epsilon_T(\omega,k)-k^2/\omega^2]}\right]_{\omega=\boldsymbol{k}\cdot\boldsymbol{v}}$$

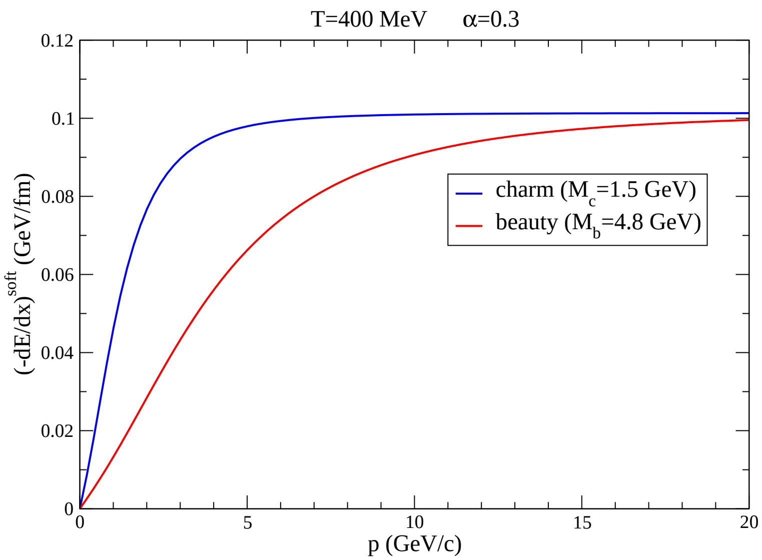
- The *energy loss* is *due to the imaginary part* of the dielectric tensor

$$-\left.\frac{dE}{dx}\right|_{\rm soft} = \frac{e^2}{v}\int \frac{d\boldsymbol{k}}{(2\pi)^3}\left[\frac{\omega\,{\rm Im}\epsilon_L(\omega,k)}{k^2|\epsilon_L(\omega,k)|^2} + \frac{\left(v^2-\frac{\omega^2}{k^2}\right){\rm Im}\epsilon_T(\omega,k)}{\omega\left|\epsilon_T(\omega,k)-\frac{k^2}{\omega^2}\right|^2}\right]_{\omega=\boldsymbol{k}\cdot\boldsymbol{v}}$$

- The generalization to the QGP case is done replacing

$$e^2 \longrightarrow C_F g^2, \qquad m_D^2 = \frac{e^2T^2}{3} \longrightarrow g^2T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right)$$

Energy loss: numerical results



- In the plot: energy loss of c and b quarks due to *soft collisions* ($q < m_D$)
- On top of this one must add the contribution of *hard collisions* ($q > m_D$):

$$\Gamma = \frac{1}{2E} \int_k \frac{n_{B/F}(k)}{2k} \int_{k'} \frac{1 \pm n_{B/F}(k')}{2k'} \int_{p'} \frac{1}{2E'} \times$$

$$\times \, (2\pi)^4 \delta^{(4)}(P+K-P'-K') \left| \overline{\mathcal{M}}_{g/q}(s,t) \right|^2$$

The link with the *Thermal Field Theory* approach

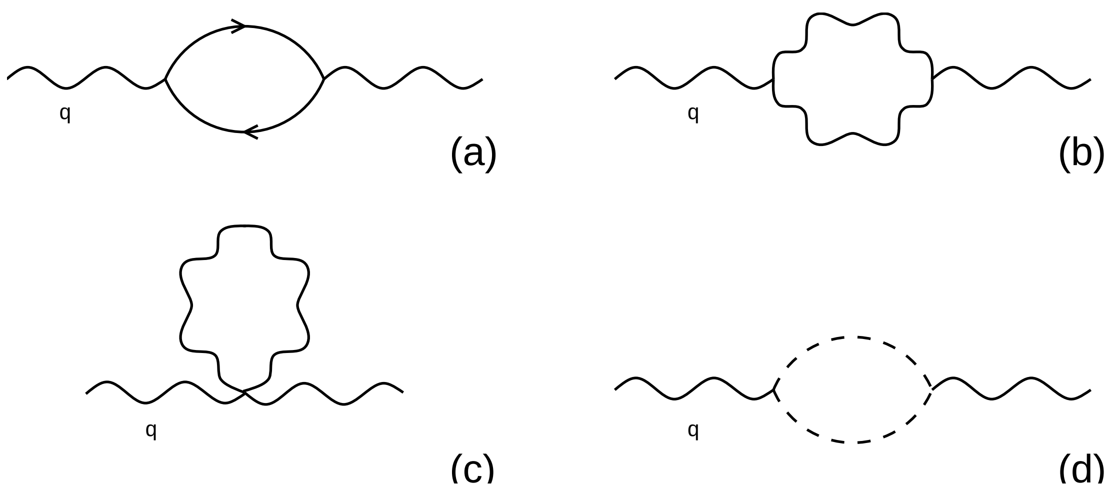
- So far the transverse/longitudinal dispersion relations where found as normal modes of the gauge field in a dielectric medium;
- We wish to provide a different perspective and show how they arise as *poles of the gluon propagator in the plasma*.

The gluon propagator in the QGP I

- The longitudinal/transverse gluon propagator in Coulomb gauge reads (q^0 being *complex*)

$$\Delta_L(q^0,q) = \frac{-1}{q^2 + \Pi_L(q^0,q)}, \quad \Delta_T(q^0,q) = \frac{-1}{(q^0)^2 - q^2 - \Pi_T(q^0,q)},$$

- $\Pi_{L/T}$ are the gluon self-energies *in the medium* and (at one-loop) are given by the following diagrams:



The gluon propagator in the QGP II

- It turns out that $\Pi_{L/T}(Q) \sim g^2T^2$, so that
  - For hard gluon momenta ($Q \sim T$) the self-energy provides a small correction to the tree-level propagator, which can be accounted for perturbatively, e.g.

$$\Delta_L = \frac{-1}{q^2} + \frac{-1}{q^2}\Pi_L\frac{-1}{q^2} + \dots$$

  - For soft gluon momenta ($Q \sim gT$) all the terms in the above expansion would be of the same order

$$\Delta_L \sim \frac{1}{g^2T^2} + \frac{1}{g^2T^2}(g^2T^2)\frac{1}{g^2T^2} + \dots$$

    and one has to keep the resummed expressions for the propagators, *no matter how small the coupling is.*

- The longitudinal/transverse excitations are obtained solving:

$$q^2 + \Pi_L(\omega_L, q) = 0, \qquad \omega_T^2 - q^2 - \Pi_T(\omega_T, q) = 0.$$

The gluon propagator in the QGP III

- For soft gluon momenta the self-energies get the dominant contribution from the integration over hard ($p \sim T$) momenta (*Hard Thermal Loop* approximation) and can be evaluated analytically ($x \equiv q^0/q$):

$$\Pi_L(x) = m_D^2 \left(1 - \frac{x}{2} \ln \frac{x+1}{x-1}\right),$$

$$\Pi_T(x) = \frac{m_D^2}{2} \left(x^2 + (1-x^2)\frac{x}{2} \ln \frac{x+1}{x-1}\right),$$

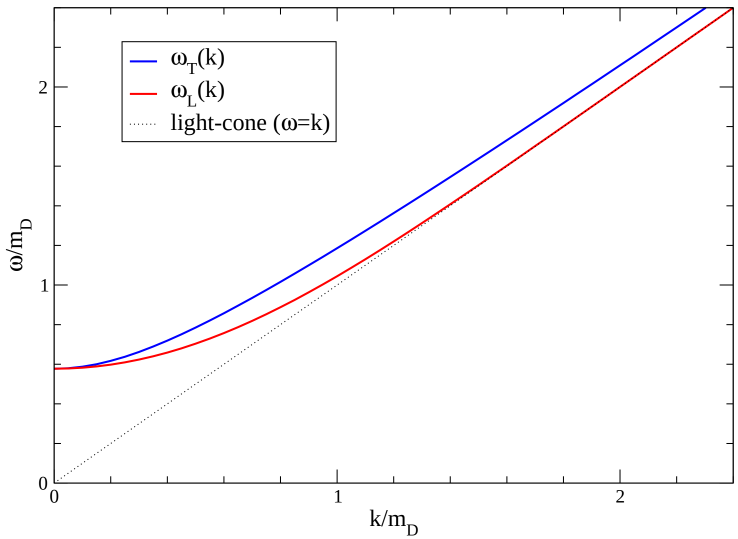
- The link with the longitudinal/transverse dielectric functions $\epsilon_{L/T}(\omega, q)$ is transparent and in fact the dispersion relations $\omega_{L/T}(q)$ one obtains are the same!

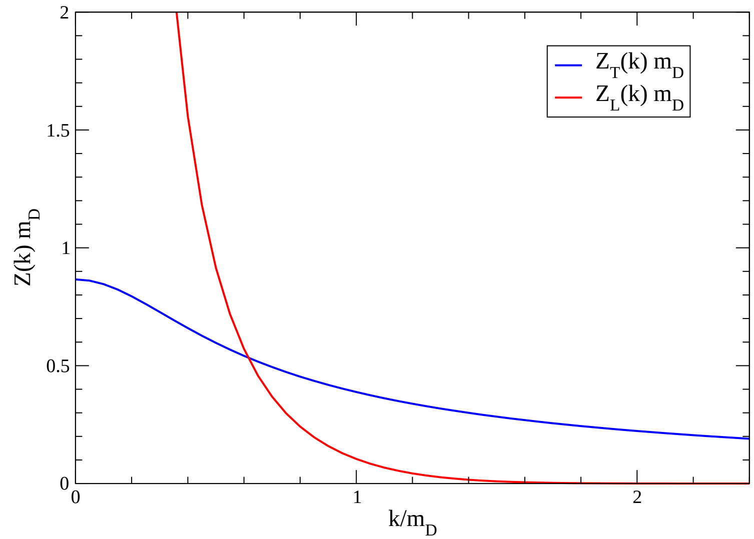
- One can expand the propagators around their *quasi-particle* poles:

$$\Delta_{L/T}(q^0, q) \underset{q^0 \to \omega_{L/T}}{\sim} \frac{-Z_{L/T}(q)}{q^0 - \omega_{L/T}(q)} + \ldots \text{(regular terms)},$$

the residues $Z_{L/T}(k)$ giving the *weight of the contribution.*

The quasi-particle spectrum





- For small momenta ($k \ll m_D$) the spectrum is dominated by the collective *plasma oscillation*;
- For large momenta ($k \gg m_D$) the longitudinal mode disappears from the spectrum and one is left only with the *physical degrees of freedom*, i.e. transverse gluons with a thermal mass m_∞.

Summary

- I tried to provide a consistent picture of a relativistic gauge plasma, relying on the assumption of a suffiently weak value of the coupling g (NB $\neq$ weakly-coupled system!);
- Several results of possible relevance for the QGP phenomenology (thermal masses, screening, energy-loss) where shown to be general plasma-physics phenomena and derived within classical kinetic theory. Such a setup requires a quite limited theoretical background and provides at the same time a quite transparent physical picture;
- The link with the *Thermal Field Theory* approach was finally presented, showing the equivalence between the kinetic setup and the so called *Hard Thermal Loop* approximation.

Some literature

- Textbooks:
  - J.I. Kapusta and C. Gale, "*Finite-Temperature Field Theory: Principles and Applications*";
  - M. Le Bellac, "*Thermal Field Theory*";
- Review articles and lecture notes:
  - J.P. Blaizot, "*Theory of the quark gluon plasma*", Lect. Notes Phys. 583, 117 (2002);
  - J.P. Blaizot and E. Iancu "*The Quark gluon plasma: Collective dynamics and hard thermal loops*", Phys. Rept. 359, 355-528 (2002);
  - S. Mrowczynski and M.H. Thoma, "*What Do Electromagnetic Plasmas Tell Us about Quark-Gluon Plasma?*", Ann. Rev. Nucl. Part. Sci. 57, 61 (2007).

Back-up material

A simple QM model

Let us consider the hamiltonian of a *perturbed* harmonic oscillator:

$$H = \frac{p^2}{2} + \frac{1}{2}\,\omega_k^2\,x^2 + \frac{\lambda}{4!}\,\omega_k^3\,x^4 \equiv H_0 + H_1$$

One introduces as usual the raising/lowering operators $a^\dagger$ and a, so that

$$H_o = \omega_k\left(a^\dagger a + \frac{1}{2}\right) \quad \text{with} \quad x \equiv \frac{1}{\sqrt{2\omega_k}}\left(a + a^\dagger\right) \quad \text{and} \quad [a, a^\dagger] = 1.$$

The thermal average in the unperturbed system is of course:

$$\langle ... \rangle_0 \equiv \frac{\mathrm{Tr}\left[\mathrm{e}^{-\beta \mathrm{H}_0} ...\right]}{\mathrm{Tr} e^{-\beta H_0}}.$$

The fluctuation of the "field" x are of particular interest:

$$\langle x^2 \rangle_0 = \frac{1}{2\omega_k}\langle \underbrace{aa + a^\dagger a^\dagger}_{=0} + \underbrace{aa^\dagger + a^\dagger a}_{1+2a^\dagger a} \rangle \quad \Rightarrow \quad \langle x^2 \rangle_0 = \frac{1 + 2N_k}{2\omega_k},$$

where $N_k \equiv \langle a^\dagger a \rangle_0 = 1/[\exp(\beta\omega_k) - 1]$.

The expectation value of H_1 is given by (the weight is gaussian!)

$$\langle H_1 \rangle_0 = \frac{\lambda}{4!}\,\omega_k^3\,\langle x^4 \rangle_0 = \frac{\lambda}{8}\omega_k^3(\langle x^2 \rangle_0)^2 = \frac{\lambda}{8}\,\omega_k^3\left(\frac{1+2N_k}{2\omega_k}\right)^2$$

to be compared with the e.v. of the unperturbed hamiltonian:

$$\langle H_0 \rangle_0 = \omega_k\left(N_k + \frac{1}{2}\right) = \omega_k^2\left(\frac{1+2N_k}{2\omega_k}\right)$$

At $T = 0$ one has:

$$\langle H_1 \rangle^{T=0} = \frac{\lambda}{32}\omega_k, \quad \langle H_0 \rangle^{T=0} = \frac{\omega_k}{2} \quad \longrightarrow \quad \langle H_1 \rangle^{T=0} \underset{\lambda \ll 1}{\ll} \langle H_0 \rangle^{T=0}$$

and if $\lambda \ll 1$ the system is *always* perturbative.

However at large T ($T \gg \omega_k$) the e.v. of H_1 can become larger then H_0:

$$\frac{\lambda}{8}\,\omega_k\left(\frac{1+2N_k}{2\omega_k}\right) \sim \frac{\lambda}{8}\frac{T}{\omega_k} > 1$$

Hence *no matter how small the coupling is* for modes with $\omega_k \lesssim \lambda T$ the system is *strongly coupled*!