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Lagrangian statistics in two-dimensional free turbulent convection

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Abstract. We discuss single-particle and two-particle statistics in two-dimensional turbulent convection in the Bolgiano–Oboukhov regime by means of high-resolution direct numerical simulations. Relative separation of two particles is found to be described well by a generalization of the Richardson diffusion model. Single-particle velocity structure functions are dominated by large-scale eddies and therefore a careful analysis based on 'exit-time' statistics is necessary to identify turbulent contributions. Because the velocity field is not intermittent, small-scale acceleration statistics is found to be in good agreement with simple dimensional predictions.

Contents

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1. Introduction

Understanding the statistical properties of particle dispersion in turbulent flows is a basic problem in turbulence research and it is of great relevance in many practical applications, being the basis for the development of Lagrangian stochastic models [1]–[3]. The subject has recently seen many improvements from the theoretical [4], numerical [5, 6] and experimental [7]–[9] points of view, mainly due to the ever-growing capabilities of experimental set-ups and computing machines.

In this paper, we present the results of the study of Lagrangian dispersion in two-dimensional Boussinesq convective turbulence from extensive direct numerical simulations. The results cover both relative dispersion of particle pairs and Lagrangian velocity increments and acceleration of single tracers. While in the first case, the theoretical prediction arising from the Bolgiano–Oboukhov scaling of velocity is found to be closely followed, in the second one, a careful analysis based on exit-time statistics is needed to disentangle the effect from spurious, large-scale contributions. Acceleration statistics is found to follow closely a simple Kolmogorov-like prediction.

The two-dimensional Boussinesq turbulent convection is described by the following set of partial differential equations [10]:

$$\begin{aligned} &\partial_t \omega + \boldsymbol{v}\cdot\nabla\omega = \nu\Delta\omega - \beta\nabla T\times\boldsymbol{g} - \alpha\omega,\\ &\partial_t T + \boldsymbol{v}\cdot\nabla T = \kappa\Delta T, \end{aligned} \tag{1}$$

where T is the temperature field and $\omega = \nabla\times\boldsymbol{v}$ the scalar vorticity, $\boldsymbol{g}$ the gravitational acceleration, β the thermal expansion coefficient and κ and ν are molecular diffusivity and viscosity; α is a linear friction coefficient needed to obtain a stationary state and to avoid energy condensation at large scales. Energy in (1) is injected by maintaining a mean temperature profile $\langle T(\boldsymbol{r},t)\rangle = \boldsymbol{G}\cdot\boldsymbol{r}$, with a constant gradient $\boldsymbol{G}$ pointing in the direction of gravity. The linear mean temperature profile is physically relevant as it is observed in the intermediate region of the convective boundary layer in the atmosphere [11].

Let us briefly recall the phenomenology of two-dimensional turbulent convection [12]. Suppose energy is mechanically injected in the system around a small scale $\ell_{\rm F}$, such that here the buoyancy force is negligible. Energy flows toward larger scales as in the usual Navier–Stokes (NS) case, until the inertial term is balanced by buoyancy. The scale $\ell_{\rm B}$ where this happens is called Bolgiano length scale and marks the beginning of the buoyancy-dominated range. For $\ell_{\rm F} \ll \ell \ll \ell_{\rm B}$, the inertial term is larger than the buoyancy force, and temperature behaves like a passive scalar. At scales $\ell \gg \ell_{\rm B}$ temperature drives the system: the buoyancy and inertial terms are balanced and energy is injected at all scales, causing a steepening in the energy spectrum.

By imposing a constant scalar flux ϵ_T and balancing between buoyancy and the nonlinear term in (1) we obtain a prediction for the scaling of velocity and temperature fluctuations in the Bolgiano regime (i.e. at scales $\ell \gg \ell_{\rm B}$)

$$\begin{aligned} &\delta_\ell v \simeq (\beta g)^{2/5}\epsilon_T^{1/5}\ell^{3/5} \simeq v_L\left(\frac{\ell}{L}\right)^{3/5},\\ &\delta_\ell T \sim (\beta g)^{-1/5}\epsilon_T^{2/5}\ell^{1/5} \simeq T_L\left(\frac{\ell}{L}\right)^{1/5}, \end{aligned} \tag{2}$$

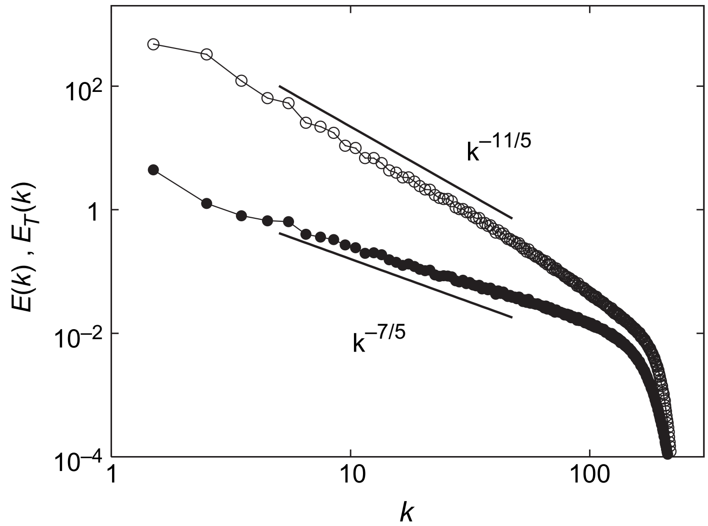

Figure 1. Kinetic energy spectrum (open circle) and temperature fluctuation spectrum (closed circle) from a direct numerical simulation of (1) at resolution 1024^2. The lines represent the two-dimensional prediction based on the Bolgiano–Oboukhov theory of turbulent convection.

where L represents the characteristic large scale, v_L and T_L are the typical large-scale fluctuations of velocity and temperature and $\epsilon_T = T_L^2 v_L/L$. Evaluating (2) at a large scale one obtains the relation $(\beta g)^2 \epsilon_T = v_L^5/L^3$.

Velocity structure functions $S_p(\ell) = \langle(\delta_\ell v)^p\rangle$ thus have scaling exponents $\zeta_p = hp$ with $h = 3/5$. The second-order structure function gives the prediction for the energy spectrum $E(k) \simeq k^{-11/5}$ which is indeed clearly observed in our numerical simulations (see figure 1 and [13]). We recall that detailed numerical simulations have shown that velocity fluctuations display self-similar statistics without intermittency corrections [14].

From the prediction (2) one can have an estimation of the Bolgiano scale ℓ_B at which buoyancy and mechanical forcing are balanced. Denoting the kinetic energy input rate by ϵ_v and equating the constant energy flux induced by this forcing with the buoyancy-induced flux obtained from (2) one obtains

$$\ell_\mathrm{B} = (\beta g)^{-3/2} \epsilon_v^{5/4} \epsilon_T^{-3/4} \,. \tag{3}$$

Because in the present setup we do not have mechanical forcing and the flow is forced by the imposed temperature gradient, we have $\epsilon_v = 0$ and the Bolgiano scale vanishes (i.e. is smaller than any other scale).

The results discussed in the present paper are obtained by direct integration of Boussinesq equations (1) by means of a standard, fully dealiased pseudospectral method in a doubly periodic square domain at resolution 1024^2. As customary in two-dimensional turbulence, in most of our simulations the viscous term in equation (1) has been replaced by a hyperviscous term of order eight to broaden the convective inertial spectrum and obtain better scalings. A set of runs with normal, Newtonian, viscosity have also been performed to check the robustness of the results on the dissipation mechanism and for the study of small-scale quantities, such as the acceleration statistics.

Lagrangian trajectories are obtained by integrating $\dot{\mathbf{x}}(t) = \boldsymbol{v}(\boldsymbol{x}(t), t)$ with the velocity at particle position obtained by linear interpolation from the nearest grid points. Some 64 000 pairs of particles are followed simultaneously, initially distributed on the integration domain homogeneously. All the results presented in the following are obtained after averaging over about 150 large-scale times in stationary conditions.

2. Two-particle dispersion

The first attempt to give a phenomenological description of particle pair separation in turbulence dates back to 1926 and the original work of Richardson [15] who first obtained from different experimental data that diffusion coefficient for relative dispersion increases with the separation as $K(r) = k_0 r^{4/3}$. He also proposed a diffusive equation for the probability density function of relative separation $p(\boldsymbol{r}, t)$ with the scale-dependent diffusivity $K(r)$ which, in the two-dimensional, isotropic case, reads

$$\frac{\partial p(\boldsymbol{r}, t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r K(r) \frac{\partial p(\boldsymbol{r}, t)}{\partial r}. \tag{4}$$

The solution to (4), with $h = 1/3$ and delta function initial condition, is the famous Richardson distribution [16]

$$p(\boldsymbol{r}, t) = \frac{A}{(k_0 t)^3} \exp\left(-\frac{9}{4}\frac{r^{2/3}}{k_0 t}\right), \tag{5}$$

where A is a normalizing number. From (5) one obtains that variance of separations grows faster than ballistic: $\langle R^2(t)\rangle \simeq (k_0 t)^3$. It was later recognized by Oboukhov [17] that the 'four-thirds' law is a simple consequence of Kolmogorov scaling in turbulence. Indeed, if the velocity field scales as $\delta v \sim r^h$, one immediately obtains $K(r) \sim r^{1+h}$ and $R(t) \sim t^{\frac{1}{1-h}}$ under the assumption that particle separation R depends only on eddies of size $r \sim R$. For $h = 1/3$, Richardson's predictions are recovered. This simple argument reveals the deep connection between two-particle Lagrangian dispersion and *spatial* velocity scaling in turbulence.

A similar argument can be developed for Bolgiano–Oboukhov scaling in turbulent convection. In this case $h = 3/5$, therefore, $K(r) = k_1 r^{8/5}$, where k_1 is a scale-independent dimensional parameter. The solution to (4), again with delta-distributed initial condition, is in this case

$$p(\boldsymbol{r}, t) = \frac{A}{(k_1 t)^5} \exp\left(-\frac{25}{4}\frac{r^{2/5}}{k_1 t}\right), \tag{6}$$

which again gives the growth law for the variance of separation $\langle R^2(t)\rangle \simeq (k_1 t)^5$, faster than in the Kolmogorov scaling case [18]. In figure 2 we show the evolution of the variance of particle separation computed in our numerical simulations. The dimensional scaling t^5 is clearly observed in the intermediate range of scales, corresponding to separation in the inertial range of scaling (2).

Probability density functions (pdfs) of particle separations are shown in figure 3 for a time inside the scaling range of figure 2. Together with prediction (6) we also plot, for a comparison, the Richardson pdf (5) which clearly deviates from numerical data. Small deviations from (6) are also observed, and are probably due to the limitation of model (4). Indeed, the possibility to describe particle pair separation as a diffusion process rests on the basic assumption that the

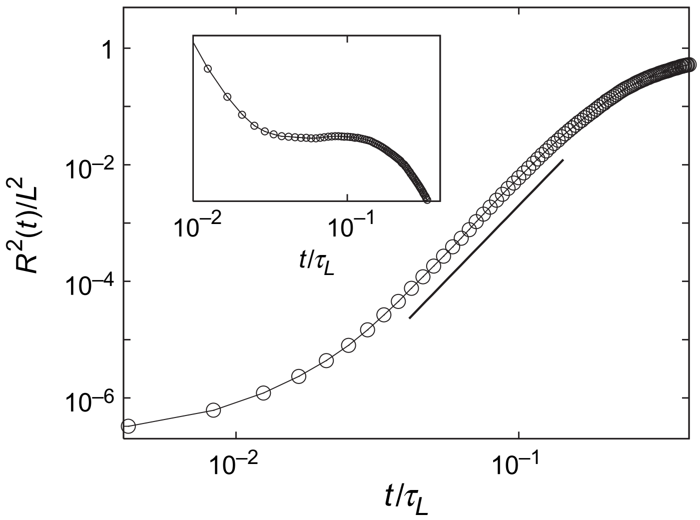

Figure 2. Variance of particle pair separation as a function of time. The line represents the dimensional prediction t^5. Inset: relative dispersion compensated with the dimensional prediction. Initial separation is $R(0) = \delta x/4$.

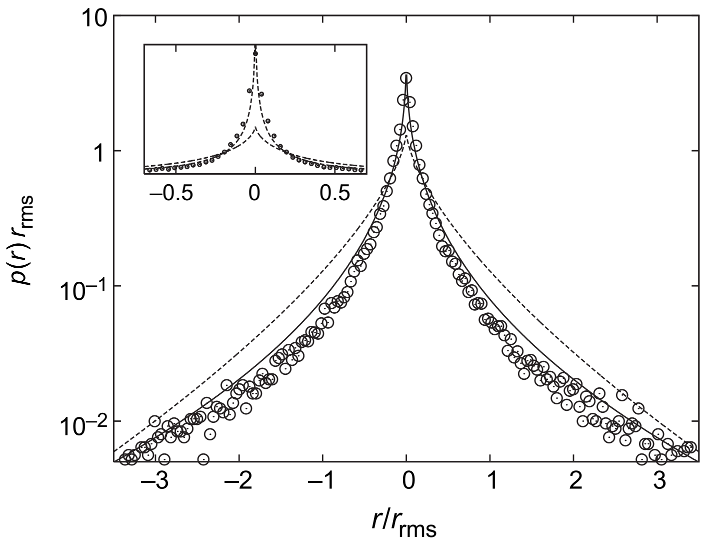

Figure 3. Probability distribution function of pair separation in lin–log plot at time $t = 0.2\tau_L$ (see figure 2). The continuous line is the prediction (6), while the dashed line is the Richardson pdf (5). Inset: linear plot.

velocity field is short-correlated in time. Of course, this is not the case for dynamically generated velocity fields, where non-trivial time correlations are present.

A recently introduced measure of time correlation in self-similar flows is given by the persistence parameter Ps [19], which may be seen as the ratio between the typical Eulerian and Lagrangian correlation times and therefore as a Kubo number. A convenient measure of Ps is given in terms of the statistics of the turning point ratio $\Psi(r_2/r_1)$, defined as the probability that the relative velocity between two particles changes sign when separation is r_2, provided a previous turning point had taken place at distance r_1. This probability can be related to persistence by means of a simple stochastic model [19] and therefore it provides a way to estimate Ps. The numerical evaluation of the distribution Ψ from our data gives $Ps \approx 0.72$ (not far from what was measured for NS turbulence [20]). Ballistic motion thus plays an important role for the relative dispersion in two-dimensional convective turbulence, nevertheless diffusion model (4) gives a good approximation to numerical data.

3. One-particle dispersion

Two-particle statistics, discussed in the previous section, is determined by the *spatial* properties of the flow. In this section, we discuss single-particle velocity increments $\delta_t \mathbf{v} = \mathbf{v}(t) - \mathbf{v}(0)$ which give information on the flow *temporal* correlation and are therefore of great importance for a complete description of a turbulent flow.

Dimensional analysis in homogeneous, isotropic turbulence predicts that $\langle \delta_t v^2 \rangle \simeq \epsilon t$, ϵ being the energy dissipation rate [16]. This diffusive-like behavior is at the basis of stochastic models of turbulent dispersion, but it is not expected to hold in the case of Bolgiano–Oboukhov turbulent convection. We have extensively studied this problem in [13]; we report here the main results for the sake of completeness.

To obtain a prediction for $\delta_t v$, we must evaluate the contributions coming from eddies of all sizes. After a time interval t, eddies with a characteristic time $\tau_r \ll t$ are effectively decorrelated and give no contribution to δv. Therefore, let us consider eddies with $\tau_r \simeq t$. The characteristic size of these eddies is expected to be $\ell \simeq L(t/\tau_L)^{1/(1-h)}$ (where h is the velocity-scaling exponent and τ_L is the large-eddy characteristic time $\tau_L \simeq L/v_L$) and their contribution to velocity increments is thus

$$\delta_t v \simeq v_L (\ell/L)^h \simeq v_L (t/\tau_L)^{h/(1-h)}. \tag{7}$$

In addition to these eddies, we have still to consider large-scale eddies (whose decorrelation time is much larger than t). Due to their relative slowness, their contribution to velocity fluctuations is differentiable, i.e. $\delta_t v \simeq (\partial_t v_L) t$.

In the end, we are left with two contributions to the velocity fluctuations, one coming from local eddies and one due to the large-scale eddies:

$$\delta_t v \simeq \tau_L (\partial_t v_L)(t/\tau_L) + v_L (t/\tau_L)^{h/(1-h)}. \tag{8}$$

When $t \ll \tau_L$ (8) will be dominated by the minimum exponent, $\min(1, h/(1-h))$. When the classical K41 theory is considered ($h = 1/3$) the dominant contribution to $\delta_t v$ is a local one, resulting in diffusive-like scaling $\delta_t v \sim t^{1/2}$. In free convection, $h/(1-h) = 3/2$ which corresponds to velocity increments 'more than smooth' and $\delta_t v$ is dominated by the infrared linear term. Therefore, standard Lagrangian structure functions $S_p^L(t) = \langle (\delta_t v)^p \rangle$ are not good statistical objects to look at, because they are unable to disentangle the Bolgiano–Oboukhov contribution from the differentiable one. Figure 4 clearly shows that only large scales are involved in this case.

Non-trivial contributions in more than smooth signals can be disentangled by using the so-called exit-time statistics (or inverse statistics) [21]. The exit-time approach is based on the time $T(\delta v)$ needed to observe a particle change its velocity of δv along its trajectory [22]. Let us consider the signal δv composed by two contributions as in equation (5). In the limit of small t, the differentiable part $\propto t$ will always dominate, except when the derivative

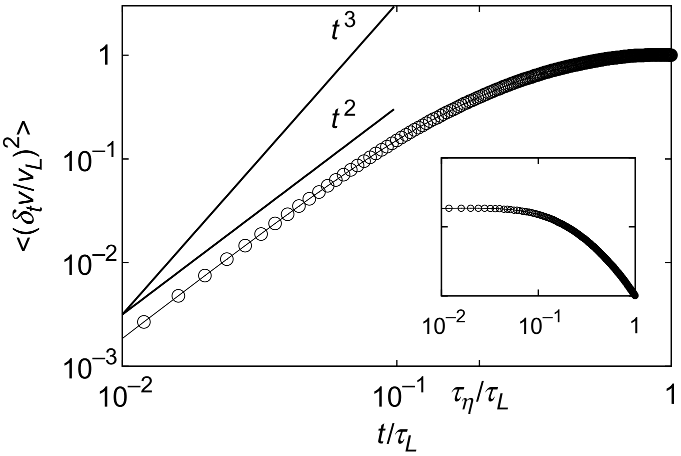

Figure 4. The second-order Lagrangian structure function versus time in log–log plot. The two lines represent the ballistic behavior (t^2) and the Bolgiano–Oboukhov scaling t^3; τ_η represents the dissipation time. Inset: the Lagrangian structure function compensated with t^2.

$\partial_t v_L$ vanishes. In this case the local part of the signal will be dominant. To correctly estimate the statistics of exit times $T(\delta v)$, we must therefore calculate the probability a tracer has to encounter a point with $\partial_t v_L = 0$. For our signal with $1 \leqslant h/(1-h) \leqslant 2$, its first derivative is a one-dimensional self-affine signal with Hölder exponent $\xi = (2h-1)/(1-h)$, which vanishes on a fractal set of dimension $D = 1-\xi$ [23]. Therefore, the probability to observe the more-than-smooth component is equal to the probability to pick a point on the fractal set of dimension D, i.e.

$$p(T \sim \delta v^{(1-h)/h}) \sim T^{1-D} \sim (\delta v)^{(2h-1)/h}. \tag{9}$$

To compute the moments of exit times $\langle T^p(\delta v)\rangle$, one must multiply the local term by the above probability p to observe such an event. The result is the following bifractal distribution:

$$\langle T^p(\delta v)\rangle \sim \delta v^{\chi(p)}, \quad \text{with} \ \ \chi(p) = \min\left(p, \frac{p(1-h)+2h-1}{h}\right). \tag{10}$$

For the Bolgiano–Oboukhov scaling ($h = 3/5$), one has $\chi(p) = \min(p, (2p+1)/3)$: low-order moments ($p \leqslant 1$) of the inverse statistics only see the differentiable part of the signal, while high-order moments ($p \geqslant 1$) are dominated by the local fluctuations.

Figure 5 shows the scaling exponents of different moments of the exit times computed from our simulations. Solid lines, representing the bifractal prediction (10), are closely followed. We note that the possibility for moments of order greater than 1 to follow (10) is due to the absence of intermittency in the velocity field of two-dimensional free convection (the temperature field is known to be strongly intermittent [14], but it is not involved directly in our measurements).

4. Statistics of acceleration

In the limit of very small time increments, single-particle velocity increments give the statistics of Lagrangian acceleration. In recent years, there have been great improvements in the study of acceleration statistics in turbulence. Experimental [7, 24] and numerical [25] investigations have demonstrated that in three-dimensional turbulence acceleration is an extremely fluctuating

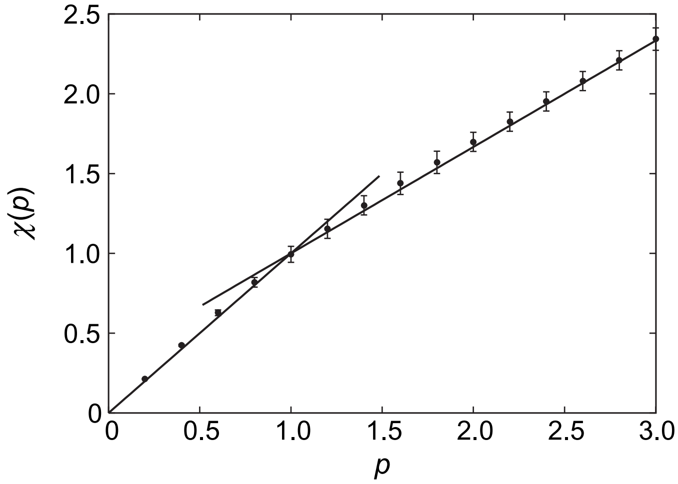

Figure 5. Exit-time scaling exponents $\chi(p)$ evaluated by fitting the p-moment of exit times versus velocity increments δv. The two lines represent the bifractal prediction (10). Error bars have been estimated by evaluating differences in the observed exponents, while changing the fitting interval.

quantity with acceleration events up to 80 times the rms values. As a consequence of these extreme fluctuations, the pdf of Lagrangian acceleration is very far from the prediction based on Kolmogorov phenomenology which does not take into account intermittency in the velocity fluctuations. On the contrary, in the case of two-dimensional, inverse cascade turbulence, where intermittency is negligible [14, 26], one may expect to observe acceleration statistics in agreement with predictions based on self-similarity of the velocity field.

Here, we consider the acceleration on Lagrangian tracers induced by the turbulent velocity field generated by (1). The usual estimate in terms of velocity fluctuations at the smallest active scale η is $a \simeq \delta_\eta v/\tau_\eta \simeq (\delta_\eta v)^2/\eta$. By using the scaling (2) one obtains $a \sim \eta^{1/5}$ which implies that the typical acceleration decreases with η, i.e. with increasing Reynolds number. This apparent paradox is removed if one considers, as in (8), also the contribution from large-scale eddies. Since the acceleration is nothing but $\delta_t v/t$ in the limit of small t, from (8) one has that the leading contribution is the first one which gives for the acceleration $a \simeq v_L^2/L$. Assuming a Gaussian distribution for the large-scale velocity fluctuations v_L (which is, *a posteriori*, well verified in our simulations), the prediction for the pdf of the acceleration is therefore simply given by

$$p(a) = Ca^{-1/2}\mathrm{e}^{-\alpha a}, \tag{11}$$

where C and α are normalizing numbers. We observe that the present derivation is substantially different from the NS turbulence (i.e. $h = 1/3$), in which the local contribution is dominant. Another important peculiarity here is that the variance of acceleration is independent of the Reynolds number, while for NS turbulence one expects $\langle a^2 \rangle \sim Re^{1/2}$. Despite these differences, the predicted form of the pdf in the two cases is close, as for NS the tails behave as $\exp(-a^{8/9})$ [16].

Figure 6 shows the pdf of one component of the acceleration of Lagrangian tracers obtained from our simulations. The line represents the exponential prediction (11) which fit very well the data for $a \geqslant 0.5a_{\mathrm{rms}}$. At very low values of a, (11) predicts a peak not observed in our data and therefore overestimates the frequency of very small accelerations. This is not surprising,

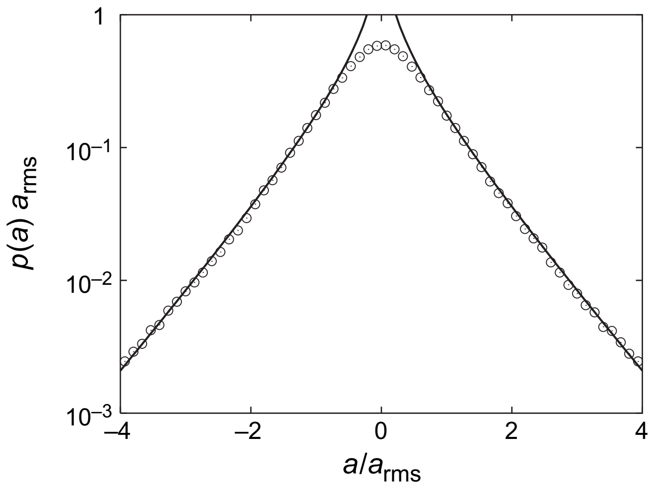

Figure 6. Probability density function of the x component of the Lagrangian acceleration normalized with the rms value. Continuous line represents prediction (11) (vertically shifted for better comparison).

as these events correspond to vanishing large-scale velocity fluctuations: in these conditions, as also explained in the previous section, one has to take into account subleading contributions.

Acceleration is numerically evaluated by computing the rhs of the Boussinesq equation written for the velocity field. A detailed analysis of the different contributions to the acceleration shows that the dominant term here is the buoyancy one, at variance with the three-dimensional NS case where acceleration is dominated by pressure gradients. Moreover, it is important to remark that this is the first observation of acceleration pdf consistent with Kolmogorov-like dimensional arguments, as in usual three-dimensional turbulence small-scale statistics is strongly affected by intermittency corrections.

5. Conclusions

We have studied single-particle and two-particle Lagrangian statistics in two-dimensional free convection turbulence. Apart from the physical relevance of convective flow, the choice of the particular system is also motivated by the fact that the Bolgiano–Oboukhov scaling gives rise to a wholly new phenomenology with respect to usual turbulence. Thanks to the absence of intermittency in the velocity field, it is possible to make quantitative predictions on Lagrangian statistical quantities on the basis of the self-similarity of the velocity field. These predictions are confirmed by numerical results obtained from high-resolution extensive simulations.

A consequence of our results, of interest more general than the present problem, is that for temporal statistics the issue of locality is more restrictive than for spatial statistics. From (8) we see that for $h > 1/2$ (as in Bolgiano scaling) temporal velocity fluctuations (and accelerations) are dominated by non-local effects, while spatial structure functions are still local. Therefore, one has to be careful when connecting spatial and temporal statistics, in particular, in the case of intermittent turbulence, in which the scaling exponent h fluctuates over a range of values.

As a final remark, we remember that the statistics of acceleration is an important ingredient for modeling droplet growth in turbulent clouds. Indeed, the droplet collision kernel is expected to be larger in regions of intense fluid acceleration as a consequence of both enhanced droplet relative velocity and enhanced droplet collision efficiency which are again functions of relative velocity [27]. Therefore, the general characterization of Lagrangian statistics in turbulent convection is a fundamental step in our understanding of cloud dynamics.

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