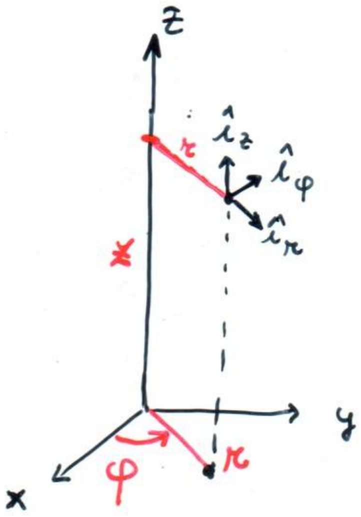
Coordinate Cilindriche



P(r, φ, z)

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \end{aligned}\right\} \Rightarrow \left.\begin{aligned} r &= (x^2+y^2)^{1/2} \\ \varphi &= \operatorname{arctg} y/x \\ z &= z \end{aligned}\right\}$$

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\cdot\frac{\partial}{\partial r} + \frac{\partial \varphi}{\partial x}\cdot\frac{\partial}{\partial \varphi}$$

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$$\left\{\begin{aligned} \frac{\partial}{\partial x} &= \cos\varphi\,\frac{\partial}{\partial r} - \frac{1}{r}\sin\varphi\,\frac{\partial}{\partial \varphi} \\ \frac{\partial}{\partial y} &= \sin\varphi\,\frac{\partial}{\partial r} + \frac{1}{r}\cos\varphi\,\frac{\partial}{\partial \varphi} \\ \frac{\partial}{\partial z} &= \frac{\partial}{\partial z} \end{aligned}\right.$$

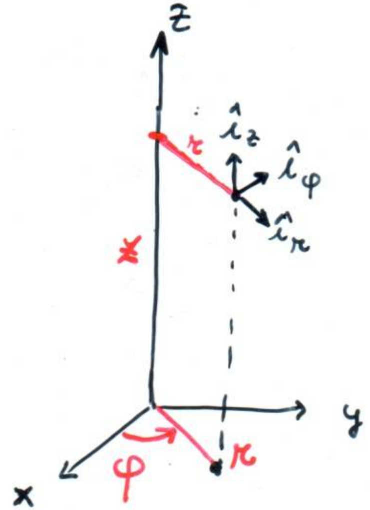
$$\begin{aligned} \hat{i}_r &= \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j} \\ \hat{i}_\varphi &= -\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j} \\ \hat{i}_z &= \hat{k} \end{aligned}$$

$$\left(\begin{aligned} \hat{i} &= \cos\varphi\,\hat{i}_r - \sin\varphi\,\hat{i}_\varphi \\ \hat{j} &= \sin\varphi\,\hat{i}_r + \cos\varphi\,\hat{i}_\varphi \end{aligned}\right)$$

COORDINATE CILINDRICHE

$$\vec{v} = \vec{i}_r v_r + \vec{i}_\varphi v_\varphi + \vec{i}_z v_z$$

$$\nabla \cdot \vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \varphi} v_\varphi + \frac{\partial}{\partial z} v_z$$



$$\nabla \times \vec{v} = \vec{i}_r \left(\frac{1}{r}\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z}\right) +$$

$$\vec{i}_\varphi \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right) +$$

$$\vec{i}_z \left(\frac{\partial}{\partial r}(r v_\varphi) - \frac{\partial v_r}{\partial \varphi}\right)\frac{1}{r}$$

- $$\nabla^2 f = \underline{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} = \underline{\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r}} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{v} = \vec{i}_r \left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\varphi}{\partial \varphi}\right) +$$

$$\vec{i}_\varphi \left(\nabla^2 v_\varphi - \frac{v_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \varphi}\right) +$$

$$\vec{i}_z \; \nabla^2 v_z$$