Funzioni di Bessel

- $$\frac{d^2R}{dx^2} + \frac{1}{x}\frac{dR}{dx} + \left(1 - \frac{\nu^2}{x^2}\right)R = 0$$ EQ. BESSEL

- $$J_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\,\Gamma(j+\nu+1)}\left(\frac{x}{2}\right)^{2j}$$

  FUNZ. BESSEL 1^ SPECIE

  $$\Gamma(z) = \int_0^\infty e^{-t}\, t^{z-1}\, dt$$ FUNZ. EULERO

- ν = INTERO $\qquad J_{-\nu}(x) = (-1)^\nu J_\nu(x)$

- FUNZ. NEUMANN

  $$N_\nu(x) = \frac{J_\nu(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

  INDIP. DA $J_\nu(x)$ $\forall\,\nu$

Funzioni di Bessel

- FUNZ. HANKEL

$$H_\nu^{(1)}(x) = J_\nu(x) + i\, N_\nu(x)$$

$$H_\nu^{(2)}(x) = J_\nu(x) - i\, N_\nu(x)$$

- $\nu =$ INTERO : FUNZ. BESSEL CILINDRICHE $(\nu = m)$

  $\nu =$ SEMINTERO : FUNZ. BESSEL SFERICHE $\left(\nu = m + \frac{1}{2}\right)$

- $$\frac{dH_0^{(i)}(x)}{dx} = -H_1^{(i)}(x)$$

Funzioni di Bessel

- COMPORTAMENTO ASINTOTICO

$$J_m(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{2m+1}{4}\pi\right) \qquad x \gg 1 \,,\quad x \gg m$$

$$N_m(x) \sim \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{2m+1}{4}\pi\right)$$

$$H_m(x) \sim \sqrt{\frac{2}{\pi x}} \exp\left[i\left(x - \frac{2m+1}{4}\pi\right)\right]$$

- FORMULA PER LE DERIVATE : $J_m(x), N_m(x), H_m^{(i)}(x) = \Omega_m(x)$

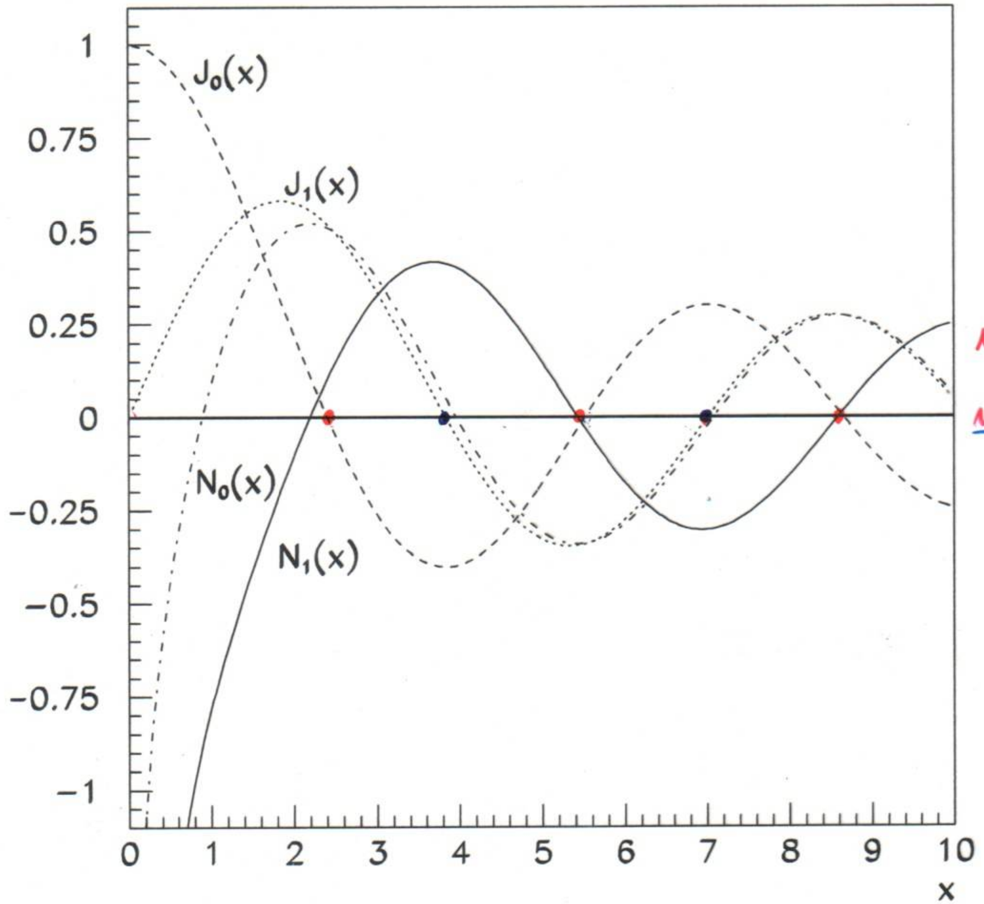
$$2 \frac{d\,\Omega_m(x)}{dx} = \Omega_{m-1}(x) - \Omega_{m+1}(x)$$

$$\rightarrow \left(\frac{dH_m^{(i)}(x)}{dx}\right)_{m=0} = \frac{dH_0^{(i)}(x)}{dx} = \left(H_{-1}^{(i)}(x) - H_1^{(i)}(x)\right)\frac{1}{2}$$

$$= -2H_1^{(i)}(x) \cdot \frac{1}{2}$$

$$= -H_1^{(i)}(x)$$

- $\Omega_{-m}(x) = (-1)^m \Omega_m(x) \qquad m = \text{INTERO}$

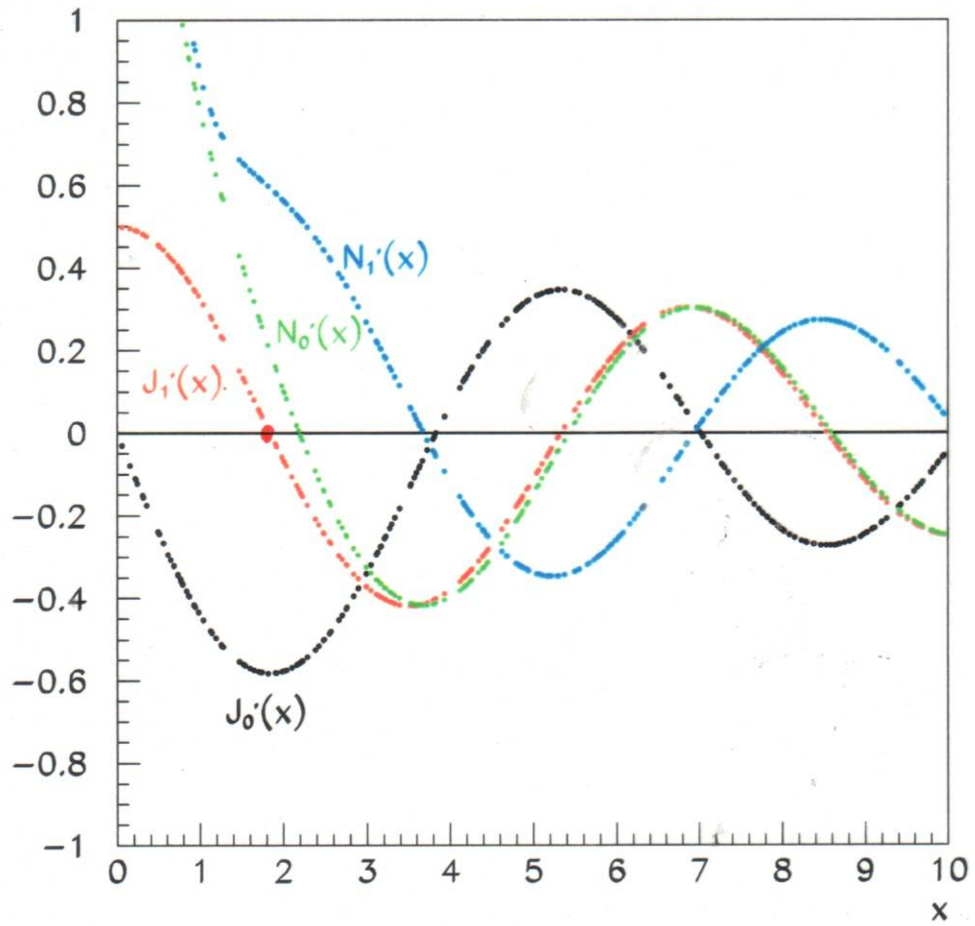


ZERI

$n=0 \quad x_{0n} = 2{,}405;\ 5{,}52;\ 8{,}654$

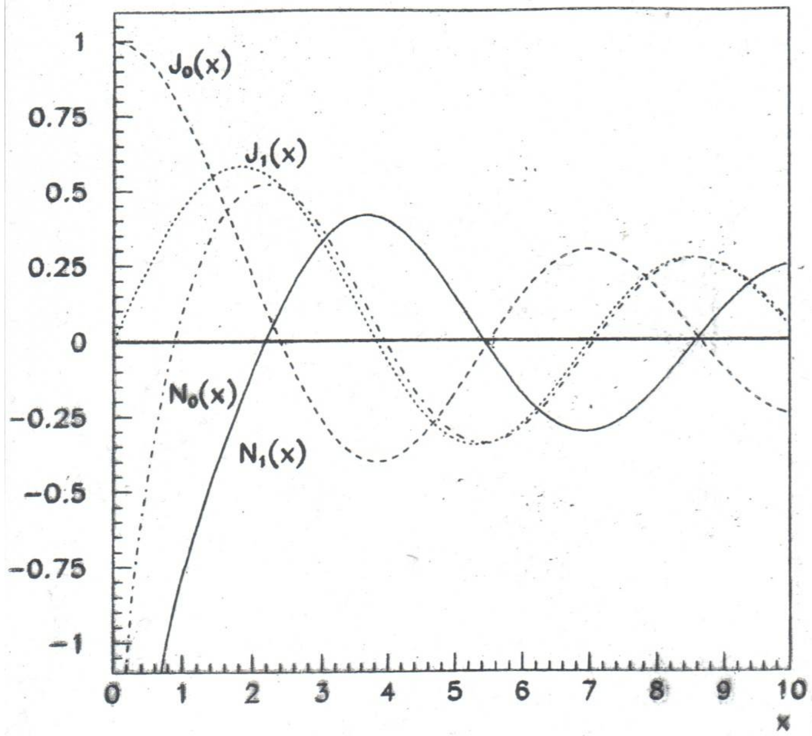
$n=1 \quad x_{1n} = 3{,}832;\ 7{,}016;\ 10{,}173$

Funzioni derivate



$\eta_{11} = 1{,}841$

Funzioni di Bessel modificate



BESSEL

NEUMANN

X : REALE

m : INTERO

BESSEL

NEUMANN

X : | IMMAGINARIO |

m : INTERO