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Waiting time statistics in solar flares

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Abstract

Solar flares activity is investigated by looking at the statistics of the waiting times between hard X-ray bursts. It is found that the distribution has a power-law tail which indicates the existence of nontrivial dynamics with long-range correlations. A shell model for MHD turbulence is capable to reproduce the power-law distribution through an identification of the intermittent bursts of dissipation with solar flares. This result suggests that the nonlinear dynamics could play a role more relevant than the particular topology associated with the field configuration. Comparisons with results from models based on self-organized criticality are made. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Solar flares are very spectacular phenomena. They consist in a sudden and transient energy release above active regions of the sun [1] in a very broad range of frequencies (from microwave to hard X-rays). Parker [2] conjectured that flares are produced by the dissipation at the many tangential discontinuities arising spontaneously in the bipolar fields of the active regions of the Sun as a consequence of random continuous motion of the footpoints of the field in the photospheric convection. Observational data

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show that probability distributions, calculated for various observed quantities, x, can be represented by power laws $P(x) = Ax^{-\alpha}$. In particular, from hard X-rays (HXR) emission, it was found that the distribution of peak flux yields $\alpha \simeq 1.7$, that of total energy associated with a single event yields $\alpha \simeq 1.5$ and finally the distribution of flare duration yields $\alpha \simeq 2$ [3].

On the basis of Parker's conjecture and the observed power-law distributions of real flares, a new way of looking at impulsive events like flares was considered. Lu et al. [4] pointed out that self-organized criticality (SOC), introduced earlier [5], could describe the main features of HXR flares. In the present paper we propose a different approach to the modelization of solar flares. In particular, a shell model for MHD turbulence is introduced to investigate the statistical features of flares, with particular attention to the statistics of the waiting times between flares.

The paper is thus organized as follows. In Section 2 the concept of SOC is reviewed and numerical results on the statistics of waiting times are presented and discussed. Section 3 is devoted to data analysis and comparison with the results of the SOC model. In Section 4 the MHD shell model is introduced and the results of numerical simulations are presented. Finally discussion and conclusions can be found in Section 5.

2. Sand-pile model for flare statistics

What is usually called SOC is a mechanism of charging and discharging, apparently without tuning parameters, which reproduces self-similarity in critical phenomena. Lu et al. [4], thus assume that the coronal magnetic field evolves in a self-organized critical state in which an "event" can give rise to other similar "events" through an avalanche process (sandpile model). They suggest that an active region on the sun could be modeled through a magnetic field B on a uniform 3D lattice.

In order to have a statistically stationary state, energy is injected into the system by adding a small magnetic field increment δB at a random site on the grid. When an avalanche takes place, the energy input is suspended until all sites become stable. In this sense the avalanches (flares) are fast phenomena, on a time scale much smaller than the injection mechanism. In the last years these models reached a great popularity because of their capability to reproduce the power-law behavior in the distribution functions of the total energy, the peak luminosity and the duration of avalanches.

However, a different statistics can be studied on solar flare signals [6]: the distribution of laminar or waiting times, i.e., the time intervals between two successive bursts. This distribution has been recently studied on solar flares HXR events [7]. Wheatland et al. [7] have also emphasized the fact that this kind of distribution is crucial from the point of view of the avalanche model. SOC models indeed are expected to display an exponential waiting time distribution $P(\tau_L) = \langle \tau_L \rangle^{-1} \exp(-\tau_L / \langle \tau_L \rangle)$, where $\langle \tau_L \rangle$, the average laminar time, depends on the parameters of the model (see Ref. [6] for details). This is confirmed by a simulation we performed with the SOC automaton proposed by

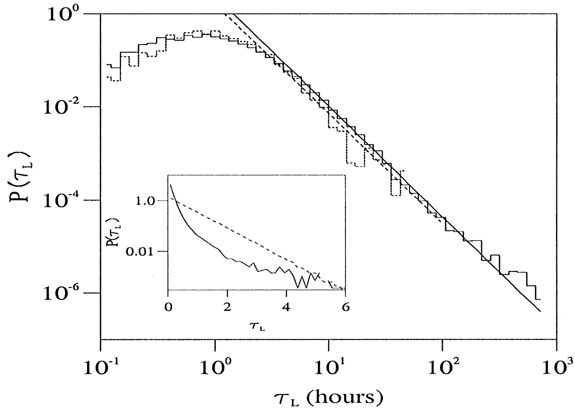

Fig. 1. Probability distribution of the laminar time $P(\tau_L)$ between two X-ray flares for dataset A (dashed line) and dataset B (full line). The straight lines are the respective power-law fits. In the inset we show, in lin–log scale, the distribution for dataset B (full line) and the distribution obtained through the SOC model (dashed line) which displays a clear exponential law. The variables shown in the inset have been normalized to the respective root-mean-square values.

Lu and Hamilton [4] (see the inset of Fig. 1). On the contrary, solar flares data [6,7], display a more or less well-defined power-law distribution.

3. Data analysis

The probability distribution function of laminar phases has been calculated using twenty years of data from National Geophysical Data Center of USA. In this database starting and ending times as well as peak times of HXR bursts associated to flares from 1976 up to 1996, measured at the Earth by satellites in the 0.1 to 0.8 nm band, are stored. We calculated the laminar times as the time differences between two successive maxima of the flares intensity recorded during periods of activity of the same instrument. Two different kind of analysis have been performed: in the first one we have built up a dataset of about 1100 samples (hence on dataset A) by calculating only the differences between the time of occurrence of flares within the same active region, as identified through the Hα flares occurrence. To build up the second dataset (dataset B), we have considered the sun as a unique physical system, and we have calculated the time differences between two successive maxima of flare intensity regardless of the position of the flare on the sun surface. In this way we get a dataset of about 32 000 samples. The analysis of these data shows that, in both cases, laminar times display a clear power-law distribution $P(\tau_L) = A\tau_L^{-\alpha}$ with $\alpha = 2.38 \pm 0.03$ in the range $6\ \mathrm{h} \leqslant \tau_L \leqslant 67\ \mathrm{h}$ (reduced $\chi^2 = 2.2$) for dataset B and $\alpha = 2.4 \pm 0.1$ (reduced $\chi^2 = 1.1$) for dataset A (Fig. 1). The exact value of the exponent can be affected by the finite length of the observation times, which underestimates the occurrence of long waiting times. However, these results allow us to consider the power-law distribution of waiting times as firmly established as the power laws observed for total energy, peak luminosity and time duration.

It is worth noting that the occurrence of a power law in the distribution of the laminar times is the analogous for the Solar flares of the Omori's law for the earthquakes [8] and represents a clear indication of the existence, in the flare dynamics of strong correlations between successive bursts, at variance with the SOC model. The unique possible origin of the correlations arises from nontrivial evolution equations of the phenomenon. In particular, we think that Parker's conjectures about solar flares dynamics could be usefully modeled and investigated in the framework of MHD turbulence.

4. MHD shell model

An alternative way to look at a turbulent MHD system is represented by the so called shell models [9]. In these models one tries to reproduce at best the nonlinear dynamics, but ignores from the beginning the details of the spatial structure and its associated topology. A single (complex) scalar variable u_n (and b_n) is considered to be representative of the velocity (magnetic) fluctuation associated to a wavenumber $k_n = k_0 2^n$ $(n = 1, \dots, N)$. The fact that the wavenumbers k_n are exponentially spaced allows to reach very large Reynolds numbers with a moderate number of degrees of freedom and then to investigate regimes of 3D MHD turbulence which are not accessible by direct numerical simulation.

The evolution equations for the dynamical variables u_n and b_n are built up by retaining only the interactions between nearest- and next-nearest-neighbor shells in the form of quadratic nonlinearities. The coupling coefficients of nonlinear terms are determined by imposing the inviscid conservation of the 3D MHD quadratic invariants [9]. The shell model we used in our simulation reads [10]

$$\begin{aligned}\frac{\mathrm{d}u_n}{\mathrm{d}t} &= -\nu k_n^2 u_n + f_n + ik_n\{(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) \\ &\quad - \tfrac{1}{4}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \tfrac{1}{8}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\}^* ,\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}b_n}{\mathrm{d}t} &= -\eta k_n^2 b_n + ik_n(1/6)\{(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) \\ &\quad + (u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + (u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\}^* ,\end{aligned}$$

where ν and η are, respectively, the viscosity and the resistivity and f_n is an external forcing term acting only on velocity fluctuations. The external forcing term has been set up in order to model the driving mechanism due to the large-scale random motion of magnetic field lines footpoints. f_n is a stochastic variable acting only on the first two shells of the velocity fluctuations. It is calculated according to the Langevin equation $\mathrm{d}f_n/\mathrm{d}t = -f_n/\tau_0 + \mu$, where τ_0 is the characteristic time of the large scales and μ is a Gaussian white noise.

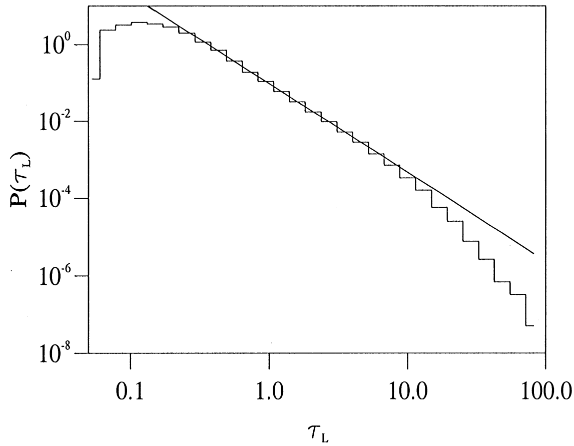

Fig. 2. The distribution of laminar times $P(\tau_L)$ for the Shell model, normalized to the root-mean-square. The straight line is the fit with a power law.

This model has been shown to be able to reproduce a dynamo-like effect (inverse cascade of magnetic helicity), which is typical of 3D MHD [10] as well as the time intermittency through the analysis of the scaling laws of the structure functions [11]. In particular time intermittency can also be observed by looking at the energy dissipation defined as $\varepsilon(t) = \nu \sum_{n=1}^{N} k_n^2 |u_n|^2 + \eta \sum_{n=1}^{N} k_n^2 |b_n|^2$. As stated above we identify the intermittent spikes of dissipation with the intermittent spikes on data (HXR, EUV, etc.) associated with flares. Then, we have performed on $\varepsilon(t)$ the same statistical analysis done for the solar flare signals. We have defined a burst of dissipation by the condition $\varepsilon(t) \geqslant \varepsilon_c$. This definition allows us to calculate the distribution functions for the peak values of the bursts, their total energy (defined as the integral of the signal above ε_c) and the duration of the bursts (defined as the time during which the dissipation is above ε_c).

It has been shown [6], that these distribution are close to power laws with exponents similar to the observational results. The further interesting result [6], is that the distribution of laminar times follows a power law. This is an indication of a nontrivial temporal dynamics with long-range interactions. In Fig. 2 the result of the analysis is reported. The threshold in this case is $\varepsilon_c = \langle \varepsilon(t) \rangle + 3\sigma$, where the average and the standard deviation are calculated considering only the background contribution to the signal. A clear power law is observed with exponent $\alpha \simeq 2.34$, close to the exponent obtained by analyzing the solar flares data. However, we do not think that the agreement is particularly significant as the model exponents depend on the value chosen for the threshold. Observational results could also depend on the particular value of the threshold selected to single out individual flares. The interesting point is that, at variance with SOC models, MHD shell models display a power-law statistics also for the laminar times, as shown in Fig. 2.

5. Conclusion

In this paper we investigated the statistical properties of solar flares. In particular, we focused on the statistics of waiting times between flares, which provides information on possible temporal correlations between subsequent events. An analysis performed on 20 years of data reveals the existence of a power-law distribution for the waiting times between flares. This behavior was not found however in a simulation done with a simple model (sand-pile) based on the concept of self-organized criticality. Sand-pile models, although capable to reproduce the power-law statistics for other quantities (energy, time duration, peak value) associated with flares, yield an exponential distribution for the waiting times. Shell models for MHD turbulence reproduce all the observed power-law distribution. The different behavior of SOC models and turbulent MHD shell models is related to the conceptually different mechanisms underlying the SOC phenomenon and the phenomenon of intermittency in fully developed turbulence. SOC models represent self-similar phenomena, while the intermittent behavior of turbulence is related to its chaotic nature [12]. The statistics of the laminar times between two bursts is linked to global properties of the system (its memory) and thus it is more relevant to characterize the dynamical behavior of the system than the statistics of the properties of the single burst. Moreover, our analysis shows that, since shell models are able to reproduce the power-law for the quiescent time, the mechanism for destabilization of the laminar phases and subsequent nonlinear restabilization is more directly related to the nonlinear dynamics than to the particular instability associated to magnetic field topology [13].

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