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(Cosmological) Relic neutrinos, from A to Z

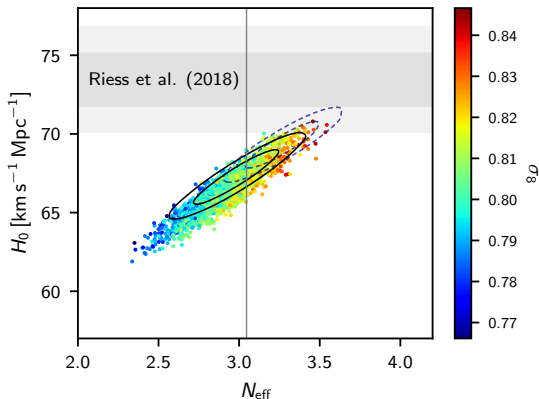
A

Active neutrinos

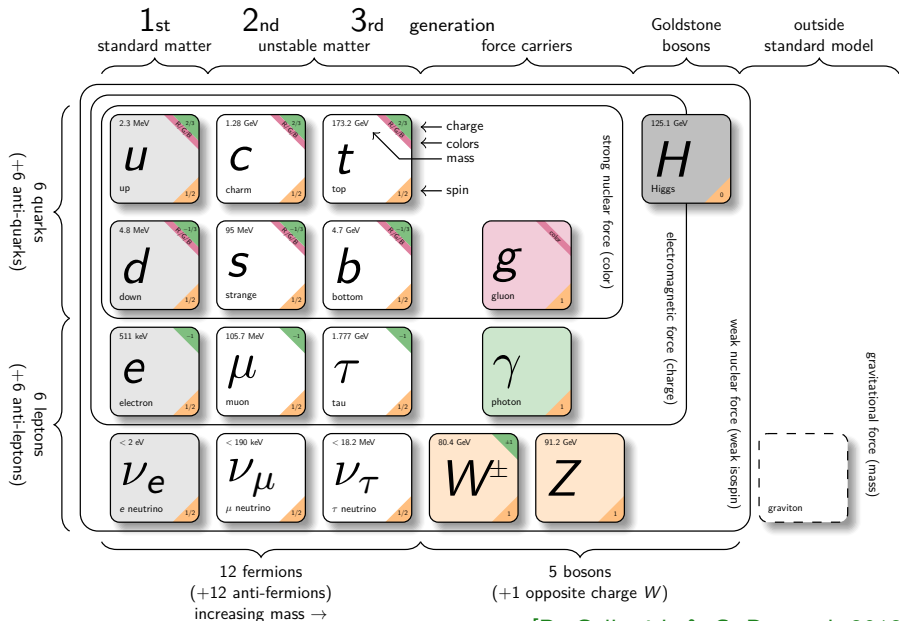
Spoiler: “Sterile” will come later

Based on:

- JHEP 02 (2021) 071 and update
- Planck 2018
- [10.1088/1475-7516/2021/04/073](https://arxiv.org/abs/10.1088/1475-7516/2021/04/073)

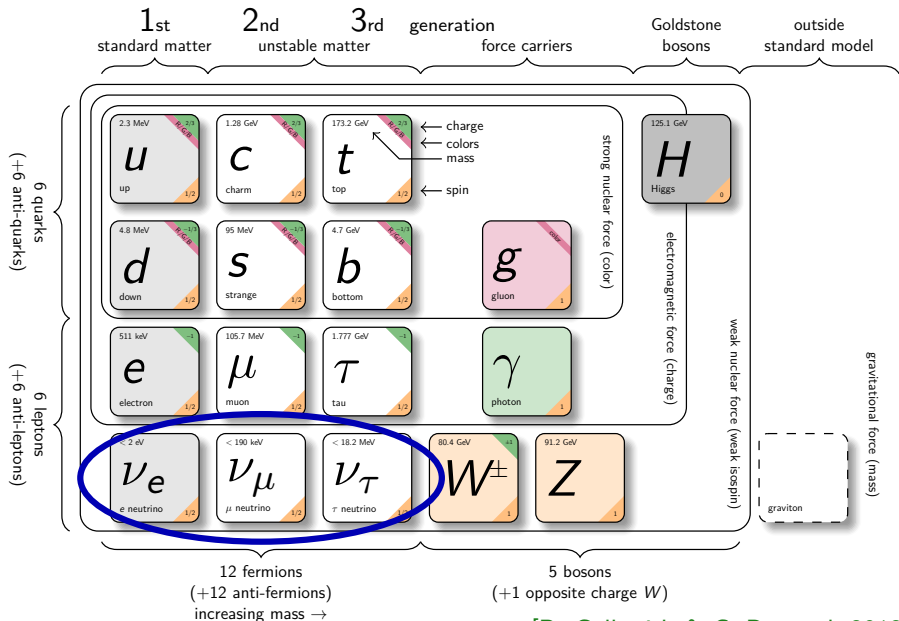


The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

The Standard Model of Particle Physics

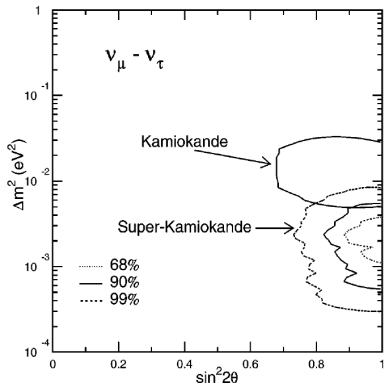


[D. Galbraith & C. Burgard, 2012]

Neutrino oscillations

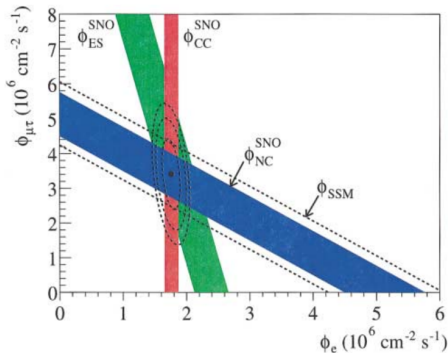
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \rightarrow \nu_\tau$
oscillations from atmospheric ν

[SNO, 2001-2002]



first discovery of $\nu_e \rightarrow \nu_\mu, \nu_\tau$
oscillations from solar ν

Nobel prize in 2015

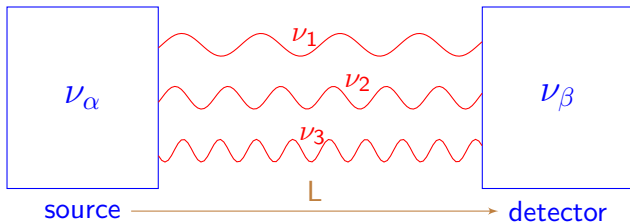
Two neutrino bases

flavor neutrinos ν_α

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1} |\nu_1\rangle + U_{\alpha 2} |\nu_2\rangle + U_{\alpha 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = |\nu_\beta\rangle = U_{\alpha 1} e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2} e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \longleftarrow \text{define} \longrightarrow t = L$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

The mixing matrix

U can be parameterized using 3 angles (θ_{12} , θ_{13} , θ_{23}) and max 3 (1 Dirac δ , 2 Majorana [\exists only for Majorana ν]) phases

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{mainly atmospheric} \\ \text{and LBL} \\ \text{accelerator} \\ \text{disappearance}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\substack{\text{mainly SBL reactors and} \\ \text{LBL accelerator} \\ \text{appearance}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{mainly solar and} \\ \text{LBL reactors}}} M$$

Majorana phases irrelevant for oscillation experiments ←

Relevant for example in neutrinoless double-beta decay

$$s_{ij} \equiv \sin \theta_{ij}; \quad c_{ij} \equiv \cos \theta_{ij}$$

SBL = short baseline; LBL = long baseline

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12} , θ_{13} , θ_{23} and one CP phase δ

Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

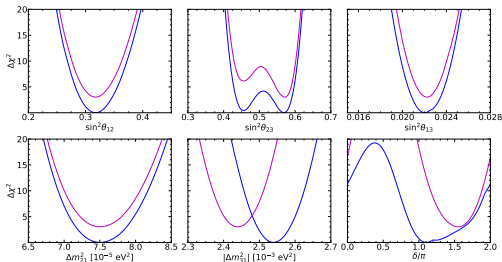
NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$

IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned} \Delta m_{21}^2 &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5} \text{ eV}^2 \\ |\Delta m_{31}^2| &= (2.54 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (NO)} \\ &= (2.44 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (IO)} \end{aligned}$$

$$\begin{aligned} 10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\ 10^2 \sin^2(\theta_{13}) &= 2.200^{+0.069}_{-0.062} \text{ (NO)} \\ &= 2.225^{+0.064}_{-0.070} \text{ (IO)} \\ 10 \sin^2(\theta_{23}) &= 4.55 \pm 0.13 \text{ U} 5.71 \pm 0.12 \text{ (NO)} \\ &= 5.71^{+0.14}_{-0.17} \text{ (IO)} \end{aligned}$$

$$\begin{aligned} \delta/\pi &= 1.10^{+0.27}_{-0.12} \text{ (NO)} \\ &= 1.54 \pm 0.14 \text{ (IO)} \end{aligned}$$

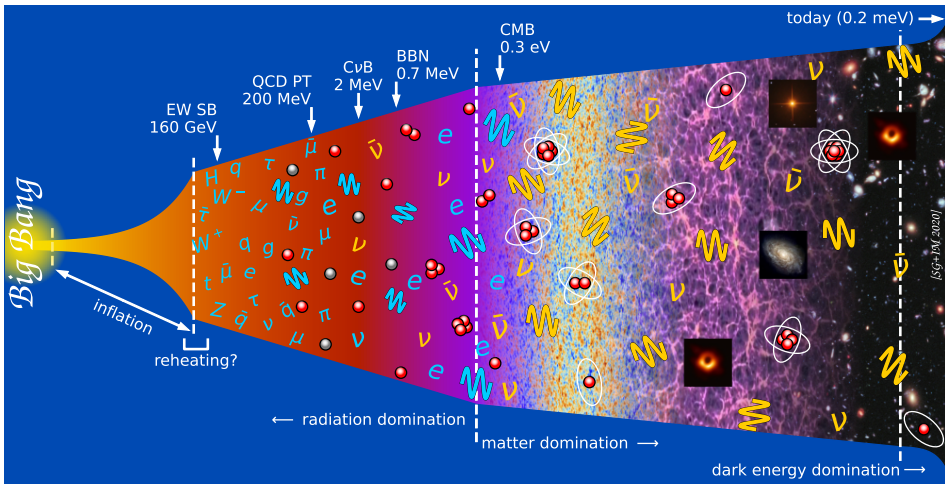


mass ordering
still unknown

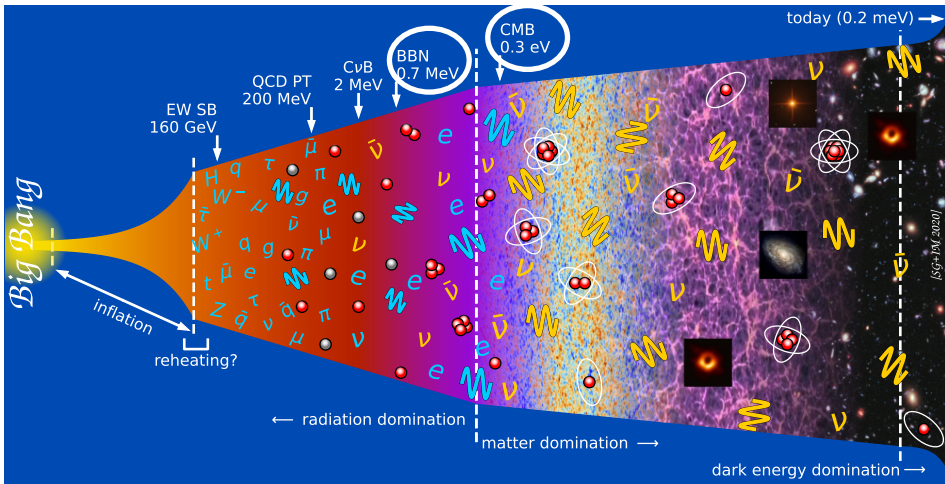
δ still unknown

see also: <http://globalfit.astroparticles.es>

History of the universe



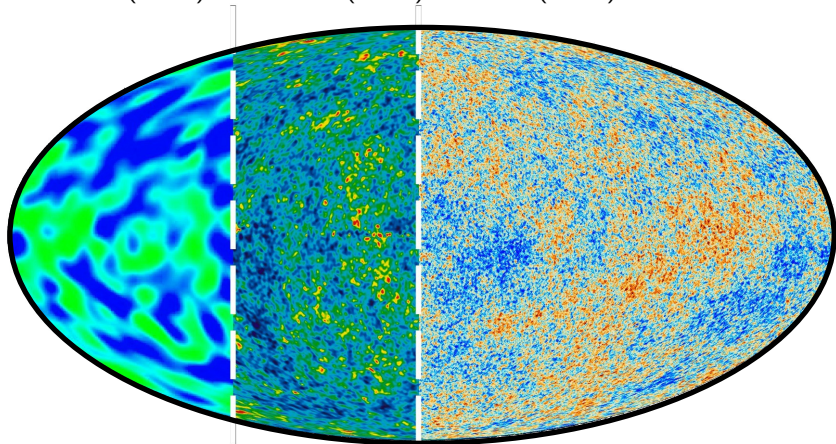
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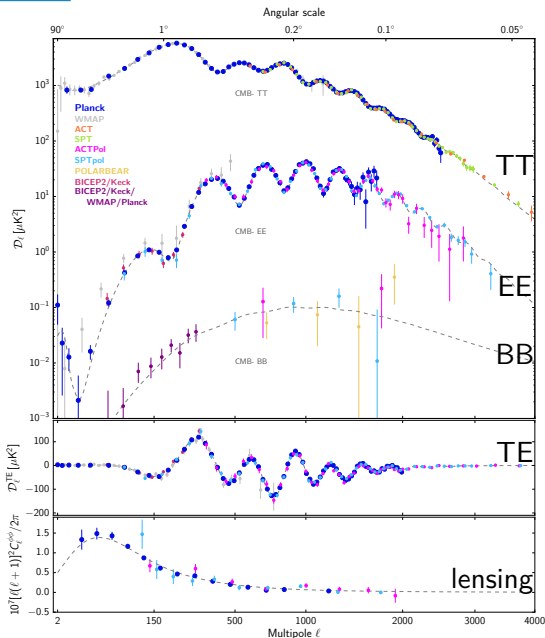
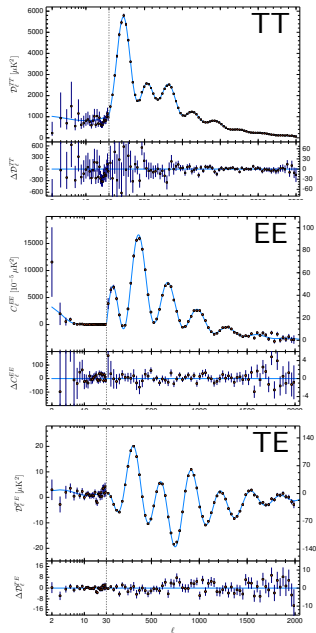


The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

COBE (1992) WMAP (2003) Planck (2013)





Big Bang Nucleosynthesis (BBN)

BBN: production of light nuclei at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

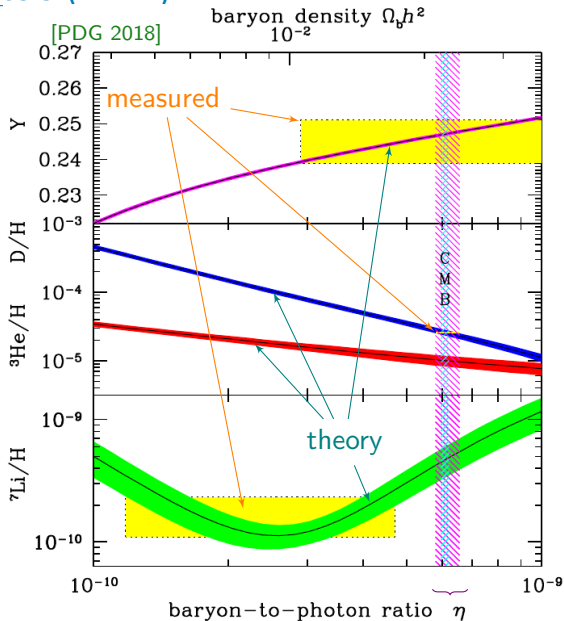
temperature $T_{fr} \simeq 1\text{ MeV}$
from nucleon freeze-out

much earlier than CMB!

strong probe for physics
before the CMB

e.g. neutrinos!

ν affect
universe expansion
and
reaction rates ($\nu_e/\bar{\nu}_e$)
at BBN time...



BBN concordance

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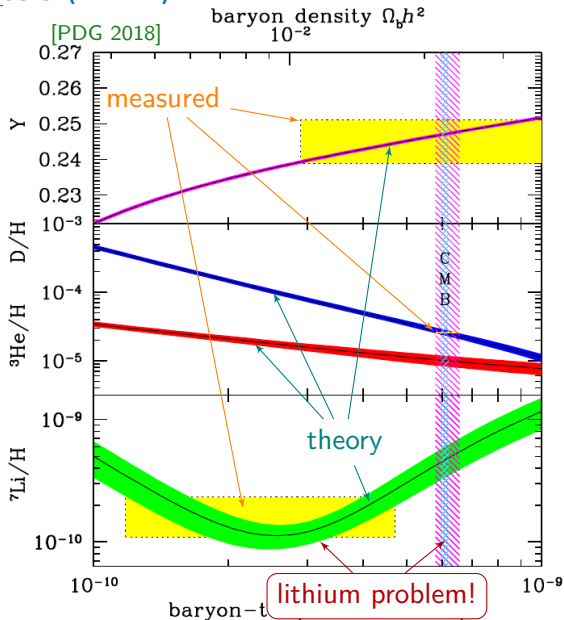
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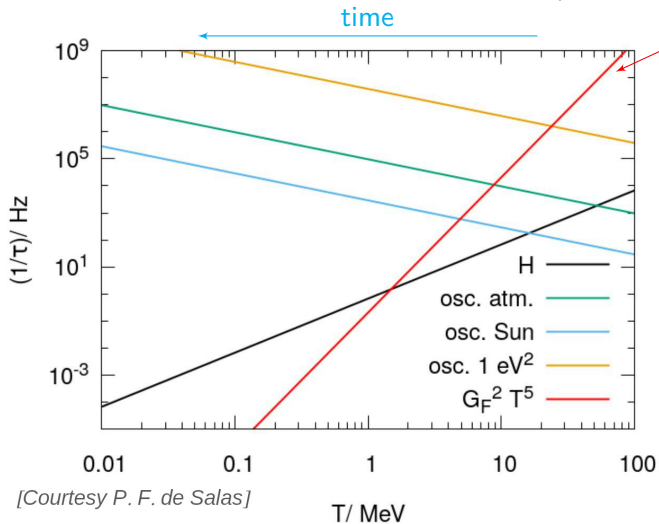
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Neutrinos in the early Universe

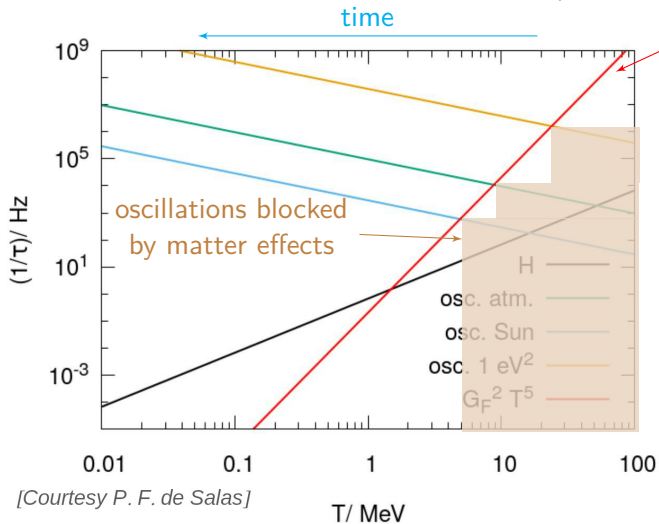
before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

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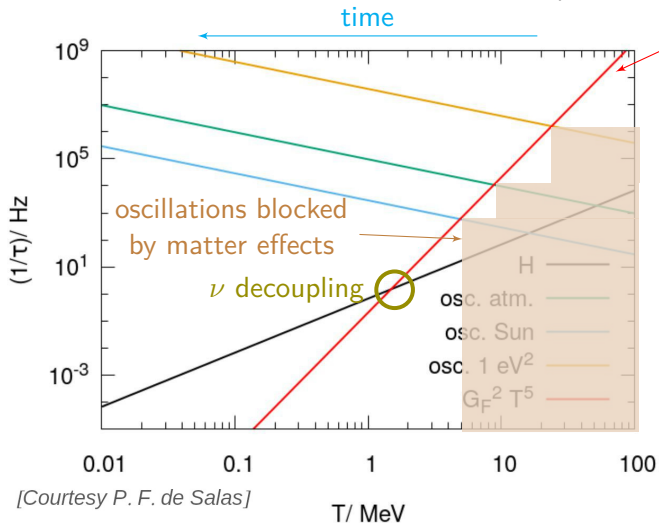
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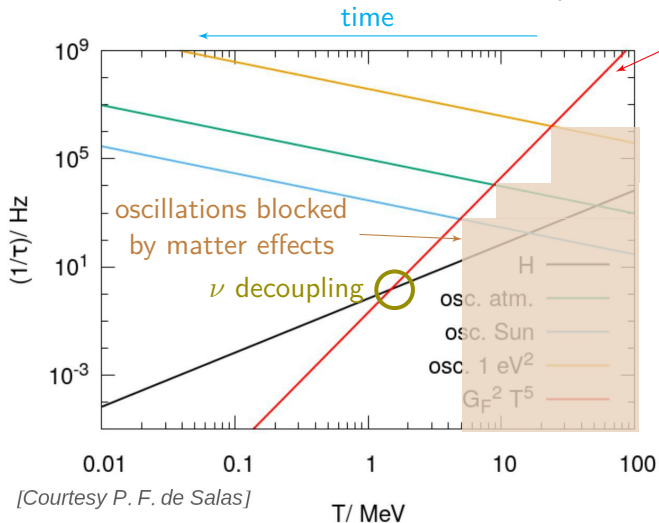
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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



$$T_\nu \simeq (4/11)^{1/3} T_\gamma$$

after $e^+ e^- \rightarrow \gamma\gamma$

f_ν : frozen Fermi-Dirac distribution

Today:

$$T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV}$$

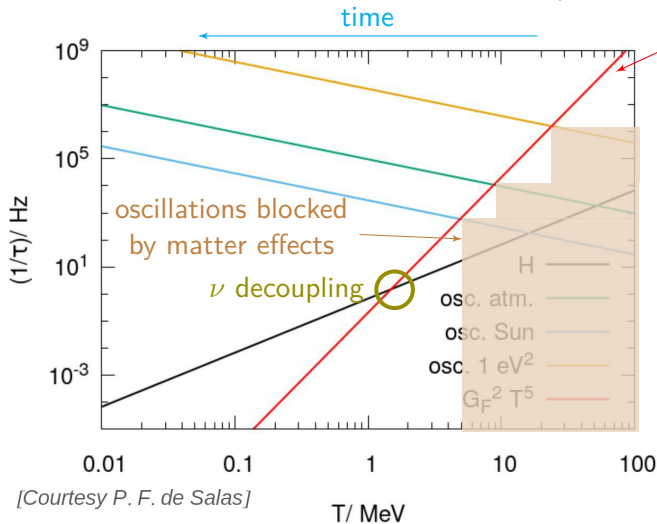
$$\langle E_\nu \rangle \simeq 3.1 T_{\nu,0} \simeq 5 \times 10^{-4} \text{ eV}$$

$$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56 \text{ cm}^{-3} \text{ per family}$$

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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!
 actually, the decoupling T is momentum dependent!

distortions to equilibrium f_ν !

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

$$\text{density matrix: } \varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$$

$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_{\text{T}}}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{M_{\text{F}}}{2y} - \frac{2\sqrt{2}G_{\text{F}}y}{x^6/m_e^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4E_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_{\text{F}}^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_{T} total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[\cdot, \cdot]$ commutator

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$$\mathbb{M}_F = \mathbf{U} \mathbf{M} \mathbf{U}^\dagger$$

$$\mathbf{M} = \text{diag}(m_1^2, m_2^2, m_3^2)$$

$$\mathbf{U} = R^{23} R^{13} R^{12}$$

$$\text{e.g. } R^{13} = \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} \\ 0 & 1 & 0 \\ -\sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix}$$

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

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$$\mathbb{M}_{\text{F}} = U \mathbb{M} U^\dagger$$

$$\mathbb{E}_\ell = \text{diag}(\rho_e, \rho_\mu, 0) \quad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho \right) S_a \quad \text{with } S_a = \text{diag}(1, 1, 1)$$

lepton densities

neutrino densities

(only for active neutrinos)

take into account matter effects in oscillations

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino–electron scattering and pair annihilation,
plus neutrino–neutrino interactions

2D integrals over momentum, take most of the computation time

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from continuity
equation
 $\dot{\rho} = -3H(\rho + P)$

$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J(r_\ell) \right] + G_1(r) - \frac{1}{2\pi^2 z^3} \int_0^\infty dy y^3 \sum_{\alpha=e}^\tau \frac{d\varrho_{\alpha\alpha}}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J(r_\ell) + Y(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

$r = x/z$, $r_\ell = m_\ell/m_e r$ $J(r)$, $Y(r)$ from non-relativistic transition of e^\pm , μ^\pm
 $G_1(r)$ and $G_2(r)$ from electromagnetic corrections

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neutrino temperature w : same equation as z , but electrons always relativistic

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neutrino temperature w : same equation as z , but electrons always relativistic
initial conditions: $\varrho_{\alpha\alpha} = \text{Fermi-Dirac}$ at $x_{\text{in}} \simeq 0.001$, with $w = z \simeq 1$

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

$$\text{density matrix: } \varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$$

$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_T}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{M_{\text{F}}}{2y} - \frac{2\sqrt{2}G_{\text{F}}y}{x^6/m_e^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4E_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_{\text{F}}^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

m_e : electron mass, ρ_T : total energy density, m_W, m_Z : mass of the W, Z bosons, G_{F} : Fermi constant, \mathcal{I} : commutator

**FORTran-Evolved Primordial Neutrino Oscillations
(FortEPiano)**

https://bitbucket.org/ahep_cosmo/fortepiano_public

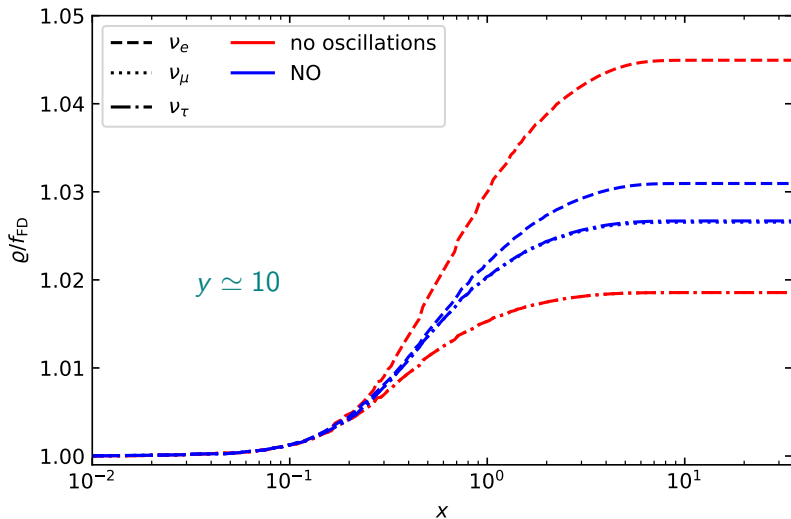
from continuity
equation

$$\dot{\rho} = -3H(\rho + P)$$

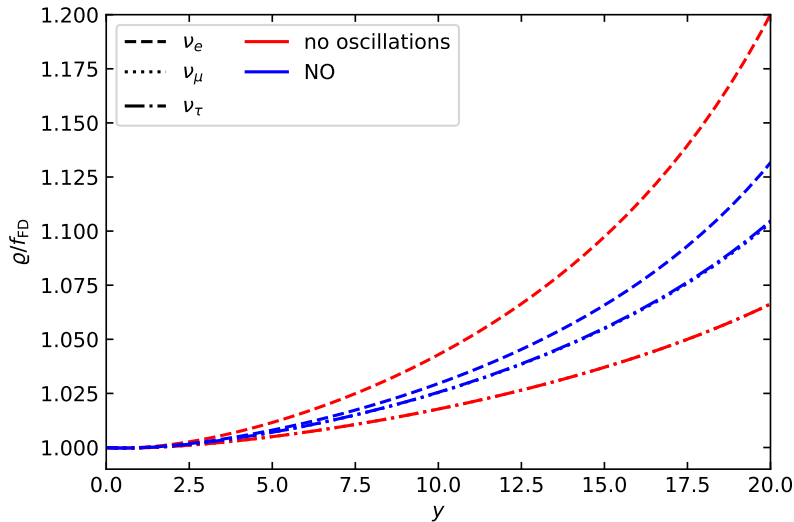
$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J(r_\ell) \right] + G_1(r) - \frac{1}{2\pi^2 z^3} \int_0^\infty dy y^3 \sum_{\alpha=e}^{\tau} \frac{d\varrho_{\alpha\alpha}}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J(r_\ell) + Y(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

neutrino temperature w : same equation as z , but electrons always relativistic
initial conditions: $\varrho_{\alpha\alpha} = \text{Fermi-Dirac}$ at $x_{\text{in}} \simeq 0.001$, with $w = z \simeq 1$

Distortion of the momentum distribution (f_{FD} : Fermi-Dirac at equilibrium)

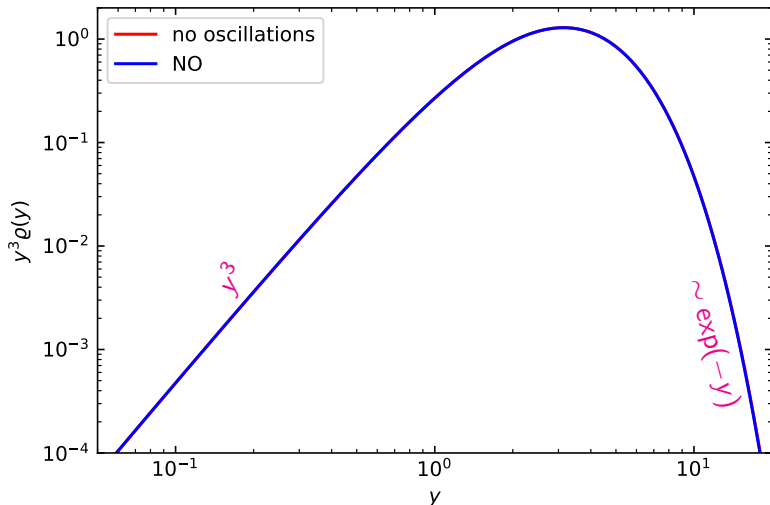


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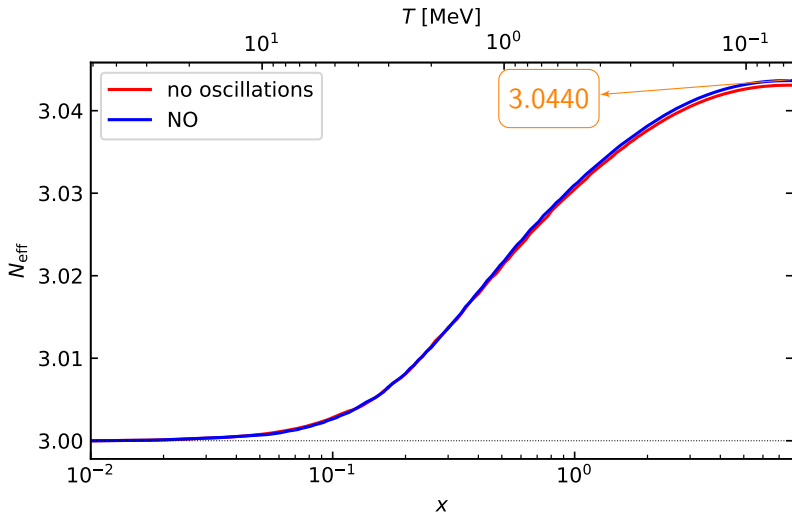


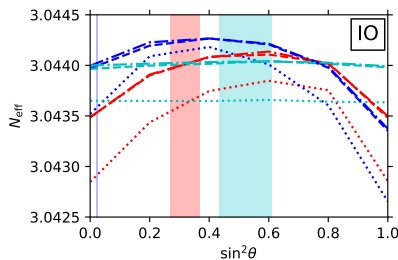
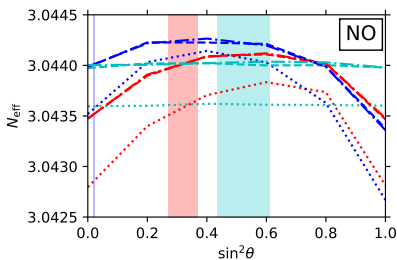
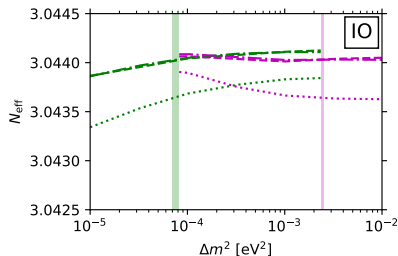
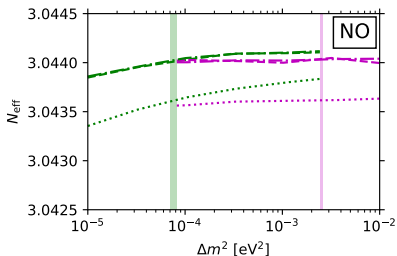
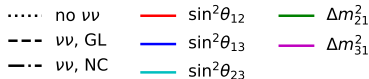
$$N_{\text{eff}}^{\text{final}} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$

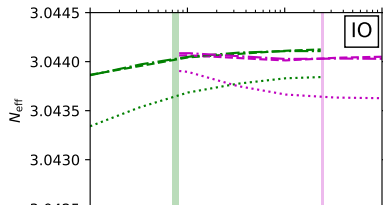
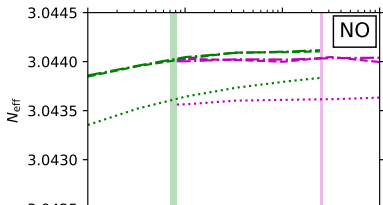
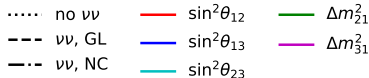
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\text{fin}}$
 $\hookrightarrow \propto y^3 g_{ii}(y)$



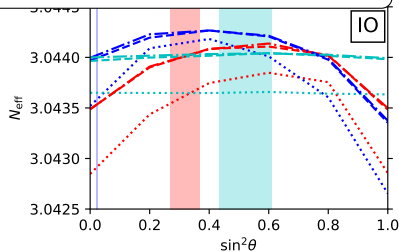
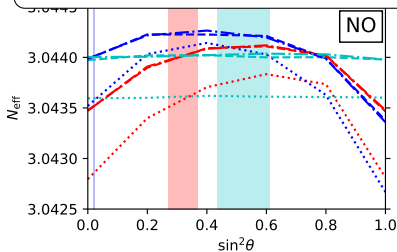
$$N_{\text{eff}}^{\text{any time}} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$







within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]
 only θ_{12} affects N_{eff} , at most by $\delta N_{\text{eff}} \approx 10^{-4}$

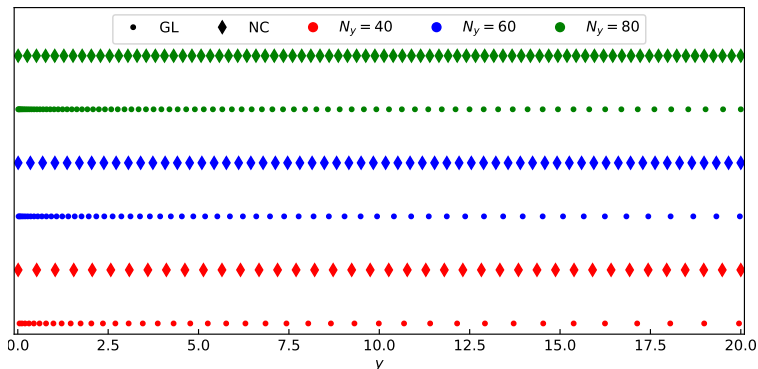


Discretize neutrino momenta to compute integrals and evolution

two sampling methods for y_i , with $i = 1, \dots, N_y$:

linear spacing,
Newton-Cotes (NC) integration

Gauss-Laguerre (GL)
optimized for computing $\int_0^\infty dy f(y)e^{-y}$



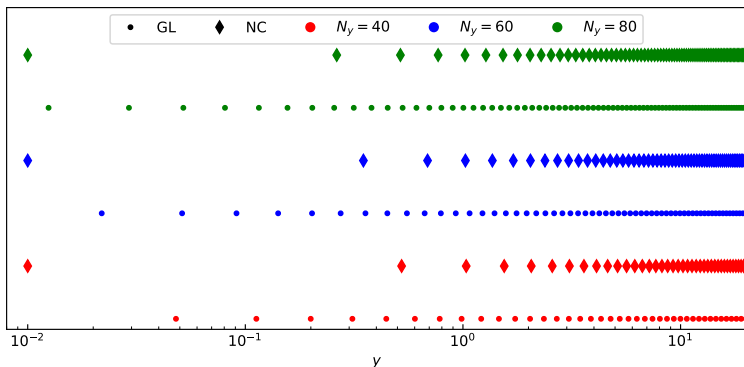
Need to define range ($y_{\min} \leq y \leq y_{\max}$) and number of nodes N_y

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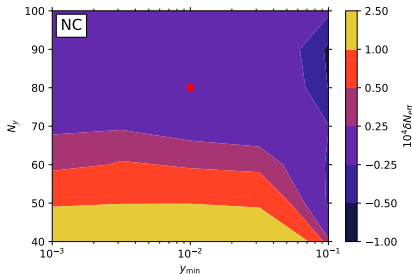
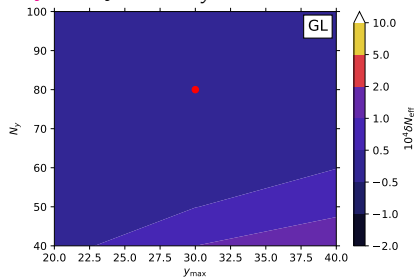
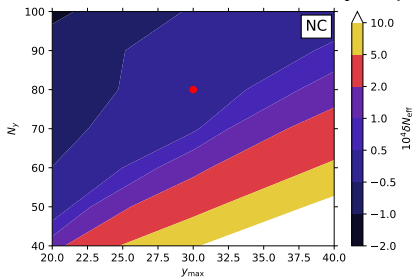
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optimized for computing $\int_0^\infty dy f(y)e^{-y}$



Need to define range ($y_{\min} \leq y \leq y_{\max}$) and number of nodes N_y

Discretize neutrino momenta to compute integrals and evolution

Results may depend on y_{\min} , y_{\max} , N_y



at same N_y ,
GL results are more stable!

GL is more efficient

$\delta N_{\text{eff}} \approx 10^{-4}$ from varying N_y , y_{\max}

How precise is $N_{\text{eff}} = 3.04\dots$?

Long list of previous works... always less than 3ν mixing

[Mangano+, 2005]: $N_{\text{eff}} = 3.046$ 1st with 3ν mixing (still most cited value)

[de Salas+, 2016]: $N_{\text{eff}} = 3.045$ updated collision terms

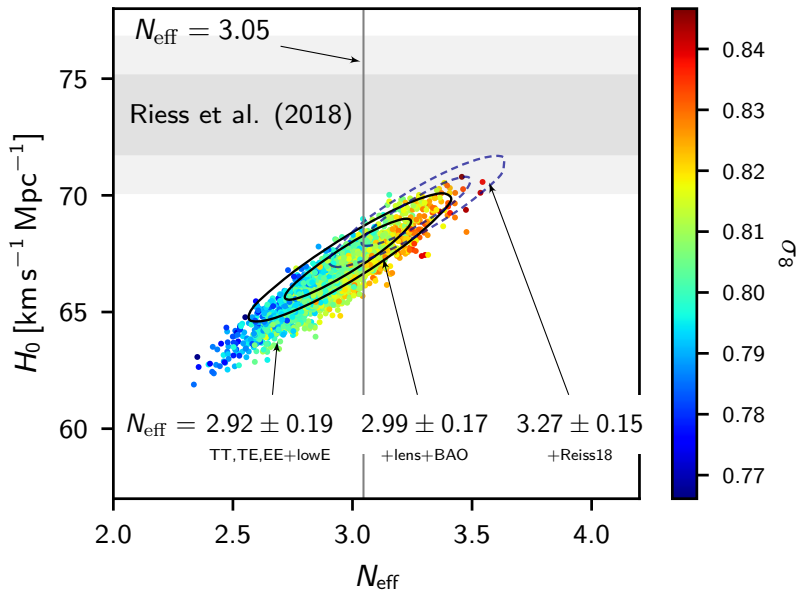
[SG+, 2019]: $N_{\text{eff}} = 3.044$ more efficient and precise code,
 $N > 3$ neutrinos allowed,
minor differences in numerical integrals

[Bennett+, 2019]: $N_{\text{eff}} = 3.043$ finite- T QED corrections at $\mathcal{O}(e^3)$!
(no full calculation) further terms should be almost negligible

[Akita+, 2020]: equations in mass and flavor basis
 $N_{\text{eff}} = 3.044 \pm 0.0005$ approximated $\nu\nu$ collisions

[Froustey+, 2020]: full $\nu\nu$ interactions
 $N_{\text{eff}} = 3.0440 \pm \mathcal{O}(10^{-4})$ 1st estimate effect of CP-violating phase

[Bennett, SG+, 2020]: 1st full discussion on effect of oscillation
 $N_{\text{eff}} = 3.0440 \pm 0.0002$ parameters, full estimation of current
numerical and physical uncertainty



N_{eff} and BBN

BBN: production of light nuclei
at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

temperature $T_{\text{fr}} \simeq 1\text{ MeV}$
from nucleon freeze-out:

$$\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5 = H \sim \sqrt{g_* G_N T^2}$$

$$T_{\text{fr}} \simeq (g_* G_N / G_F^4)^{1/6}$$

enters

$$n/p = \exp(-Q/T_{\text{fr}})$$

which controls element abundances

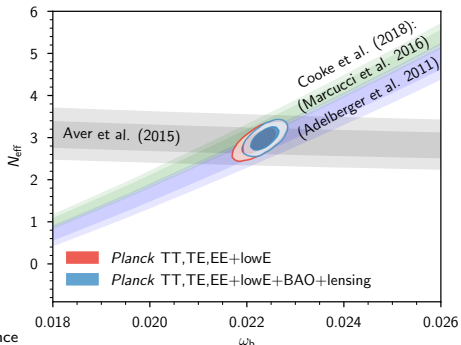
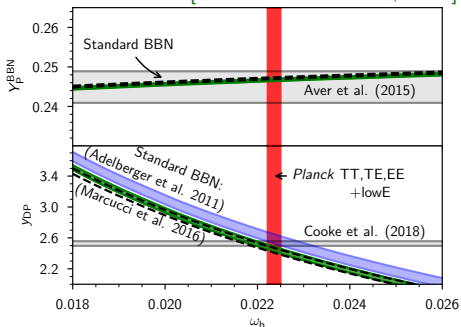
$$g_* \text{ depends on } N_{\text{eff}}$$

abundances depend on N_{eff}

G_F Fermi constant n, p : neutron, proton density number
 G_N Newton constant $Q = 1.293\text{ MeV}$ neutron-proton mass difference

S. Gariazzo "(Cosmological) Relic neutrinos, from A to Z"

[Planck Collaboration, 2018]



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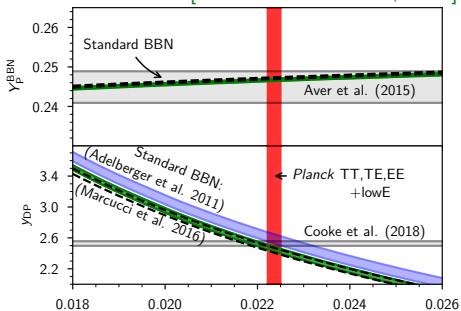
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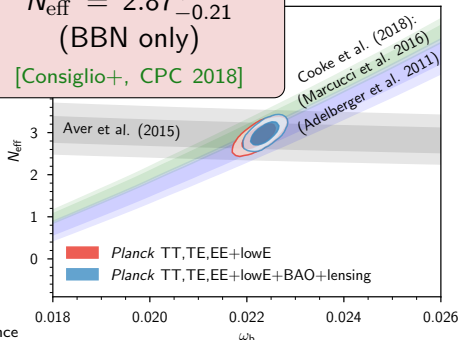
[Planck Collaboration, 2018]

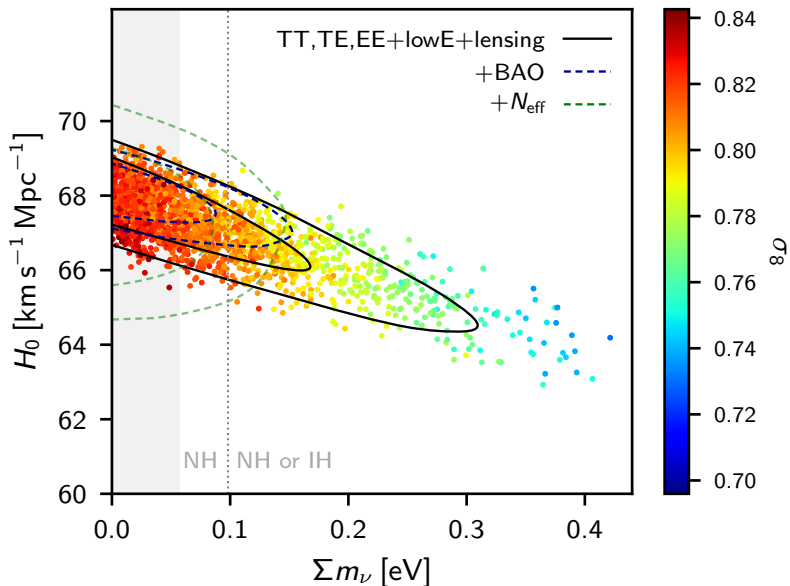


$$N_{\text{eff}} = 2.87^{+0.24}_{-0.21}$$

(BBN only)

[Consiglio+, CPC 2018]





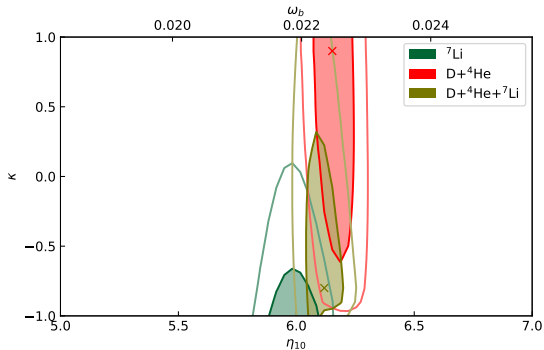
B

Bosonic neutrinos

(?!? what?)

Based on:

- JCAP 03 (2018) 050



Motivation

Neutrinos are fermions \longrightarrow they obey Fermi-Dirac statistics

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Do they obey Fermi-Dirac statistics?

No experimental confirmation of spin-statistics theorem for neutrinos!

Can we find violations of the Pauli exclusion principle?

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Can we find violations of the Pauli exclusion principle?

electrons

no violations for atomic electrons
e.g. look for anomalous X -rays from
atomic decays

[Goldhaber&Scharff-Goldhaber, 1948]

[Fischbach&Kirsten&Schaeffer, 1968]

[Reines&Sobel, 1974]

...

nucleons

no violations for protons/neutrons
e.g. look for anomalous star (Sun)
dynamics or transitions in nuclei

[Plaga, 1989]

[Miljanić+, 1990]

[Borexino, 2004]

...

see detailed discussion in [Dolgov&Smirnov, PLB 2005]

The neutrino case

important: since spin-statistics relation confirmed for electrons,
difficult to imagine large deviation for neutrinos

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for example the two-neutrino double beta decay,
 $A \rightarrow A' + 2\bar{\nu} + 2e^-$ or $A \rightarrow A' + 2\nu + 2e^+$

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Fermi-Bose parameter κ_ν [Dolgov+, JCAP 2005]

$$f_\nu(E) = \frac{1}{\exp(E/T) + \kappa_\nu}$$

“mixed”
distribution!

BE $\leftarrow \kappa_\nu = -1$ $\xleftrightarrow[\text{MB}]{\kappa_\nu = 0}$ $\kappa_\nu = +1 \rightarrow$ FD

[Barabash+, NPB 2007]: $\kappa_\nu \gtrsim -0.2$

100% violation excluded [Barabash+, NPB 2007],
but still 50% admixture of bosonic component allowed

what can cosmology say about κ_ν ?

different $f_\nu(p)$ affects BBN!

statistics factor becomes $(1 - \kappa_\nu f_\nu)$

$(1 + f_\nu) \rightarrow$ Bose enhancement,

$(1 - f_\nu) \rightarrow$ Pauli blocking

Constraints on κ_ν from BBN

[de Salas, SG+, JCAP 03 (2018) 050]

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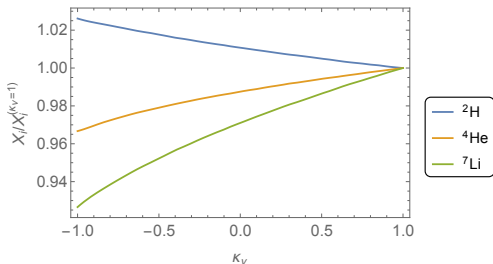
$(1 - f_\nu) \rightarrow$ Pauli blocking



change of n/p ratio at BBN

[Dolgov+, JCAP 2005]

less He, more D, less Li



deviation from $\kappa_\nu = 1$
obtained with a modified version
of PARthENoPE

[Coniglio+, CPC 2018]

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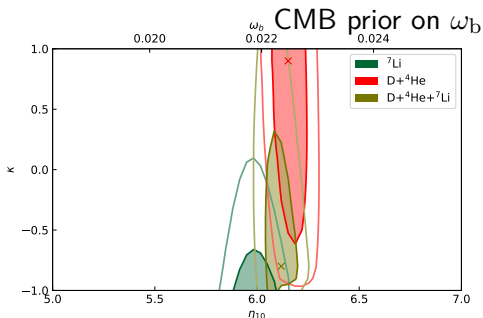
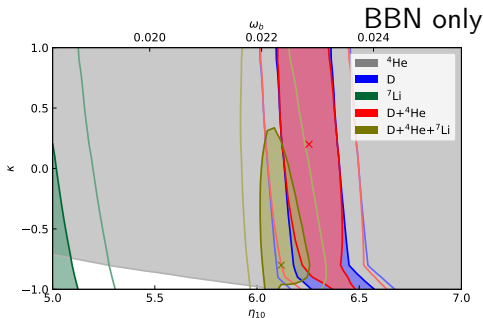
less He, more D, less Li

He or D alone cannot constrain κ_ν

Li problem drives ω_b down

and κ_ν to -1

also when prior on ω_b is included



$$f_\nu(E) = \frac{1}{\exp(E/T) + \kappa_\nu}$$

κ_ν affects

background evolution:

$$\rho_\nu^{\text{rel}} \simeq \frac{g_\nu}{2\pi^2} \int_0^\infty dp p^3 f_\nu(p)$$

bosons:

$$\frac{\pi^2}{30} g_i T^4$$

fermions:

$$\frac{7}{8} \frac{\pi^2}{30} g_i T^4$$

$$\rho_\nu^{\text{nr}} \simeq m_\nu \frac{g_\nu}{2\pi^2} \int_0^\infty dp p^2 f_\nu(p)$$

bosons:

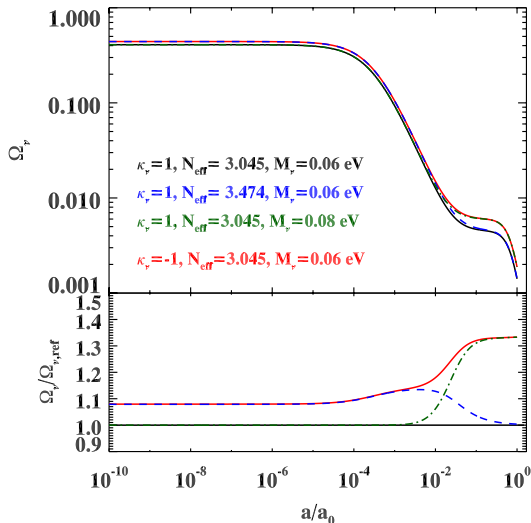
$$\frac{\zeta(3)}{\pi^2} m_\nu g_i T^3$$

fermions:

$$\frac{3}{4} \frac{\zeta(3)}{\pi^2} m_\nu g_i T^3$$

changing κ_ν “mimics” altering N_{eff} or Σm_ν (at late or early times)

partial degeneracies with N_{eff} and Σm_ν



need to cover $\kappa_\nu - \Sigma m_\nu$ degeneracy:
vary both!

degeneracy affects
mostly CMB only bounds

with BAO, bound on Σm_ν is stronger

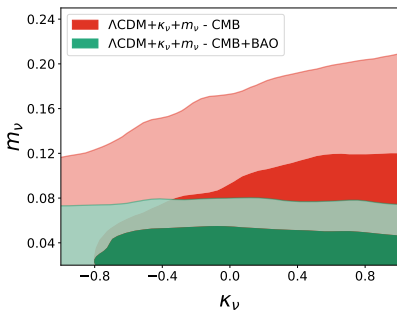
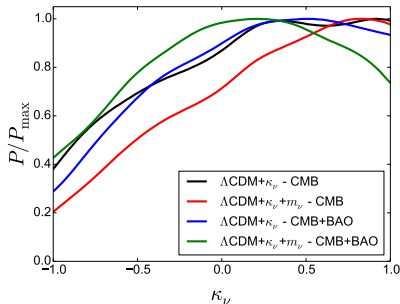
adding radiation (through κ_ν) and Ω_Λ alters
 H_0 and compensates a bit the larger mass

bounds: $\kappa_\nu \gtrsim -0.1$ at 68%

$-1 \leq \kappa_\nu \leq 1$ at 95%

$\kappa_\nu = -1$ corresponds to
 $N_{\text{eff}} \simeq 3.47$ at early times

inside Planck 2σ region!
reasonably it's not excluded

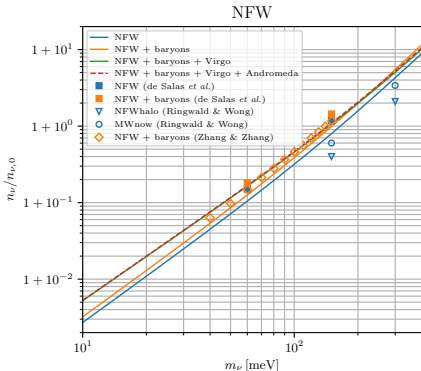


C

Clustering in the local Universe

Based on:

- JCAP 09 (2017) 034
- JCAP 01 (2020) 015



ν clustering with N-one-body simulations

Relic neutrinos are **slow!** [$c_\nu \sim 160(1+z)(1 \text{ eV}/m_\nu) \text{ km s}^{-1}$]

Can be trapped in the gravitational potential of the Milky Way and neighbours

$f_c(m_i) = n_i/n_{i,0}$ clustering factor \rightarrow How to compute it?

Idea from [Ringwald & Wong, 2004] \rightarrow **N-one-body** = $N \times$ single ν simulations

\rightarrow each ν evolved from initial conditions at $z = 3$

\rightarrow spherical symmetry, coordinates (r, θ, p_r, l)

\rightarrow need $\rho_{\text{matter}}(z) = \rho_{\text{DM}}(z) + \rho_{\text{baryon}}(z)$

Assumptions:

ν s are independent

only gravitational interactions

ν s do not influence matter evolution

$(\rho_\nu \ll \rho_{\text{DM}})$

how many ν s is "N"?

\rightarrow must sample all possible r, p_r, l

\rightarrow must include all possible ν s that reach the MW

(fastest ones may come from
several (up to $\mathcal{O}(100)$) Mpc!)

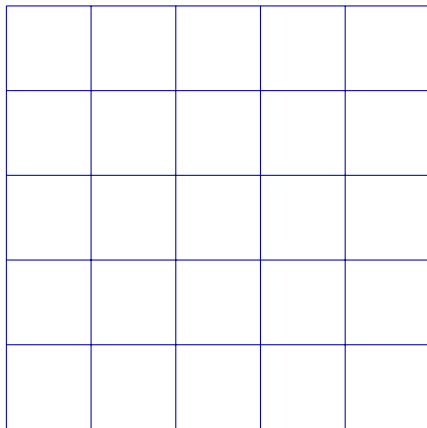
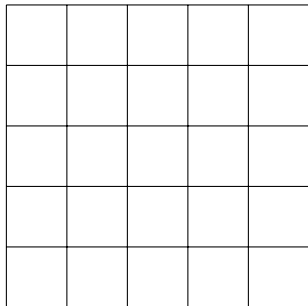
given N ν :

\rightarrow weigh each neutrinos

\rightarrow reconstruct final density profile with kernel method from [Merritt & Tremblay, 1994]

Forward-tracking and back-tracking

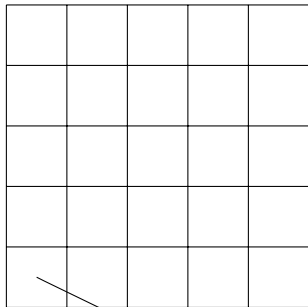
initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



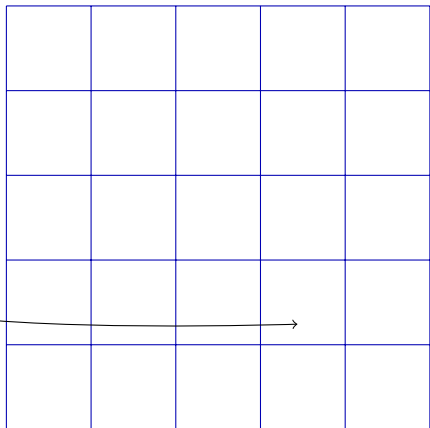
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



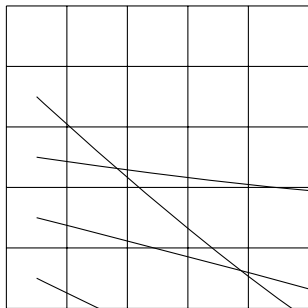
compute final position of each particle



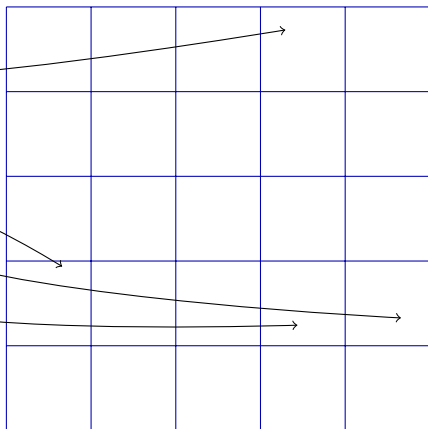
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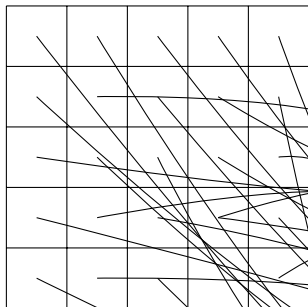
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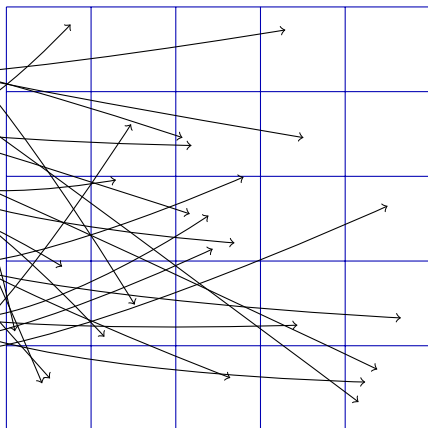
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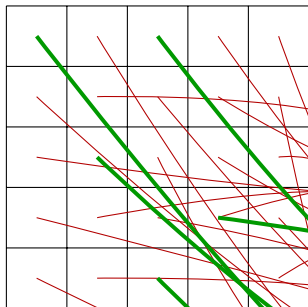
use positions to find neutrino distribution today



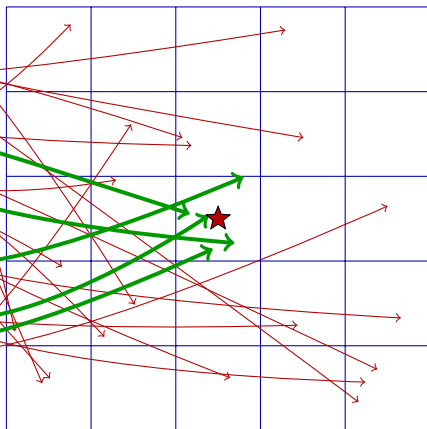
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only interested in overdensity at Earth? ★

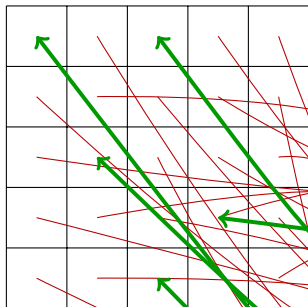


a lot of time is wasted!

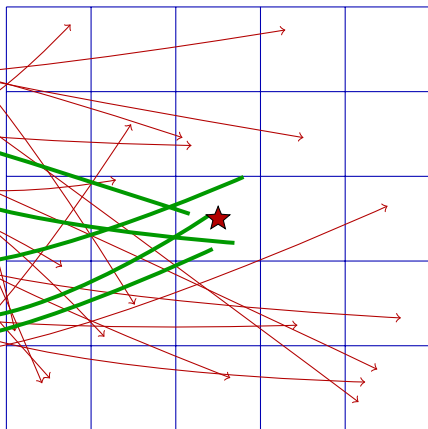
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★



a lot of time is wasted!

smarter way: track backwards
only interesting particles!

final phase space, $z = 0$

Advantages of tracking back

First advantage is in computational terms: much less points to compute

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Second advantage: no need to use spherical symmetry!

Forward-tracking

initial conditions need to sample
1D for position + 2D for momentum
when using spherical symmetry

with full grid would re-
quire 3+3 dimensions!

Impossible to relax
spherical symmetry!

Back-tracking

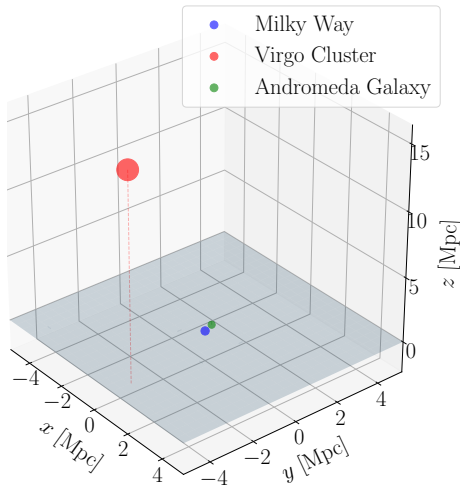
“Initial” conditions only described
by 3D in momentum
(position is fixed, apart for checks)

can do the calculation with
any astrophysical setup

Advantages of tracking back

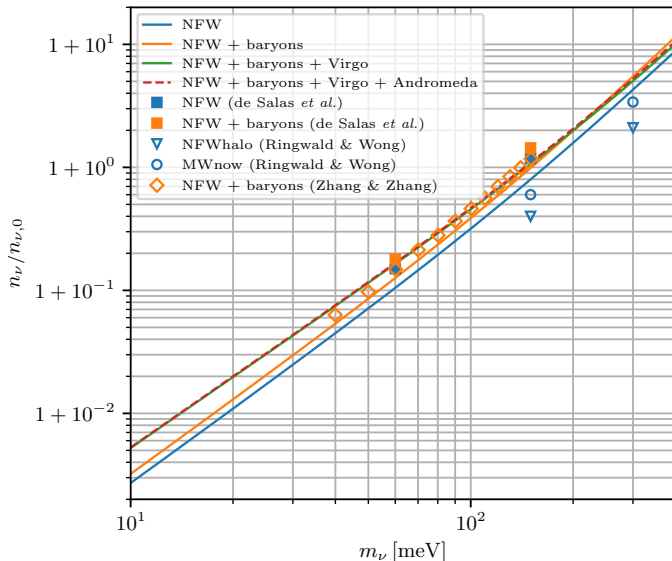
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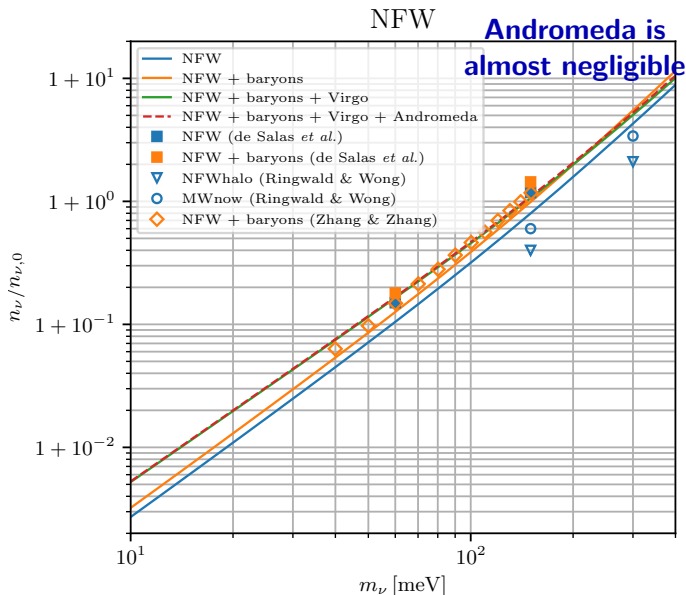


In comparison with previous results:

NFW

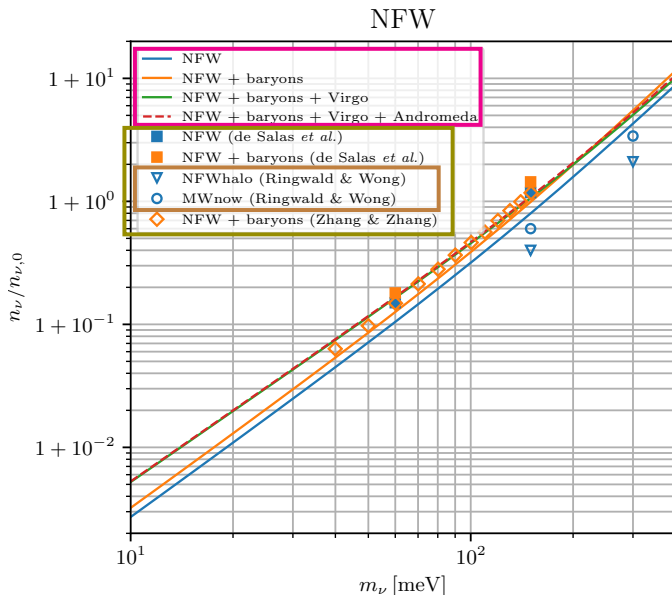


In comparison with previous results:



Clustering results with back-tracking

In comparison with previous results:



Warning: NFW
is not the same
for all the cases!

[de Salas+, 2017]

and

[Zhang², 2018]

use $\gamma \neq 1$,
now we have

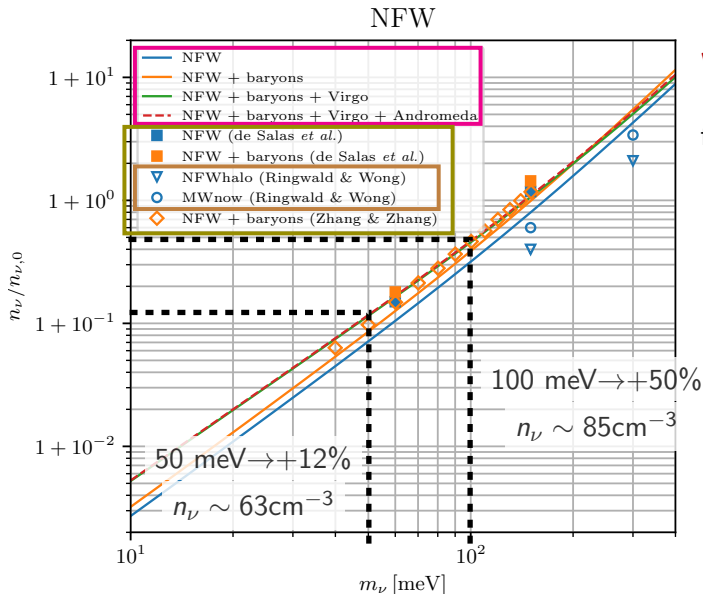
$$\gamma = 1$$

[Ringwald&Wong,

2004] uses old
parameters

Clustering results with back-tracking

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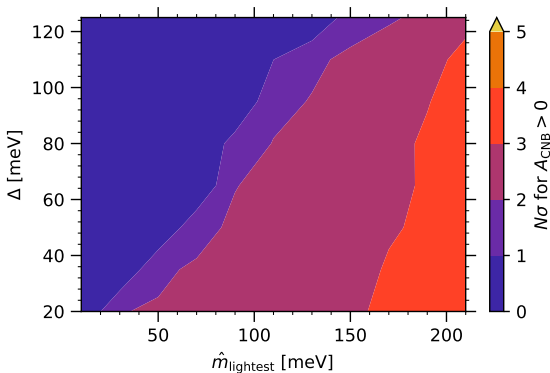
2004] uses old
parameters

D Direct Detection

i.e. currently science-fiction, but in few years...

Based on:

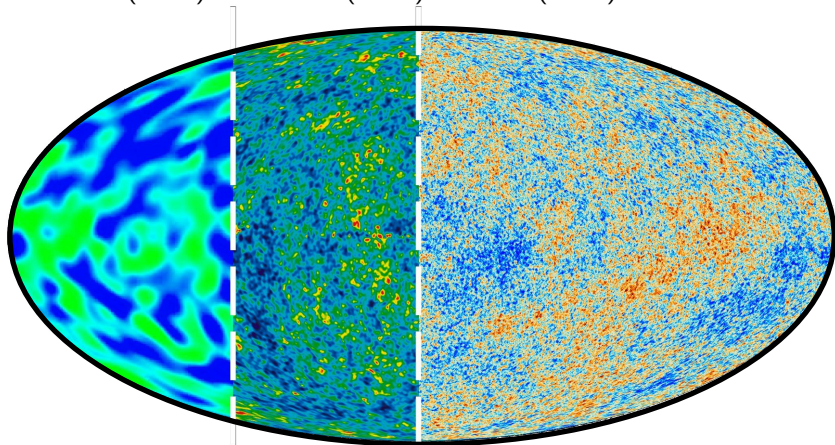
- [arxiv:1808.01892](https://arxiv.org/abs/1808.01892)
- JCAP 07 (2019) 047



The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

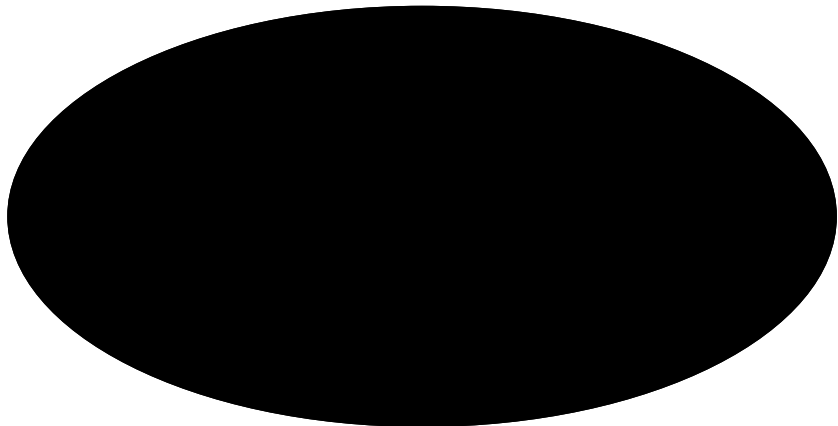
COBE (1992) WMAP (2003) Planck (2013)



The oldest picture of the Universe

The Cosmic Neutrino Background, generated at $t \simeq 1$ s

... → 2022 → ...



How to directly detect non-relativistic neutrinos?

Remember that
 $\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today



a process without energy
 threshold is necessary

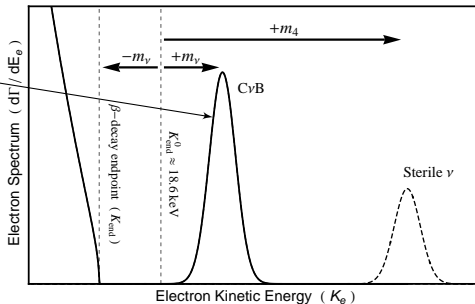
[Weinberg, 1962]: neutrino capture in β -decaying nuclei $\nu + n \rightarrow p + e^-$

Main background: β decay $n \rightarrow p + e^- + \bar{\nu}$!

signal is a peak at $2m_\nu$
 above β -decay endpoint

only with a lot of material

need a very good energy resolution



$$\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi\sigma}} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2\sigma^2}}$$

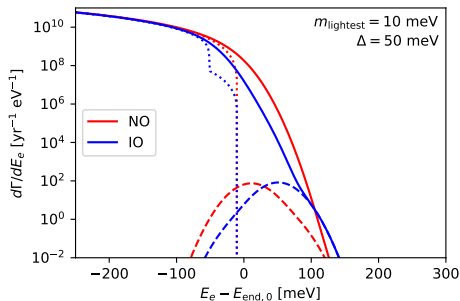
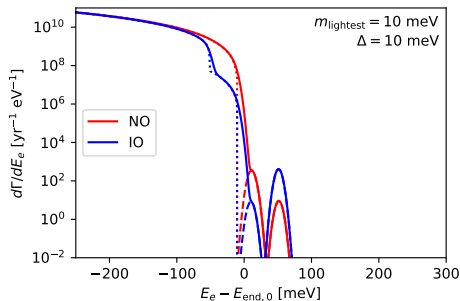
$$\frac{d\Gamma_\beta}{dE_e} = \frac{\bar{\sigma}}{\pi^2} N_T \sum_{i=1}^{N_\nu} |U_{ei}|^2 H(E_e, m_i)$$

$$\frac{d\tilde{\Gamma}_\beta}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} dx \frac{d\Gamma_\beta}{dE_e}(x) \exp\left[-\frac{(E_e - x)^2}{2\sigma^2}\right]$$

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Pontecorvo Tritium Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1 \text{ eV?}$
 0.05 eV?

can probe $m_\nu \simeq 1.4\Delta \simeq 0.1 \text{ eV}$

built mainly for CNB

$M_T = 100 \text{ g}$ of atomic ${}^3\text{H}$

$$\Gamma_{\text{CNB}} = \sum_{i=1}^3 |U_{ei}|^2 [n_i(\nu_{hR}) + n_i(\nu_{hL})] N_T \bar{\sigma} \sim \mathcal{O}(10) \text{ yr}^{-1}$$

N_T number of ${}^3\text{H}$ nuclei in a sample of mass M_T $\bar{\sigma} \simeq 3.834 \times 10^{-45} \text{ cm}^2$ n_i number density of neutrino i

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enhancement from
 ν clustering in the galaxy?

enhancement from
 other effects?

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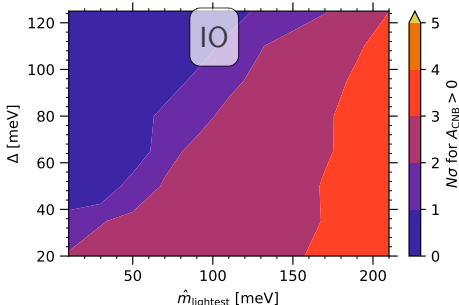
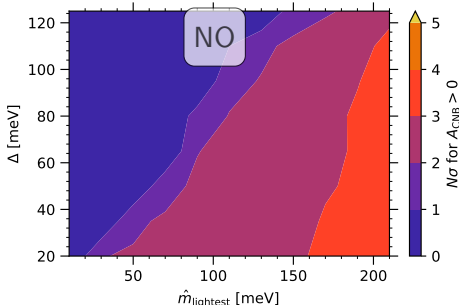
using the definition:

$$N_{\text{th}}^i(\theta) = A_{\beta} N_{\beta}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + \mathbf{A}_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + N_b$$

if $\mathbf{A}_{\text{CNB}} > 0$ at $N\sigma$, direct detection of CNB accomplished at $N\sigma$

statistical only!

significance on $A_{\text{CNB}} > 0$
as a function of $\hat{m}_{\text{lightest}}$, Δ





(skipping. . .)

seriously, I cannot go
through the entire alphabet in 50 minutes!

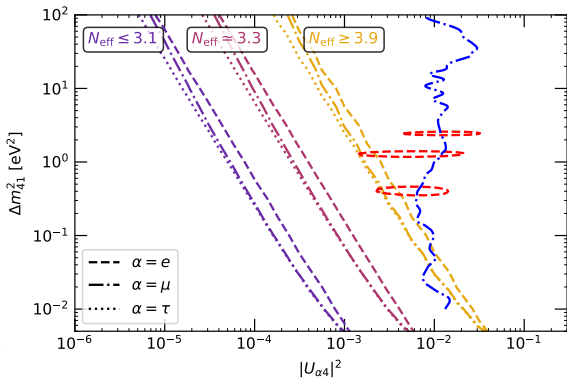
S

(Light) Sterile neutrinos

let's pretend they exist

Based on:

- JPG 43 (2016) 033001
- JHEP 06 (2017) 135
- PLB 782 (2018) 13-21
- in preparation
- JCAP 07 (2019) 014
- PRD 104 (2021) 123524
- JCAP 07 (2019) 047



Problem: **anomalies**
in SBL experiments

→ { errors in flux calculations?
deviations from 3- ν description?

A short review:

LSND search for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, with $L/E = 0.4 \div 1.5$ m/MeV. Observed a 3.8σ excess of $\bar{\nu}_e$ events [Aguilar et al., 2001]

Reactor re-evaluation of the expected anti-neutrino flux \Rightarrow disappearance of $\bar{\nu}_e$ events compared to predictions ($\sim 3\sigma$) with $L < 100$ m [Mention et al, 2011], [Azabajan et al, 2012]

Gallium calibration of GALLEX and SAGE Gallium solar neutrino experiments give a 2.7σ anomaly (disappearance of ν_e) [Giunti, Laveder, 2011]

MiniBooNE

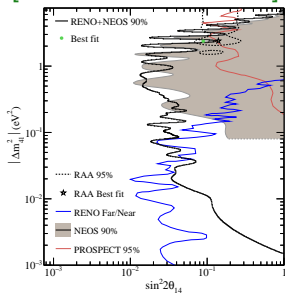
See next

Possible explanation:

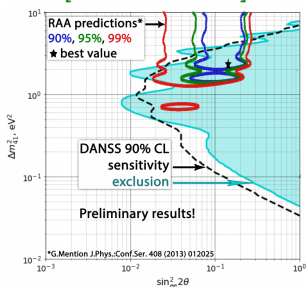
Additional squared mass
difference $\Delta m_{\text{SBL}}^2 \simeq 1 \text{ eV}^2$

ν_s at reactors in 2020

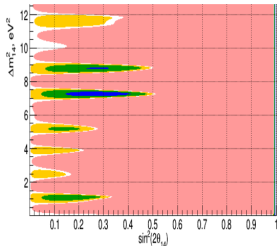
[RENO+NEOS, 2020]



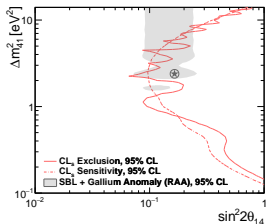
[DANSS, 2020]



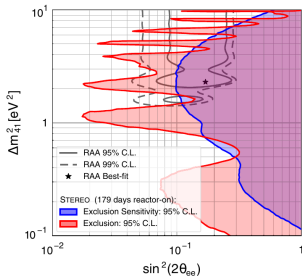
[Neutrino-4, PZETF 2020]



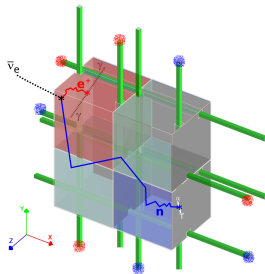
[PROSPECT, PRD 2020]

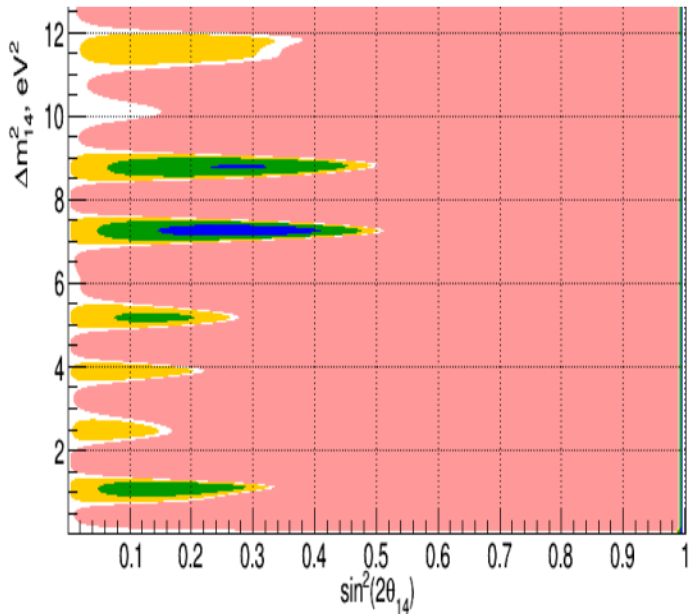


[STEREO, PRD 2020]



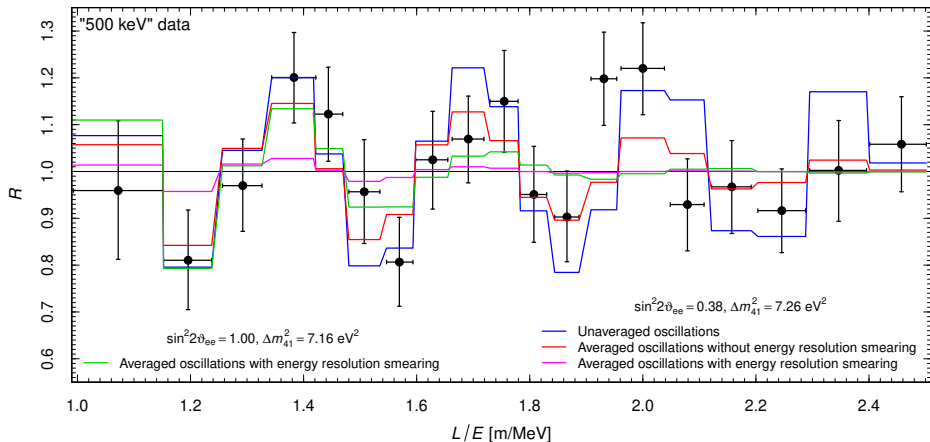
[SoLiD, JINST 2021]



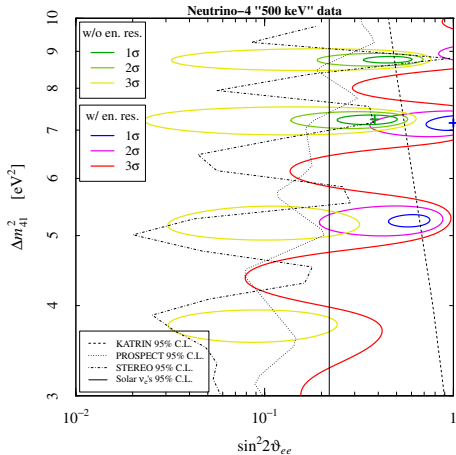


claimed $> 3\sigma$
preference for
 $3+1$ over 3ν case

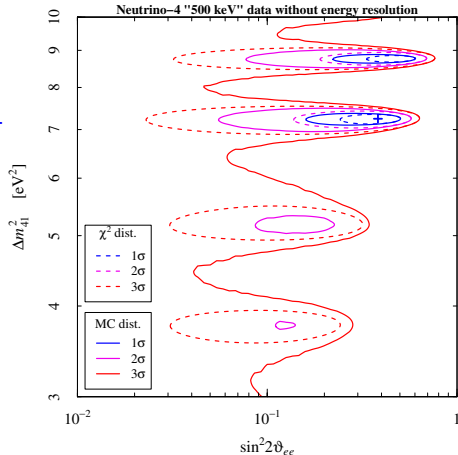
best fit
incompatible
with other
reactor
experiments



energy resolution smearing not properly taken into account?



proper energy resolution treatment
moves best-fit $\rightarrow \sin^2 2\vartheta \simeq 1$



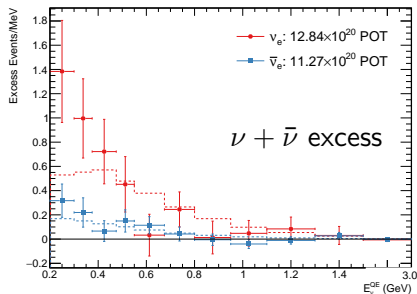
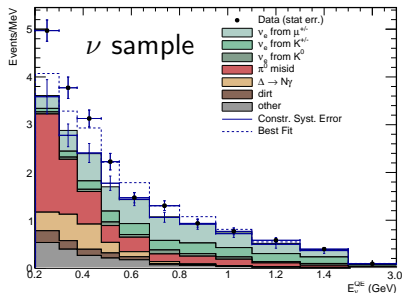
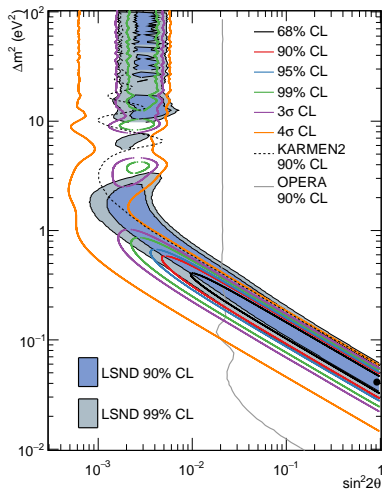
need to take into account
violation of Wilk's theorem

↓
relaxed constraints

purpose: check LSND signal

$L \simeq 541$ m, $200 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$

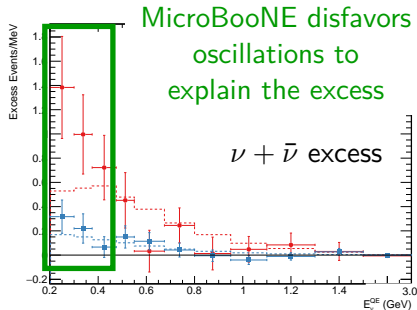
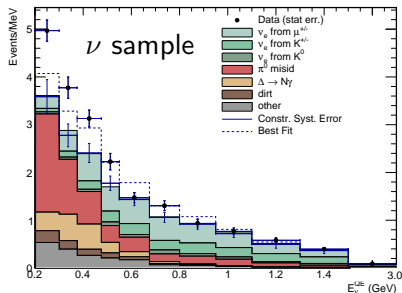
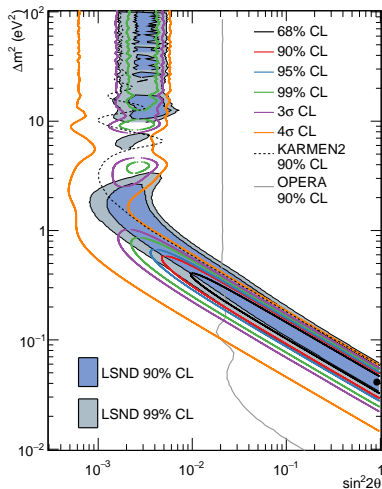
no money, no near detector

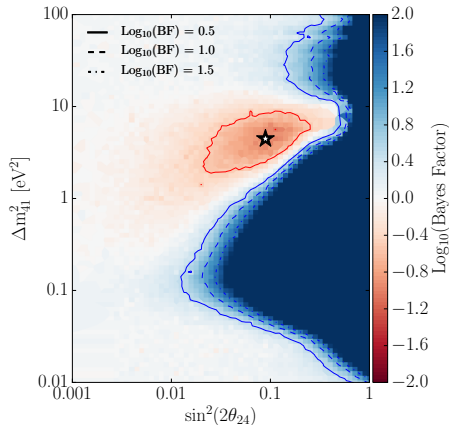
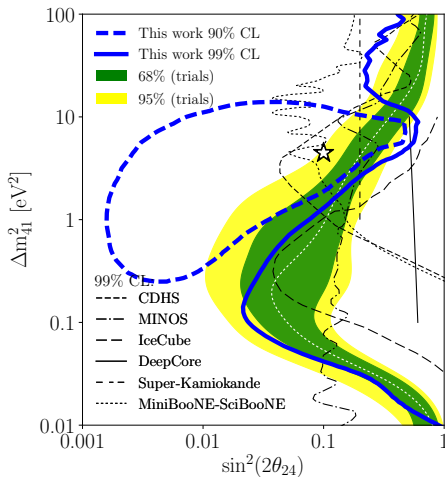


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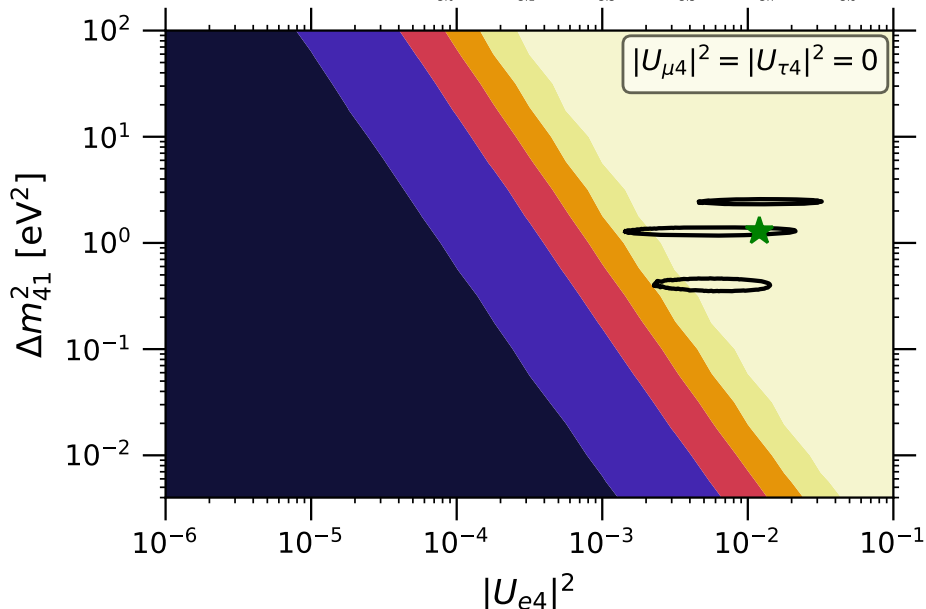
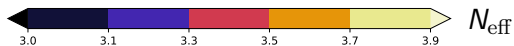


first indication in favor of sterile from ν_μ DIS!

although rather weak: $\log_{10} BF \simeq 1$ (weak preference)
 or compatible with no oscillations at p -value of 8%

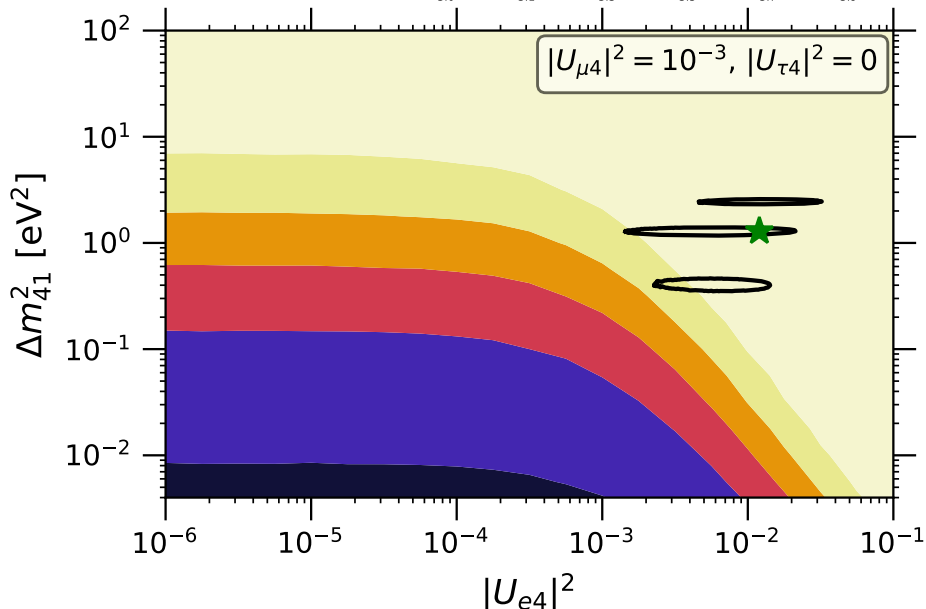
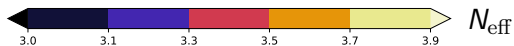
N_{eff} and the new mixing parameters

We can vary more than one angle:



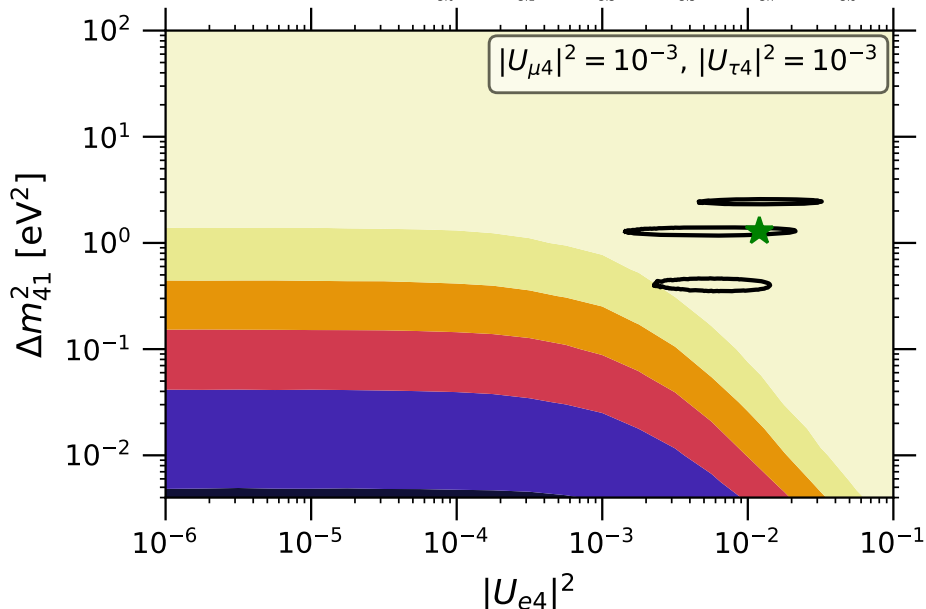
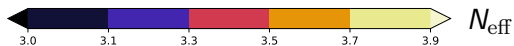
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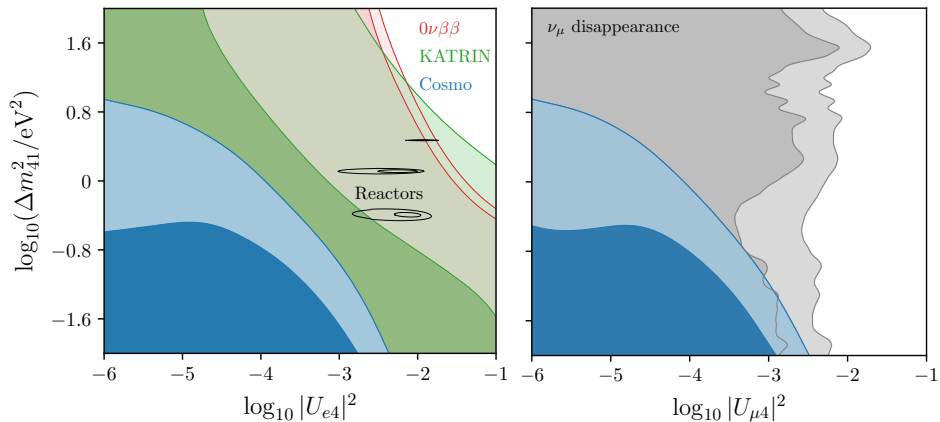
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Cosmological constraints are stronger than most other probes

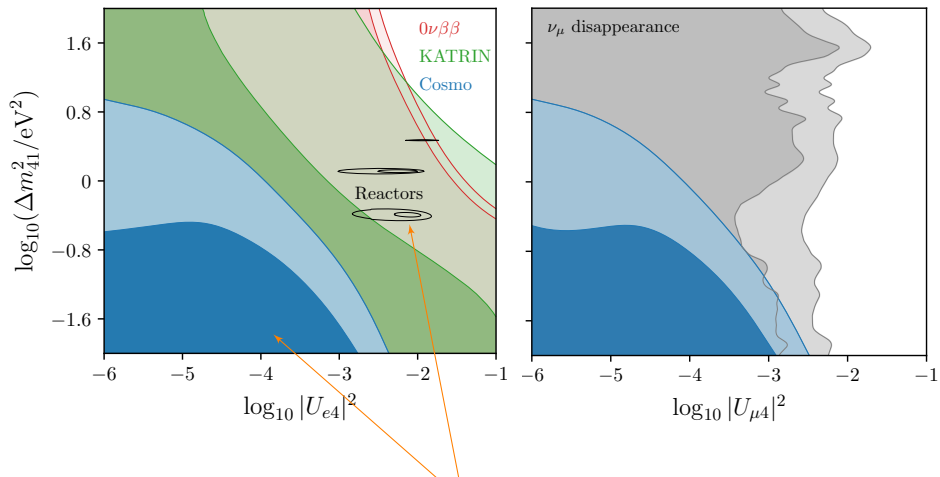
But much more model dependent (as all the cosmological constraints)!



Comparing constraints

Cosmological constraints are stronger than most other probes

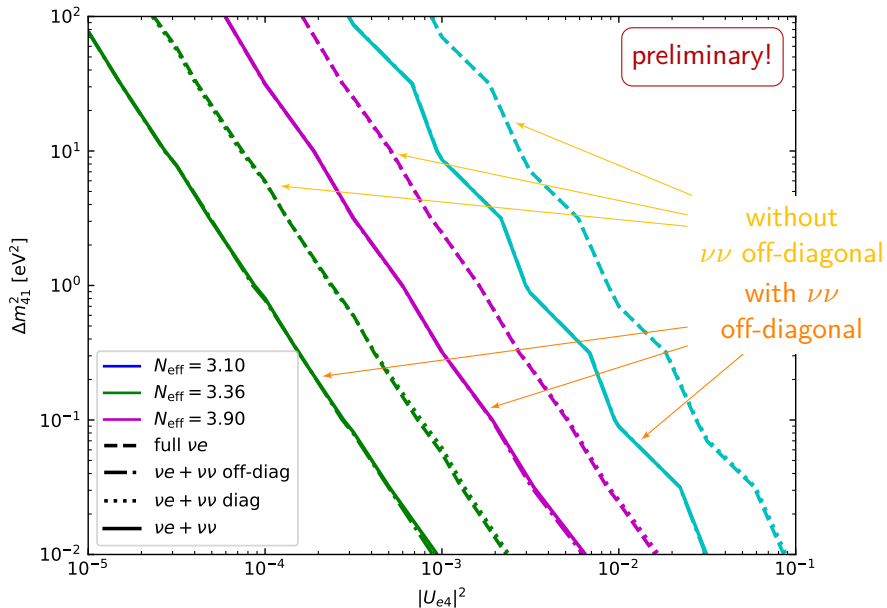
But much more model dependent (as all the cosmological constraints)!



Warning: tension between reactor experiments and CMB bounds!

Collision terms with sterile neutrinos

Full collision integrals can be computed also with sterile neutrinos



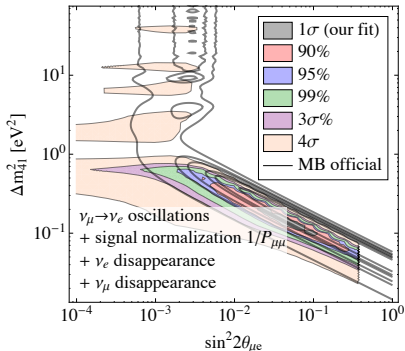
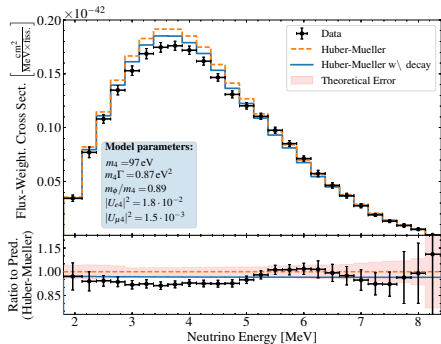
Can new physics solve the anomalies and tensions?

Many attempts to explain LSND/MiniBooNE anomalies,
 APP vs DIS, oscillations vs cosmo tensions with new physics

one recent example: [Dentler+, 2019]

$$\mathcal{L} \supset -g\bar{\nu}_s\nu_s\phi \quad \text{with } \mathcal{O}(\text{eV}) \lesssim m_4 \lesssim \mathcal{O}(100 \text{ keV}) \text{ and } m_\phi \lesssim m_4$$

new interactions with scalar ϕ and ν_s decay



see also: [de Gouvea+, 2019], [Moulay+, 2019], [Fischer+, 2019], [Diaz+, 2019], ...

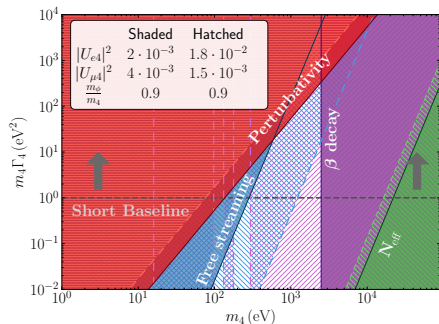
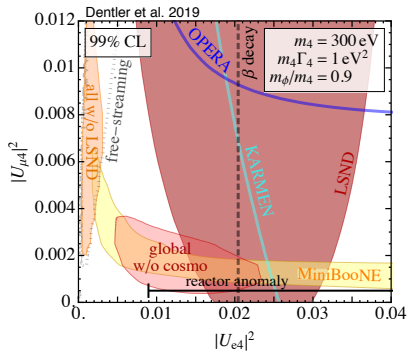
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$$\mathcal{L}_{\text{CC-NSI}} = -2\sqrt{2}G_F\epsilon_{\alpha\beta}^{ff'C}[\bar{\nu}_\beta\gamma^\rho P_L\ell_\alpha] [\bar{f}'\gamma_\rho P_C f]$$

Non-standard interactions (NSI) involving ν_s

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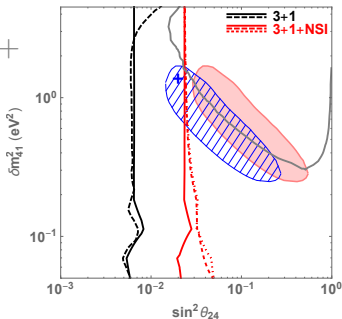
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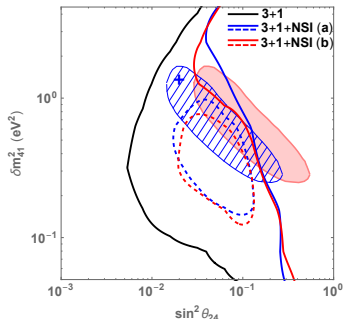
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Non-standard interactions (NSI) involving ν_s

MINOS+
vs APP



IceCube/
DeepCore
vs APP



Scenarios with low reheating temperature

Reheating: phase ending inflation

during inflation, the inflaton (non-rel. scalar) dominates the energy density

during reheating: inflaton decays into standard model particles

⇒ photons, electrons, ... are populated directly

radiation domination begins after reheating

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Low reheating temperature: when reheating occurs at $T_{\text{rh}} \lesssim 20$ MeV

notice: if $T_{\text{rh}} \lesssim 3$ MeV, BBN is broken!

3 neutrino oscillations start to be affected when $T_{\text{rh}} \lesssim 8$ MeV

what about sterile neutrinos?

N_{eff} with low reheating

Need to edit equations for **inflaton** energy density and its contribution:

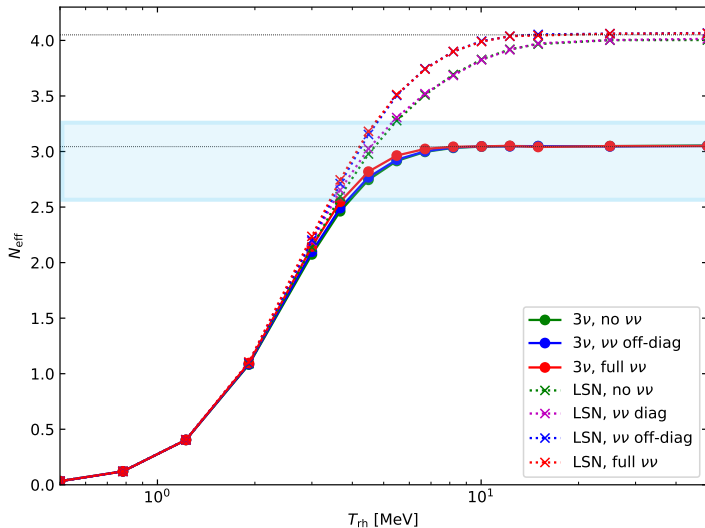
$$\frac{d\rho(y)}{dx} = \text{unchanged}$$

$$\frac{d\rho_\phi}{dx} = -\frac{x\rho_\phi\Gamma_\phi}{m_e^2} \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_{\text{tot}}}}$$

$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J_2(r_\ell) \right] + G_1(r) - \frac{1}{2z^3} \sum_{\alpha=e}^s \frac{d\rho_{\nu_\alpha}}{dx} - \frac{x}{2z^3} \frac{d\rho_\phi}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J_2(r_\ell) + J_4(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

$$\rho_{\text{tot}} = \sum_{i=\gamma,\nu_j,e,\mu} \rho_i + \delta\rho(x,z) + x\rho_\phi$$

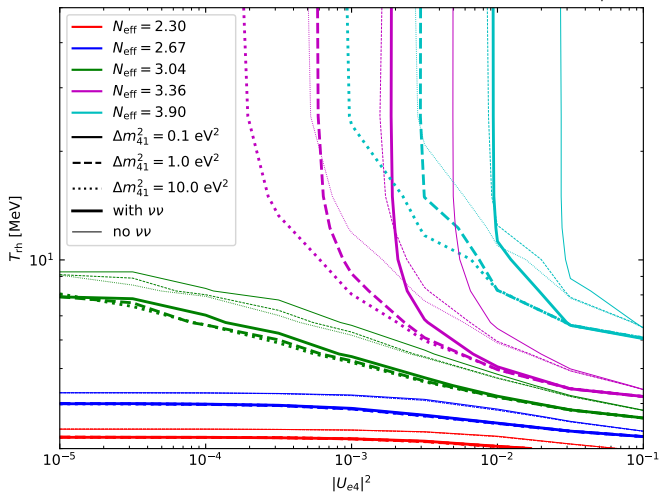
$$\Gamma_\phi \simeq \left(\frac{T_{\text{rh}}}{0.7\text{MeV}} \right)^2 \text{sec}^{-1}$$

N_{eff} with low reheating N_{eff} as a function of T_{rh} (3 or 3+1 neutrinos):

Planck constraint: $N_{\text{eff}} = 2.92^{+0.36}_{-0.37}$ (95%, TT, TE, EE+lowE)

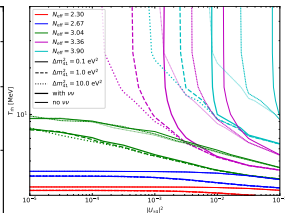
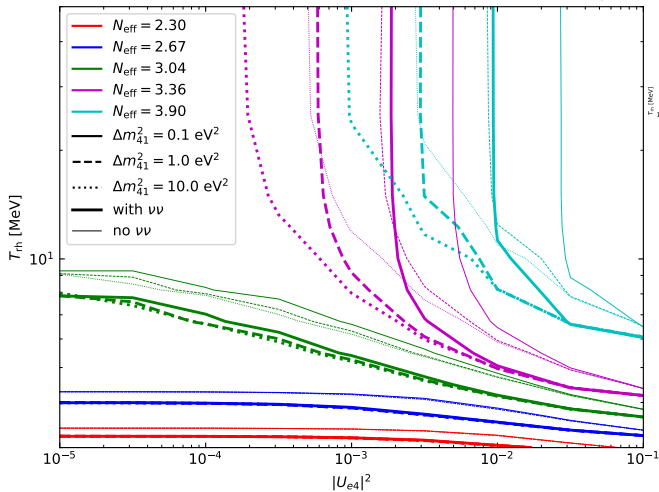
N_{eff} with low reheating

sterile case with varying mixing angle/mass splitting:

for low T_{rh} , mixing parameters are irrelevantfor higher Δm_{41}^2 , T_{rh} has more impact

N_{eff} with low reheating

sterile case with varying mixing angle/mass splitting:



almost the
same for $|U_{\tau 4}|^2$

for low T_{rh} , mixing parameters are irrelevant

for higher Δm_{41}^2 , T_{rh} has more impact



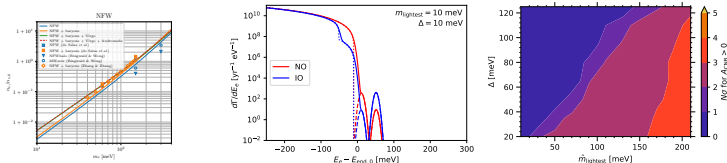
Z

Conclusions

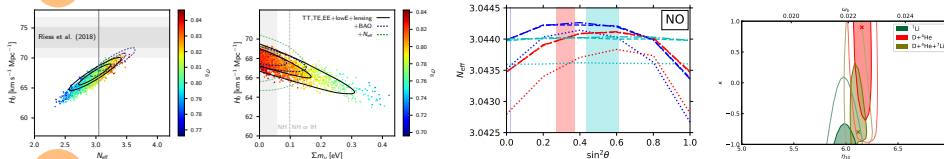
almost there!

What do we learn from relic neutrinos?

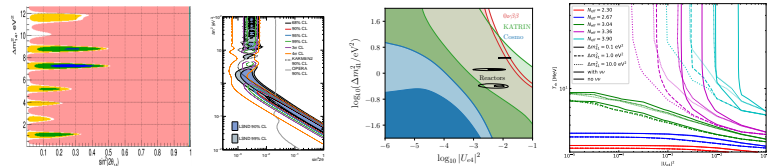
D Direct detection - wonderful opportunities for the future



I Indirect probes - what we have now, it's a lot and it will improve

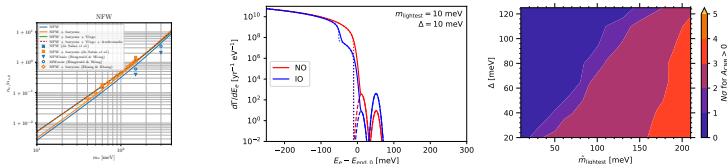


N New physics - beyond the corner? neutrinos will help us find it!

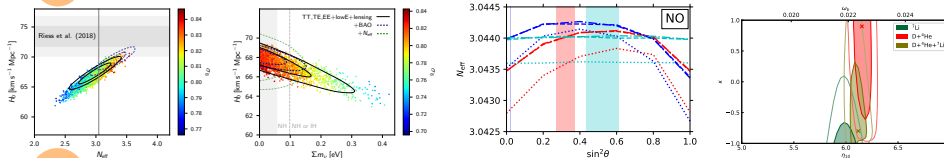


What do we learn from relic neutrinos?

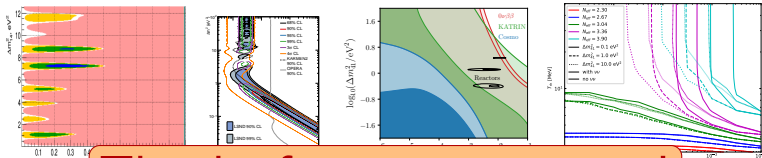
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Thanks for your attention!