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Horizon 2020
European Union funding
for Research & Innovation

Theoretical introduction to neutrino masses

Tematic afternoon on neutrino masses, Sapienza Univ. (online), 8/07/2020

1 *Neutrino masses*

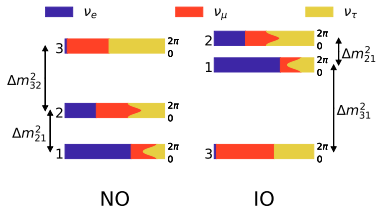
2 *Cosmological bounds*

3 *β decay*

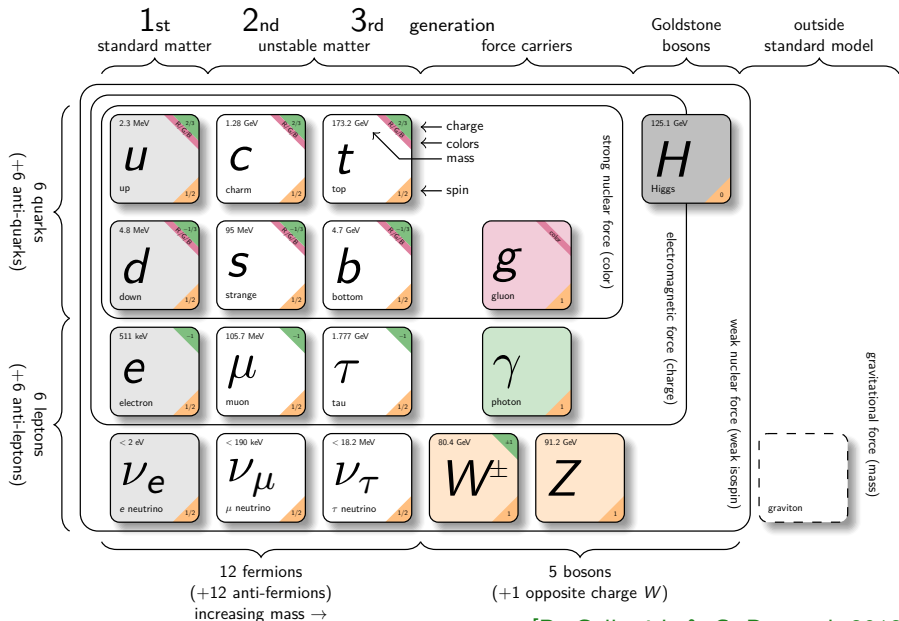
4 *Neutrinoless double β decay*

5 *Beyond the standard: light sterile neutrinos*

6 *Conclusions*

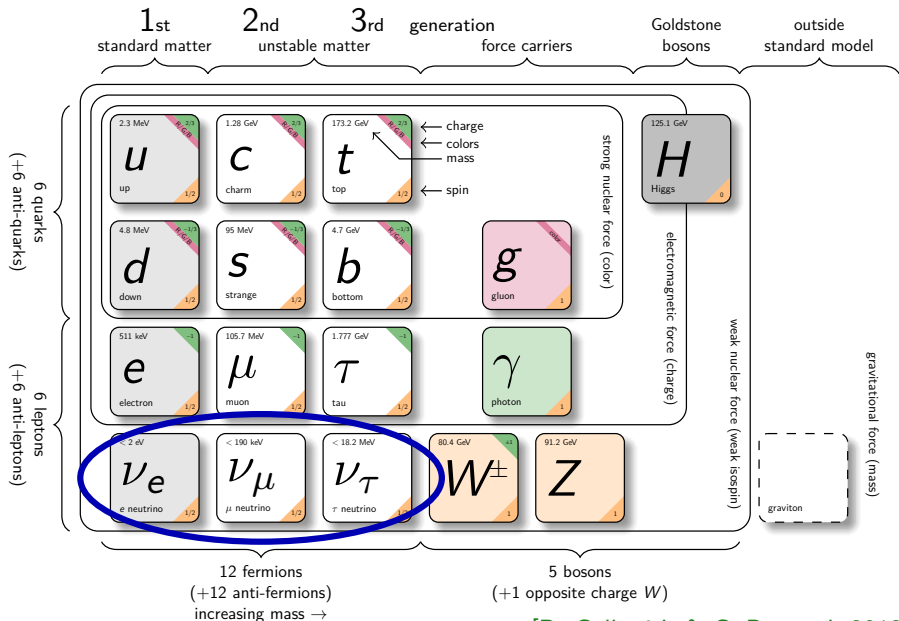


The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

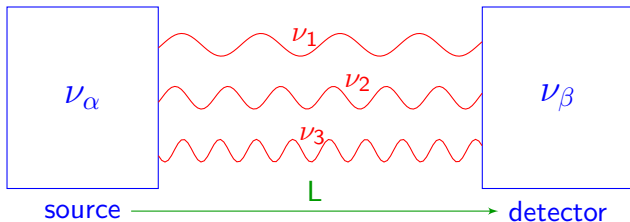
Two neutrino bases

flavor neutrinos ν_α

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1} |\nu_1\rangle + U_{\alpha 2} |\nu_2\rangle + U_{\alpha 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = |\nu_\beta\rangle = U_{\alpha 1} e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2} e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \longleftarrow \text{define} \longrightarrow t = L$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12} , θ_{13} , θ_{23} and one CP phase δ

Current knowledge of the 3 active ν mixing: [arxiv:2006.11237]

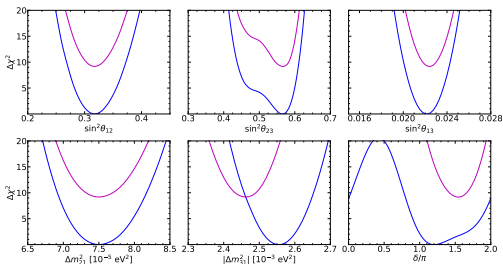
NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$

IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned} \Delta m_{21}^2 &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5} \text{ eV}^2 \\ |\Delta m_{31}^2| &= (2.56^{+0.03}_{-0.04}) \cdot 10^{-3} \text{ eV}^2 \text{ (NO)} \\ &= (2.46 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (IO)} \end{aligned}$$

$$\begin{aligned} 10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\ 10^2 \sin^2(\theta_{13}) &= 2.225^{+0.055}_{-0.078} \text{ (NO)} \\ &= 2.250^{+0.056}_{-0.076} \text{ (IO)} \\ 10 \sin^2(\theta_{23}) &= 5.66^{+0.16}_{-0.22} \text{ (NO)} \\ &= 5.66^{+0.18}_{-0.23} \text{ (IO)} \end{aligned}$$

$$\begin{aligned} \delta/\pi &= 1.20^{+0.23}_{-0.14} \text{ (NO)} \\ &= 1.54 \pm 0.13 \text{ (IO)} \end{aligned}$$



see also: <http://globalfit.astroparticles.es>

Normal ordering (NO)

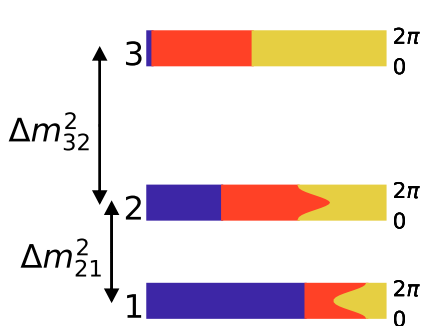
$$m_1 < m_2 < m_3$$

$$\sum m_k \gtrsim 0.06 \text{ eV}$$

 ν_e

 ν_μ

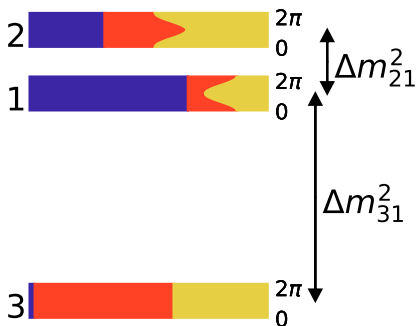
 ν_τ



Inverted ordering (IO)

$$m_3 < m_1 < m_2$$

$$\sum m_k \gtrsim 0.1 \text{ eV}$$



Absolute scale unknown!

Can we constrain the mass ordering using bounds on $\sum m_\nu$?

1 *Neutrino masses*

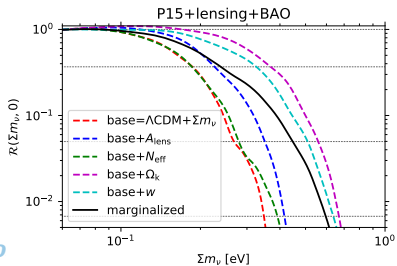
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4 *Neutrinoless double β decay*

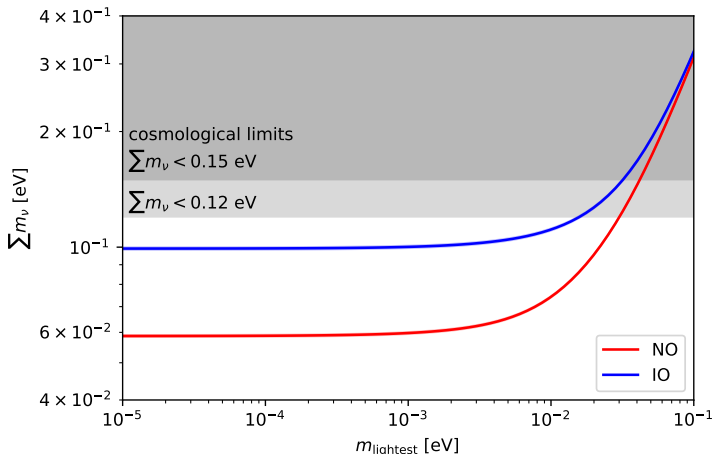
5 *Beyond the standard: light sterile neutrino*

6 *Conclusions*



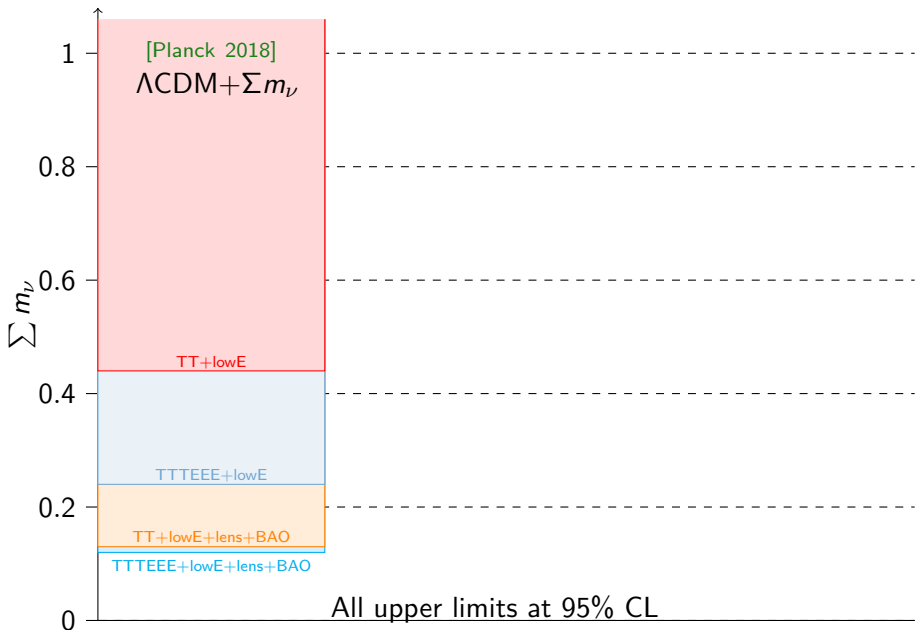
Warning: model dependent content!

How the limit change when considering extensions of the Λ CDM model?

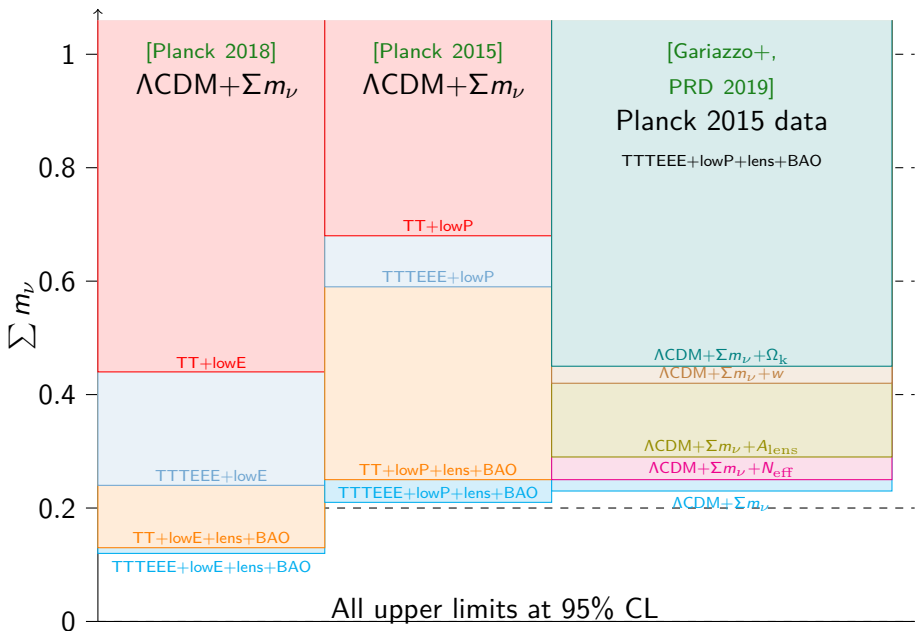


Warning: $\sum m_\nu \lesssim 0.1$ eV at 95% CL
does not mean IO disfavored at 95% CL!

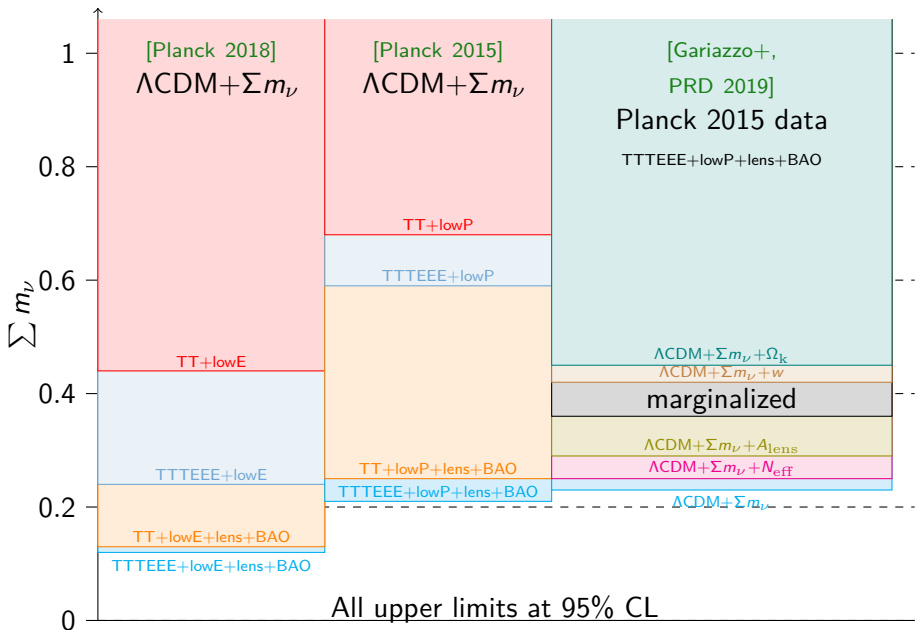
Cosmological neutrino mass bounds



Cosmological neutrino mass bounds



Cosmological neutrino mass bounds



Playing with priors

Bayes theorem:

$$p(\theta|d, \mathcal{M}) = \mathcal{L}(\theta) \frac{\pi(\theta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

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[Planck 2018]: prior

$$0 < \Sigma m_{\nu} < \mathcal{O}(1) \text{ eV}$$

strongest upper limit (95%):

$$\Sigma m_{\nu} < 113 \text{ meV}$$

(CMB+lens+BAO+SN)

corresponding to

$$\Sigma m_{\nu} < 53.6 \text{ meV (68\%)}$$

below minimum for NO!
does it make sense?

Playing with priors

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posterior depends on prior!

Different limits if you consider simply $\Sigma m_{\nu} > 0$ or you take into account oscillation results...

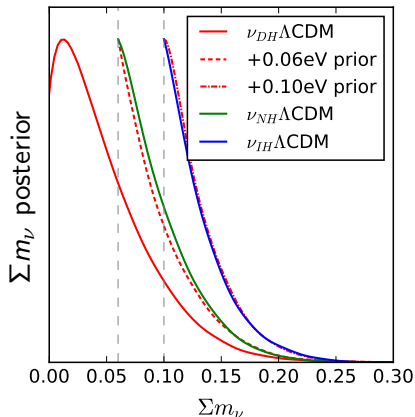
[Wang+, 2017]

degenerate (DH)

vs normal (NH)

vs inverted (IH) hierarchy

(i.e. change the prior lower bound)



An example with Planck 2018

*relative belief
updating ratio*

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

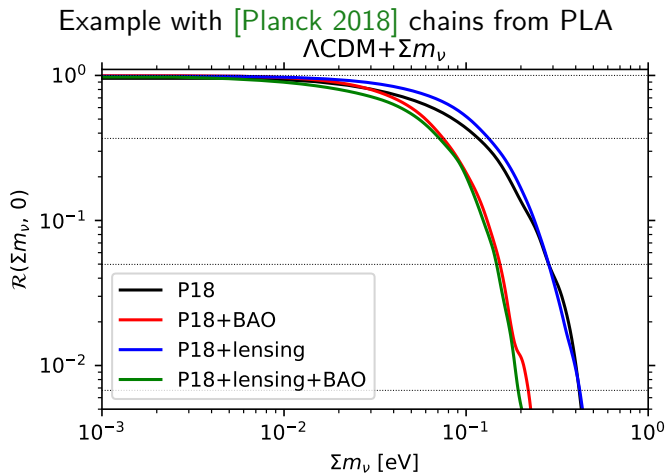
Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

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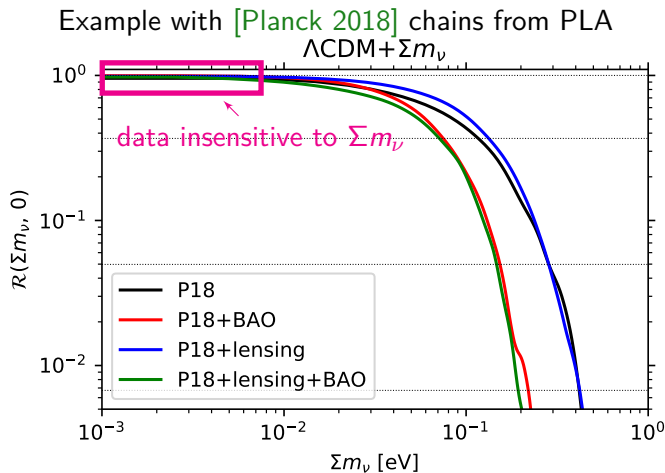


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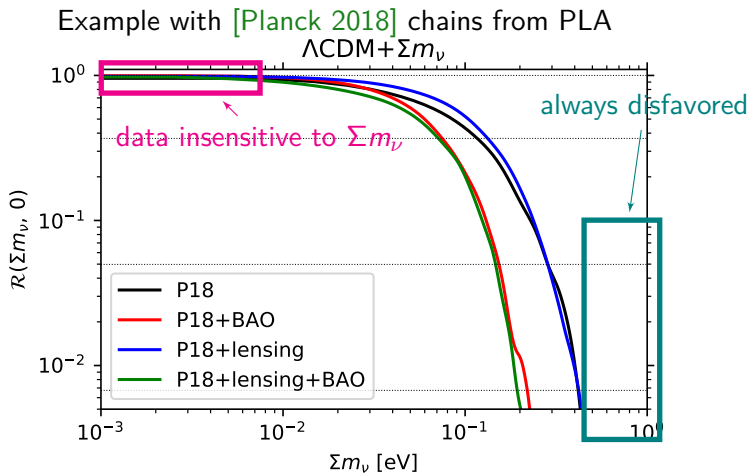


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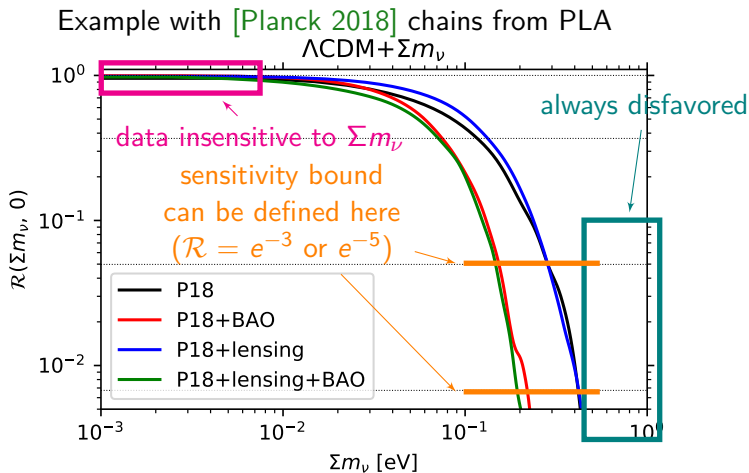


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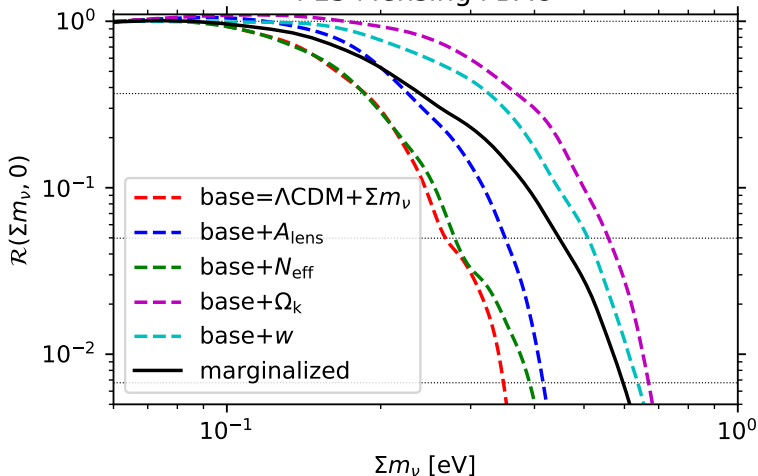


relative belief
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Example with [Planck 2015] chains from [SG+, PRD 99 (2019) 021301]

P15+lensing+BAO

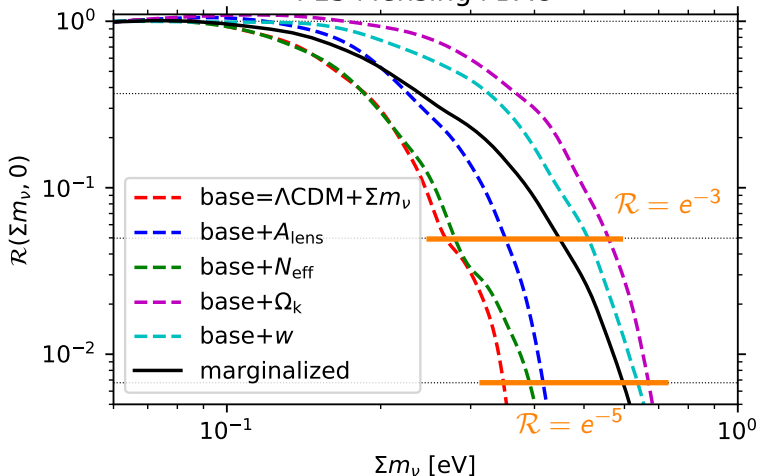


A model-marginalized example

relative belief
updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Example with [Planck 2015] chains from [SG+, PRD 99 (2019) 021301]
P15+lensing+BAO



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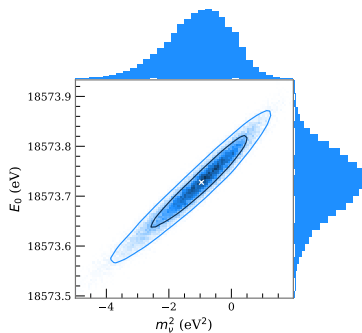
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β decay



$$Q_\beta = M_i - M_f - m_e$$

total available energy

$$E_\nu = Q_\beta - T = Q_\beta - (E_e - m_e)$$

neutrino energy

notice that max electron energy is:

$$T_{\max} = Q_\beta - m_{\bar{\nu}_e}$$

β decay



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Kurie function: (degenerate ν masses)

$$K(T) = \left[(Q_\beta - T) \sqrt{(Q_\beta - T)^2 - m_{\bar{\nu}_e}^2} \right]^{1/2}$$

Useful to describe
the e^- spectrum
near the endpoint

notice: flavor neutrinos have no definite mass!

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Useful to describe
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$$m_{\bar{\nu}_e}^2 = \sum |U_{ei}|^2 m_i^2$$

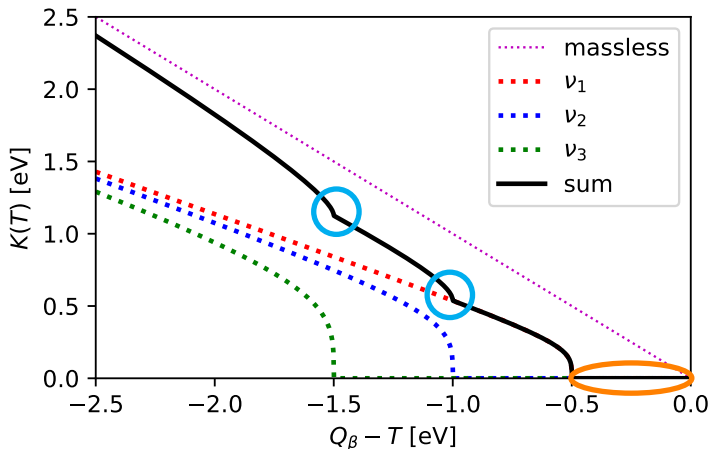
Full expression:

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$

N_ν neutrinos
with different
masses m_i

mixing angles
enter ($|U_{ei}|^2$)

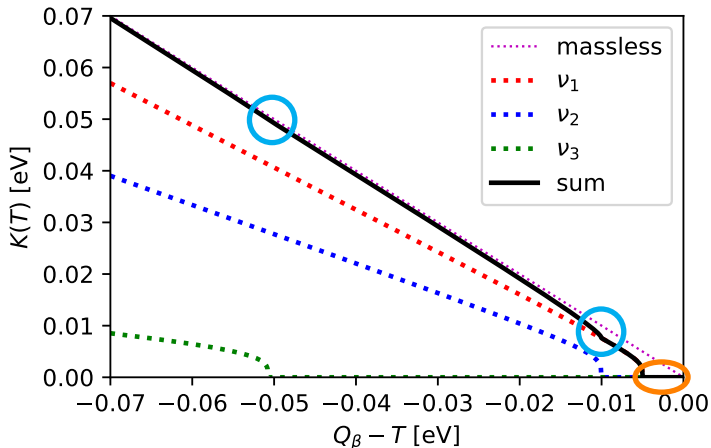
$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



Fake case:
3 neutrinos
masses:
 $m_i = i \cdot 0.5$ eV,
mixings:
 $|U_{ei}|^2 = 1/3$

endpoint shifted + one kink for each mass eigenstate

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



Realistic case:

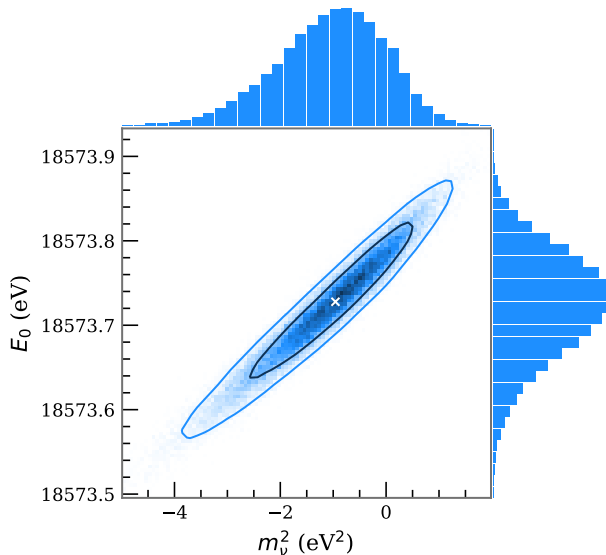
3 neutrinos,
normal
ordering

masses: $m_i =$
[5, 10, 51] meV,

mixings:
 $|U_{ei}|^2 =$
[0.67, 0.31, 0.02]

Much harder to see the endpoint shift and kinks!

strongest bound on $m_\nu (\equiv m_{\bar{\nu}_e})$ are from KATRIN



$$m_\nu^2 = -1.0_{-1.1}^{+0.9} \text{ eV}^2$$

Upper limit 90%:

$$m_\nu < 1.1 \text{ eV}$$

Feldman-Cousins 90%:

$$m_\nu < 0.8 \text{ eV}$$

statistics dominated!

expected final
sensitivity (90%):

$$m_\nu \lesssim 0.2 \text{ eV}$$

1 *Neutrino masses*

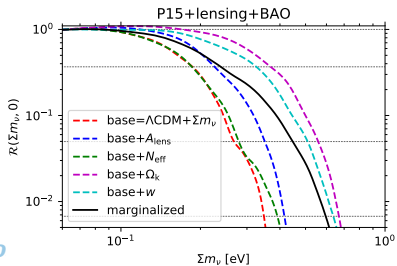
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Neutrino masses from neutrinoless double β decay

(if neutrino is Majorana)

[Schechter&Valle, 1982]

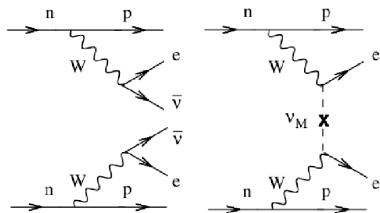
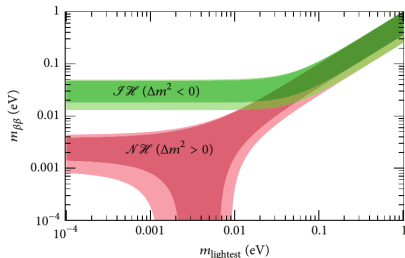
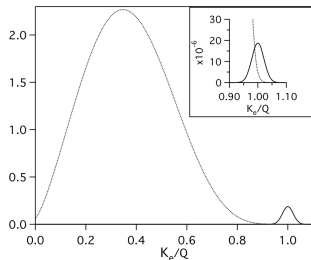


figure from [NEXT] webpage



[Dell'Oro et al., 2016]

Measure $T_{1/2}^{0\nu}$

m_e electron mass,
 $G_{0\nu}$ phase space,
 $\mathcal{M}'^{i\nu}$ matrix element

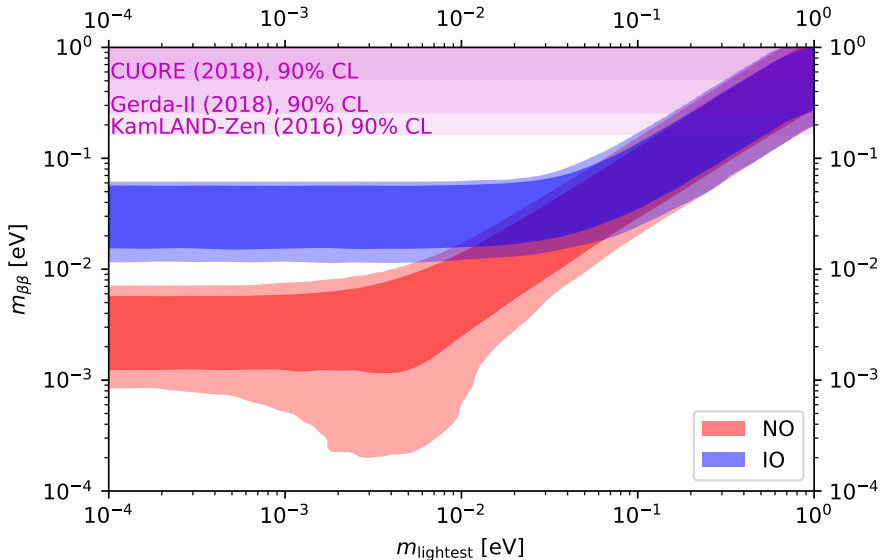
convert into
$$m_{\beta\beta} = \frac{m_e}{\mathcal{M}'^{i\nu} \sqrt{G_{0\nu} T_{1/2}^{0\nu}}}$$

and then use
$$m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} U_{ek}^2 m_k \right|$$

α_k Majorana phases

Constraints on $m_{\beta\beta}$

$$m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} U_{ek}^2 m_k \right|$$



1 *Neutrino masses*

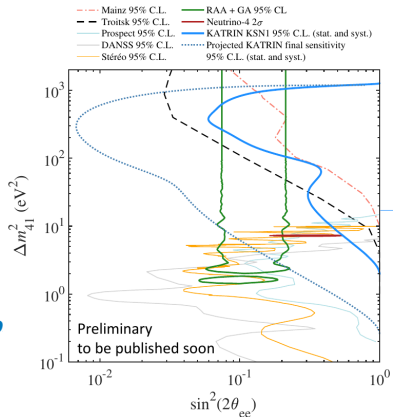
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A large family

In principle, previous discussion is valid for N neutrinos

only constraint: there are exactly three flavor neutrinos in the SM

[LEP, Phys. Rept. 427 (2006) 257,
arXiv:hep-ex/0509008]

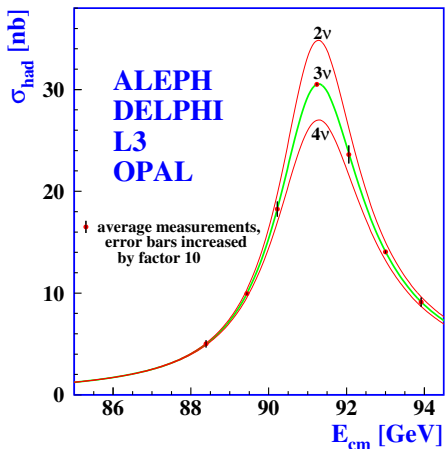
$$N_{\nu}^{(Z)} = 2.9840 \pm 0.0082$$

through the measurement
of the Z resonance

$$e^+ e^- \rightarrow Z \rightarrow \sum_{a=e,\mu,\tau} \nu_a \bar{\nu}_a$$

neutrinos $\alpha > 3$ must be sterile

sterile neutrino = SM singlet: no couplings with other SM particles



A large family

In principle, previous discussion is valid for N neutrinos

$N \times N$ mixing matrix, N flavor neutrinos, N massive neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_{s1}\rangle \\ \dots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \vdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \\ U_{s11} & U_{s12} & U_{s13} & U_{s14} & \\ \dots & & & & \ddots \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ \dots \end{pmatrix}$$

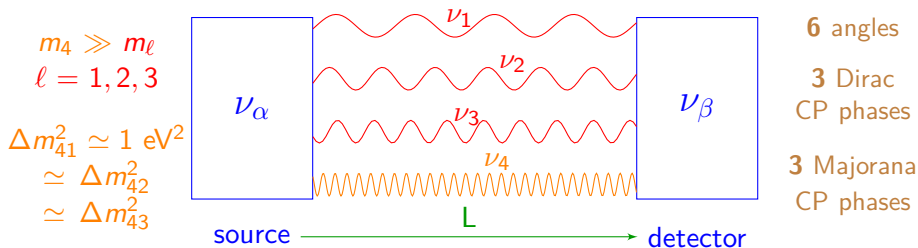
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Our case will be 3 (active)+1 (sterile), a perturbation of 3 neutrinos case



3+1 Neutrino Model

new $\Delta m_{\text{SBL}}^2 \Rightarrow 4$ neutrinos!

\downarrow
 ν_4 with $m_4 \simeq 1$ eV,
no weak interactions

\downarrow
light sterile neutrino (LS ν)

3 (active) + 1 (sterile) mixing:

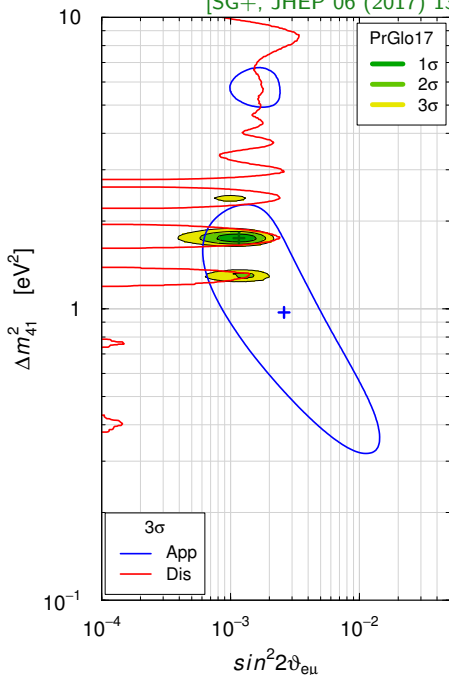
$$\nu_\alpha = \sum_{k=1}^{3+1} U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau, s)$$

ν_s is mainly ν_4 :

$$m_s \simeq m_4 \simeq \sqrt{\Delta m_{41}^2} \simeq \sqrt{\Delta m_{\text{SBL}}^2}$$

assuming $m_4 \gg m_i$ ($i = 1, 2, 3$)

[SG+, JHEP 06 (2017) 135]



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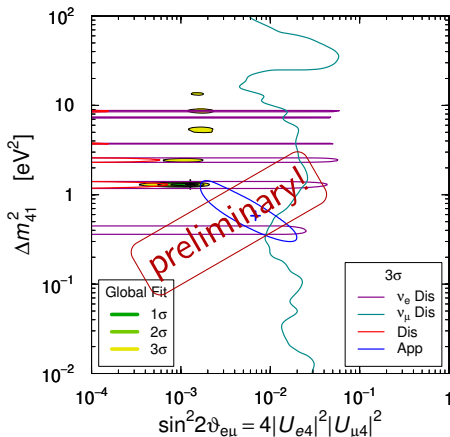
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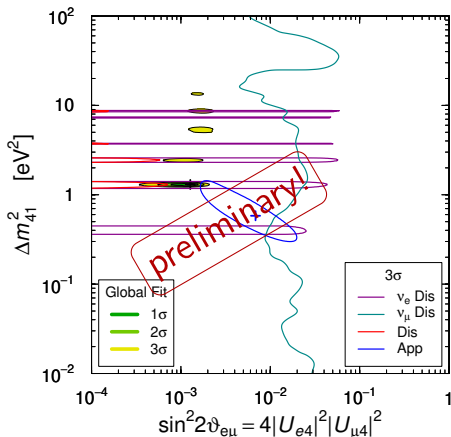
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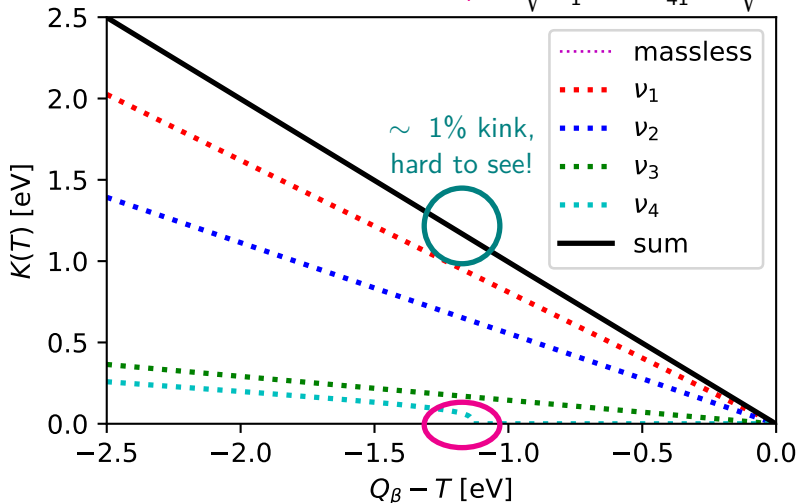
constraints from
mass measurements?

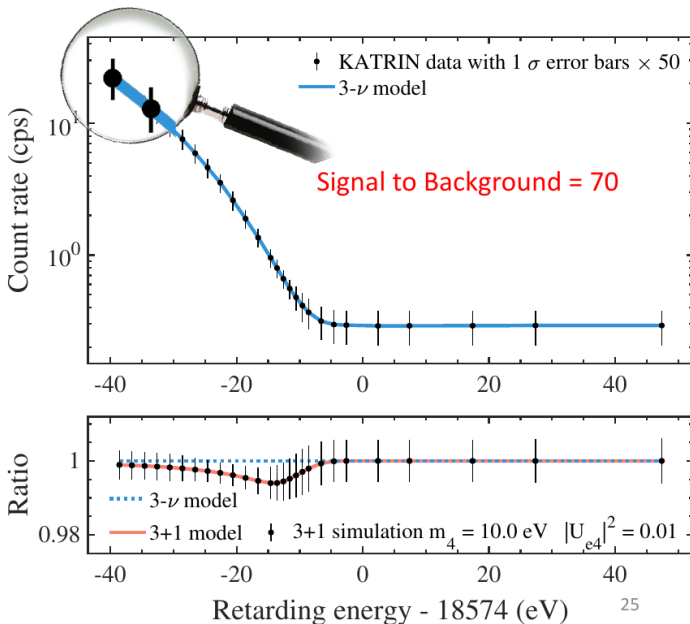
Sterile neutrino in β decay

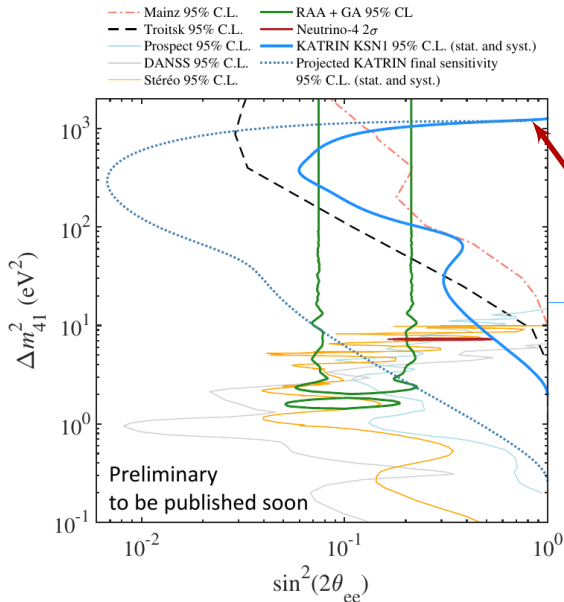
$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$

$$|U_{e4}|^2 \sim 0.01$$

$$m_4 = \sqrt{m_1^2 + \Delta m_{41}^2} \sim \sqrt{\Delta m_{41}^2}$$







final sensitivity will test
several oscillation results!

search for keV states
needs to measure the
spectrum much further
from the endpoint...

1 *Neutrino masses*

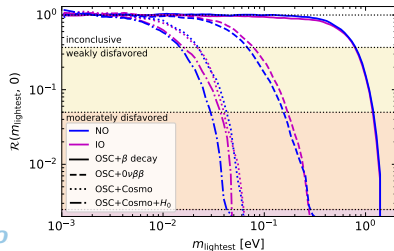
2 *Cosmological bounds*

3 *β decay*

4 *Neutrinoless double β decay*

5 *Beyond the standard: light sterile neutrino*

6 *Conclusions*



Mass ordering in 2020

Bayes theorem for models:

$$p(\mathcal{M}|d) \propto Z_{\mathcal{M}}\pi(\mathcal{M})$$

Bayesian evidence:

$$Z_{\mathcal{M}} = \int_{\Omega_{\mathcal{M}}} \mathcal{L}(\theta) \pi(\theta) d\theta$$

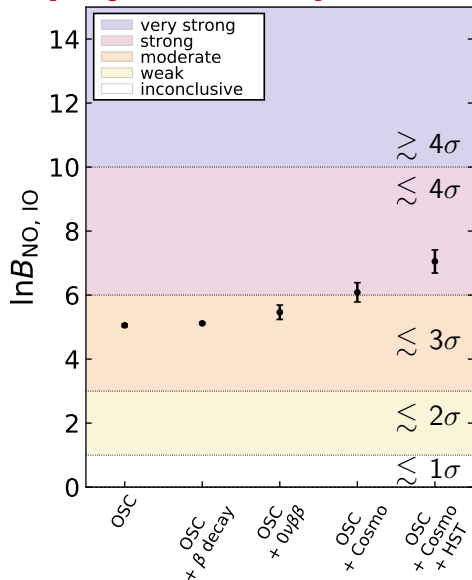
Bayes factor NO vs IO:

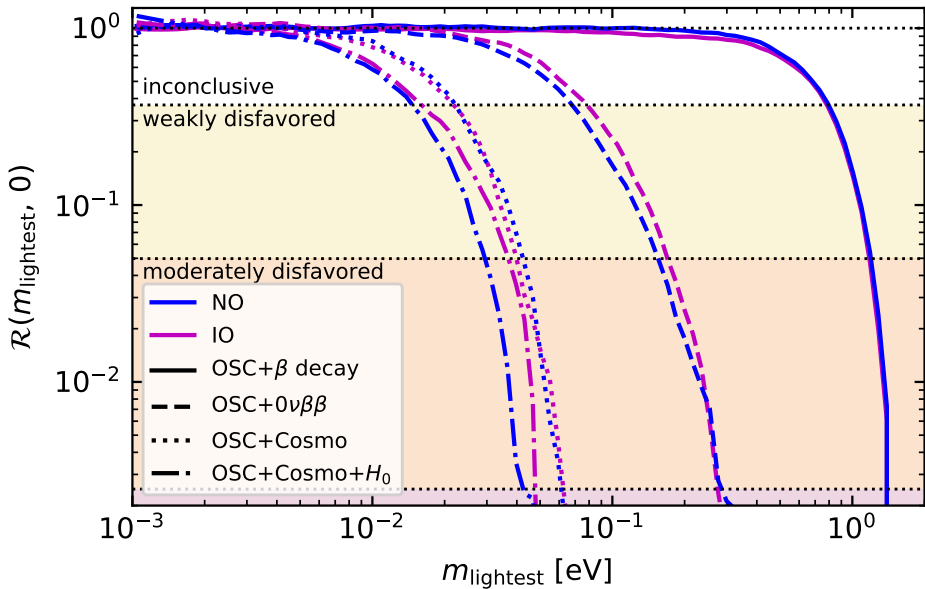
$$B_{\text{NO,IO}} = Z_{\text{NO}}/Z_{\text{IO}}$$

Posterior probability:

$$P_{\text{NO}} = B_{\text{NO,IO}} / (B_{\text{NO,IO}} + 1)$$
$$P_{\text{IO}} = 1 / (B_{\text{NO,IO}} + 1)$$

$$N\sigma \text{ from } P_{\text{NO}} = \text{erf}(N/\sqrt{2})$$

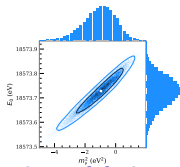
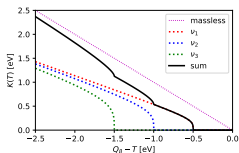




What do we learn on neutrino masses?

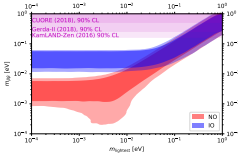
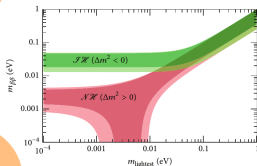
β

β decay



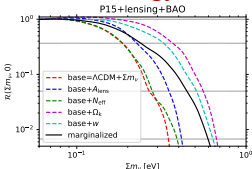
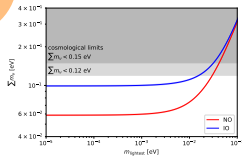
$\beta\beta$

Majorana only: neutrinoless $\beta\beta$ decay



Cosmo

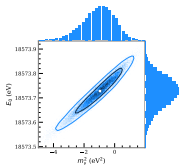
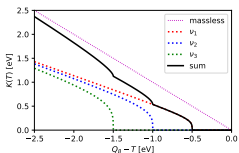
Model dependent: cosmology



What do we learn on neutrino masses?

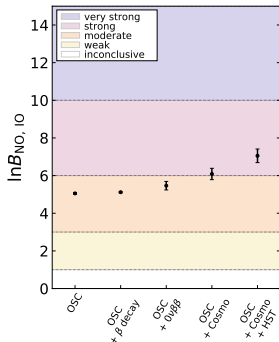
β

β decay



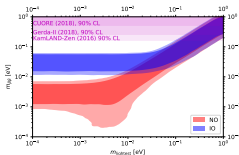
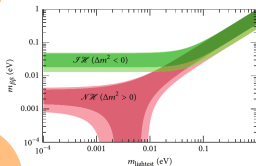
+

global fit



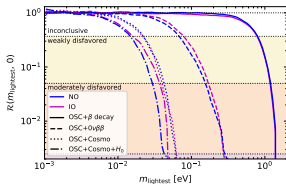
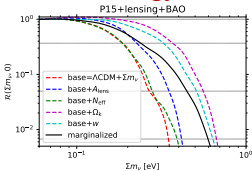
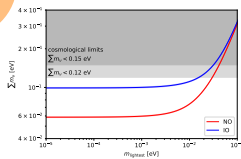
$\beta\beta$

Majorana only: neutrinoless $\beta\beta$ decay



Cosmo

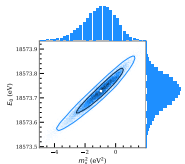
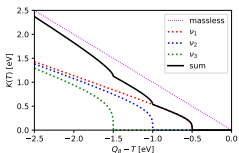
Model dependent: cosmology



What do we learn on neutrino masses?

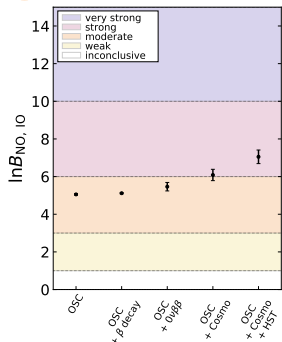
β

β decay



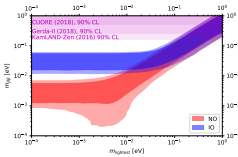
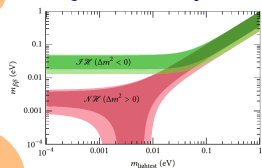
+

global fit



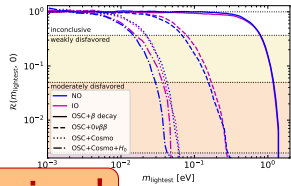
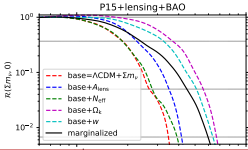
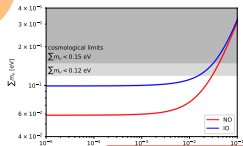
$\beta\beta$

Majorana only: neutrinoless $\beta\beta$ decay



Cosmo

Model dependent: cosmology



Thanks for the attention!