Relative dispersion in turbulence

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ABSTRACT – *Il problema della dispersione relativa in turbolenza pienamente sviluppata è studiato per mezzo di simulazioni numeriche dirette ad alta risoluzione. I risultati della statistica Lagrangiana sono compatibili con la descrizione di Richardson, sebbene si osservino delle piccole deviazioni. Il valore della costante di Richardson è stimato* $C_2 \simeq 0.55$, *in ottimo accordo con recenti risultati sperimentali. Per mezzo della statistica dei tempi di uscita, si mostra che le deviazioni dalla legge di Richardson sono una conseguenza dell'intermittenza.*

Relative dispersion in fully developed turbulence is investigated by means of high resolution direct numerical simulations. Lagrangian statistics is found to be compatible with Richardson description although small systematic deviations are found. The value of the Richardson constant is estimated as $C_2 \simeq 0.55$, *in a close agreement with recent experimental findings. By means of exit-time statistics it is shown that the deviations from Richardson's law are a consequence of Eulerian intermittency.*

The statistics of two particle dispersion is historically the first issues which has been quantitatively addressed in the study of fully developed turbulence. This was done by Richardson, in a pioneering work on the properties of dispersion in the atmosphere in 1926[1], 15 years before the theoretical development by Kolmogorov and Obukhov[2]. Despite this fact, there are still few experimental studies on turbulent Lagrangian dispersion. This is essentially due to the difficulties to obtain Lagrangian trajectories in fully developed turbulent flow. The problem has been recently approached in laboratory experiments[3] but the results are still not conclusive. Therefore, relative dispersion in turbulence is a natural candidate for numerical studies where Lagrangian statistics is easily obtained. Numerical studies are based on direct numerical simulation of fully developed turbulence at high Reynolds numbers, a challenging numerical task requiring high performance computers.

1 L.F. Richardson, Proc. Roy. Soc. A **110**, 709 (1926).
2 A. Monin and A. Yaglom, Statistical Fluid Mechanics (MIT Press, Cambridge, MA, 1975), Vol. 2.
3 S. Ott and J. Mann J. Fluid Mech. **422**, 207 (2000).

Richardson's original description of relative dispersion is based on a diffusion equation for the probability density function of pair separation $p(\mathbf{r},t)$ which in the isotropic case can be written as

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 K(r) \frac{\partial p(\mathbf{r},t)}{\partial r}. \tag{1}$$

The turbulent eddy diffusivity was empirically established by Richardson to follow the "four-thirds law" $K(r) = k_0 \varepsilon^{1/3} r^{4/3}$. The solution of (1) for δ-distribution initial condition has the form

$$p(\mathbf{r},t) = \frac{A}{(k_0 t)^3 \varepsilon} \exp\left(-\frac{9r^{2/3}}{4k_0 \varepsilon^{1/3} t}\right) \tag{2}$$

where A is a normalizing factor. The most peculiar feature of the Richardson distribution (2) is non-Gaussianity with a very pronounced peak at the origin and fat tails. In the past, alternative distributions have been proposed, in particular Gaussian.

According to (2), turbulent dispersion is self-similar in time, i.e. the scaling exponents of the moments of the separation

$$R^{2n}(t) \equiv \langle r^{2n}(t) \rangle = C_{2n} \varepsilon^n t^{\alpha_{2n}} \tag{3}$$

have the values $\alpha_{2n} = 3n/2$, as follows from dimensional analysis. All the dimensionless coefficients C_{2n} are given in terms of the constant k_0 and a single number, such as

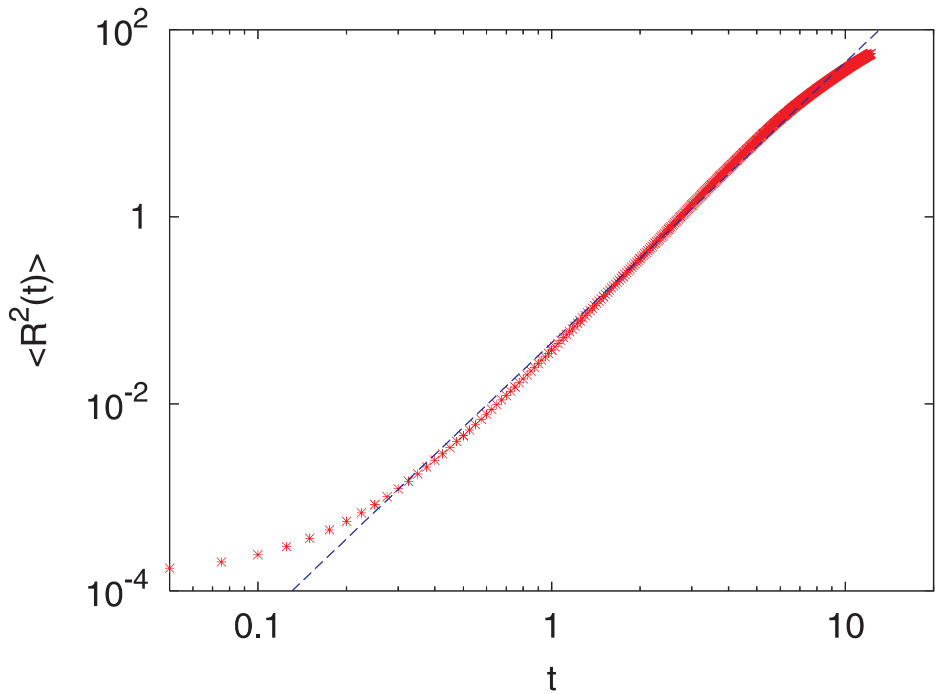

FIGURE 1. – *Relative dispersion $R^2(t)$ versus time t. The blue line is the Richardson t^3 law.*

the Richardson constant C_2, is sufficient to parameterize turbulent dispersion. There is still a large uncertainty on the value of C_2, ranging from $O(10^{-2}) - O(10^{-1})$ for kinematic simulations to $O(1)$ or more in the case of closure predictions. A recent experimental investigation gives the value $C_2 = 0.5$ [3].

We now turn to our numerical procedure. The turbulent velocity field is generated by direct numerical integration of Navier-Stokes equation in a periodic box of size $L = 2\pi$. The integration was done on the Cray T3E at CINECA by means of a fully parallel pseudo-spectral code at resolution 256^3 with $Re_\lambda \simeq 200$. Passive tracer trajectories are obtained by simultaneous integration of about 3×10^5 Lagrangian tracer pairs advected by the turbulent flow and starting from initial separation $R(0) = L/256$. The reported results are obtained after averaging over 10 independent runs.

The movie shows the evolution of blobs of 10^4 particles released in the center of the box. The remarkable formation of long filaments (with respect to standard diffusive behavior) is a consequence of the accelerated nature of turbulent dispersion as expressed by (3).

In Fig. 1 we plot the second moment of relative dispersion $R^2(t)$: the Richardson t^3 law is clearly observable. From an independent measure of the energy flux ε, it is possible to determine the value of the Richardson constant $C_2 \simeq 0.55$, in close agreement with recent experimental data [3].

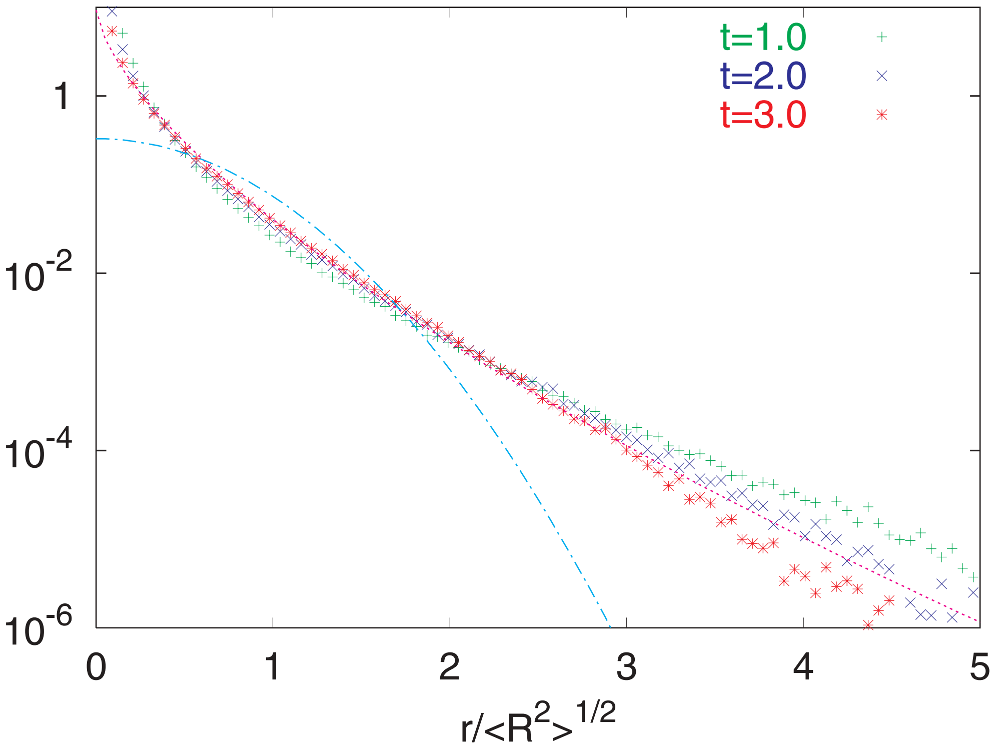

FIGURE 2. – *Probability distribution function of relative separations at three different times. The purple line is the Richardson prediction, the blue line is the Gaussian distribution.*

The distribution of relative separations is plotted in Fig. 2 for three different times within the t^3 range. The form of the pdf is very close to the Richardson prediction (2) and excludes other distributions. Our result is the first direct numerical evidence of the substantial validity of Richardson's equation.

In order to investigate possible deviations from self-similar behavior (3) in relative dispersion we have implemented an *exit-time* statistics which enhances extreme events[4]. We have measured the exit-time $T(R)$ for a pair separation to grow from R to ρR (with $\rho \geq 1$). The outstanding advantage of exit-time statistics, as opposite to a fixed time one, is that it removes crossover effects and thus a better determination of scaling exponents can be achieved. Richardson scaling (3) dimensionally corresponds to exit-time scaling

$$\left\langle \frac{1}{T^p(R)} \right\rangle \simeq R^{\beta_p} \tag{4}$$

with $\beta_p = -2p/3$. In Fig. 3 we plot the first moments of inverse doubling time compensated with best fit exponents β_p. The remarkable quality of scaling allows a

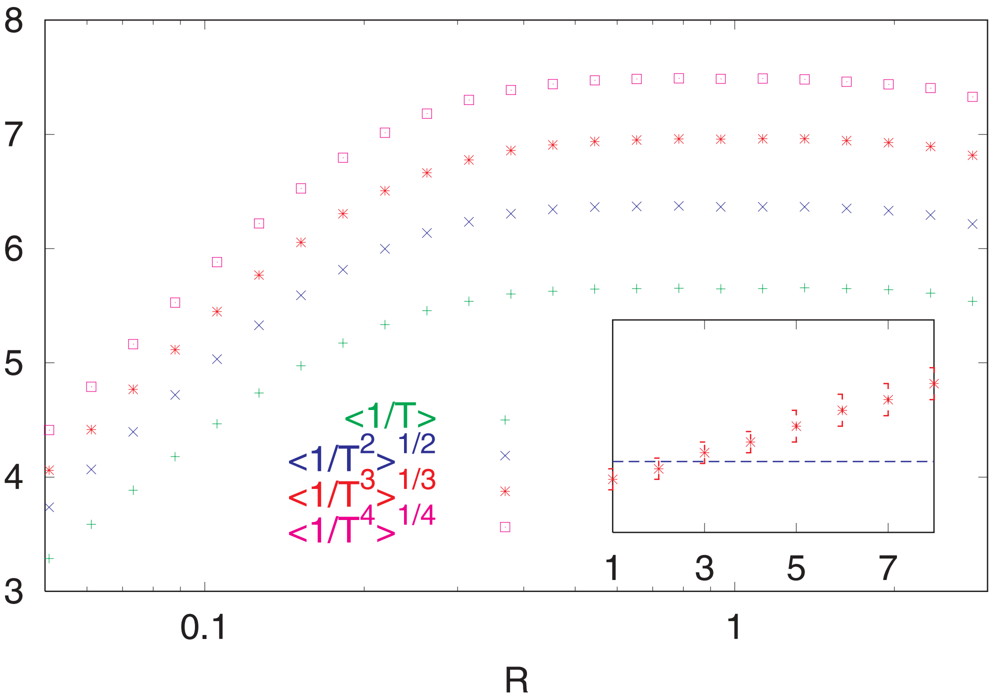

FIGURE 3. – *First moments of the inverse doubling time* $\langle 1/T^p(R) \rangle$ *compensated with best fit exponent* β_p. *In the inset we plot the scaled exponent* $-\beta_p/p$ *compared with the self-similar value* 2/3 *(blue line).*

[4] V. Artale, G. Boffetta, A. Celani, M. Cencini and A. Vulpiani, Phys. Fluids A **9**, 3162 (1997).

precise determination of scaling exponents β_p which deviates from self-similar prediction $-2p/3$. This is the first indication of Lagrangian intermittency in two-particle dispersion in turbulence. The values of the Lagrangian scaling exponents are compatible with the Eulerian structure function exponents according to a theoretical model of Lagrangian intermittency [1].

In the next future it will be probably possible to have experimental Lagrangian trajectories in high Reynolds number flows[5]. It would be extremely interesting to check our findings in real fluid turbulence.

Publications

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[5] A. La Porta, G.A. Voth, A.M. Crawford, J. Alexander and E. Bodenschatz, Nature **409**, 1017 (2001).