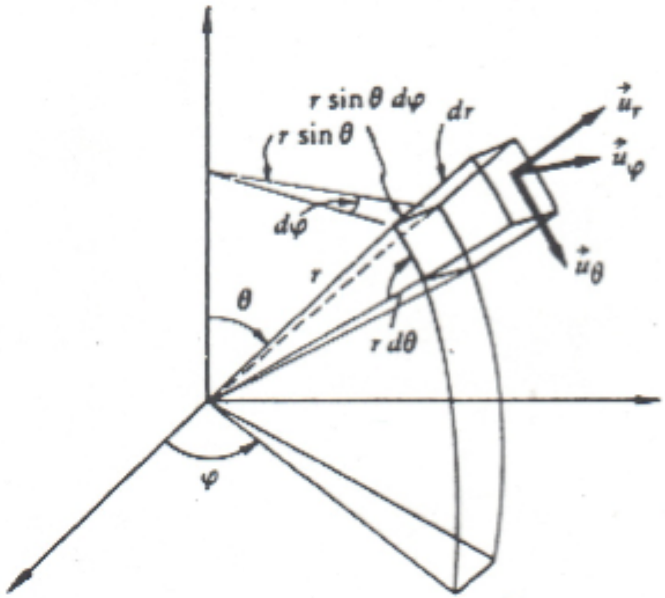
Coordinate Sferiche



$$x = r \sin\theta \cos\varphi$$

$$y = r \sin\theta \sin\varphi$$

$$z = r \cos\theta$$

$$\vec{i}_r = \sin\theta \cos\varphi \, \vec{i} + \sin\theta \sin\varphi \, \vec{j} + \cos\theta \, \vec{k}$$

$$\vec{i}_\theta = \cos\theta \cos\varphi \, \vec{i} + \cos\theta \sin\varphi \, \vec{j} - \sin\theta \, \vec{k}$$

$$\vec{i}_\varphi = -\sin\varphi \, \vec{i} + \cos\varphi \, \vec{j}$$

$$\vec{i}_r \times \vec{k} = \vec{i} \sin\theta \sin\varphi - \vec{j} \sin\theta \cos\varphi$$

$$= - \vec{i}_\varphi \sin\theta$$

$$\vec{i}_r \times \vec{i}_\varphi = - \vec{i}_\theta$$

Operatori differenziali in Coordinate Sferiche

$P(r,\vartheta,\varphi)$ $\qquad$ $\vec{v} = \vec{i}_r v_r + \vec{i}_\theta v_\theta + \vec{i}_\varphi v_\varphi$

- $\nabla f(r,\vartheta,\varphi) = \vec{i}_r \frac{\partial f}{\partial r} + \vec{i}_\theta \frac{1}{r}\frac{\partial f}{\partial \theta} + \vec{i}_\varphi \frac{1}{r\sin\vartheta}\frac{\partial f}{\partial \varphi}$

- $\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\vartheta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$

- $$\nabla \times \vec{v} = \vec{i}_r \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(\sin\vartheta\, v_\varphi\right) - \frac{\partial v_\theta}{\partial \varphi}\right] +$$
$$\vec{i}_\vartheta \frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(r v_\varphi\right)\right] +$$
$$\vec{i}_\varphi \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r v_\vartheta\right) - \frac{\partial v_r}{\partial \theta}\right]$$

- $$\nabla^2 f = \underbrace{\frac{1}{r}\frac{\partial^2}{\partial r^2}(r f)}_{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$

- $$\nabla^2\vec{v} = \vec{e}_r\left[\nabla^2 v_r - \frac{2}{r^2}v_r - \frac{2}{r^2\sin^2\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{\partial v_\varphi}{\partial\varphi}\right)\right] + \vec{e}_\theta\left[\nabla^2 v_\theta - \frac{v_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\varphi}{\partial\varphi}\right] + \vec{e}_\varphi\left[\nabla^2 v_\varphi - \frac{v_\varphi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\varphi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\varphi}\right]$$

- $$\nabla\times(f\vec{v}) = \nabla f\times\vec{v} + f\,\nabla\times\vec{v}$$

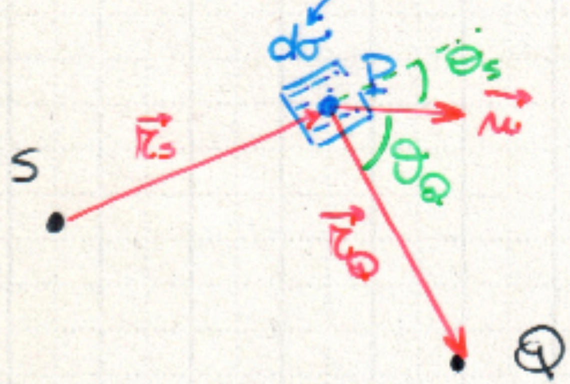
HUYGHENS - FRESNEL

- PRINCIPIO DI HUYGHENS (1629 - 1695)

  → ONDA REGRESSIVA!

- FRESNEL (1788 - 1827) : FORMULAZIONE MATEMATICA

ELEMENTO d. FRONTE D'ONDA O DI SUPERFICIE: ORIENTAZIONE DATA DA $\vec{n}$



$r_S \gg \lambda$, $r_Q \gg \lambda$ (OTTICO!)

(MISURA AMPIEZZA ONDA SECONDARIA EMESSA DA P)

– $d\sigma$: ELEMENTO DI UNA SUP. CHIUSA Σ CHE CONTIENE LA SORGENTE .

- ONDA PIANA (SOVRAPPOSIZIONE!) :

$$\rightarrow \quad E(P,t) = R_0 \frac{E_0}{r_s} \cos(K r_s - \omega t) \qquad E_0 \text{ A } R_0 \quad (\text{COMPONENTE DI } \vec{E})$$

IN Q:

$$\omega(t - r_p/c)$$

AMPIEZZA VS r_Q

$$\rightarrow \quad E(Q,t) = \frac{R_0}{\lambda} \int_{\Sigma} \boxed{\frac{E_0}{r_s r_Q}} \boxed{K(\theta)} \cos\left[K(r_s + r_p) - \omega t - \frac{\pi}{2}\right] d\sigma$$

DIP. ANGOLARE

ANTICIPO.

$$\boxed{K(\theta)} = \frac{1}{2}\left(\vec{n} \cdot \vec{r}_s + \vec{n} \cdot \vec{r}_Q\right) = \frac{1}{2}(\cos\vartheta_s + \cos\vartheta_Q)$$

FATTORE DI OBLIQUITÀ

SE Σ = FRONTE D'ONDA :

- ONDE SECONDARIE IN FASE TRA LORO
- ANTICIPO DI $T/4$ VS ONDA IN ARRIVO
- $\cos\vartheta_s = 1 \rightarrow K(\theta) = \frac{1}{2}(1 + \cos\vartheta_Q)$

$\rightarrow$ EMISSIONE IN AVANTI.

- IPOTESI DI FRESNEL ($E(Q)$ VS r_Q, ϑ_Q) GIUSTIFICATA DA KIRCHHOFF (1824 - 1887) → FORMULA DI DIFFRAZIONE DI FRESNEL - KIRCHHOFF (λ PICCOLO)
- CASO PIÙ GENERALE: FORMULA DI KIRCHHOFF → DIFFRAZIONE (KIT)