

# CP Physics with



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## **Summary:**

- **Mixing of neutral B mesons**
- **Formalism for Coherent B  $\bar{B}$  states**
- **CP violation in B decays**
- **CP violation in the Standard Model**
- **Experimental Requirements to Search for CP violation**
- **Hadronic vs  $e^+e^-$  colliders**
- **BaBar detector**
- **Performances of BaBar**
- **Example of CP analysis from BaBar**

# Introduction

**CP violation has been so far observed only in the K system**

**The K-decay observations, together with other measurements, place constraints on the parameters of the CKM matrix**

**Many CP-violating effects are expected in B decays: some cleanly predicted by the Standard Model.**

**If enough independent observations of CP violation in B decays can be made, it will be possible to test the Standard Model predictions for CP violation.**

**CP violation can be related to the disappearance of antimatter from the Universe**

**Charge Conjugation C interchanges particles and antiparticles**

**Parity P sends  $(t, \mathbf{x}) \rightarrow (t, -\mathbf{x})$**

**Electromagnetic and strong interactions are symmetric w.r.t.  
C and P separately**

**Weak interactions violate C and P separately**

## CP transformation properties:

term	$\bar{\psi}_i \psi_j$	$i\bar{\psi}_i \gamma^5 \psi_j$	$\bar{\psi}_i \gamma^\mu \psi_j$	$\bar{\psi}_i \gamma^\mu \gamma^5 \psi_j$
CP-transformed term	$\bar{\psi}_j \psi_i$	$-i\bar{\psi}_j \gamma^5 \psi_i$	$-(-1)^\mu \bar{\psi}_j \gamma^\mu \psi_i$	$-(-1)^\mu \bar{\psi}_j \gamma^\mu \gamma^5 \psi_i$

term	$H$	$A$	$W^{\pm\mu}$	$\partial_\mu$
CP-transformed term	$H$	$-A$	$-(-1)^\mu W^{\mp\mu}$	$(-1)^\mu \partial_\mu$

$(-1)^\mu = 1$  for  $\mu=0$ ,  $(-1)^\mu = -1$  for  $\mu=1,2,3$

Each combination of fields and derivatives that appears in the Lagrangian **transforms under CP to its hermitian conjugate**

There are coefficients in front of these expressions which represent either coupling constants or particle masses

If any of these quantities are complex, then the **coefficients in front of CP-related terms are complex conjugates of each other.**

In such a case, CP is not necessarily a good symmetry of the Lagrangian.

There can be CP-violating effects, namely **rate differences between pairs of CP conjugate processes.**

# Mixing of Neutral B Mesons

An arbitrary linear combination of the neutral B-meson flavor eigenstates

$$a|B^0\rangle + b|\bar{B}^0\rangle,$$

$$i\frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = H \begin{pmatrix} a \\ b \end{pmatrix} \equiv (M - \frac{i}{2}\Gamma) \begin{pmatrix} a \\ b \end{pmatrix}$$

is governed by a time-dependent Schrodinger equation  
M and  $\Gamma$  are 2x2 Hermitian matrices

**The light  $B_L$  and heavy  $B_H$  mass eigenstates are given by:**

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$$|q|^2 + |p|^2 = 1.$$

**The mass difference  $\Delta m_B$  and width difference  $\Delta\Gamma_B$  between the neutral B mesons are:**

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L$$



**Finding the eigenvalues one gets:**

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = 4(|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2)$$

$$\Delta m_B \Delta\Gamma_B = 4 \mathcal{R}e(M_{12}\Gamma_{12}^*).$$

$$\frac{q}{p} = -\frac{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}{2(M_{12} - \frac{i}{2}\Gamma_{12})} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}$$

$\Delta\Gamma_B$  has never been measured. Expected  $O(10^{-2})$

From B mixing:

$$\Delta m_B / \Gamma_B \sim 0.7$$

One can assume:

$$\Delta\Gamma_B \ll \Delta m_B$$

and the expressions of the previous slide become:

$$\Delta m_B = 2|M_{12}|, \quad \Delta\Gamma_B = 2 \mathcal{R}e(M_{12}\Gamma_{12}^*) / |M_{12}|$$

$$q/p = -|M_{12}|/M_{12}.$$

**The time evolution of a state that at  $t=0$  is a pure  $B^0$  ( $\bar{B}^0$ ) is:**

$$|B_{\text{phys}}^0(t)\rangle = g_+(t)|B^0\rangle + (q/p)g_-(t)|\bar{B}^0\rangle,$$

$$|\bar{B}_{\text{phys}}^0(t)\rangle = (p/q)g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle,$$

$$g_+(t) = e^{-iMt}e^{-\Gamma t/2} \cos(\Delta m_B t/2),$$

$$g_-(t) = e^{-iMt}e^{-\Gamma t/2} i \sin(\Delta m_B t/2),$$

$$M = 1/2(M_H + M_L)$$

# Formalism for Coherent $B\bar{B}$ states

At  $e^+ e^-$  collider, at the energy of the  $Y(4S)$ , the  $B^0$  and  $\bar{B}^0$  mesons are produced in a **coherent  $L=1$  state**

**Until one decays, there is exactly one  $B^0$  and one  $\bar{B}^0$**

**Once one particle decays, the other continues to evolve and events with two  $B^0$  (or two  $\bar{B}^0$ ) decays are possible with probability governed by the time between the two decay**

**Two B, coming from Y decay, are identified by the angle  $\theta$  that they make with the  $e^-$  direction in the Y rest frame:**

$$S(t_f, t_b) = \frac{1}{\sqrt{2}} \{ B_{\text{phys}}^0(t_f, \theta, \phi) \bar{B}_{\text{phys}}^0(t_b, \pi - \theta, \phi + \pi) - \bar{B}_{\text{phys}}^0(t_f, \theta, \phi) B_{\text{phys}}^0(t_b, \pi - \theta, \phi + \pi) \} \sin(\theta)$$

**Using the expressions for B evolution:**

$$S(t_f, t_b) = \frac{1}{\sqrt{2}} e^{-(\Gamma/2 + iM)(t_f + t_b)} \{ \cos[\Delta m_B(t_f - t_b)/2] (B_f^0 \bar{B}_b^0 - \bar{B}_f^0 B_b^0) - i \sin[\Delta m_B(t_f - t_b)/2] (\frac{p}{q} B_f^0 B_b^0 - \frac{q}{p} \bar{B}_f^0 \bar{B}_b^0) \} \sin(\theta_f),$$

**where:**

**$B_f$  is moving forward ( $\theta_f < \pi/2, \Phi$ )**

**$B_b$  is moving backward ( $\pi - \theta_f, \Phi + \pi$ )**

**The amplitude for one B decaying to the state  $f_1$  at time  $t_1$  and the other B decaying to the state  $f_2$  at time  $t_2$  is :**

$$A(t_1, t_2) = \frac{1}{\sqrt{2}} e^{-(\Gamma/2 + iM)(t_1 + t_2)} \zeta(t_1, t_2) \left\{ \cos[\Delta m_B(t_1 - t_2)/2] (A_1 \bar{A}_2 - \bar{A}_1 A_2) \right. \\ \left. - i \sin[\Delta m_B(t_1 - t_2)/2] \left( \frac{p}{q} A_1 A_2 - \frac{q}{p} \bar{A}_1 \bar{A}_2 \right) \right\} \sin(\theta_1),$$

**where:**

**$A_i$  is the amplitude for a B to decay to the state  $f_i$**

**$\bar{A}_i$  is the amplitude for a  $\bar{B}$  to decay to the same state  $f_i$**

$$\zeta(t_1, t_2) = \begin{cases} +1 & t_1 = t_f, t_2 = t_b, \\ -1 & t_1 = t_b, t_2 = t_f \end{cases}$$

**The time dependent rate can be written as:**

$$\begin{aligned}
 R(t_1, t_2) = & C e^{-\Gamma(t_1+t_2)} \{ (|A_1|^2 + |\bar{A}_1|^2)(|A_2|^2 + |\bar{A}_2|^2) - 4 \operatorname{Re}\left(\frac{q}{p} A_1^* \bar{A}_1\right) \operatorname{Re}\left(\frac{q}{p} A_2^* \bar{A}_2\right) \\
 & - \cos(\Delta m_B(t_1 - t_2)) [ (|A_1|^2 - |\bar{A}_1|^2)(|A_2|^2 - |\bar{A}_2|^2) + 4 \operatorname{Im}\left(\frac{q}{p} A_1^* \bar{A}_1\right) \operatorname{Im}\left(\frac{q}{p} A_2^* \bar{A}_2\right) ] \\
 & + 2 \sin(\Delta m_B(t_1 - t_2)) [ \operatorname{Im}\left(\frac{q}{p} A_1^* \bar{A}_1\right) (|A_2|^2 - |\bar{A}_2|^2) - (|A_1|^2 - |\bar{A}_1|^2) \operatorname{Im}\left(\frac{q}{p} A_2^* \bar{A}_2\right) ] \}.
 \end{aligned}$$

**To search for CP violation, one looks for event in which:**

- **one B decays to a CP eigenstate  $f_{\text{CP}}$  at time  $t_{\text{fCP}}$**
- **the other B decays to a tagging mode with  $A_2 = 0$  or  $\bar{A}_2 = 0$  at time  $t_{\text{tag}}$**

If we take a tagging mode with  $A_2 = 0$  and  $\bar{A}_2 = \bar{A}_{\text{tag}}$ , the other  $B$  is identified as a  $B^0$  at time  $t_2 = t_{\text{tag}}$ . True also if  $t_{fCP} < t_{\text{tag}}$

The rate reduces to:

$$R(t_{\text{tag}}, t_{fCP}) = C e^{-\Gamma(t_{\text{tag}} + t_{fCP})} |\bar{A}_{\text{tag}}|^2 |A_{fCP}|^2 \{1 + |\lambda_{fCP}|^2 + \cos[\Delta m_B(t_{fCP} - t_{\text{tag}})](1 - |\lambda_{fCP}|^2) - 2 \sin[\Delta m_B(t_{fCP} - t_{\text{tag}})] \mathcal{I}m(\lambda_{fCP})\}$$

where:

$$\lambda_{fCP} \equiv \frac{q}{p} \frac{\bar{A}_{fCP}}{A_{fCP}} = \eta_{fCP} \frac{q}{p} \frac{\bar{A}_{\bar{fCP}}}{A_{fCP}}.$$

$$\bar{A}_{fCP} = \eta_{fCP} \bar{A}_{\bar{fCP}},$$

$\eta_{CP}$  is the CP eigenvalues of the state  $f_{CP}$



For the case with  $\bar{A}_2 = 0$  and  $A_2 = A_{\text{tag}}$ , the other B is identified as a  $\bar{B}^0$  at time  $t_{\text{tag}}$  and the sign of the cosine and sine terms are reversed in the expression above.

The time dependent CP asymmetry :

$$a_{f_{CP}} = \frac{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}.$$

can be written as:

$$a_{f_{CP}} = \frac{(1 - |\lambda_{f_{CP}}|^2) \cos(\Delta m_B t) - 2 \mathcal{I}m \lambda_{f_{CP}} \sin(\Delta m_B t)}{1 + |\lambda_{f_{CP}}|^2} ;$$

# CP Violation in B Decays

The possible manifestations of CP violation can be classified in a model-independent way:

- CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP conjugate process have different magnitudes
- CP violation in mixing, which occurs when the two neutral mass eigenstates cannot be chosen to be CP eigenstates
- CP violation in the interference between decays with and without mixing, which occurs in decays into final states that are common to  $B^0$  and  $\bar{B}^0$

# CP Violation in Decay

Each contribution to  $A$  can be written in three parts: its magnitude  $A$ , its weak-phase term  $e^{i\phi}$ , and its strong phase term  $e^{i\delta}$

If several amplitudes contribute to  $B^0 \rightarrow f$ , the amplitude  $A_f$  and the CP conjugate amplitude  $\bar{A}_{\bar{f}}$  are given by:

$$A_f = \sum_i A_i e^{i(\delta_i + \phi_i)}, \quad \bar{A}_{\bar{f}} = e^{2i(\xi_f - \xi_B)} \sum_i A_i e^{i(\delta_i - \phi_i)}$$

where:

$$\text{CP } |B^0\rangle = e^{2i\zeta_B} |\bar{B}^0\rangle$$

$$\text{CP } |\bar{B}^0\rangle = e^{-2i\zeta_B} |B^0\rangle$$

$$\text{CP } |f\rangle = e^{2i\zeta_f} |\bar{f}\rangle$$

$$\text{CP } |\bar{f}\rangle = e^{-2i\zeta_f} |f\rangle$$

The interesting quantity is:

$$\left| \frac{\bar{A}_f}{A_f} \right| = \left| \frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}} \right|.$$

$$|\bar{A}_f/A_f| \neq 1 \implies CP \text{ violation.}$$

This type of CP violation is called **CP violation in decay** or **direct CP violation**.

It results from the CP-violating interference among various terms in the decay amplitude.

# CP Violation in Mixing

A quantity that is independent of phase conventions and physically meaningful is:

$$\left| \frac{q}{p} \right|^2 = \left| \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right|.$$

When CP is conserved, the mass eigenstates must be CP eigenstates. In that case the relative phase between  $M_{12}$  and  $\Gamma_{12}$  vanishes

$$|q/p| \neq 1 \implies CP \text{ violation.}$$

This type of CP violation is here called **CP violation in mixing**; it is often referred to as **indirect CP violation**.

It results from the mass eigenstates being different from the CP eigenstates. **CP violation in mixing has been observed unambiguously in the neutral kaon system.**

**For the neutral B system, this effect could be observed through the asymmetries in semileptonic decays:**

$$a_{\text{sl}} = \frac{\Gamma(\overline{B}_{\text{phys}}^0(t) \rightarrow \ell^+ \nu X) - \Gamma(B_{\text{phys}}^0(t) \rightarrow \ell^- \nu X)}{\Gamma(\overline{B}_{\text{phys}}^0(t) \rightarrow \ell^+ \nu X) + \Gamma(B_{\text{phys}}^0(t) \rightarrow \ell^- \nu X)}.$$

**Expected to be small**

# CP Violation in the Interference Between Decays With and Without Mixing

CP eigenstates,  $f_{CP}$ , are accessible in both  $B^0$  and  $\bar{B}^0$  decays.

Let's consider:

$$\lambda = \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

$$\lambda \neq \pm 1 \implies CP \text{ violation}$$

**It is possible that, to a good approximation,  $|q/p| = 1$  and  $|A/\bar{A}| = 1$  and still have CP violation:**

$$|\lambda| = 1, \quad \text{Im } \lambda \neq 0.$$

**This type of CP violation is called CP violation in the interference between decays with and without mixing**

**The asymmetry :**

$$a_{f_{CP}} = \frac{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}.$$



**that usually is:**

$$a_{f_{CP}} = \frac{(1 - |\lambda_{f_{CP}}|^2) \cos(\Delta m_B t) - 2 \mathcal{I}m \lambda_{f_{CP}} \sin(\Delta m_B t)}{1 + |\lambda_{f_{CP}}|^2} ;$$

**reduces to:**

$$a_{f_{CP}} = - \mathcal{I}m \lambda_{f_{CP}} \sin(\Delta m_B t)$$

**When it occurs with no CP violation in decay, CP violation in the interference between decays with and without mixing can be cleanly related to Lagrangian parameters**

# CP violation in the Standard Model

SM accounts for flavor changing quark transition through the coupling of the V-A charged current operator to a W boson:

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}}(\mathcal{J}^\mu W_\mu^+ + \mathcal{J}^{\mu\dagger} W_\mu^-),$$

where:

$$\mathcal{J}^\mu = \sum_{i,j} V_{ij} J_{ij}^\mu = \sum_{i,j} \bar{u}_i \gamma^\mu \frac{1}{2}(1 - \gamma_5) V_{ij} d_j.$$

$V_{ij}$  are the elements of the CKM matrix  
i, j run on the three quark generations

Amplitudes for  $d_j \rightarrow W^- u_i$  or  $\bar{u}_i \rightarrow W^- d_j$  are proportional to  $V_{ij}$

Amplitudes for  $\bar{d}_j \rightarrow W^+ \bar{u}_i$  or  $u_i \rightarrow W^+ d_j$  are proportional to  $V_{ij}^*$

CKM matrix can be regarded as a rotation from the quark mass eigenstates (d, s, b) to a set of new states (d', s', b') with diagonal coupling to u, c, t

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

**A  $n \times n$  complex matrix has  $2n^2$  real parameters**

**Unitarity gives  $n$  constraints for the normalization of each column and  $n(n-1)$  constraints from the orthogonality between each pair of columns. A unitary matrix has  $n^2$  independent real parameters**

**Of these  $1/2 \times n(n-1)$  are real rotation angles, the others are phases**

**The freedom to select the phases of the quark field can be used to absorb  $2n - 1$  phases of the CKM matrix**

**The number of physical parameters is  $n^2 - (2n - 1) = (n-1)^2$**

**With 3 generations: 3 real rotation angles and 1 phase**

## Conditions to have CP violation:

angles different from  $0, \pi/2$ ;

phase different from  $0, \pi$

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \neq 0$$

at least 3 generations

**J** defined by:

$$\mathcal{I}m[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J \sum_{m,n=1}^3 \epsilon_{ikm}\epsilon_{jln}$$

must be different from zero

A standard parametrisation of CKM matrix uses the set of angles  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$  and  $\delta_{13}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

where:

$$c_{12} = \cos\theta_{12}, s_{12} = \sin\theta_{12}, \dots$$

**The phase  $\delta_{13}$  produces the CP violation**

**In this parametrization:**

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13} \sin \delta.$$

## Wolfenstein parametrization:

expansion in the parameter  $\lambda = \sin\theta_C \sim 0.22$

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \end{aligned}$$

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)$$

In this parametrization:

$$J = A^2 \eta \lambda^6$$

**Unitarity conditions coming from inner products between the columns, can be written as:**

$$\begin{aligned}
 V_{td}V_{ts}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* &= 0 \\
 V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \\
 V_{ts}V_{tb}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* &= 0.
 \end{aligned}$$

**and can be represented as triangles in the complex plane**  
**Expressing them in powers of  $\lambda$**

$$\begin{aligned}
 \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) &= 0 \\
 \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) &= 0 \\
 \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) &= 0.
 \end{aligned}$$



## The Unitary Triangles:

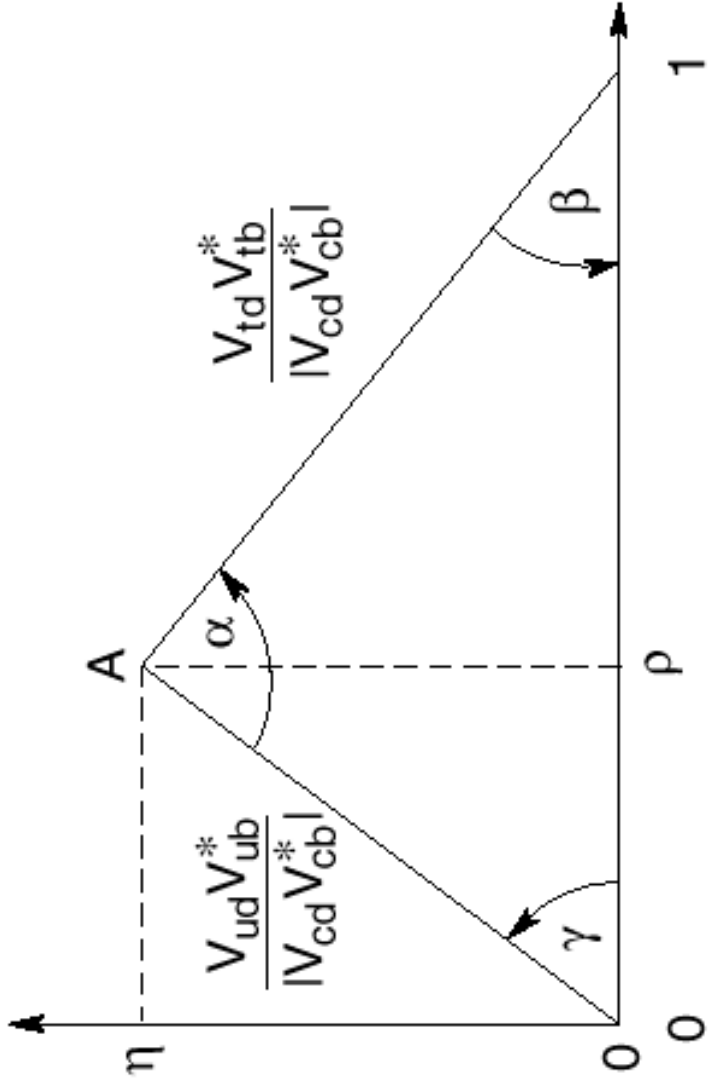
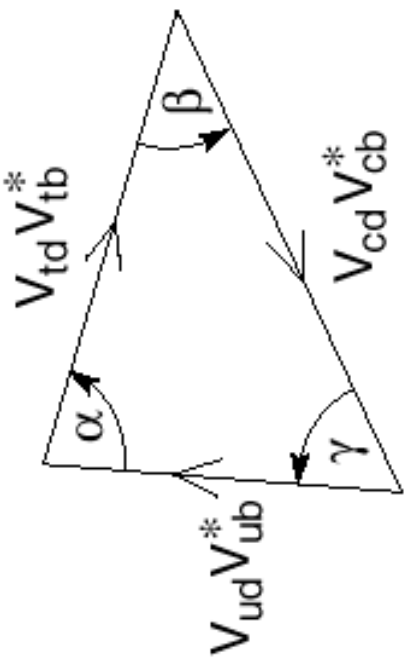
- are equal in area
- the area is  $|J|/2$

In the first and in the third triangles, **one side is much shorter than the other two.**

CP violation is small in the leading K decays (the first triangle) and in the leading  $B_s$  decays (the third triangle).

The most exciting physics of CP violation lies in the B system, related to the second triangle. **The openness of this triangle predicts large CP asymmetries in B decays.**

It is usually referred as “**the Unitarity Triangle**”



**The rescaled Unitarity Triangle is derived**

- **choosing a phase convention such that  $(V_{cd}V_{cb}^*)$  is real**
- **dividing the lengths of all sides by  $|V_{cd}V_{cb}^*|$**

**Two vertices of the rescaled Unitarity Triangle are thus fixed at  $(0,0)$  and  $(1,0)$ .**

**The coordinates of the remaining vertex are  $(\rho,\eta)$**

**The angles are:**

$$\alpha \equiv \arg \left[ -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right], \quad \beta \equiv \arg \left[ -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$$

$$\gamma \equiv \arg \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \equiv \pi - \alpha - \beta.$$

## Three ways to determine the CKM elements:

- direct measurements
- indirect measurements
- unitarity condition

### Direct measurement:

- Nuclear beta decays  $\rightarrow |V_{ud}|$
- Semileptonic kaons and hyperon decays  $\rightarrow |V_{us}|$
- $\nu$  production of charm  $\rightarrow |V_{cd}|$
- Semileptonic D decays  $\rightarrow |V_{cs}|$
- Semileptonic B decays  $\rightarrow |V_{cb}|$
- Spectrum endpoint in semileptonic B decays  $\rightarrow |V_{ub}/V_{cd}|$

### Indirect measurements:

- $B^0 \bar{B}^0$  mixing  $\rightarrow |V_{tb}^* V_{td}|$

**Taking into account all information:**

$$\lambda = 0.2205 \pm 0.0018, \quad A = 0.826 \pm 0.041.$$

$$-0.15 \leq \rho \leq +0.35, \quad +0.20 \leq \eta \leq +0.45,$$

$$0.4 \leq \sin 2\beta \leq 0.8, \quad -0.9 \leq \sin 2\alpha \leq 1.0, \quad 0.23 \leq \sin^2 \gamma \leq 1.0.$$

**From CP asymmetries it's possible, in principle,  
to extract  $\alpha$ ,  $\beta$ ,  $\gamma$**

# CP violation and B decay amplitude

Most b decay amplitude have contributions from Tree and Penguin diagrams

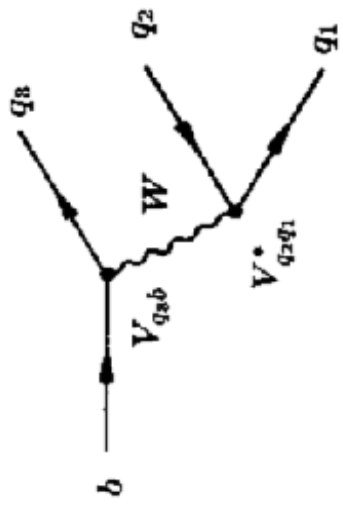
$$A(c\bar{c}s) = (T_{c\bar{c}s} + P_s^c - P_s^t)V_{cb}V_{cs}^* + (P_s^u - P_s^t)V_{ub}V_{us}^*$$

$$A(u\bar{u}s) = (P_s^c - P_s^t)V_{cb}V_{cs}^* + (T_{u\bar{u}s} + P_s^u - P_s^t)V_{ub}V_{us}^*$$

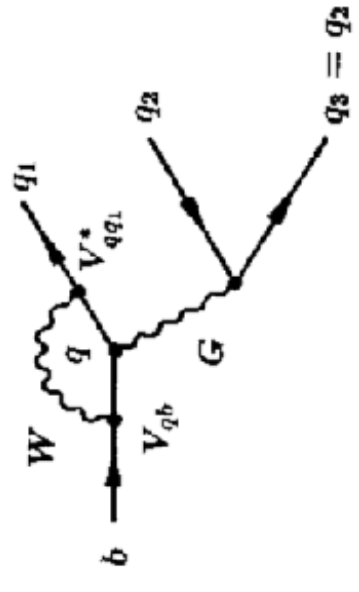
$$A(c\bar{c}d) = (P_d^t - P_d^u)V_{tb}V_{td}^* + (T_{c\bar{c}d} + P_d^c - P_d^u)V_{cb}V_{cd}^*$$

$$A(u\bar{u}d) = (P_d^t - P_d^c)V_{tb}V_{td}^* + (T_{u\bar{u}d} + P_d^u - P_d^c)V_{ub}V_{ud}^*$$

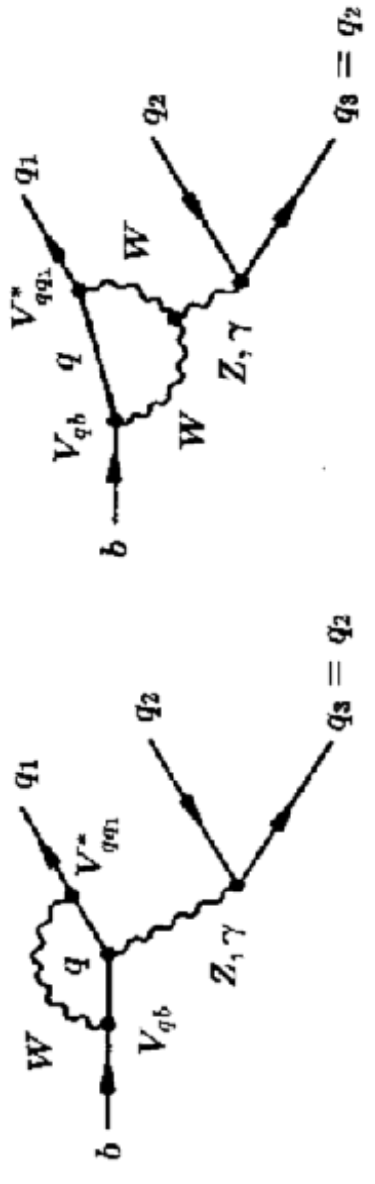
Tree Diagrams:



QCD Penguin Diagrams:



EW Penguin Diagrams:



**In the B mesons:**

- **CP violation in decay is expected to be small and can be ignored in hadronic decays**
- **Direct CP violation requires at least two contribution with different weak and strong phases**
- **CP violation from interference of decays with and without mixing has clear relationship between asymmetries and CKM parameters**

**In many decays direct CP violation and CP violation from interference of decays with and without mixing are both present.**

**Decays with small direct CP violation are of great interest**

**According to their amplitude structure, B decays can be classified in different classes:**



**Decays dominated by a single term:  $b \rightarrow c\bar{c}s$ ,  $b \rightarrow s\bar{s}s$**

**SM predicts small direct CP violation effect**

$B \rightarrow J/\psi K$ ,  $B \rightarrow \phi K$

**Decays with a small second term:  $b \rightarrow c\bar{c}d$ ,  $b \rightarrow u\bar{u}d$**

**Depending on the relative size of Tree and Penguin contributions, direct CP violation can play a non negligible role**

$B \rightarrow D D$ ,  $B \rightarrow \pi\pi$

**Decays with suppressed tree contribution:  $b \rightarrow u\bar{u}s$**

**Tree suppressed by CKM matrix element**

$B \rightarrow \rho K$

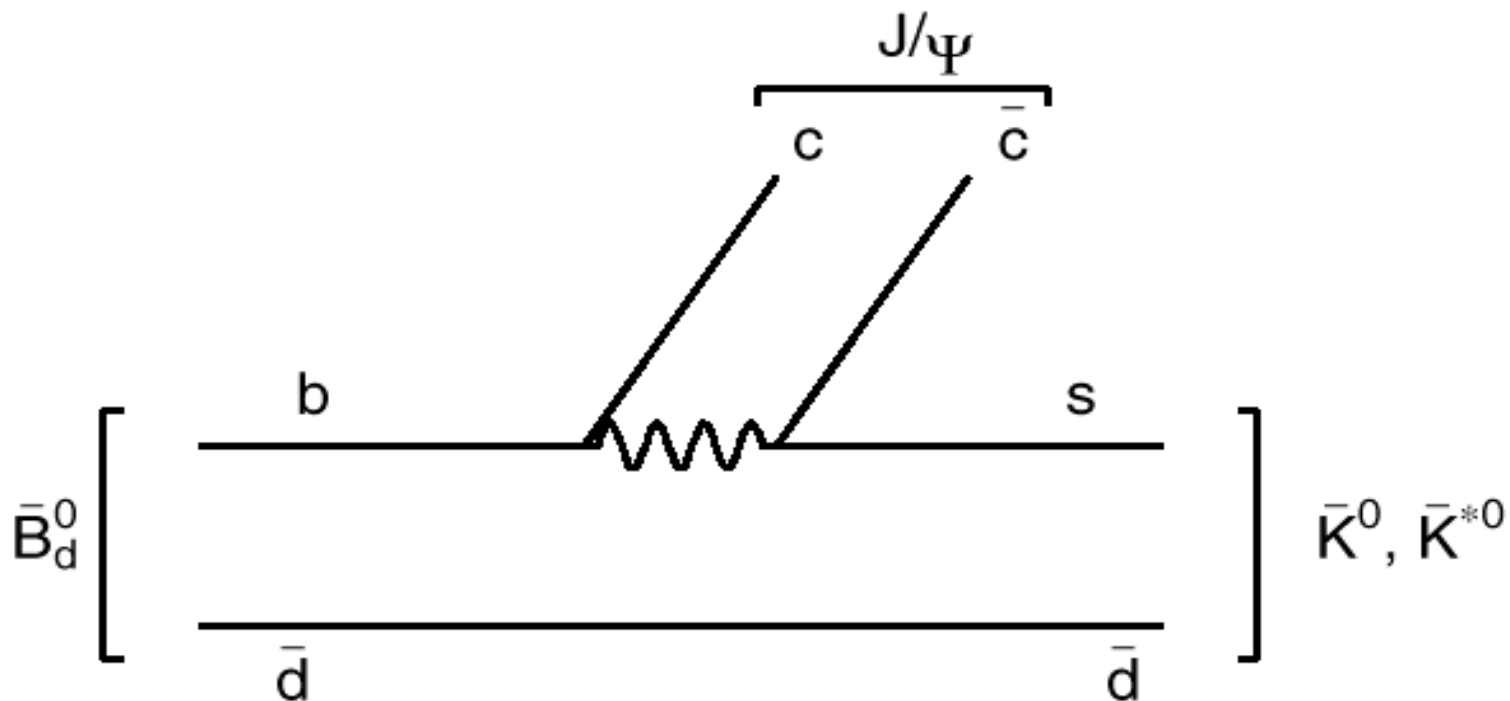
**Decays with no Tree contribution:  $b \rightarrow s\bar{s}d$**

**Interference between Penguin with quark of different charge in the loop  $B \rightarrow KK$**

# Modes that can be used to measure $\beta$

$B \rightarrow J/\psi K^0$ ,  $B \rightarrow J/\psi K^*$  (with  $K^* \rightarrow K^0 \pi^0$ )

$\text{Br}(B \rightarrow J/\psi K^0) \sim 5 \cdot 10^{-4}$ ,  $\text{Br}(B \rightarrow J/\psi K^*) \sim 1.4 \cdot 10^{-3}$



Dominant penguin has the same weak phase as the tree. The term with different phase is a Cabibbo-suppressed penguin. **Small Direct CP violation. Clean extraction of angle  $\beta$**

For example for  $B \rightarrow J/\psi K_S$  ( the “golden mode” )

$$\lambda(B \rightarrow \psi K_S) = - \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left( \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right)$$

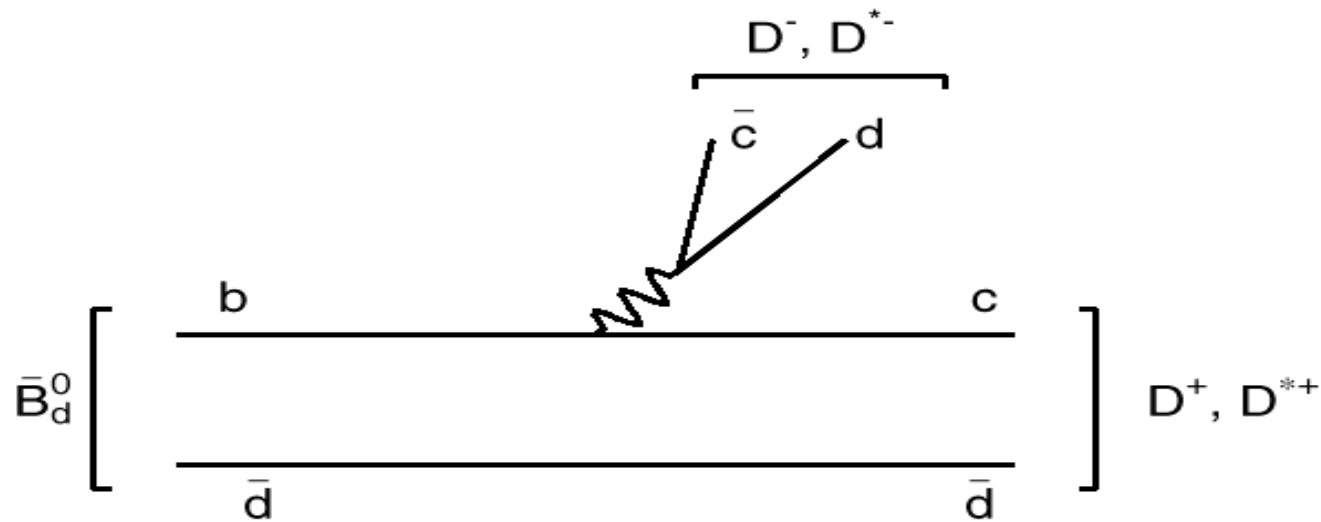
$$\text{Im} \lambda_{\psi K_S} = \sin(2\beta).$$

The CP of the two body state depends on relative angular momentum as  $(-1)^L$

For  $B \rightarrow J/\psi K^*$  the final state is a mixture of odd and even CP. **An angular analysis is needed to separate amplitudes of definite CP**

$B \rightarrow D^+ D^-$ ,  $B \rightarrow D^* D^*$  ( with  $D^* \rightarrow D^0 \pi$  )

$\text{Br}(B \rightarrow D^+ D^-) \sim 4 \cdot 10^{-4}$ ,  $\text{Br}(B \rightarrow D^* D^*) \sim 10^{-3}$



**Tree amplitude is Cabibbo suppressed. Penguin Diagrams contribution (with different phase) is potentially significant.**

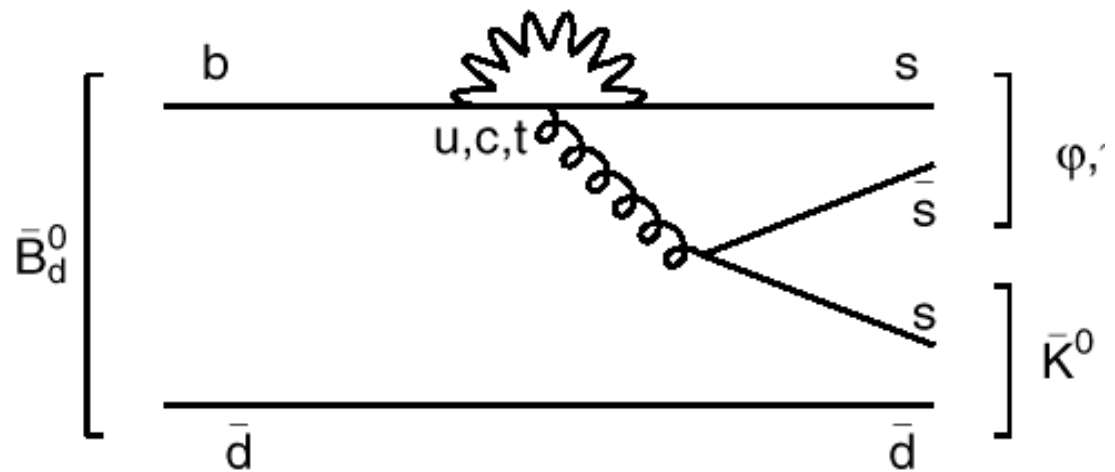
**Direct CP violation can play a role**

**In addition, for the VV case (  $D^* D^*$  ), an angular analysis is needed to separate amplitude of definite CP**

$B \rightarrow \phi K^0, B \rightarrow \phi K^*$

$\text{Br}(B \rightarrow \phi K^0),$

$\text{Br}(B \rightarrow \phi K^*) < 2 \cdot 10^{-5}$



**No Tree contribution. Two groups of Penguins with different phases, one group is Cabibbo-suppressed. Ignoring the Cabibbo-suppressed term:**

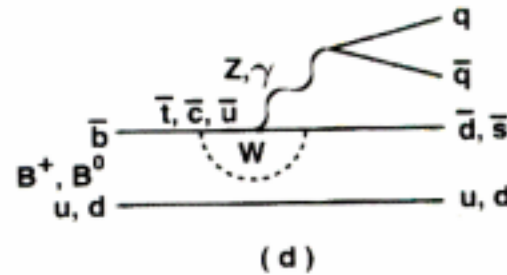
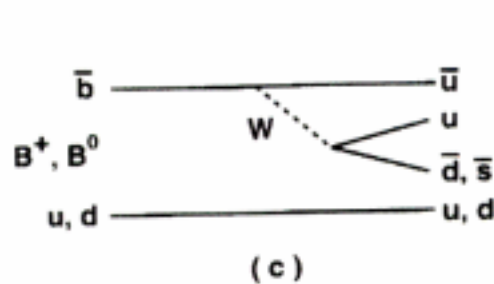
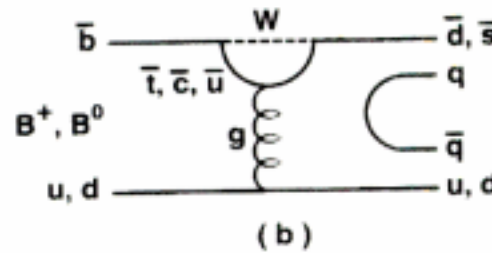
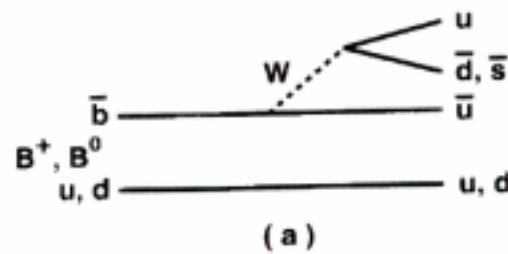
$$\lambda(B_d \rightarrow \phi K_S) \cong - \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*} \right) \left( \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right)$$

$$\text{Im} \lambda(B_d \rightarrow \phi K_S) \cong \sin 2\beta .$$

# Modes that can be used to measure $\alpha$

$B \rightarrow \pi^+ \pi^-$

$\text{Br}(B \rightarrow \pi^+ \pi^-) \sim 5 \cdot 10^{-6}$



**Penguin terms have phase different from the Tree contributions**  
**Neglecting penguins:**

$$\lambda(B \rightarrow \pi^+ \pi^-) = \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*} \right) \implies \mathcal{I}m \lambda_{\pi\pi} = \sin(2\alpha)$$

**Recent CLEO results suggest that Penguin contribution is significant**

**Possible solution: measure rates for Isospin related channels:**

$$B^+ \rightarrow \pi^+ \pi^0, B^0 \rightarrow \pi^0 \pi^0$$

**and perform an Isospin Analysis**

## Basic idea:

Isospin amplitudes  $A_{\Delta I, I_f}$  can be labelled by  $\Delta I$  value of quark decay and by  $I_f$  of final state

Gluon is pure  $I=0$ , so the dominant gluonic penguin ( $b \rightarrow d$ ) is pure  $\Delta I=1/2$

Tree level  $b \rightarrow u\bar{u}d$  have both  $\Delta I=3/2$  and  $\Delta I=1/2$

If the  $\Delta I=3/2$  piece can be isolated, then the tree contribution, that contains the weak phase to be measured, is isolated

$$\begin{aligned} A(B^+ \rightarrow \pi^+ \pi^0) &= \frac{\sqrt{3}}{2} A_{3/2,2} \\ \frac{1}{\sqrt{2}} A(B^0 \rightarrow \pi^+ \pi^-) &= \frac{1}{\sqrt{12}} A_{3/2,2} - \sqrt{\frac{1}{6}} A_{1/2,0} \\ A(B^0 \rightarrow \pi^0 \pi^0) &= \frac{1}{\sqrt{3}} A_{3/2,2} + \sqrt{\frac{1}{6}} A_{1/2,0} \end{aligned}$$

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$B \rightarrow \rho\pi$  ( $\pi^+\pi^-\pi^0$  final state)

$B \rightarrow \rho\rho$ ,  $B \rightarrow a_1\pi$  ( $4\pi$  final state)

**Decay modes not yet observed (expected Br  $\sim 10^{-5}$ )**

**Require Isospin analysis**

**$\rho\rho$  requires also angular analysis to separate CP even and CP odd amplitude**

# Modes that can be used to measure $\gamma$

**Direct measurements of  $\gamma$  are difficult**

$B^+ \rightarrow DK^+$

**Relation between amplitude in  $B_{u,d} \rightarrow K\pi$**

**Partial reconstruction of  $B_d \rightarrow D^{(*)}\pi$  to extract  $\sin(2\beta+\gamma)$**

# Experimental Requirements to Search for CP violation

Interesting modes have small branching ratio  $O(10^{-4}) - O(10^{-6})$ :

- you need a lot of B

$B \rightarrow f_{CP}$  decays must be fully reconstructed:

- high tracking efficiency
- good vertexing to have good mass resolution
- good particle identification capability to distinguish among different modes and reject background
- $\gamma$  and  $\pi^0$  reconstruction
- identify neutral hadron ( $K_L$ )

the other B must be tagged as  $B^0$  or  $\bar{B}^0$

- you need particle ID capability

To study the time dependence, one need to measure the distance  $\Delta z$  between the two B decay:

- very good vertexing

You have to process a lot of data (BaBar: 300KB/event at ~100 Hz):

- a robust computing model
- well designed and well coded software