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Stefano Gariazzo

INFN, Turin section
Turin (IT)

Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

gariazzo@to.infn.it
http://personalpages.to.infn.it/~gariazzo/

Neutrino oscillations in the early universe with three or four neutrinos: precision calculations

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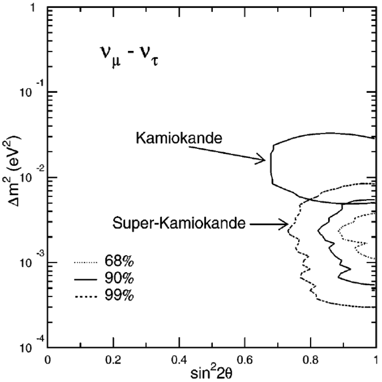
Neutrino oscillations

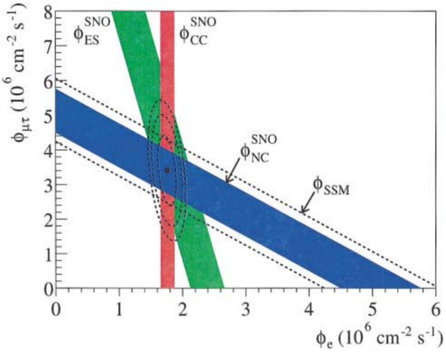
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \to \nu_\tau$ oscillations from atmospheric ν

[SNO, 2001-2002]

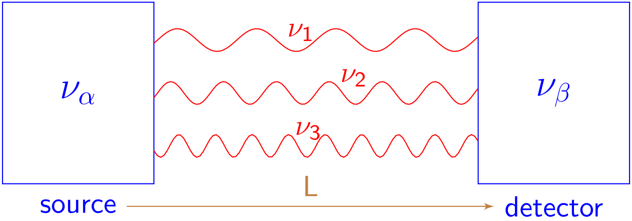


first discovery of $\nu_e \to \nu_\mu, \nu_\tau$ oscillations from solar ν

Nobel prize in 2015

Two neutrino bases

interaction	U mixing matrix	propagation
flavor neutrinos ν_α	$\vert\nu_\alpha\rangle = \sum_k U_{\alpha k}\vert\nu_k\rangle$	massive neutrinos ν_k



Transition probability between source and detector:

$$P_{\nu_\alpha\to\nu_\beta}(L) = |\langle\nu_\alpha|\nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k}U^*_{\alpha k}U^*_{\beta j}U_{\alpha j}\exp\left(-i\frac{\Delta m^2_{kj}L}{2E}\right)$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12}, θ_{13}, θ_{23} and one CP phase δ

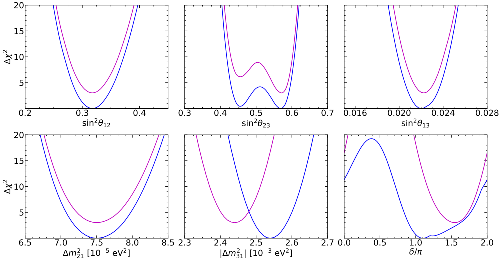
Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$ IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned}
\Delta m^2_{21} &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2 \\
|\Delta m^2_{31}| &= (2.54 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{NO}) \\
&= (2.44 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{IO})
\end{aligned}$$

$$\begin{aligned}
10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\
10^2 \sin^2(\theta_{13}) &= 2.200^{+0.069}_{-0.062}\ (\mathrm{NO}) \\
&= 2.225^{+0.064}_{-0.070}\ (\mathrm{IO}) \\
10 \sin^2(\theta_{23}) &= 4.55 \pm 0.13 \cup 5.71 \pm 0.12\ (\mathrm{NO}) \\
&= 5.71^{+0.14}_{-0.17}\ (\mathrm{IO})
\end{aligned}$$

$$\begin{aligned}
\delta/\pi &= 1.10^{+0.27}_{-0.12}\ (\mathrm{NO}) \\
&= 1.54 \pm 0.14\ (\mathrm{IO})
\end{aligned}$$



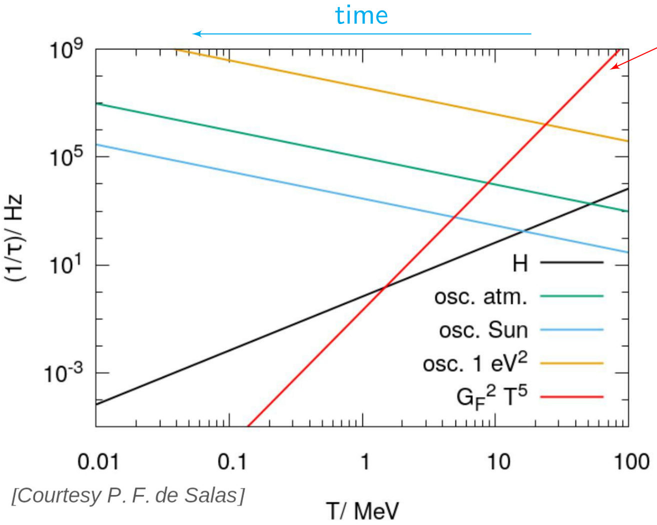
mass ordering still unknown

δ still unknown

see also: `http://globalfit.astroparticles.es`

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

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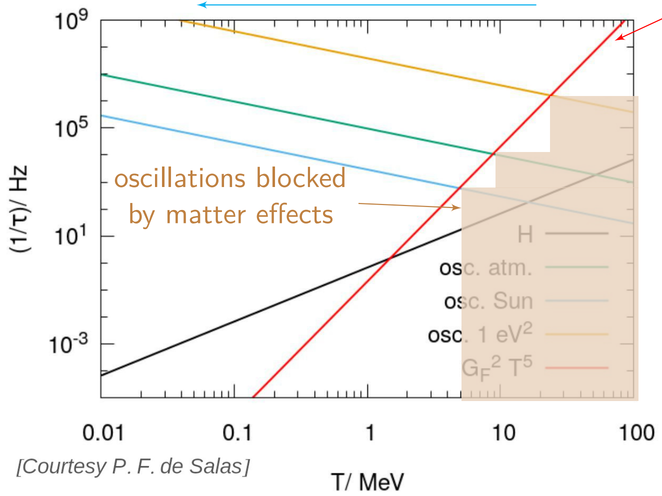
time



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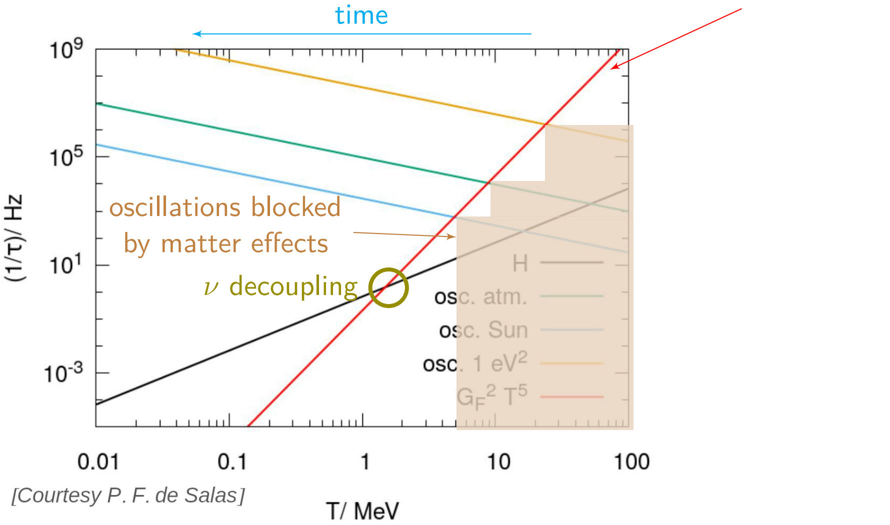

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ν decouple mostly before $e^+e^- \to \gamma\gamma$ annihilation!

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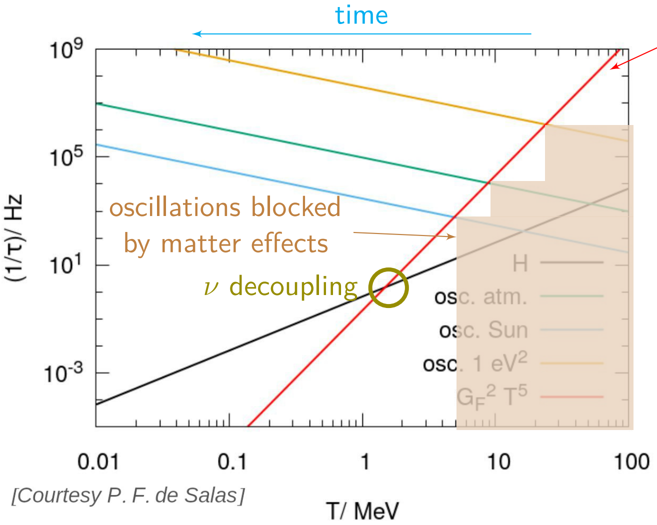

$T_\nu \simeq (4/11)^{1/3} T_\gamma$ after $e^+e^- \to \gamma\gamma$

f_ν: frozen Fermi-Dirac distribution

Today:

$T_{\nu,0} = 1.945$ K $\simeq 1.676 \times 10^{-4}$ eV

$\langle E_\nu \rangle \simeq 3.1\, T_{\nu,0} \simeq 5 \times 10^{-4}$ eV

$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56$ cm^{-3} per family

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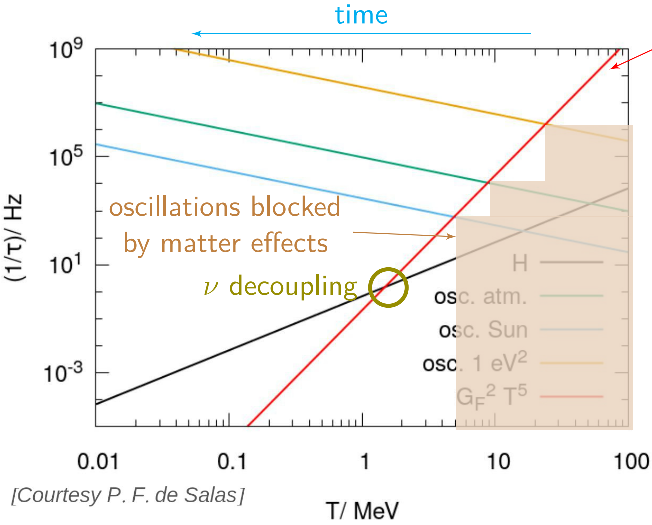

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actually, the decoupling T is momentum dependent!

distortions to equilibrium f_ν!

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$

$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\rm Pl}^2}{8\pi\rho_{\rm T}}}\left\{-i\,\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\rm F}}{2y} - \frac{2\sqrt{2}G_{\rm F}\,y}{x^6/m_e^6}\left(\frac{\mathbb{E}_\ell+\mathbb{P}_\ell}{m_W^2}+\frac{4\mathbb{E}_\nu}{3m_Z^2}\right),\varrho\right] + \frac{m_e^3 G_{\rm F}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho)\right\}$$

$m_{\rm Pl}$ Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – $G_{\rm F}$ Fermi constant – $[.,.]$ commutator

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m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

$$\mathbb{M}_{\mathrm{F}} = U\mathbb{M}U^\dagger$$

$$\mathbb{M} = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

$$U = R^{23}R^{13}R^{12}$$

e.g. $R^{13} = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$

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$\mathbb{E}_\ell = \mathrm{diag}(\rho_e, \rho_\mu, 0) \qquad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho \right) S_a \quad$ with $S_a = \mathrm{diag}(1,1,1)$

lepton densities — neutrino densities — (only for active neutrinos)

take into account matter effects in oscillations

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino-electron scattering and pair annihilation, plus neutrino–neutrino interactions

2D integrals over momentum, take most of the computation time

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from continuity equation

$\dot\rho = -3H(\rho + P)$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\sum_{\ell=e,\mu}\left[\frac{r_\ell^2}{r}J(r_\ell)\right] + G_1(r) - \frac{1}{2\pi^2 z^3}\int_0^\infty dy\, y^3 \sum_{\alpha=e}^{\tau}\frac{\mathrm{d}\varrho_{\alpha\alpha}}{\mathrm{d}x}}{\sum_{\ell=e,\mu}\left[r_\ell^2 J(r_\ell) + Y(r_\ell)\right] + G_2(r) + \frac{2\pi^2}{15}}$$

$r = x/z$, $r_\ell = m_\ell/m_e\, r$ $\qquad J(r)$, $Y(r)$ from non-relativistic transition of $e^\pm$, $\mu^\pm$

$G_1(r)$ and $G_2(r)$ from electromagnetic corrections

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neutrino temperature w: same equation as z, but electrons always relativistic

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initial conditions: $\varrho_{\alpha\alpha}$ = Fermi-Dirac at $x_{\mathrm{in}} \simeq 0.001$, with $w = z \simeq 1$

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FORTran-Evolved PrimordIAl Neutrino Oscillations (FortEPiaNO)

https://bitbucket.org/ahep_cosmo/fortepiano_public

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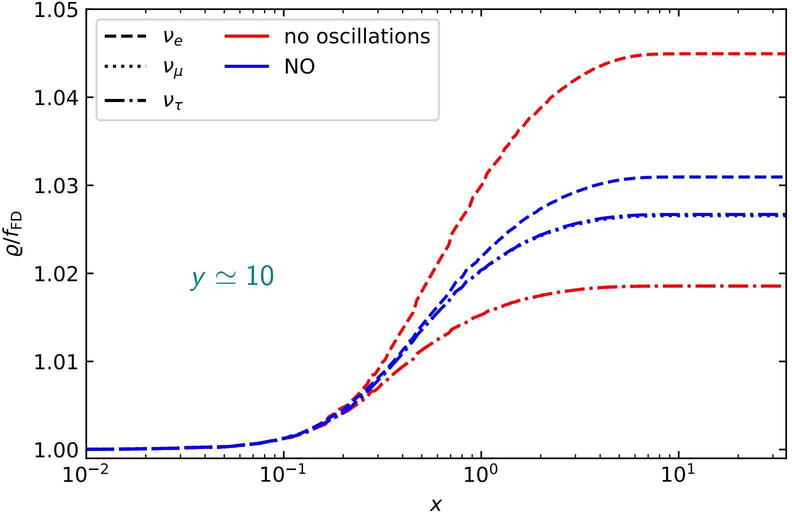
Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, JCAP 2021]

Distortion of the momentum distribution ($f_{\rm FD}$: Fermi-Dirac at equilibrium)



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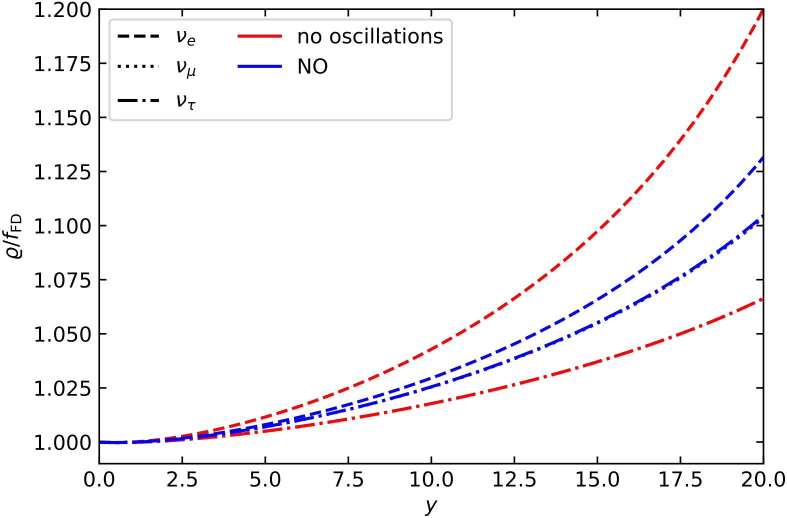

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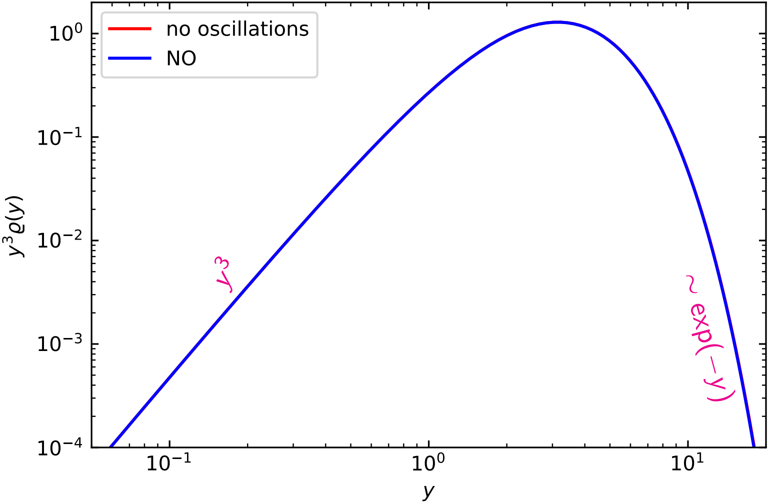
$$N_{\rm eff}^{\rm final} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{1}{\rho_\gamma}\sum_i g_i \int \frac{d^3p}{(2\pi)^3} E(p)\, f_{\nu,i}(p)$$

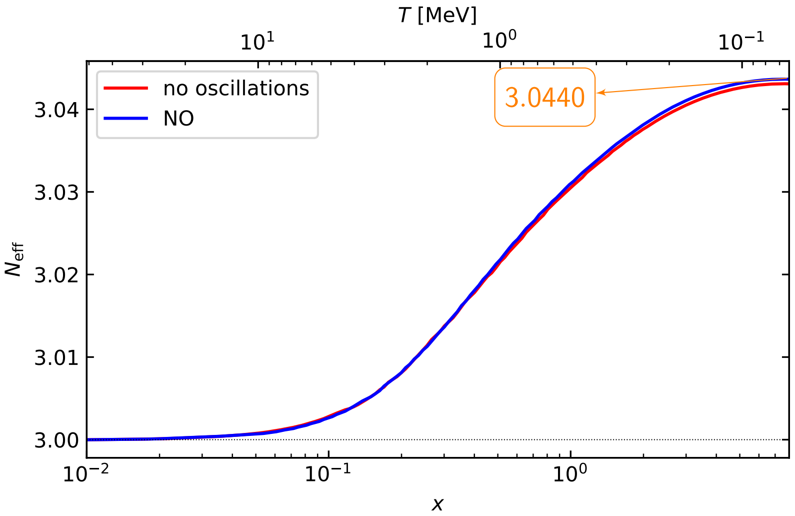
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\rm fin}$

$\hookrightarrow \propto y^3 \varrho_{ii}(y)$



Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, JCAP 2021]

$$N_{\rm eff}^{\rm any\ time} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{1}{\rho_\gamma}\sum_i g_i \int \frac{d^3p}{(2\pi)^3} E(p)\, f_{\nu,i}(p)$$



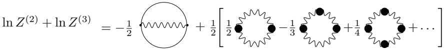
Finite temperature QED

[Bennett+, JCAP 2020]

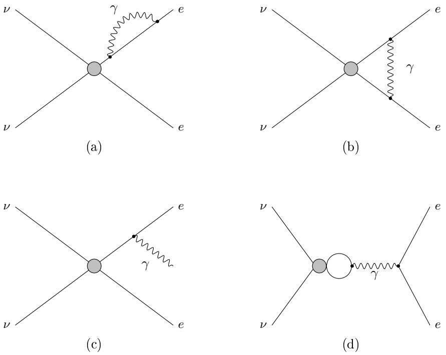
ν decoupling strongly depends on interactions occurring at $T \gtrsim 1$ MeV

→ finite temperature effects! ←

altered equation of state of QED plasma



corrections to weak rates



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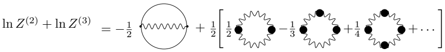
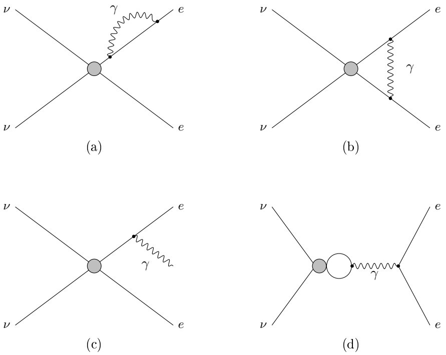


corrections to weak rates



Leading contribution $\mathcal{O}(e^2)$
gives $\delta N_{\rm eff} \sim 0.01$!
[Fornengo+, 1997]

Finite temperature QED

[Bennett+, JCAP 2020]

ν decoupling strongly depends on interactions occurring at $T \gtrsim 1$ MeV

finite temperature effects!

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)
Finite-temperature QED corrections		
$(2)\not{\ln}$	3.04361	3.04458
$(2)\not{\ln} + (2)\ln$	3.04358	3.04452
$(2)\not{\ln} + (3)$	3.04264	3.04361
$(2)\not{\ln} + (2)\ln + (3)$	3.04263	3.04360

[Bennett, SG+, 2020]

$\mathcal{O}(e^2) \sim 0.01$ and $\mathcal{O}(e^3) \sim -0.001$ are important!

Logarithmic term and following orders affect less than numerical parameters for configuring the y_i grid

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ and $4\overset{(-)}{\nu}$ interactions

first calculations by [Sigl&Raffelt, 1993]

computationally expensive

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ and $4\overset{(-)}{\nu}$ interactions

annihilation: $\nu(p_1) + \bar{\nu}(p_2) \leftrightarrow e^-(p_3) + e^+(p_4)$ gives:
[de Salas+, JCAP 2016]

$$
\begin{aligned}
\mathcal{I}_{\nu\bar{\nu}\to e^-e^+} = & \frac{1}{2}\frac{2^5 G_{\rm F}^2}{2|\vec{p}_1|}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2|\vec{p}_2|}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\\
& \times \Big\{4(p_1\cdot p_4)(p_2\cdot p_3)F_{\rm ann}^{LL}(\nu^{(1)},\bar{\nu}^{(2)},e^{(3)},\bar{e}^{(4)})\\
& + 4(p_1\cdot p_3)(p_2\cdot p_4)F_{\rm ann}^{RR}(\nu^{(1)},\bar{\nu}^{(2)},e^{(3)},\bar{e}^{(4)})\\
& + 2(p_1\cdot p_2)m_e^2\left(F_{\rm ann}^{RL}(\nu^{(1)},\bar{\nu}^{(2)},e^{(3)},\bar{e}^{(4)}) + F_{\rm ann}^{LR}(\nu^{(1)},\bar{\nu}^{(2)},e^{(3)},\bar{e}^{(4)})\right)\Big\}\,,
\end{aligned}
$$

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ and $4\overset{(-)}{\nu}$ interactions

scattering: $\nu(p_1) + e^\pm(p_2) \leftrightarrow \nu(p_3) + e^\pm(p_4)$ gives:
[de Salas+, JCAP 2016]

$$\mathcal{I}_{\nu e^- \to \nu e^-} = \frac{1}{2}\frac{2^5 G_F^2}{2|\vec{p}_1|}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2|\vec{p}_3|}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$
$$\times \Big\{4(p_1\cdot p_4)(p_2\cdot p_3)F_{\rm sc}^{RR}(\nu^{(1)},e^{(2)},\nu^{(3)},e^{(4)})$$
$$+4(p_1\cdot p_2)(p_3\cdot p_4)F_{\rm sc}^{LL}(\nu^{(1)},e^{(2)},\nu^{(3)},e^{(4)})$$
$$-2(p_1\cdot p_3)m_e^2\left(F_{\rm sc}^{RL}(\nu^{(1)},e^{(2)},\nu^{(3)},e^{(4)})+F_{\rm sc}^{LR}(\nu^{(1)},e^{(2)},\nu^{(3)},e^{(4)})\right)\Big\},$$

$$\mathcal{I}_{\nu e^+ \to \nu e^+} = \frac{1}{2}\frac{2^5 G_F^2}{2|\vec{p}_1|}\int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2|\vec{p}_3|}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$
$$\times \Big\{4(p_1\cdot p_4)(p_2\cdot p_3)F_{\rm sc}^{LL}(\nu^{(1)},\bar{e}^{(2)},\nu^{(3)},\bar{e}^{(4)})$$
$$+4(p_1\cdot p_2)(p_3\cdot p_4)F_{\rm sc}^{RR}(\nu^{(1)},\bar{e}^{(2)},\nu^{(3)},\bar{e}^{(4)})$$
$$-2(p_1\cdot p_3)m_e^2\left(F_{\rm sc}^{RL}(\nu^{(1)},\bar{e}^{(2)},\nu^{(3)},\bar{e}^{(4)})+F_{\rm sc}^{LR}(\nu^{(1)},\bar{e}^{(2)},\nu^{(3)},\bar{e}^{(4)})\right)\Big\}.$$

And so on for neutrino–neutrino terms

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$ and $4 \overset{(-)}{\nu}$ interactions

switch to comoving coordinates, you get:

$$\begin{aligned}
\mathcal{I}[\varrho(y)] &= \frac{G_F^2}{(2\pi)^3 y^2} \left(\mathcal{I}^u_{\rm sc} + \mathcal{I}^u_{\rm ann} + \mathcal{I}^u_{\nu\nu} + \mathcal{I}^u_{\nu\bar{\nu}}\right)\\
\mathcal{I}^u_{\rm sc} &= \int {\rm d}y_2 {\rm d}y_3 \frac{y_2}{E_2} \left\{\left(\Pi^s_2(y, y_4) + \Pi^s_2(y, y_2)\right) \left[F^{LL}_{\rm sc}(\ldots) + F^{RR}_{\rm sc}(\ldots)\right] - 2x^2 \Pi^s_1 \left[F^{RL}_{\rm sc}(\ldots) + F^{LR}_{\rm sc}(\ldots)\right]\right\}\\
\mathcal{I}^u_{\rm ann} &= \int {\rm d}y_2 {\rm d}y_3 \frac{y_3}{E_3} \left\{\Pi^a_2(y, y_4) F^{LL}_{\rm ann}(\ldots) + \Pi^a_2(y, y_3) F^{RR}_{\rm ann}(\ldots) + x^2 \Pi^a_1 \left[F^{RL}_{\rm ann}(\ldots) + F^{LR}_{\rm ann}(\ldots)\right]\right\}\\
\mathcal{I}^u_{\nu\nu} &= \frac{1}{4} \int {\rm d}y_2 {\rm d}y_3\, \Pi^\nu_2(y, y_2) F_{\nu\nu}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right)\\
\mathcal{I}^u_{\nu\bar{\nu}} &= \frac{1}{4} \int {\rm d}y_2 {\rm d}y_3\, \Pi^\nu_2(y, y_4) F_{\nu\bar{\nu}}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right)
\end{aligned}$$

Π functions are combinations of (y, y_2, y_3, y_4) that emerge from $\int {\rm d}^3\vec{p}$

See literature for their expressions

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ and $4\overset{(-)}{\nu}$ interactions

F functions encode phase space distributions:

$$
\begin{aligned}
F^{ab}_{\rm sc}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right) &= f_e^{(4)}(1-f_e^{(2)})\left[G^a\Phi_1^{3,b,1}+\Phi_2^{1,b,3}G^a\right] - f_e^{(2)}(1-f_e^{(4)})\left[\Phi_1^{1,b,3}G^a+G^a\Phi_2^{3,b,1}\right]\\
F^{ab}_{\rm ann}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right) &= f_e^{(3)}f_e^{(4)}\left[G^a\Phi_4^{2,b,1}+\Phi_4^{1,b,2}G^a\right] - (1-f_e^{(3)})(1-f_e^{(4)})\left[G^a\Phi_3^{2,b,1}+\Phi_3^{1,b,2}G^a\right]\\
F_{\nu\nu}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right) &= \Phi_2^{1,S,3}G_S\left[\Phi_2^{2,S,4}G_S+{\rm Tr}(\cdots)\right] - \Phi_1^{1,S,3}G_S\left[\Phi_1^{2,S,4}G_S+{\rm Tr}(\cdots)\right] + {\rm h.c.}\\
F_{\nu\bar{\nu}}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right) &= \Phi_4^{1,S,2}G_S\left[\Phi_3^{4,S,3}G_S+{\rm Tr}(\cdots)\right] - \Phi_3^{1,S,2}G_S\left[\Phi_4^{4,S,3}G_S+{\rm Tr}(\cdots)\right]\\
&+ \Phi_2^{1,S,3}G_S\left[\Phi_1^{4,S,2}G_S+{\rm Tr}(\cdots)\right] - \Phi_1^{1,S,3}G_S\left[\Phi_2^{4,S,2}G_S+{\rm Tr}(\cdots)\right] + {\rm h.c.}
\end{aligned}
$$

$\varrho^{(i)} = \varrho(y_i)$ – $f_e^{(i)} = f_{\rm FD}(y_i, z)$ – ${\rm Tr}(\cdots)$ is the trace of the term immediately before it

Convenient definitions:

$$
\begin{aligned}
\Phi_1^{\alpha,i,\beta} &= \varrho^{(\alpha)}G^i(1-\varrho^{(\beta)})\\
\Phi_2^{\alpha,i,\beta} &= (1-\varrho^{(\alpha)})G^i\varrho^{(\beta)}\\
\Phi_3^{\alpha,i,\beta} &= \varrho^{(\alpha)}G^i\varrho^{(\beta)}\\
\Phi_4^{\alpha,i,\beta} &= (1-\varrho^{(\alpha)})G^i(1-\varrho^{(\beta)})
\end{aligned}
$$

Interaction strenghts ($a, b \in [L, R]$):

$$
\begin{aligned}
G^R &= {\rm diag}(g_R, g_R, g_R)\\
G^L &= {\rm diag}(g_L, \tilde{g}_L, \tilde{g}_L)\\
G^S &= {\rm diag}(1, 1, 1)
\end{aligned}
$$

$$
\begin{aligned}
g_R &= \sin^2\theta_W\\
g_L &= \sin^2\theta_W + 1/2\\
\tilde{g}_L &= \sin^2\theta_W - 1/2
\end{aligned}
$$

θ_W weak mixing angle

Collision terms

[Bennett, SG+, JCAP 2021]

Encode the effect of $\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ and $4\overset{(-)}{\nu}$ interactions

Sometimes one can avoid integrals: damping approximations!

$$\mathcal{I}^u_{\alpha\beta}(\varrho) \quad = \quad -D^u_{\alpha\beta}\varrho_{\alpha\beta}$$

i.e. the collision term is proportional to the density matrix

$$\begin{aligned}
\{D^u(y)\}_{\alpha\beta} &= \frac{1}{2}\left[\{R^u(y)\}_\alpha + \{R^u(y)\}_\beta\right] \\
\{R^u_{\nu\nu}(y)\}_\alpha &= 2\int \mathrm{d}y_2\mathrm{d}y_3\ [\Pi^\nu_2(y,y_2) + 2\Pi^\nu_2(y,y_4)] \times \Big([1-f_2]f_3f_4 + f_2[1-f_3][1-f_4]\Big) \\
&\equiv \mathcal{D}^u(y,z) \\
\{R^u_{\nu e}(y)\}_\alpha &= \frac{1}{4}\left[(2\sin^2\theta_W \pm 1)^2_\alpha + 4\sin^4\theta_W\right]\mathcal{D}^u(y,z)
\end{aligned}$$

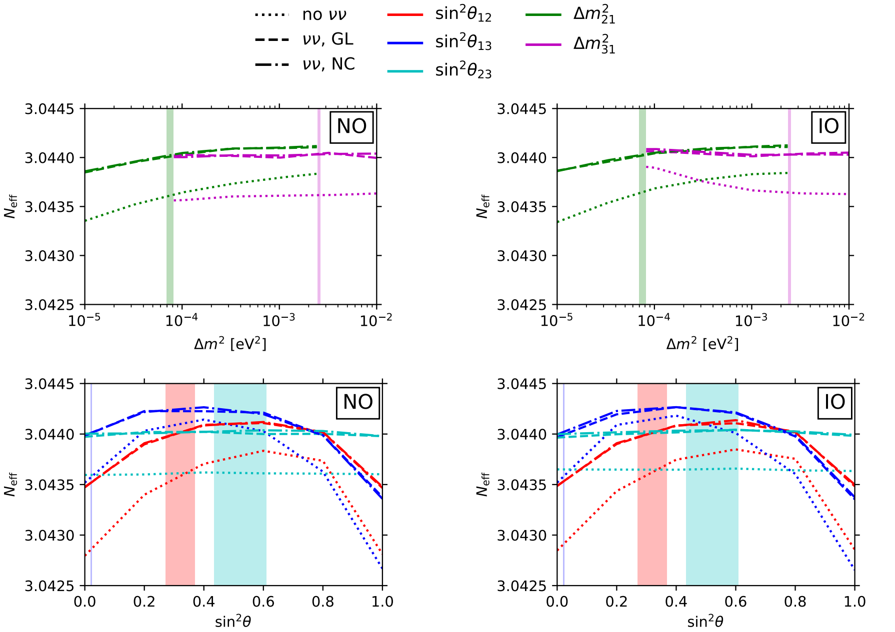
"$+$" for $\alpha = e$ and "$-$" for $\alpha = \mu,\tau$

$f_i \equiv f_{\rm eq}(y_i)$

For relativistic Fermi–Dirac distributions: $\mathcal{D}^u(y,z) = 2y^3z^4 d(y/z)$

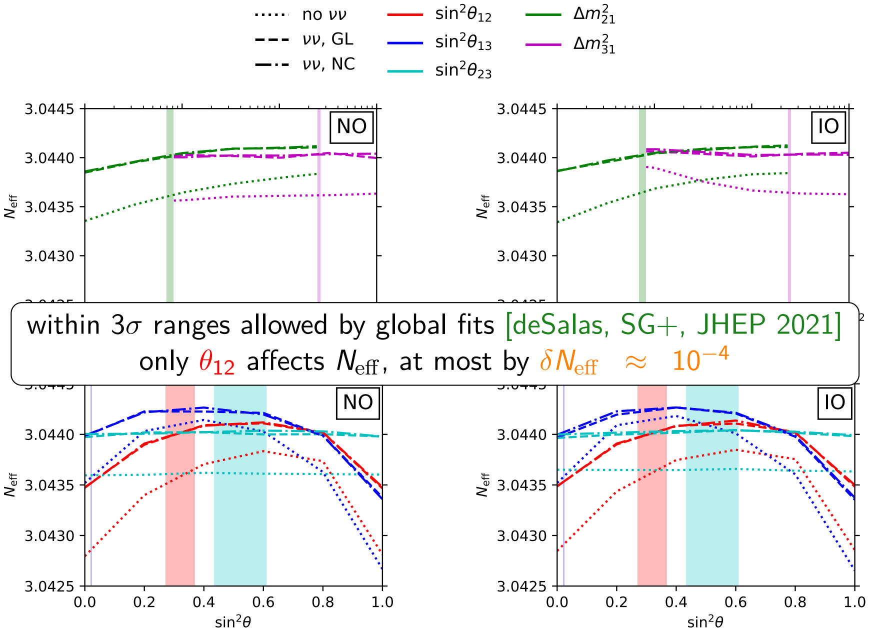
$$d(s) \approx d_0e^{-1.01s} + d_\infty(1-e^{-0.01s}) + (e^{-0.01s} - e^{-1.01s})\left[\frac{a_0 + a_1\ln(s) + a_2\ln^2(s)}{1 + b_1\ln(s) + b_2\ln^2(s)}\right]$$

Effect of neutrino oscillations



Effect of neutrino oscillations

[Bennett, SG+, JCAP 2021]



within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]
only θ_{12} affects $N_{\rm eff}$, at most by $\delta N_{\rm eff} \approx 10^{-4}$

Sampling the y momenta

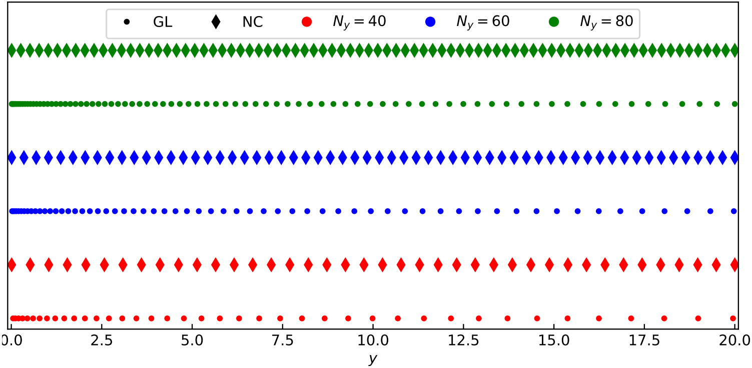
[Bennett, SG+, JCAP 2021]

Discretize neutrino momenta to compute integrals and evolution

two sampling methods for y_i, with $i = 1, \ldots, N_y$:

linear spacing,
Newton-Cotes (NC) integration

Gauss-Laguerre (GL)
optimized for computing $\int_0^\infty dy\, f(y) e^{-y}$



Need to define range ($y_{\rm min} \leq y \leq y_{\rm max}$) and number of nodes N_y

Sampling the y momenta

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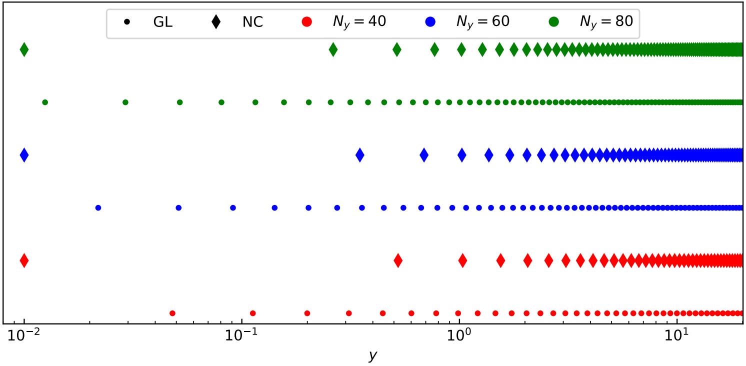


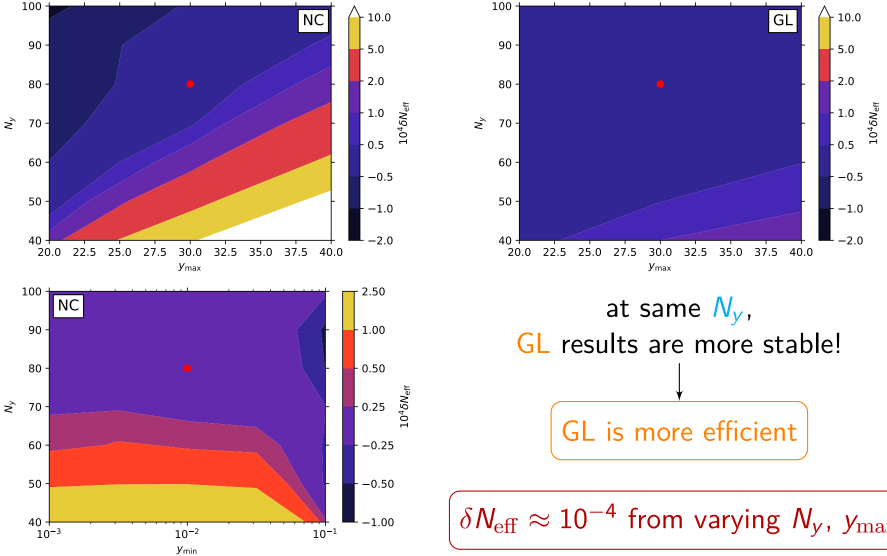
Need to define range ($y_{\min} \le y \le y_{\max}$) and number of nodes N_y

Sampling the y momenta

[Bennett, SG+, JCAP 2021]

Discretize neutrino momenta to compute integrals and evolution

Results may depend on $y_{\rm min}$, $y_{\rm max}$, N_y



at same N_y,
GL results are more stable!

GL is more efficient

$\delta N_{\rm eff} \approx 10^{-4}$ from varying N_y, $y_{\rm max}$

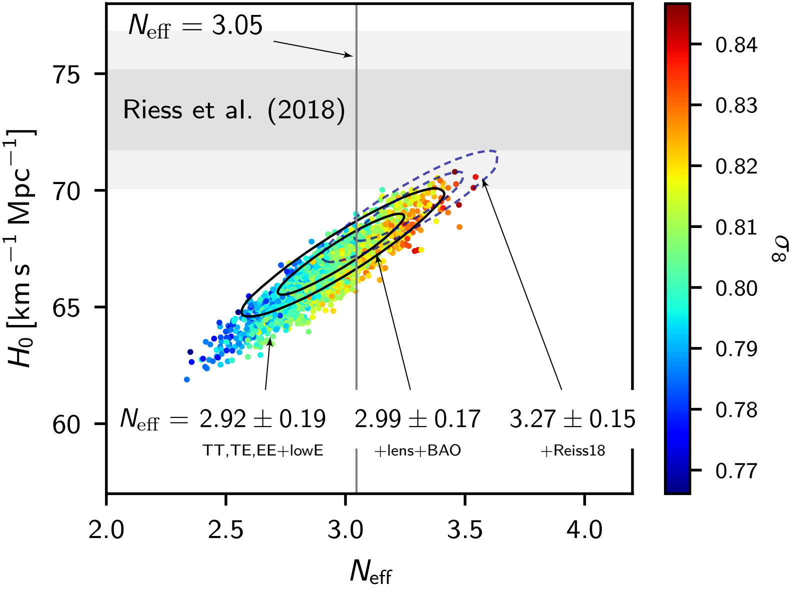
How precise is $N_{\rm eff} = 3.04\ldots$?

Long list of previous works. . . always less than 3ν mixing

[Mangano+, 2005]: $N_{\rm eff} = 3.046$	1st with 3ν mixing (still most cited value)
[de Salas+, 2016]: $N_{\rm eff} = 3.045$	updated collision terms
[SG+, 2019]: $N_{\rm eff} = 3.044$ FortEPiaNO code	more efficient and precise code, $N > 3$ neutrinos allowed, minor differences in numerical integrals
[Bennett+, 2019]: $N_{\rm eff} = 3.043$ (no full calculation)	finite-T QED corrections at $\mathcal{O}(e^3)$! further terms should be almost negligible
[Akita+, 2020]: $N_{\rm eff} = 3.044 \pm 0.0005$	equations in mass and flavor basis approximated $\nu\nu$ collisions
[Froustey+, 2020]: $N_{\rm eff} = 3.0440 \pm \mathcal{O}(10^{-4})$	full $\nu\nu$ interactions 1st estimate effect of CP-violating phase
[Bennett, SG+, 2020]: $N_{\rm eff} = 3.0440 \pm 0.0002$ FortEPiaNO improved	1st full discussion on effect of oscillation parameters, full estimation of current numerical and physical uncertainty

N_{eff} and CMB

[Planck Collaboration, 2018]



Short Baseline (SBL) anomalies

[SG+, JPG 43 (2016) 033001]

Do three-neutrino oscillations explain all experimental results?

Short Baseline (SBL) anomalies

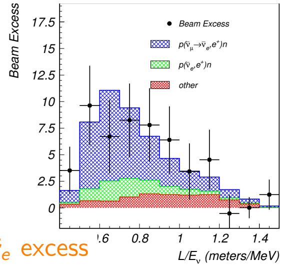
[SG+, JPG 43 (2016) 033001]

Do three-neutrino oscillations explain all experimental results?

LSND

3.8σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess



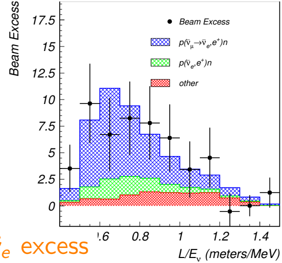
[Aguilar+, 2001]

Short Baseline (SBL) anomalies

[SG+, JPG 43 (2016) 033001]

Do three-neutrino oscillations explain all experimental results?

LSND

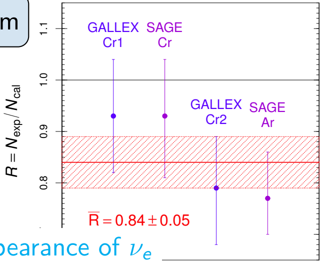


[Aguilar+, 2001]

3.8σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Gallium



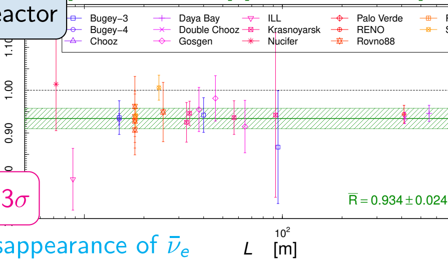
[Giunti, Laveder, 2011]

2.7σ

disappearance of ν_e

[Mention+, 2011] [Azabajian+, 2012]

Reactor



$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

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[SG+, JPG 43 (2016) 033001]

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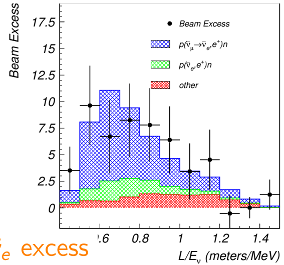

[Aguilar+, 2001]

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$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

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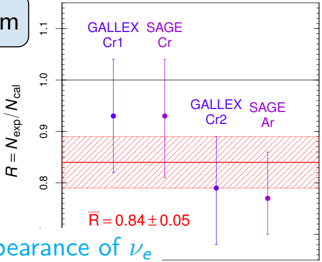
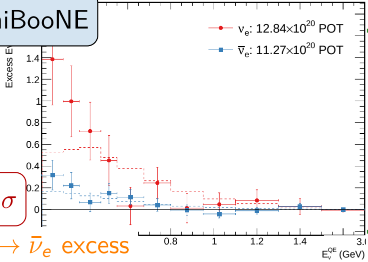


[Giunti, Laveder, 2011]

2.7σ

disappearance of ν_e

MiniBooNE



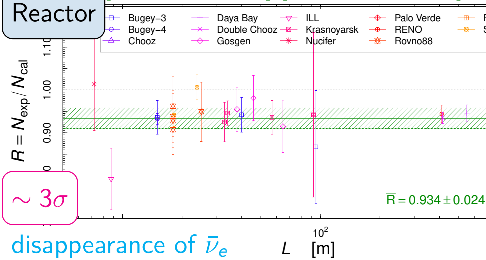
[Aguilar+, 2008-2018]

$\sim 5\sigma$

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Reactor

[Mention+, 2011] [Azabajian+, 2012]



$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

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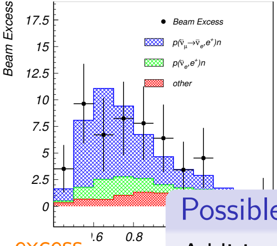
[SG+, JPG 43 (2016) 033001]

Do three-neutrino oscillations explain all experimental results?

LSND

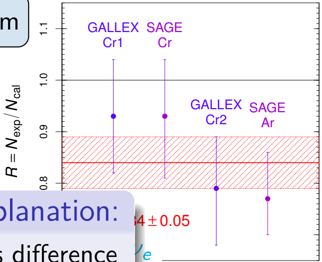


[Aguilar+, 2001]

3.8σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Gallium



[Giunti, Laveder, 2011]

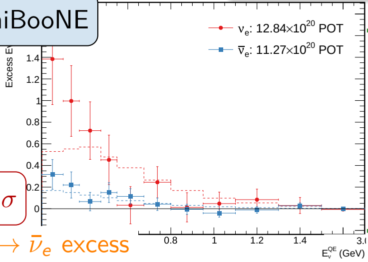
Possible common explanation:

Additional squared mass difference

$\Delta m^2_{SBL} \simeq 1\ \mathrm{eV}^2$

[Mention+, 2011] [Azabajian+, 2012]

MiniBooNE

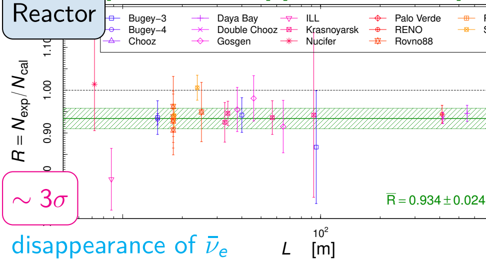


[Aguilar+, 2008-2018]

∼ 5σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Reactor



∼ 3σ

disappearance of $\bar{\nu}_e$

ν oscillations in the early universe with ν_s [SG+, JCAP 07 (2019) 014]

We need to update the equations to include the additional sterile neutrino!

density matrix: $$\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\rho_{\mathrm{T}}}} \left\{ -i\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\mathrm{F}}}{2y} - \frac{2\sqrt{2}G_{\mathrm{F}}y}{x^6/m_e^6}\left(\frac{\mathbb{E}_\ell + \mathbb{P}_\ell}{m_W^2} + \frac{4\mathbb{E}_\nu}{3m_Z^2}\right), \varrho\right] + \frac{m_e^3 G_{\mathrm{F}}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

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$$\mathbb{M}_{\mathrm{F}} = U\mathbb{M}U^\dagger$$

$$\mathbb{M} = \mathrm{diag}(m_1^2,\ldots,m_4^2)$$

$$U = R^{34}R^{24}R^{14}R^{23}R^{13}R^{12}$$

e.g. $$R^{13} = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|U|^2 = \begin{pmatrix} \ldots & \ldots & \ldots & \sin^2\theta_{14} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\sin^2\theta_{24} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\cos^2\theta_{24}\sin^2\theta_{34} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\cos^2\theta_{24}\cos^2\theta_{34} \end{pmatrix}$$

ν oscillations in the early universe with ν_s [SG+, JCAP 07 (2019) 014]

We need to update the equations to include the additional sterile neutrino!

density matrix: $$\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

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$\mathbb{E}_\ell = \mathrm{diag}(\rho_e, \rho_\mu, 0, 0)$ $\quad$ $\mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho \right) S_a$ $\quad$ with $S_a = \mathrm{diag}(1,1,1,0)$

lepton densities $\quad$ neutrino densities $\quad$ (only for active neutrinos)

take into account matter effects in oscillations

ν oscillations in the early universe with ν_s [SG+, JCAP 07 (2019) 014]

We need to update the equations to include the additional sterile neutrino!

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino-electron scattering and pair annihilation, plus neutrino–neutrino interactions

sterile neutrino never take part into interactions

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from continuity equation $\dot{\rho} = -3H(\rho + P)$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\sum_{\ell=e,\mu}\left[\frac{r_\ell^2}{r}J(r_\ell)\right] + G_1(r) - \frac{1}{2\pi^2 z^3}\int_0^\infty dy\, y^3 \sum_{\alpha=e}^{s}\frac{\mathrm{d}\varrho_{\alpha\alpha}}{\mathrm{d}x}}{\sum_{\ell=e,\mu}\left[r_\ell^2 J(r_\ell) + Y(r_\ell)\right] + G_2(r) + \frac{2\pi^2}{15}}$$

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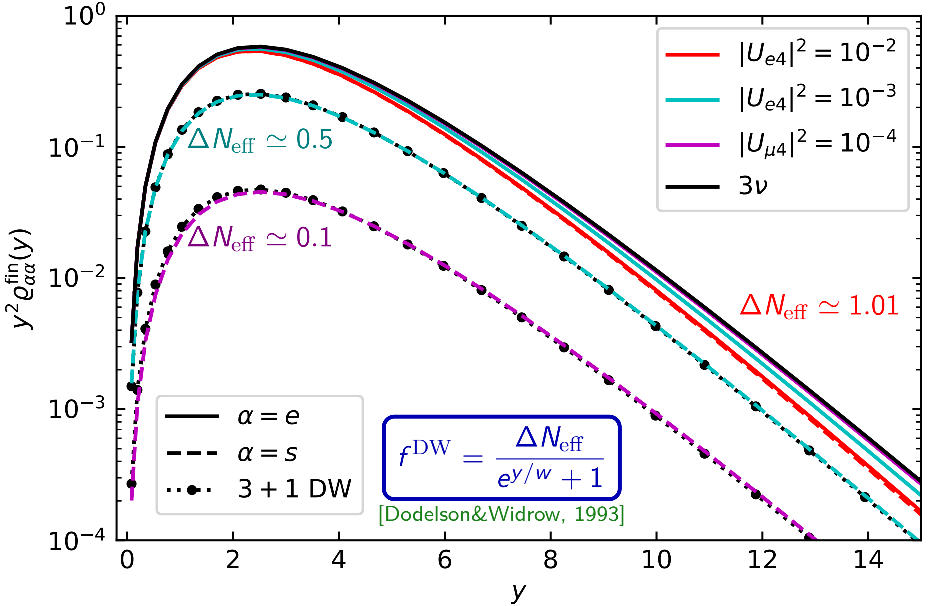
initial conditions: $\varrho_{\alpha\alpha}(z_{\mathrm{in}})$ = FD for active neutrinos, zero for steriles

Momentum distributions

[SG+, JCAP 07 (2019) 014]

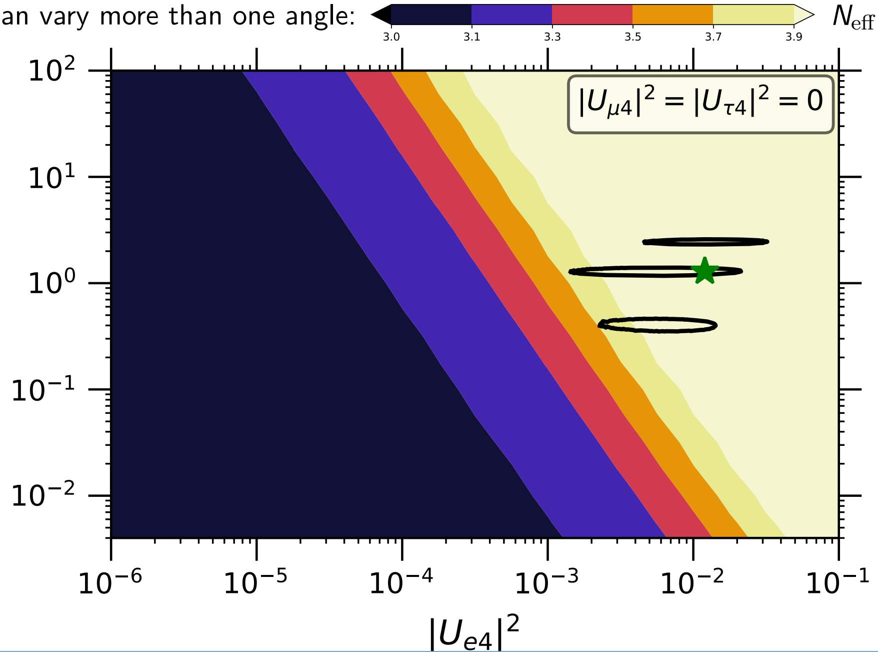
$\Delta m^2_{41} = 1.29$ eV^2, other $|U_{\beta 4}|^2 = 0$, $\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm active}$



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

We can vary more than one angle:



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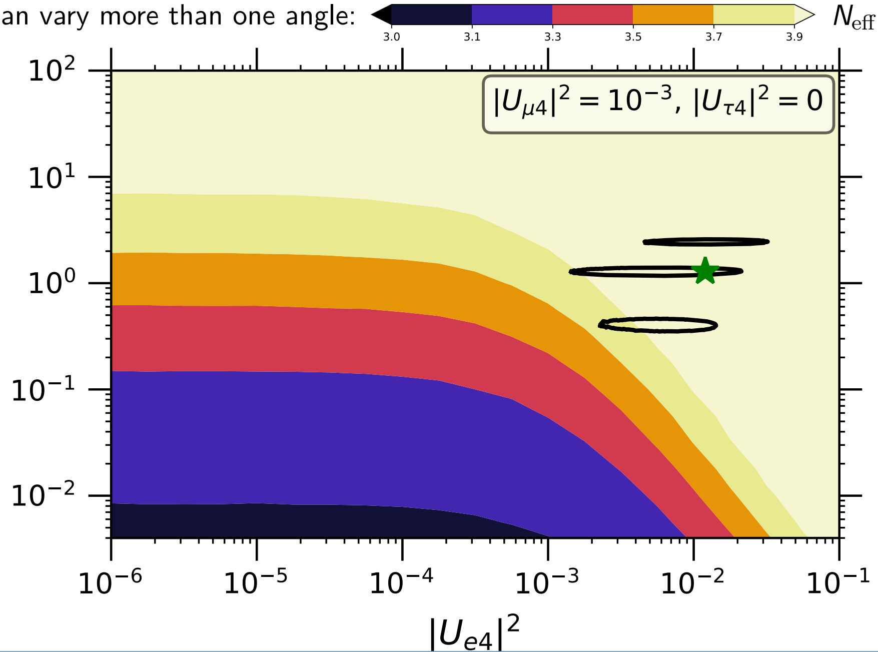

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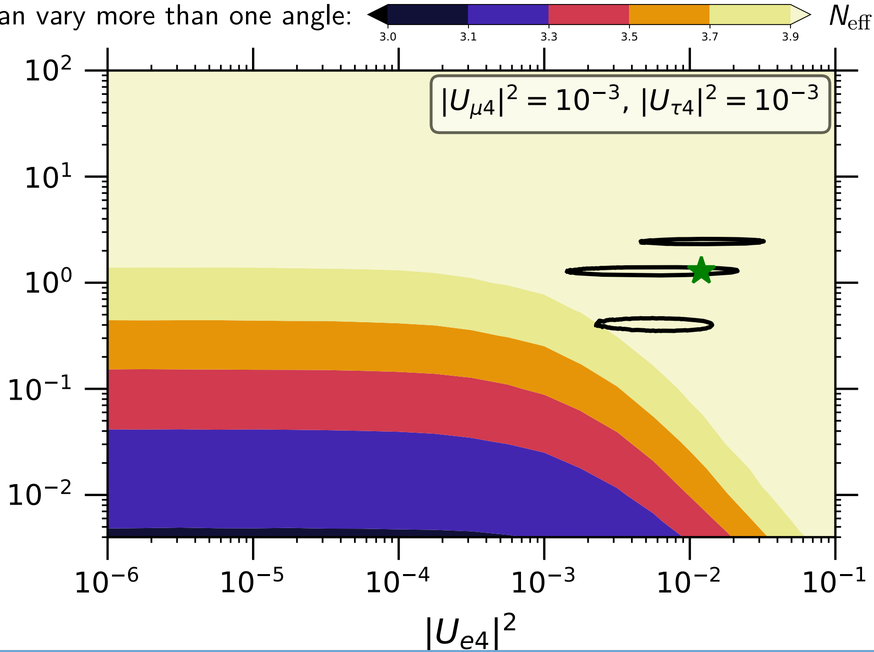
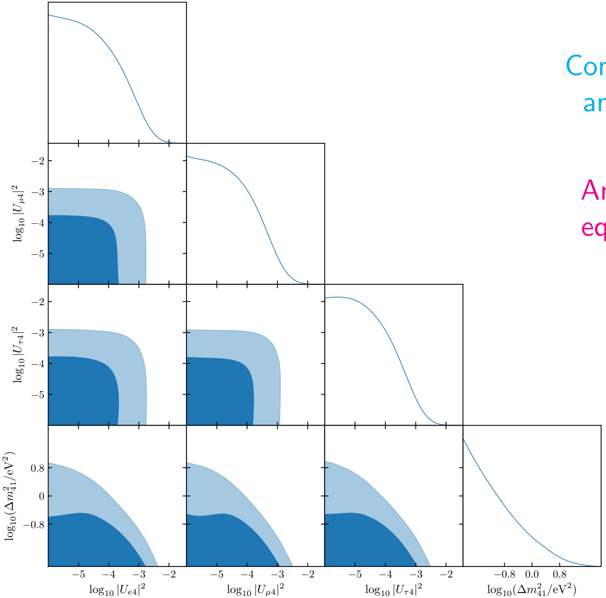


Cosmological constraints on $|U_{\alpha 4}|^2$

[arxiv:2003.02289]

Use multi-angle results from `FortEPiaNO` to derive constraints on $|U_{\alpha 4}|^2$:



Constraints come from $N_{\rm eff}$ and late-time density Ω_s

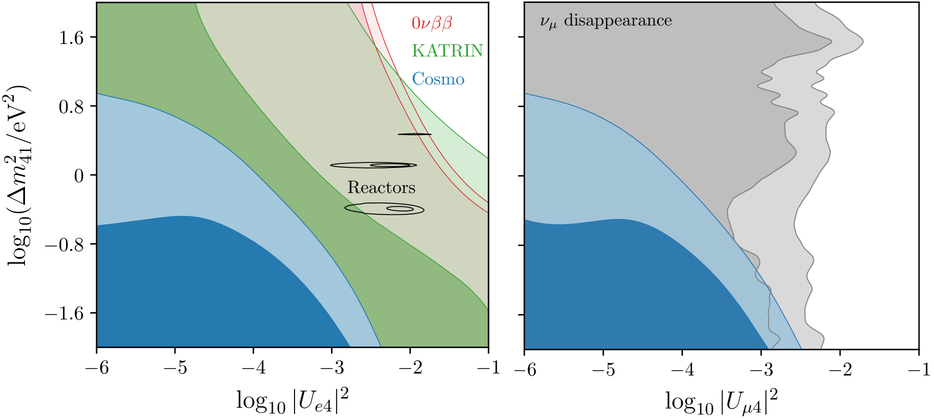
Angles $|U_{\alpha 4}|^2$ are almost equivalent for cosmology

Comparing constraints

[arxiv:2003.02289]

Cosmological constraints are stronger than most other probes

But much more model dependent (as all the cosmological constraints)!



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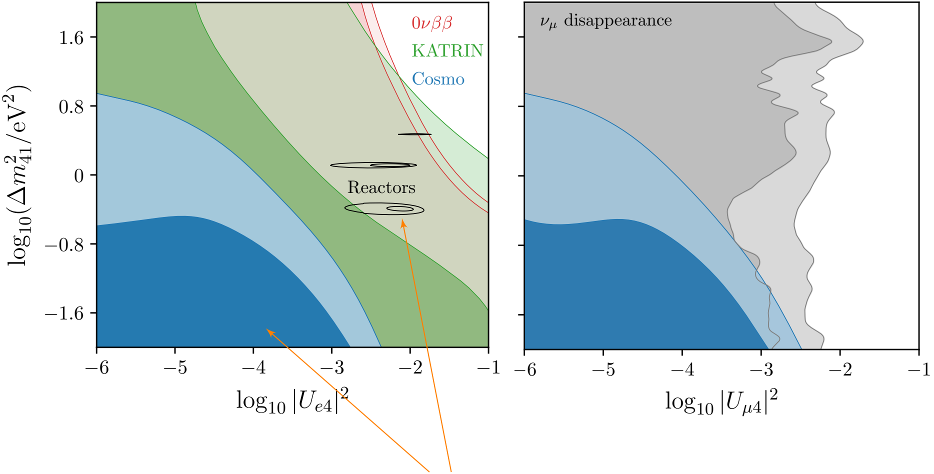


Warning: tension between reactor experiments and CMB bounds!

Collision terms with sterile neutrinos

[in preparation]

Full collision integrals can be computed also with sterile neutrinos

Equations unchanged – except phase space F, where the couplings enter:

$$
\begin{aligned}
F^{ab}_{\rm sc}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right) &= f_e^{(4)}(1-f_e^{(2)})\left[G^a\Phi_1^{3,b,1}+\Phi_2^{1,b,3}G^a\right] - f_e^{(2)}(1-f_e^{(4)})\left[\Phi_1^{1,b,3}G^a + G^a\Phi_2^{3,b,1}\right]\\
F^{ab}_{\rm ann}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right) &= f_e^{(3)}f_e^{(4)}\left[G^a\Phi_4^{2,b,1}+\Phi_4^{1,b,2}G^a\right] - (1-f_e^{(3)})(1-f_e^{(4)})\left[G^a\Phi_3^{2,b,1} + \Phi_3^{1,b,2}G^a\right]\\
F_{\nu\nu}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right) &= \Phi_2^{1,S,3}G_S\left[\Phi_2^{2,S,4}G_S+\mathrm{Tr}(\cdots)\right] - \Phi_1^{1,S,3}G_S\left[\Phi_1^{2,S,4}G_S+\mathrm{Tr}(\cdots)\right] + \mathrm{h.c.}\\
F_{\nu\bar\nu}\left(\varrho^{(1)}, \varrho^{(2)}, \varrho^{(3)}, \varrho^{(4)}\right) &= \Phi_4^{1,S,2}G_S\left[\Phi_3^{4,S,3}G_S+\mathrm{Tr}(\cdots)\right] - \Phi_3^{1,S,2}G_S\left[\Phi_4^{4,S,3}G_S+\mathrm{Tr}(\cdots)\right]\\
&+ \Phi_2^{1,S,3}G_S\left[\Phi_1^{4,S,2}G_S+\mathrm{Tr}(\cdots)\right] - \Phi_1^{1,S,3}G_S\left[\Phi_2^{4,S,2}G_S+\mathrm{Tr}(\cdots)\right] + \mathrm{h.c.}
\end{aligned}
$$

Interaction strenghts ($a, b \in [L, R]$):

$$
\begin{aligned}
G^R &= \mathrm{diag}(g_R, g_R, g_R, 0)\\
G^L &= \mathrm{diag}(g_L, \tilde g_L, \tilde g_L, 0)\\
G^S &= \mathrm{diag}(1, 1, 1, 0)
\end{aligned}
$$

Remember also:

$$
\begin{aligned}
\Phi_1^{\alpha,i,\beta} &= \varrho^{(\alpha)}G^i(1-\varrho^{(\beta)})\\
\Phi_2^{\alpha,i,\beta} &= (1-\varrho^{(\alpha)})G^i\varrho^{(\beta)}\\
\Phi_3^{\alpha,i,\beta} &= \varrho^{(\alpha)}G^i\varrho^{(\beta)}\\
\Phi_4^{\alpha,i,\beta} &= (1-\varrho^{(\alpha)})G^i(1-\varrho^{(\beta)})
\end{aligned}
$$

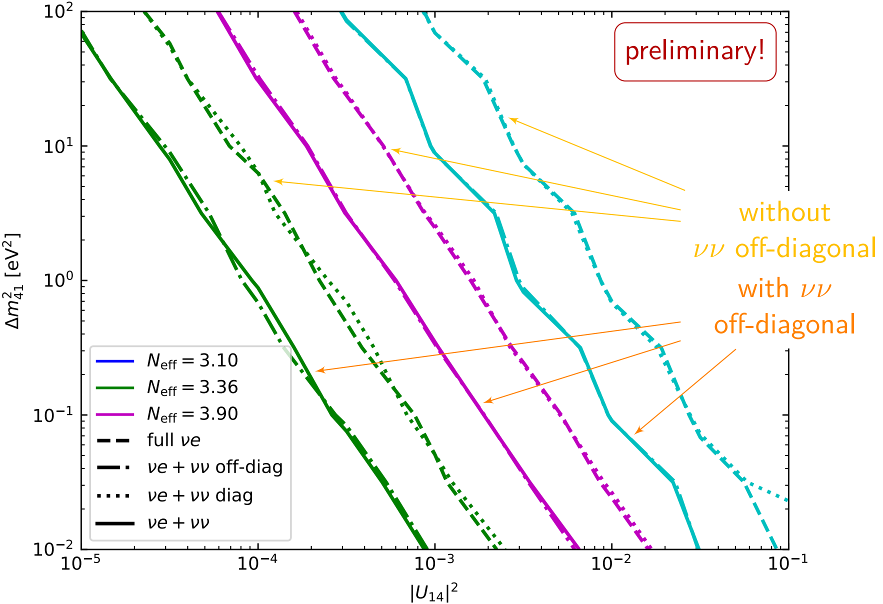
Damping approximations also affected!

$$
\left\{D^u(y)\right\}_{\alpha\beta} = \frac{1}{2}\left[\left\{R^u(y)\right\}_\alpha + \left\{R^u(y)\right\}_\beta\right], \text{ but } \left\{R^u_{\nu\nu}(y)\right\}_s = \left\{R^u_{\nu e}(y)\right\}_s = 0
$$

Collision terms with sterile neutrinos

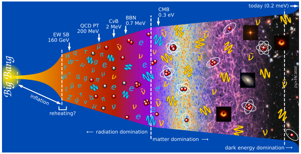
[in preparation]

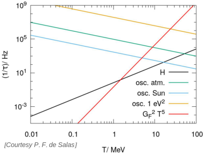
Full collision integrals can be computed also with sterile neutrinos

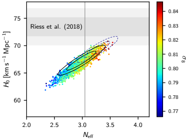


Conclusions

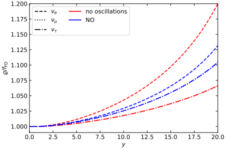
1 Neutrinos in the early universe – probe lowest energies

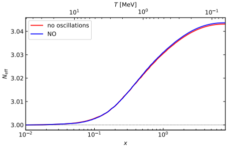


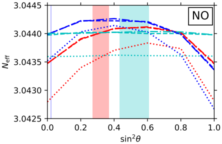




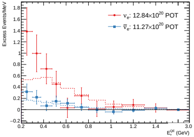
2 Active neutrinos – precision

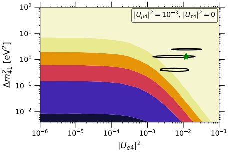


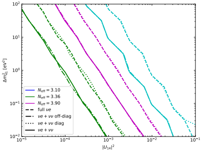


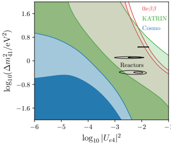


3 Sterile neutrino hints – new physics?









Conclusions

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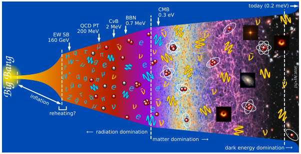
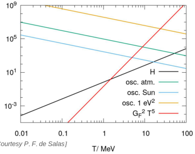
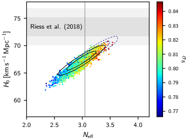




[Courtesy P. F. de Salas]



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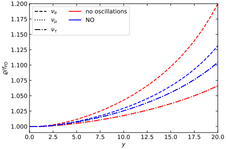

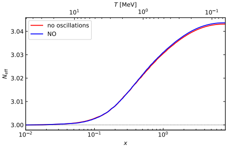

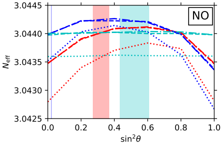

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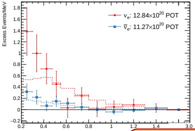

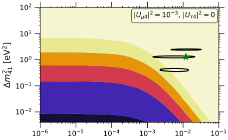

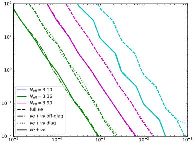

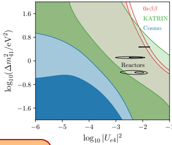

Thanks for your attention!