H2020 MSCA COFUND
G.A. 754496

Stefano Gariazzo

INFN, Turin section
Turin (IT)

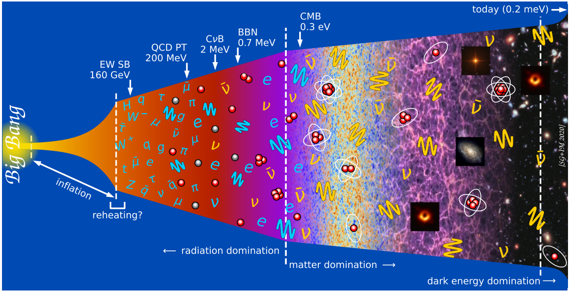
gariazzo@to.infn.it
http://personalpages.to.infn.it/~gariazzo/

Introduction on neutrino cosmology

Applications of Quantum Information in Astrophysics and Cosmology,
Cape Town (ZA), 25/04/2023

Outline

Universe history



A flash on general relativity

use metric $g_{\mu\nu}$ to define measure: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

x^μ coordinates, $u^\mu = \frac{dx^\mu}{d\lambda}$ velocity, $P^\mu = m u^\mu$ momentum

short notation for derivatives: $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

A flash on general relativity

use metric $g_{\mu\nu}$ to define measure: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

x^μ coordinates, $u^\mu = \frac{dx^\mu}{d\lambda}$ velocity, $P^\mu = mu^\mu$ momentum

short notation for derivatives: $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

Christoffel symbols (not tensors!): $\Gamma^\mu_{\nu\rho} = \frac{g^{\mu\sigma}}{2} \left(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho}\right)$

Ricci tensor: $R_{\mu\nu} = \partial_\sigma \Gamma^\sigma_{\mu\nu} - \partial_\nu \Gamma^\sigma_{\mu\sigma} + \Gamma^\sigma_{\rho\sigma}\Gamma^\rho_{\mu\nu} - \Gamma^\sigma_{\rho\nu}\Gamma^\rho_{\mu\sigma}$

Ricci scalar: $R = R_{\mu\nu} g^{\mu\nu}$

Einstein equations:

$$\boxed{G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}}$$

$T_{\mu\nu}$ stress-energy tensor, symmetric, must satisfy $\nabla_\mu T^{\mu\nu} = 0$

A flash on general relativity

use metric $g_{\mu\nu}$ to define measure: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

x^μ coordinates, $u^\mu = \frac{dx^\mu}{d\lambda}$ velocity, $P^\mu = m u^\mu$ momentum

short notation for derivatives: $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

Christoffel symbols (not tensors!): $\Gamma^\mu_{\nu\rho} = \frac{g^{\mu\sigma}}{2} \left(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho}\right)$

Ricci tensor: $R_{\mu\nu} = \partial_\sigma \Gamma^\sigma_{\mu\nu} - \partial_\nu \Gamma^\sigma_{\mu\sigma} + \Gamma^\sigma_{\rho\sigma}\Gamma^\rho_{\mu\nu} - \Gamma^\sigma_{\rho\nu}\Gamma^\rho_{\mu\sigma}$

Ricci scalar: $R = R_{\mu\nu} g^{\mu\nu}$

Einstein equations:

$$\boxed{G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}}$$

$T_{\mu\nu}$ stress-energy tensor, symmetric, must satisfy $\nabla_\mu T^{\mu\nu} = 0$

given lagrangian $\mathcal{L}(\phi_\alpha) \longrightarrow T^{\mu\nu}_{(\phi_\alpha)} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_\alpha)} \partial^\nu \phi_\alpha - g^{\mu\nu} \mathcal{L}$

ϕ_α set of fields

Homogeneous and isotropic universe

Metric defines the structure of the universe

One of the simplest assumptions: universe is

Homogeneous	Isotropic
universe properties do not change with position	universe properties do not change with direction

Friedmann-Lemaître-Robertson-Walker (FLRW) metric (polar coordinates):

$$g_{\mu\nu}dx^\mu dx^\nu \; = \; -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

a scale factor, encodes expansion of the space-time

k spatial curvature of the universe (0→flat, ±1 → curved)

Homogeneous and isotropic universe

Metric defines the structure of the universe

One of the simplest assumptions: universe is

Homogeneous

universe properties do
not change with position

Isotropic

universe properties do
not change with direction

FLRW metric using conformal time $\eta = \int \frac{dt}{a(t)}$:

$$g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left(-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right)$$

a scale factor, encodes expansion of the space-time

k spatial curvature of the universe (0→flat, ±1 → curved)

Homogeneous and isotropic universe

Metric defines the structure of the universe

One of the simplest assumptions: universe is

Homogeneous

universe properties do not change with position

Isotropic

universe properties do not change with direction

FLRW metric using conformal time $\eta = \int \frac{dt}{a(t)}$:

$$g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left(-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

Perfect fluid (energy density ρ, pressure P): $T^{\mu\nu} = (\rho + P) u^\mu u^\nu - P g^{\mu\nu}$

in fluid rest frame: $u^\mu = (1,0,0,0) \longrightarrow T^0_0 = -\rho \qquad T^i_j = P\delta^i_j$

Homogeneous and isotropic universe

Metric defines the structure of the universe

One of the simplest assumptions: universe is

Homogeneous

universe properties do
not change with position

Isotropic

universe properties do
not change with direction

FLRW metric using conformal time $\eta = \int \frac{dt}{a(t)}$:

$$g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta)\left(-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

Perfect fluid (energy density ρ, pressure P): $T^{\mu\nu} = (\rho + P)u^\mu u^\nu - Pg^{\mu\nu}$

Use Einstein equations to obtain **Friedmann equations**:

$$\boxed{H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}} \qquad \boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}}$$

expansion rate $H \equiv \dot{a}/a$

expansion depends on universe content!

Background evolution of the universe

conservation of stress-energy tensor:

$$\nabla_\mu T^{\nu\mu} \;\equiv\; \partial_\mu T^{\nu\mu} + \Gamma^\nu_{\mu\rho} T^{\mu\rho} + \Gamma^\mu_{\mu\rho} T^{\nu\rho} \;=\; 0$$

for a perfect fluid this leads to continuity equation:

$$\boxed{\dot\rho + 3H(\rho + P) = 0}$$

define w equation of state, so that $P = w\rho$:

continuity equation solved by $\rho(a) \;=\; a^{-3(1+w)}$

radiation	matter	Λ
(relativistic fluid)	(non-rel. fluid)	(dark energy)
$w = 1/3$, $\rho_\mathrm{R} \propto a^{-4}$	$w = 0$, $\rho_\mathrm{M} \propto a^{-3}$	$w = -1$, $\rho_\Lambda = const$

Background evolution of the universe

conservation of stress-energy tensor:

$$\nabla_\mu T^{\nu\mu} \quad \equiv \quad \partial_\mu T^{\nu\mu} + \Gamma^\nu_{\mu\rho} T^{\mu\rho} + \Gamma^\mu_{\mu\rho} T^{\nu\rho} \quad = \quad 0$$

for a perfect fluid this leads to continuity equation:

$$\boxed{\dot\rho + 3H(\rho + P) = 0}$$

define w equation of state, so that $P = w\rho$:

continuity equation solved by $\rho(a) \quad = \quad a^{-3(1+w)}$

radiation	matter	Λ
(relativistic fluid)	(non-rel. fluid)	(dark energy)
$w = 1/3$, $\rho_\mathrm{R} \propto a^{-4}$	$w = 0$, $\rho_\mathrm{M} \propto a^{-3}$	$w = -1$, $\rho_\Lambda = const$

Consider Friedmann equation: $H^2 = \dfrac{8\pi G}{3}\rho_\mathrm{tot} + \dfrac{\Lambda}{3} - \dfrac{k}{a^2}$

$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G}$

$\rho_k \equiv \frac{3k}{8\pi G a^2}$

If one component dominates ($\rho_\mathrm{tot} \simeq \rho_i$, with $i \in [\mathrm{R}, \mathrm{M}, k, \Lambda, \ldots]$), we have:

$a(t) \;=\; t^{2/(3(1+w))}$ for $w \;\neq\; -1$

$a(t) = e^{Ht}$ for $w = -1$

Background evolution of the universe

conservation of stress-energy tensor:

$$\nabla_\mu T^{\nu\mu} \equiv \partial_\mu T^{\nu\mu} + \Gamma^{\nu}_{\mu\rho} T^{\mu\rho} + \Gamma^{\mu}_{\mu\rho} T^{\nu\rho} = 0$$

for a perfect fluid this leads to continuity equation:

$$\boxed{\dot{\rho} + 3H(\rho + P) = 0}$$

define w equation of state, so that $P = w\rho$:

continuity equation solved by $\rho(a) = a^{-3(1+w)}$

radiation
(relativistic fluid)
$w = 1/3$, $\rho_{\rm R} \propto a^{-4}$

matter
(non-rel. fluid)
$w = 0$, $\rho_{\rm M} \propto a^{-3}$

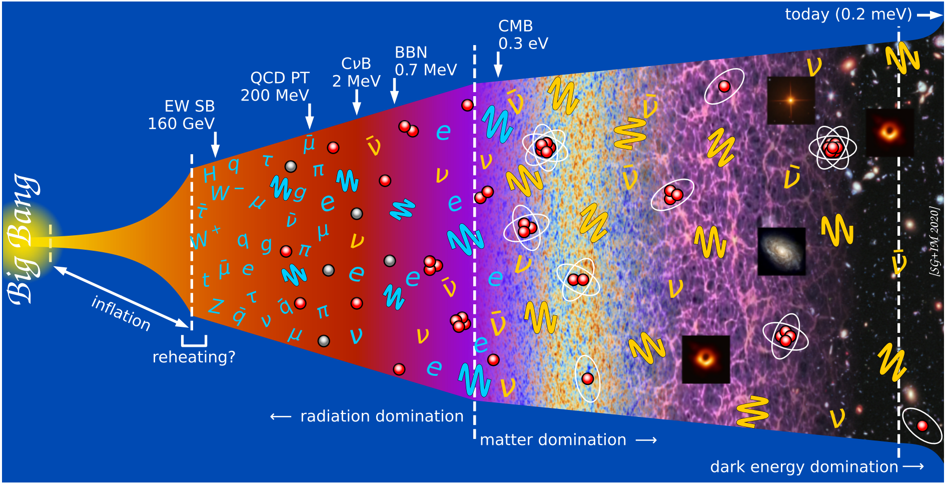
Λ
(dark energy)
$w = -1$, $\rho_\Lambda = const$

define critical density: $\rho_{\rm cr} \equiv \dfrac{3H^2}{8\pi G}$

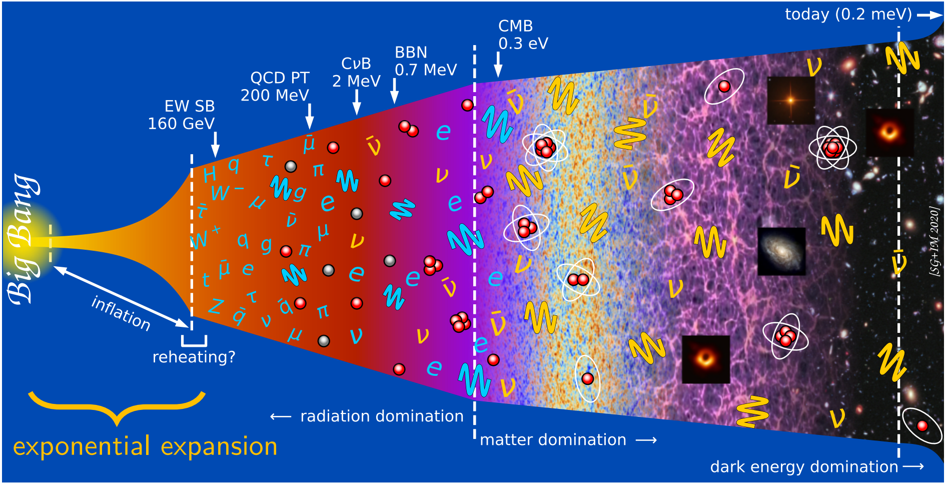
define fractional energy densities: $\Omega_i = \rho_{i,0}/\rho_{\rm cr,0}$ 0→today

Friedmann equation: $H(a)^2/H_0^2 = \Omega_{\rm R} a^{-4} + \Omega_{\rm M} a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$

Short history of the universe



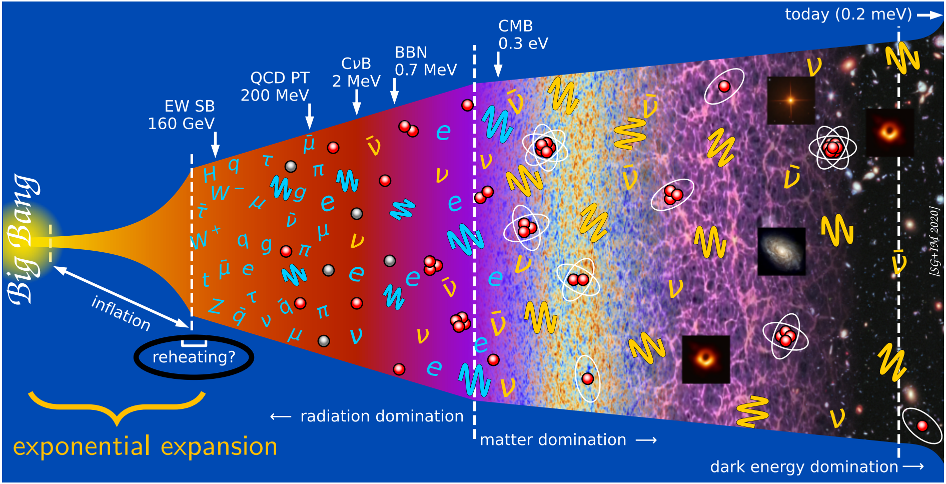
Short history of the universe



exponential expansion achieved with $w = -1$ $\longrightarrow$ by scalar field? inflaton

inflation is needed to solve: flatness problem horizon problem

Short history of the universe

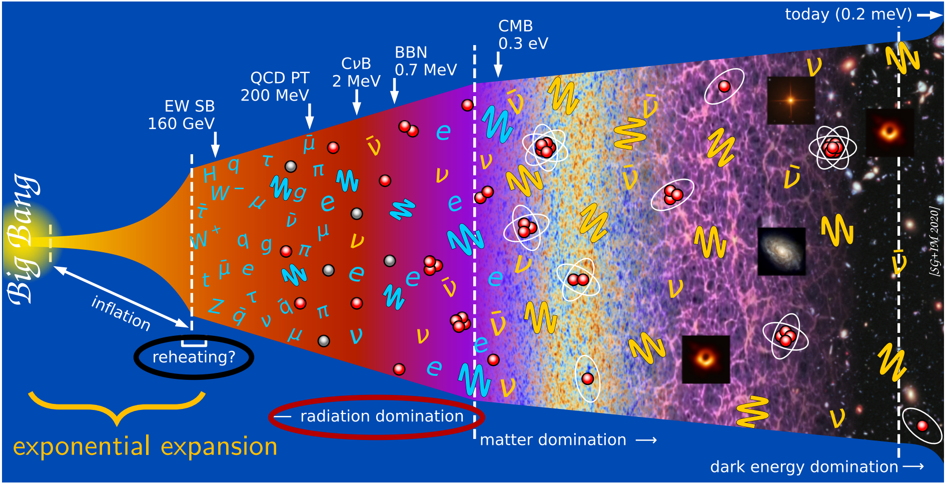


Reheating: inflation ends with energy transfer

from inflaton to (relativistic) standard model particles

which later reach thermal equilibrium

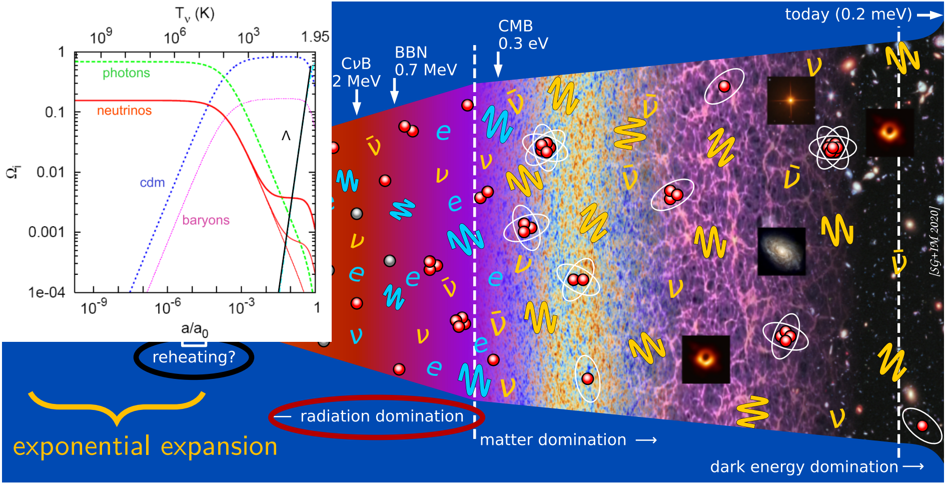
Short history of the universe



after **reheating**, relativistic particles (= radiation) start to dominate

while temperature decreases, several particles become non relativistic

Short history of the universe

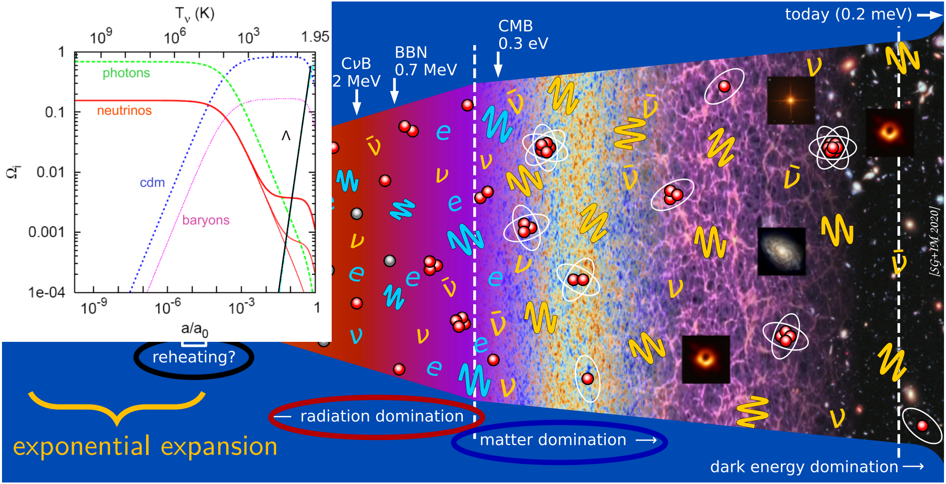


after **reheating**, relativistic particles (= radiation) start to dominate

while temperature decreases, several particles become non relativistic

last particles to remain in equilibrium are photons, electrons, neutrinos

Short history of the universe

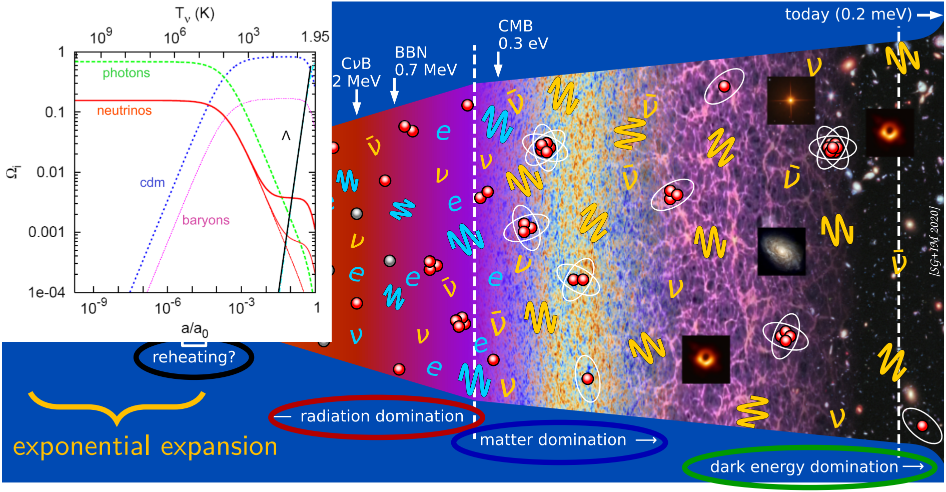


at some point, $\Omega_{\rm R}\, a^{-4}$ becomes smaller than $\Omega_{\rm M}\, a^{-3}$

matter domination!

gravity start to be stronger than radiation pressure ⟶ growth of structures!

Short history of the universe



Finally, $\Omega_\mathrm{M} a^{-3}$ becomes smaller than Ω_Λ

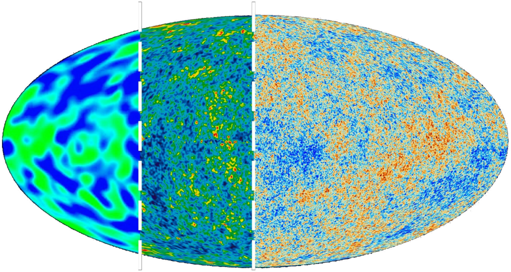
dark energy domination!

expansion starts to (exponentially) accelerate again

C Cosmic Microwave Background

Based on:

- Lesgourgues+, Neutrino Cosmology
- Planck Collaboration, 2018



Photon decoupling

Photons in equilibrium have $f_\gamma(q) = [\exp(q/T) - 1]^{-1}$

T fluid/photon temperature, q photon momentum

while electrons (e) are free, γ scatter and cannot move freely

when e and protons (p) form H atoms, γs can break atomic bound

H binding energy: $B_{\rm H} = m_e + m_p - m_{\rm H} \simeq 13.6$ eV

γs start to move freely when they cannot break H bound anymore

Notice: this depends on photon momentum distribution!

Photon decoupling

Photons in equilibrium have $f_\gamma(q) = [\exp(q/T) - 1]^{-1}$

T fluid/photon temperature, q photon momentum

while electrons (e) are free, γ scatter and cannot move freely

when e and protons (p) form H atoms, γs can break atomic bound

H binding energy: $B_{\rm H} = m_e + m_p - m_{\rm H} \simeq 13.6$ eV

γs start to move freely when they cannot break H bound anymore

generic Saha equation: $$\frac{n_c n_d}{n_a n_b} = \frac{\int d^3q e^{-E_c/T} \int d^3q e^{-E_d/T}}{\int d^3q e^{-E_a/T} \int d^3q e^{-E_b/T}}$$

(chemical equilibrium condition)

n_i number densities, E_i energies, T fluid temperature, q momenta

Photon decoupling

Photons in equilibrium have $f_\gamma(q) \;=\; [\exp(q/T)-1]^{-1}$

T fluid/photon temperature, q photon momentum

while electrons (e) are free, γ scatter and cannot move freely

when e and protons (p) form H atoms, γs can break atomic bound

H binding energy: $B_\mathrm{H} \;=\; m_e + m_p - m_\mathrm{H} \;\simeq\; 13.6$ eV

γs start to move freely when they cannot break H bound anymore

Saha equation applied to $e+p\leftrightarrow\gamma+H$:

$$\boxed{\frac{n_p n_e}{n_\mathrm{H}} = \left(\frac{m_e T}{2\pi}\right)^{3/2}\exp\left(-\frac{B_\mathrm{H}}{T}\right)}$$

Photon decoupling

Photons in equilibrium have $f_\gamma(q) = [\exp(q/T) - 1]^{-1}$

T fluid/photon temperature, q photon momentum

while electrons (e) are free, γ scatter and cannot move freely

when e and protons (p) form H atoms, γs can break atomic bound

H binding energy: $B_{\rm H} = m_e + m_p - m_{\rm H} \simeq 13.6$ eV

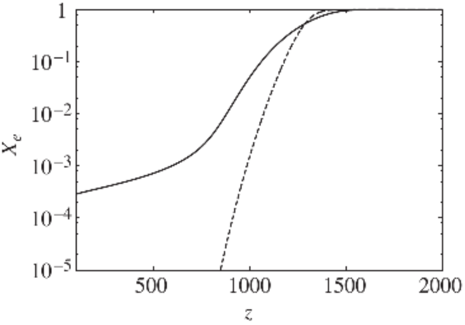
γs start to move freely when they cannot break H bound anymore

define $X_e \equiv \frac{n_e}{n_e + n_{\rm H}}$, use $Y_p \equiv \frac{m_{\rm He} n_{\rm He}}{m_{\rm N} n_{\rm B}} \sim 0.25$, $\eta_{\rm B} \equiv \frac{n_{\rm B} - n_{\bar{\rm B}}}{n_\gamma} \sim 6 \times 10^{-10}$

$$\boxed{\frac{{X_e}^2}{1 - X_e} = \frac{1}{\eta_{\rm B}(1 - Y_p)} \left(\frac{m_e}{T}\right)^{3/2} \frac{\sqrt{\pi}}{2^{5/2}\zeta(3)} \exp\left(-\frac{B_{\rm H}}{T}\right)}$$

Y_p ^{4}He mass fraction, $\eta_{\rm B}$ baryon-to-photon ratio, $\zeta(3) \simeq 1.202\ldots$

Photon decoupling



Photons i $T) - 1]^{-1}$

while electr move freely

when e and pr atomic bound

H binding 13.6 eV

γs start to mo ound anymore

define $X_e \equiv \frac{n_e}{n_e+}$ $\frac{n_{\bar{\mathrm{B}}}}{\gamma} \sim 6 \times 10^{-10}$

$$\boxed{\frac{{X_e}^2}{1-X_e} = \frac{1}{\eta_{\mathrm{B}}(1-Y_p)} \left(\frac{m_e}{T}\right)^{3/2} \frac{\sqrt{\pi}}{2^{5/2}\zeta(3)} \exp\left(-\frac{B_{\mathrm{H}}}{T}\right)}$$

Y_p ^{4}He mass fraction, η_{B} baryon-to-photon ratio, $\zeta(3) \simeq 1.202\ldots$

For $T \simeq B_{\mathrm{H}}$, X_e is close to 1: too many high-E γs break H!

Fraction of free electrons decreases rapidly at $T \simeq 0.3$ eV ($z \sim 1100$)

At that point (last scattering) photons start to move freely!

Cosmology with perturbations

[see also: Ma&Bertschinger, 1995]

Beyond homogeneous and isotropic universe: add perturbations!

$$\text{metric:}\quad g_{\mu\nu} \quad = \quad \bar{g}_{\mu\nu} \ + \ \delta g_{\mu\nu}$$

extend FLRW: $$ds^2 = a^2(\eta)[-(1+2\psi(\eta,\vec{x}))d\eta^2 + (1-2\phi(\eta,\vec{x}))d\vec{x}^2]$$

Newtonian gauge: ψ (Newtonian potential), ϕ metric perturbations

only scalar, no vector/tensor perturbations!

Cosmology with perturbations

[see also: Ma&Bertschinger, 1995]

Beyond homogeneous and isotropic universe: add perturbations!

$$\text{metric: } g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

extend FLRW: $$ds^2 = a^2(\eta)[-(1+2\psi(\eta,\vec{x}))d\eta^2 + (1-2\phi(\eta,\vec{x}))d\vec{x}^2]$$

$$\text{stress-energy tensor: } T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

4 scalars define the T perturbations:

$\delta = \delta\rho/\bar{\rho}$ density contrast

δP pressure perturbations

θ related to bulk velocity divergence

σ anisotropic stress

Cosmology with perturbations

[see also: Ma&Bertschinger, 1995]

Beyond homogeneous and isotropic universe: add perturbations!

metric: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$

extend FLRW: $ds^2 = a^2(\eta)[-(1+2\psi(\eta,\vec{x}))d\eta^2 + (1-2\phi(\eta,\vec{x}))d\vec{x}^2]$

stress-energy tensor: $T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$

4 scalars define the T perturbations:

$\delta = \delta\rho/\bar{\rho}$ density contrast

θ related to bulk velocity divergence

δP pressure perturbations

σ anisotropic stress

Einstein equations (Fourier space):

$$k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right) = -4\pi G a^2 \sum_i \delta\rho_i \quad \text{and} \quad k^2(\phi-\psi) = 12\pi G a^2 \sum_i(\bar{\rho}_i + \bar{p}_i)\sigma_i$$

Cosmology with perturbations

[see also: Ma&Bertschinger, 1995]

Beyond homogeneous and isotropic universe: add perturbations!

$$\text{metric: } g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

extend FLRW: $$ds^2 = a^2(\eta)[-(1+2\psi(\eta,\vec{x}))d\eta^2 + (1-2\phi(\eta,\vec{x}))d\vec{x}^2]$$

$$\text{stress-energy tensor: } T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

4 scalars define the T perturbations:

$\delta = \delta\rho/\bar{\rho}$ density contrast

δP pressure perturbations

θ related to bulk velocity divergence

σ anisotropic stress

Einstein equations (Fourier space):

$$k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right) = -4\pi G a^2 \sum_i \delta\rho_i \quad \text{and} \quad k^2(\phi - \psi) = 12\pi G a^2 \sum_i (\bar{\rho}_i + \bar{p}_i)\sigma_i$$

Perturbed photon distribution:

$$f_\gamma(\eta,\vec{x},\vec{p}) = \left[\exp\left(\frac{y}{a(\eta)\bar{T}(\eta)\{1+\Theta_\gamma(\eta,\vec{x},\hat{n})\}}\right) - 1\right]^{-1}$$

$$\Theta'_\gamma + \hat{n}\cdot\vec{\nabla}\Theta_\gamma - \phi' + \hat{n}\cdot\vec{\nabla}\psi = a n_e \sigma_T(\Theta_{\gamma 0} - \Theta_\gamma + \hat{n}\cdot\vec{v}_\mathrm{B})$$

Cosmic Microwave Background (CMB)

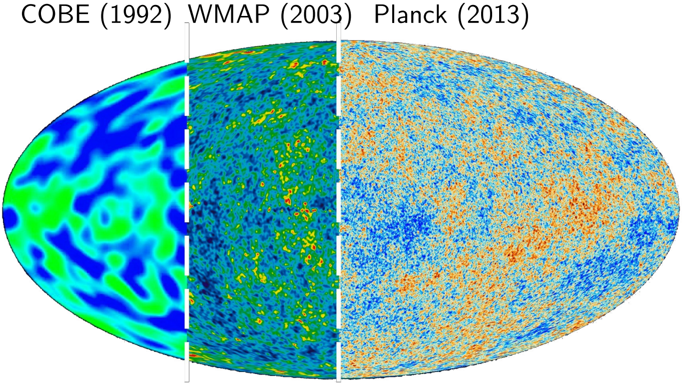
Predicted in 1948 [Alpher, Herman]: blackbody background radiation at $T \simeq 5$ K

Discovery (accidental): [Penzias, Wilson 1964] ⟶ Nobel prize 1978

perfect black body spectrum at $T_{\rm CMB} = 2.72548 \pm 0.00057$ K [Fixsen, 2009]

Anisotropies at the level of 10^{-5}: very high precision measurements are needed.

Improvement of the CMB experiments in 20 years:



Power spectrum

Simplest assumption: only Gaussian fluctuations in the Early Universe

linear theory preserves gaussianity

all Gaussian fluctuations can be described by two-point correlation function

$$\langle A(\eta,\vec{k})A^*(\eta,\vec{k}')\rangle$$

Power spectrum

Simplest assumption: only Gaussian fluctuations in the Early Universe

linear theory preserves gaussianity

all Gaussian fluctuations can be described by two-point correlation function

$$\langle A(\eta, \vec{k}) A^*(\eta, \vec{k}') \rangle$$

stochastic gaussian field → uncorrelated wavevectors

→ Fourier transform equal $\delta^{(3)}(\vec{k} - \vec{k}')$ times power spectrum P_A

Also defined as: $\mathcal{P}_A(k) = \frac{k^3}{2\pi^2} P_A(k)$

Power spectrum

Simplest assumption: only Gaussian fluctuations in the Early Universe

linear theory preserves gaussianity

all Gaussian fluctuations can be described by two-point correlation function

$$\langle A(\eta,\vec{k})A^*(\eta,\vec{k}')\rangle$$

stochastic gaussian field → uncorrelated wavevectors

→ Fourier transform equal $\delta^{(3)}(\vec{k}-\vec{k}')$ times power spectrum P_A

Also defined as: $\mathcal{P}_A(k)=\frac{k^3}{2\pi^2}P_A(k)$

Curvature perturbations: $\mathcal{R}=\psi-\frac{1}{3}\frac{\delta\rho_{\rm tot}}{\bar\rho_{\rm tot}+\bar P_{\rm tot}}$

Inflation predicts $\mathcal{P}_{\mathcal{R}}(k)=A_s(k/k_0)^{n_s-1}$ as initial spectrum

Power spectrum

Simplest assumption: only Gaussian fluctuations in the Early Universe

linear theory preserves gaussianity

all Gaussian fluctuations can be described by two-point correlation function

$$\langle A(\eta,\vec{k})A^*(\eta,\vec{k}')\rangle$$

stochastic gaussian field → uncorrelated wavevectors

→ Fourier transform equal $\delta^{(3)}(\vec{k}-\vec{k}')$ times power spectrum P_A

Also defined as: $\mathcal{P}_A(k)=\frac{k^3}{2\pi^2}P_A(k)$

Curvature perturbations: $\mathcal{R}=\psi-\frac{1}{3}\frac{\delta\rho_{\rm tot}}{\bar{\rho}_{\rm tot}+\bar{P}_{\rm tot}}$

Inflation predicts $\mathcal{P}_\mathcal{R}(k)=A_s(k/k_0)^{n_s-1}$ as initial spectrum

Expression for the power spectrum of photon temperature perturbations:

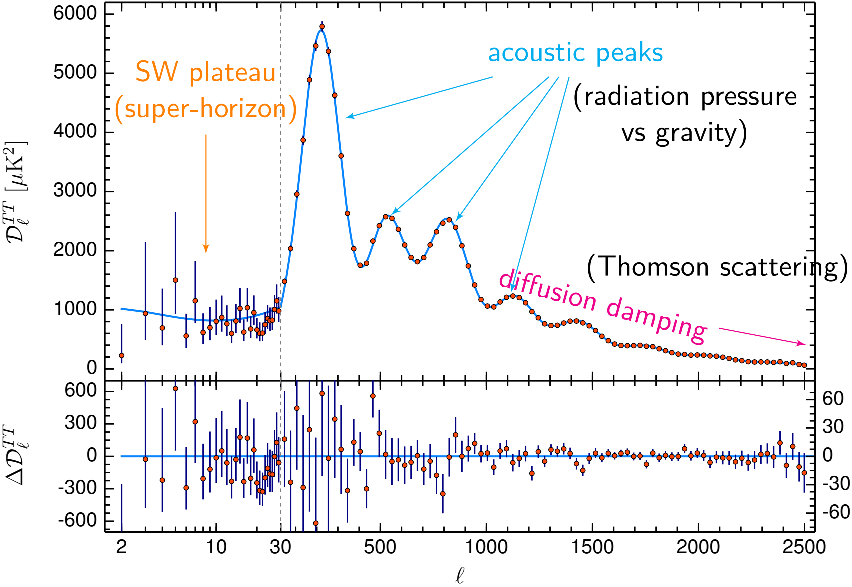
$$\boxed{\langle\Theta_{\gamma l}(\eta,\vec{k})\Theta^*_{\gamma l}(\eta,\vec{k}')\rangle=\frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{R}(k)[\Theta_{\gamma l}(\eta,k)]^2\delta^{(3)}(\vec{k}-\vec{k}')}$$

$\Theta_{\gamma l}(\eta,k)\equiv[\Theta_{\gamma l}(\eta,\vec{k})/\mathcal{R}(\eta_{\rm in},\vec{k})]$ transfer function

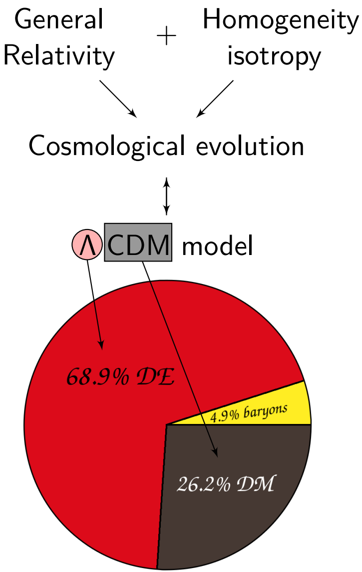
Planck DR3 results - Temperature

[Planck Collaboration, 2018]

Planck legacy temperature auto-correlation power spectrum:



Cosmological parameters



General Relativity + Homogeneity isotropy

Cosmological evolution

ΛCDM model

[Planck Collaboration, 2018]

ΛCDM model described by 6 base parameters:

$\omega_b = \Omega_b h^2$ baryon density today;

$\omega_c = \Omega_c h^2$ CDM density today;

τ optical depth to reionization;

θ angular scale of acoustic peaks;

n_s tilt and

A_s amplitude of the power spectrum of initial curvature perturbations.

Other quantities can be studied:

H_0 Hubble parameter today;

σ_8 mean matter fluctuations at small scales;

$\dots$

Cosmological parameters

General Relativity + Homogeneity isotropy

↓

Cosmological evolution

↕

ΛCDM model

[Planck Collaboration, 2018]

ΛCDM model described by 6 base parameters:

- $\omega_b = \Omega_b h^2$ baryon density today;
- $\omega_c = \Omega_c h^2$ CDM density today;
- τ optical depth to reionization;
- θ angular scale of acoustic peaks;
- n_s tilt and
- A_s amplitude of the power spectrum of initial curvature perturbations.

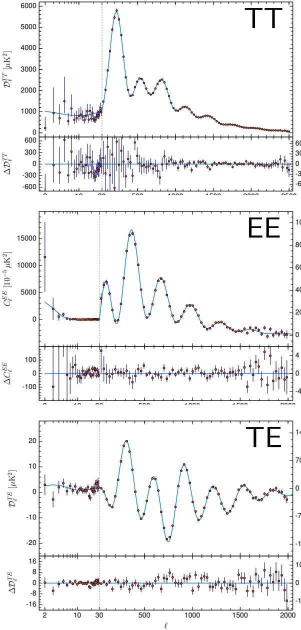
Other quantities can be studied:

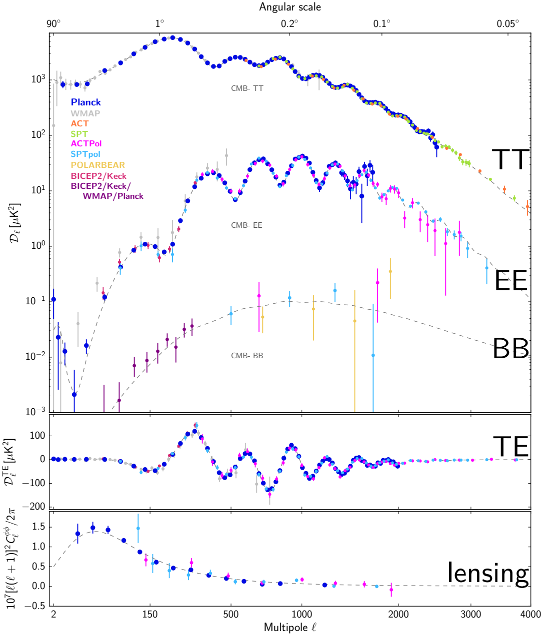
- H_0 Hubble parameter today;
- σ_8 mean matter fluctuations at small scales;

$\dots$

CMB spectra as of 2018

[Planck Collaboration, 2018]

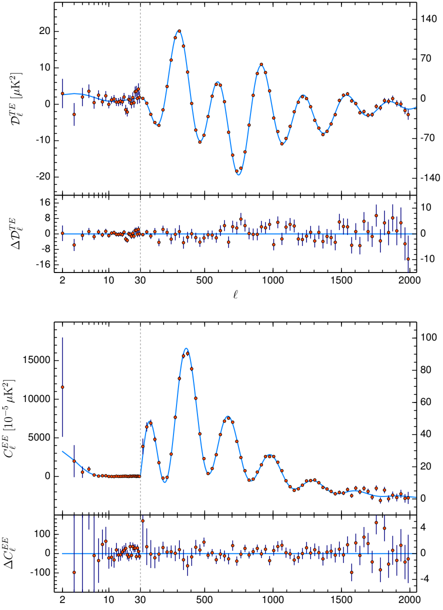




Planck DR3 results - Polarization

[Planck Collaboration, 2018]

- TE cross-correlation and EE auto-correlation measured with high precision;
- ΛCDM explains very well the data;
- Note: in the plots, the red curve is the prediction based on the TT only best-fit for ΛCDM model → very good consistency between temperature and polarization spectra.

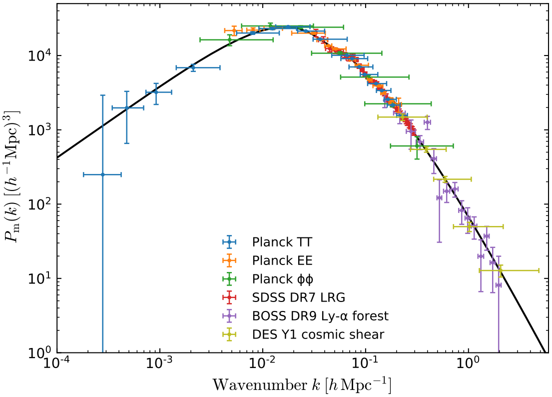


O Other observables

matter power spectrum, H_0, σ_8, BBN

Based on:

- Lesgourgues+, Neutrino Cosmology
- Planck Collaboration, 2018
- PDG (BBN review)



Matter perturbations

What about evolution of matter density perturbations?

$$\langle\delta(\eta,\vec{k})\delta^*(\eta,\vec{k}')\rangle \quad = \quad \delta^{(3)}(\vec{k} \ - \ \vec{k}')P(\eta,k)$$

goal: determine matter power spectrum

Matter perturbations

What about evolution of matter density perturbations?

$$\langle\delta(\eta,\vec{k})\delta^*(\eta,\vec{k}')\rangle \quad = \quad \delta^{(3)}(\vec{k} - \vec{k}')P(\eta,k)$$

goal: determine matter power spectrum

fluctuations with wavelengths k smaller or larger
than the casual horizon behave differently!

large scales
small k

superhorizon

grow with expansion of the universe (no gravity effect)

sub-horizon

small scales
large k

growth from gravitational collapse

balance between expansion
and gravitational interactions

Matter perturbations

What about evolution of matter density perturbations?

$$\langle\delta(\eta,\vec{k})\delta^*(\eta,\vec{k}')\rangle = \delta^{(3)}(\vec{k} - \vec{k}')P(\eta,k)$$

goal: determine matter power spectrum

fluctuations with wavelengths *k* smaller or larger
than the casual horizon behave differently!

large scales
small *k*

superhorizon

grow with expansion of the universe (no gravity effect)

sub-horizon

small scales
large *k*

growth from gravitational collapse

balance between expansion
and gravitational interactions

moreover: evolution is different during RD, MD, ΛD

Matter perturbations

What about evolution of matter density perturbations?

$$\langle\delta(\eta,\vec{k})\delta^*(\eta,\vec{k}')\rangle \quad = \quad \delta^{(3)}(\vec{k}\ -\ \vec{k}')P(\eta,k)$$

goal: determine matter power spectrum

fluctuations with wavelengths k smaller or larger
than the casual horizon behave differently!

large scales
small k

superhorizon

grow with expansion of the universe (no gravity effect)

sub-horizon

small scales
large k

growth from gravitational collapse

balance between expansion
and gravitational interactions

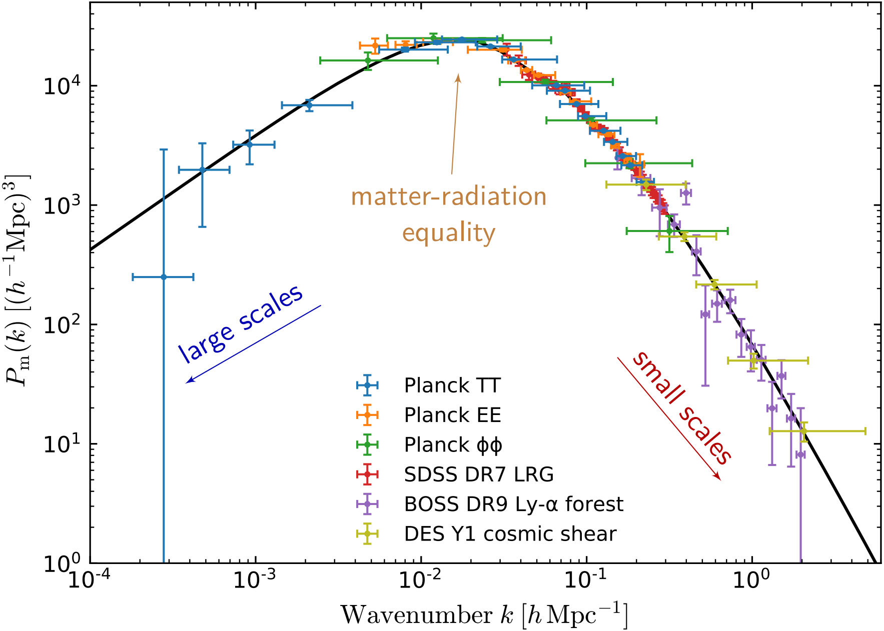
moreover: evolution is different during RD, MD, ΛD

approximated $P(a,k)$ with negligible baryon fraction:

$$P(a,k) = \left(\frac{a}{a_0}\frac{a_M\,\delta_C(a,k)}{a\,\delta_C(a_M,k)}\right)^2 \frac{k\mathcal{P}_\mathcal{R}(k)}{(\Omega_m a_0^2 H_0^2)^2} \times \begin{cases} \frac{8\pi^2}{25} & (a_0 H_0 < k < k_\mathrm{eq}) \\ \frac{k_\mathrm{eq}^4}{2k^4}\left(\alpha+\beta\ln\left(\frac{k}{k_\mathrm{eq}}\right)\right)^2 & (k > k_\mathrm{eq}) \end{cases}$$

(Linear) matter power spectrum

[Planck Collaboration, 2018]



Tension I: the Hubble parameter H_0

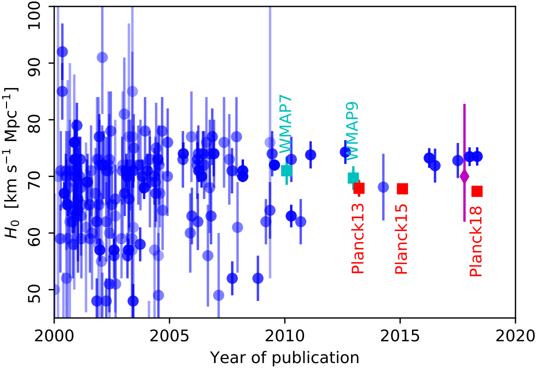
[Planck Collaboration, 2018]

$$\boxed{\begin{array}{c} v \;=\; H_0 d, \\ \text{with } H_0 = H(z=0) \end{array}}$$

Local measurements:
$H(z=0)$,
local and independent on evolution (model independent, but **systematics?**)

CMB measurements
(probe $z \simeq 1100$):
H_0 from the cosmological evolution (model dependent, well controlled systematics)

68% CL error bars



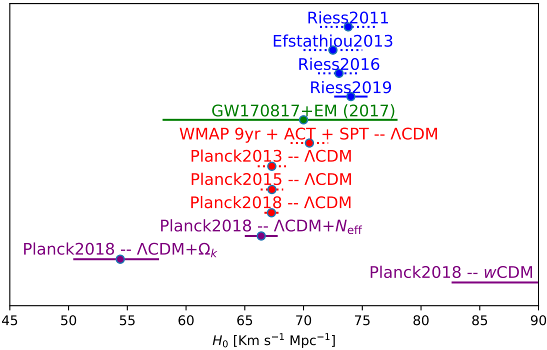
Tension I: the Hubble parameter H_0

$$\boxed{\begin{array}{c} v \;=\; H_0 d, \\ \text{with } H_0 = H(z=0) \end{array}}$$

Local measurements:
$H(z=0)$,
local and independent on evolution (model independent, but **systematics?**)

CMB measurements
(probe $z \simeq 1100$):
H_0 from the cosmological evolution (model dependent, well controlled systematics)

68% CL error bars



Using HST Cepheids:
[Efstathiou 2013] $H_0 = 72.5 \pm 2.5\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Riess+, 2019] $H_0 = 74.03 \pm 1.42\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

GW: [Abbott+, 2017] $H_0 = 70^{+12}_{-8}\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

(ΛCDM model - CMB data only)
[Planck 2013]: $H_0 = 67.3 \pm 1.2\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Planck 2018]: $H_0 = 67.27 \pm 0.60\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

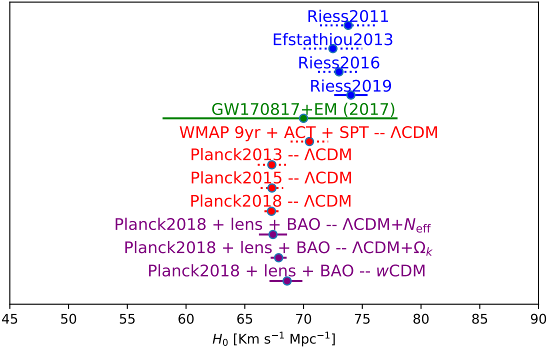
Tension I: the Hubble parameter H_0

$$\boxed{\begin{array}{c} v \;=\; H_0 d, \\ \text{with } H_0 = H(z=0) \end{array}}$$

Local measurements:
$H(z=0)$,
local and independent on evolution (model independent, but **systematics?**)

CMB measurements
(probe $z \simeq 1100$):
H_0 from the cosmological evolution (model dependent, well controlled systematics)

68% CL error bars



Using HST Cepheids:
[Efstathiou 2013] $H_0 = 72.5 \pm 2.5\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Riess+, 2019] $H_0 = 74.03 \pm 1.42\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

GW: [Abbott+, 2017] $H_0 = 70^{+12}_{-8}\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

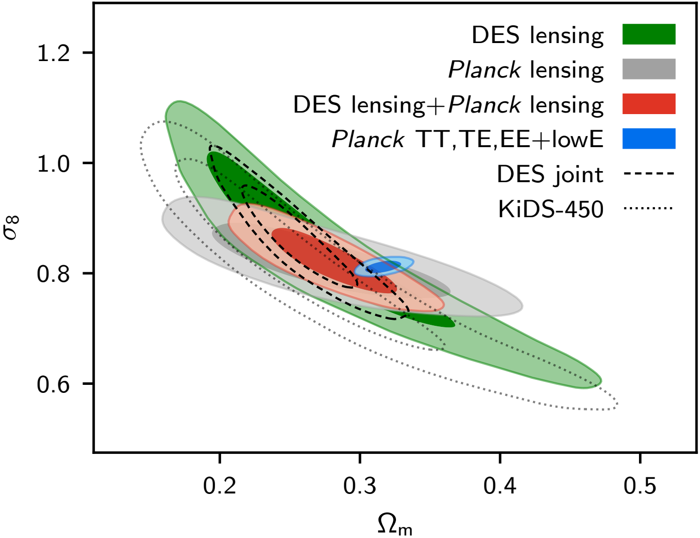
(ΛCDM model - CMB data only)
[Planck 2013]: $H_0 = 67.3 \pm 1.2\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Planck 2018]: $H_0 = 67.27 \pm 0.60\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

Tension II (?): the matter distribution at small scales

Assuming ΛCDM model:

σ_8: rms fluctuation in total matter (baryons + CDM + neutrinos) in $8h^{-1}$ Mpc spheres, today;

Ω_m: total matter density today divided by the critical density



[Planck collaboration, 2018]

Big Bang Nucleosynthesis (BBN)

BBN: production of light nuclei at $t \sim 1$s to $t \sim \mathcal{O}(10^2)$s

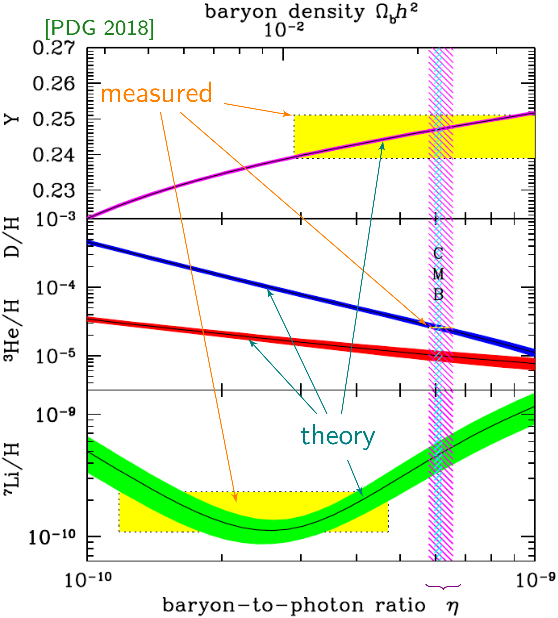
temperature $T_{fr} \simeq 1$ MeV
from nucleon freeze-out

much earlier than CMB!

strong probe for physics
before the CMB

e.g. neutrinos!

ν affect
universe expansion
and
reaction rates $(\nu_e/\bar{\nu}_e)$
at BBN time. . .



[PDG 2018]

BBN concordance

Big Bang Nucleosynthesis (BBN)

BBN: production of light nuclei at $t \sim 1$s to $t \sim \mathcal{O}(10^2)$s

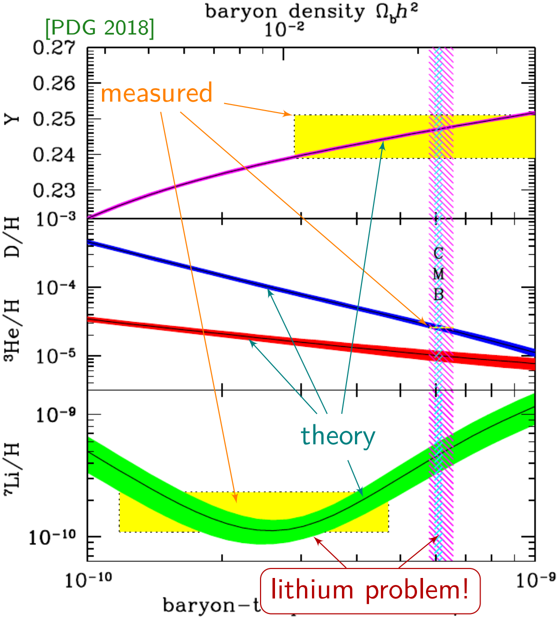
temperature $T_{fr} \simeq 1$ MeV
from nucleon freeze-out

much earlier than CMB!

strong probe for physics
before the CMB

e.g. neutrinos!

ν affect
universe expansion
and
reaction rates $(\nu_e/\bar{\nu}_e)$
at BBN time...



[PDG 2018]

measured

theory

lithium problem!

BBN concordance

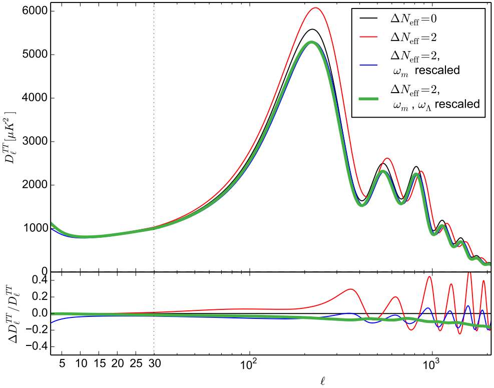
N

Neutrinos in cosmology

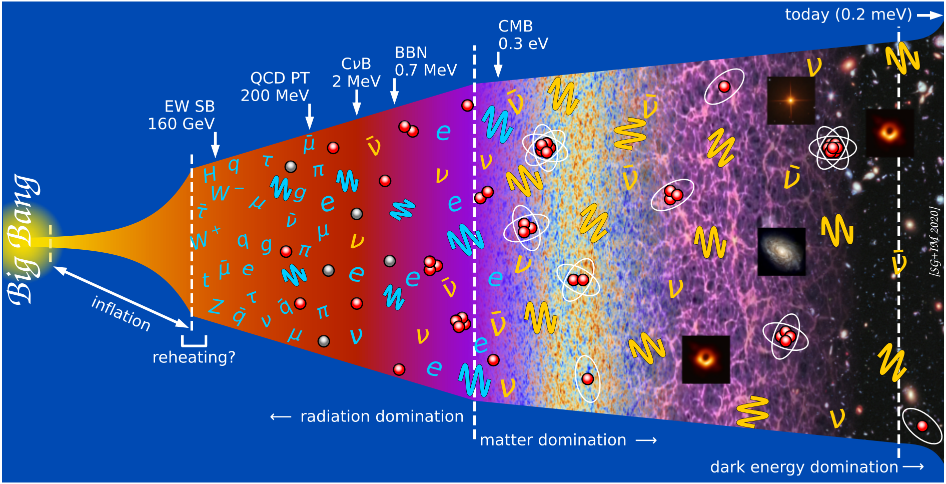
Impact of neutrinos, what do we learn?

Based on:

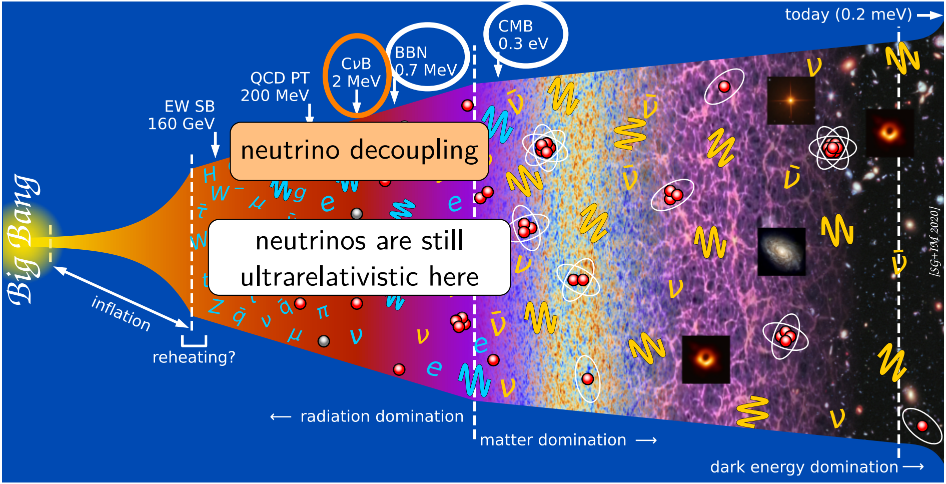
- Lesgourgues+, Neutrino Cosmology
- Bennett+, JCAP 2021
- di Valentino+, PRD 106 (2022)
- SG+, JCAP 10 (2022)
- SG+, arxiv:2302.14159



History of the universe



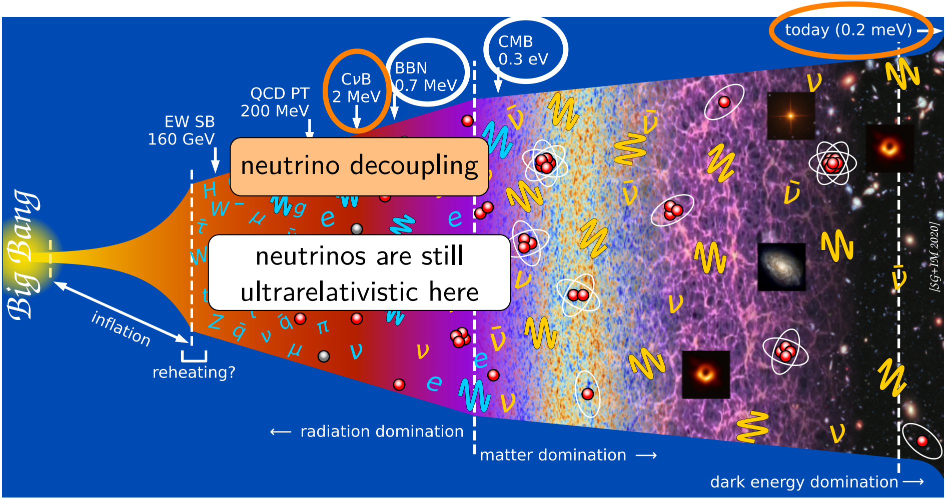
History of the universe



neutrino decoupling

neutrinos are still ultrarelativistic here

History of the universe



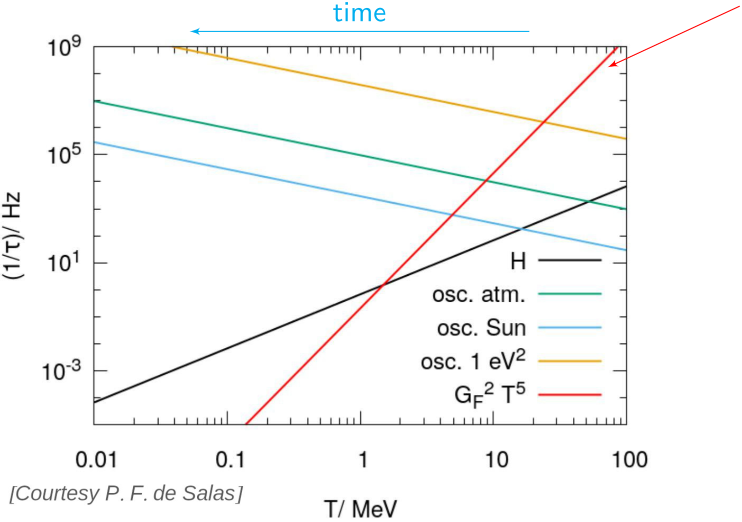
neutrino decoupling

neutrinos are still ultrarelativistic here

∃ at least 2 mass eigenstates with $m_i \gtrsim 8$ meV $\left(=\sqrt{\Delta m^2_{\rm sol}}\right) > \langle E_\nu \rangle$ → many relic neutrinos are non-relativistic today!

Neutrinos in the early Universe

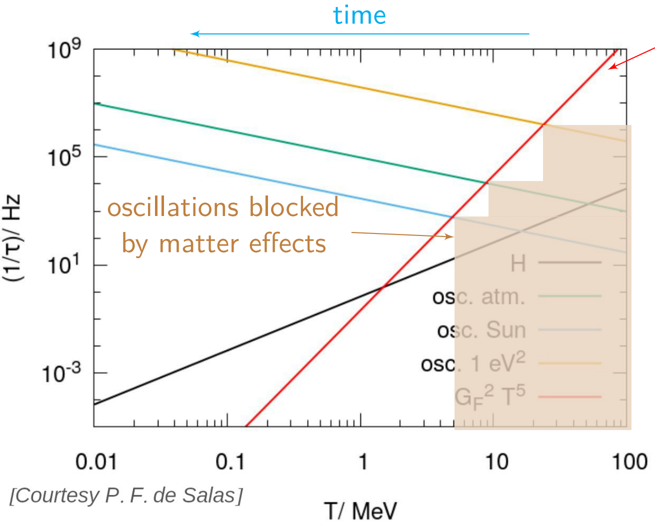
before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

Neutrinos in the early Universe

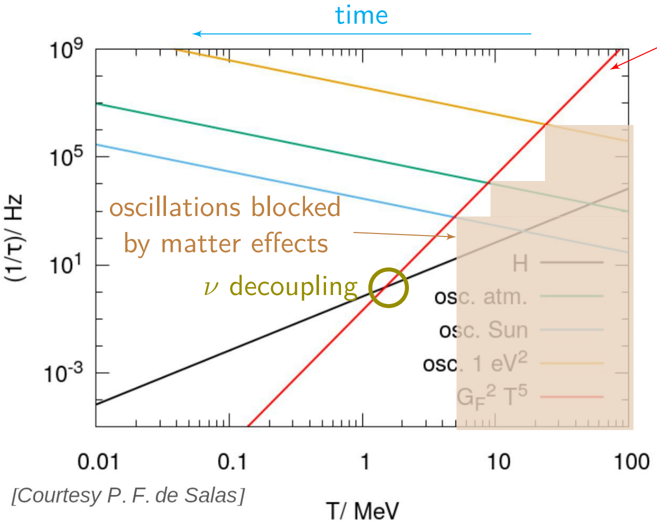
before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)

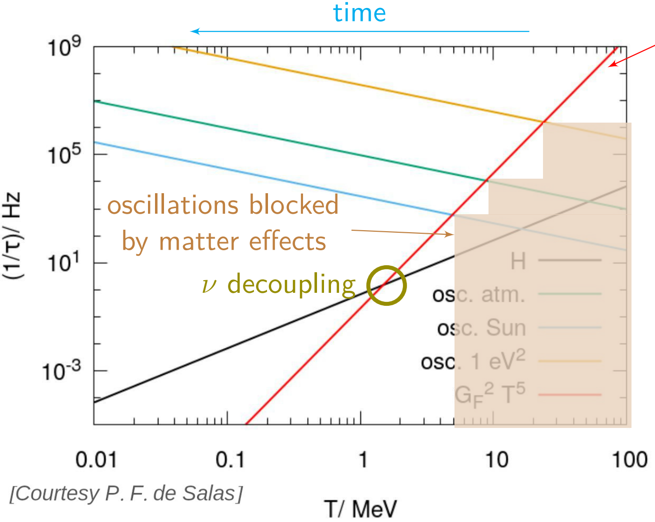


[Courtesy P. F. de Salas]

ν decouple mostly before $e^+e^- \to \gamma\gamma$ annihilation!

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)



time

oscillations blocked by matter effects

ν decoupling

[Courtesy P. F. de Salas]

$T_\nu \simeq (4/11)^{1/3}\,T_\gamma$
after $e^+e^- \to \gamma\gamma$

f_ν: frozen Fermi-Dirac distribution

Today:

$T_{\nu,0} = 1.945$ K $\simeq$
1.676×10^{-4} eV

$\langle E_\nu \rangle \simeq 3.1\,T_{\nu,0} \simeq$
5×10^{-4} eV

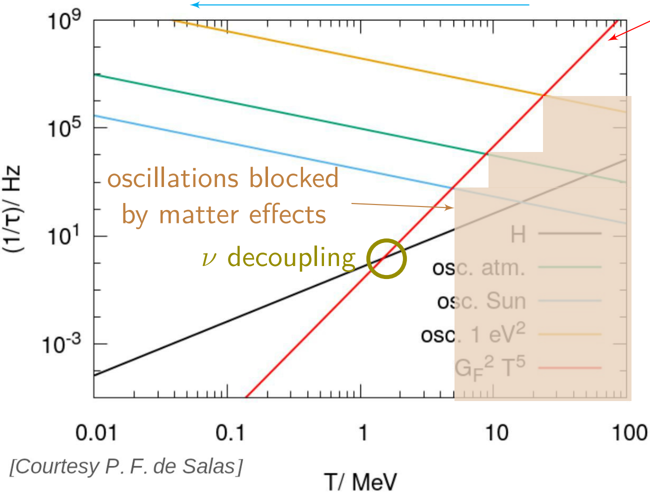
$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq$
$56\ \text{cm}^{-3}$ per family

ν decouple mostly before $e^+e^- \to \gamma\gamma$ annihilation!

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)

time ←



oscillations blocked by matter effects

ν decoupling

[Courtesy P. F. de Salas]

$T_\nu \simeq (4/11)^{1/3} T_\gamma$ after $e^+ e^- \to \gamma\gamma$

f_ν: frozen Fermi-Dirac distribution

Today:

$T_{\nu,0} = 1.945$ K $\simeq 1.676 \times 10^{-4}$ eV

$\langle E_\nu \rangle \simeq 3.1\, T_{\nu,0} \simeq 5 \times 10^{-4}$ eV

$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56$ cm^{-3} per family

ν decouple mostly before $e^+ e^- \to \gamma\gamma$ annihilation!

actually, the decoupling T is momentum dependent! →

distortions to equilibrium f_ν!

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]
[Sigl, Raffelt, 1993]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $\varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$

$\propto \langle a_j^\dagger(p,t)\, a_i(p,t) \rangle$

off-diagonals to take into account coherency in the neutrino system

ϱ evolution from $x H \dfrac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = -ia[\mathcal{H}_{\mathrm{eff}}, \varrho] + b\mathcal{I}$

H Hubble factor → expansion (depends on universe content)

effective Hamiltonian $\mathcal{H}_{\mathrm{eff}} = \dfrac{\mathbb{M}_{\mathrm{F}}}{2y} - \dfrac{2\sqrt{2} G_{\mathrm{F}}\, y\, m_e^6}{x^6} \left(\dfrac{\mathbb{E}_\ell + \mathbb{P}_\ell}{m_W^2} + \dfrac{4}{3} \dfrac{\mathbb{E}_\nu}{m_Z^2} \right)$

vacuum oscillations ← → matter effects

$\mathcal{I}$ collision integrals

take into account ν–e scattering and pair annihilation, ν–ν interactions

2D integrals over momentum, take most of the computation time

solve together with z evolution, from $x \frac{\mathrm{d}\rho(x)}{\mathrm{d}x} = \rho - 3P$

ρ, P total energy density and pressure, also take into account FTQED corrections

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]
[Sigl, Raffelt, 1993]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$

$\propto \langle a_j^\dagger(p,t)\, a_i(p,t)\rangle$

off-diagonals to take into account coherency in the neutrino system

ϱ evolution from $x H \frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = -ia[\mathcal{H}_{\mathrm{eff}}, \varrho] + b\mathcal{I}$

`FORTran-Evolved PrimordIAl Neutrino Oscillations`
`(FortEPiaNO)`
`https://bitbucket.org/ahep_cosmo/fortepiano_public`

vacuum oscillations ←

→ matter effects

$\mathcal{I}$ collision integrals

take into account ν–e scattering and pair annihilation, ν–ν interactions

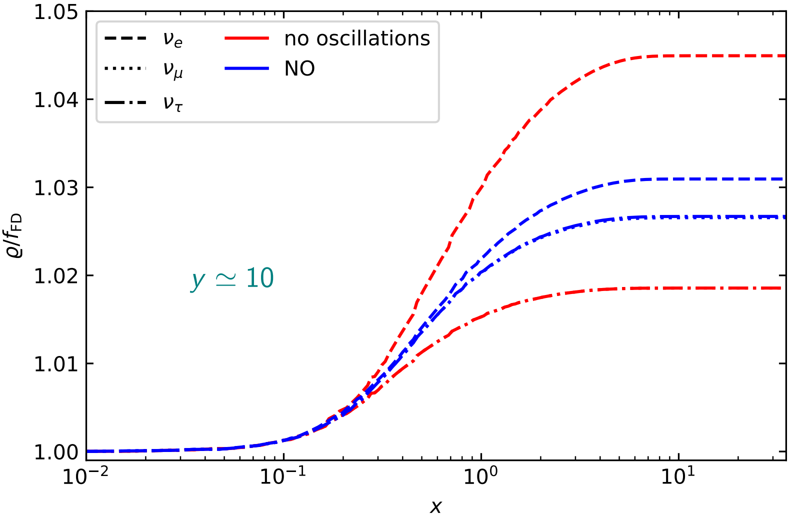
2D integrals over momentum, take most of the computation time

solve together with z evolution, from $x\frac{\mathrm{d}\rho(x)}{\mathrm{d}x} = \rho - 3P$

ρ, P total energy density and pressure, also take into account FTQED corrections

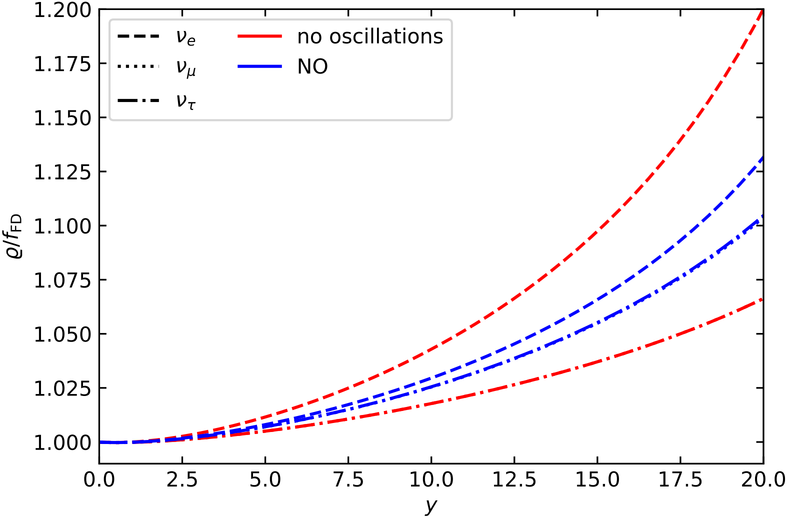
Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, JCAP 2021]

Distortion of the momentum distribution ($f_{\rm FD}$: Fermi-Dirac at equilibrium)



Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, JCAP 2021]

Distortion of the momentum distribution ($f_{\rm FD}$: Fermi-Dirac at equilibrium)

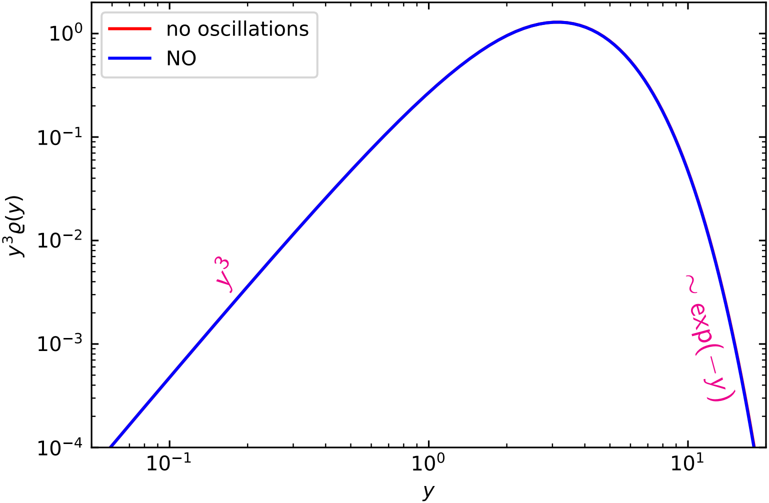


Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, JCAP 2021]

$$N_{\rm eff}^{\rm final} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{1}{\rho_\gamma}\sum_i g_i\int\frac{d^3p}{(2\pi)^3}E(p)\,f_{\nu,i}(p)$$

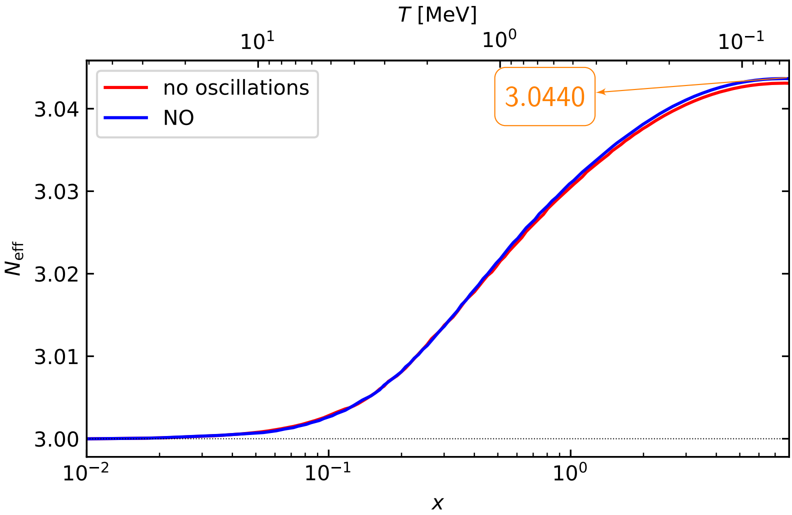
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\rm fin}$

$\hookrightarrow \propto y^3\varrho_{ii}(y)$



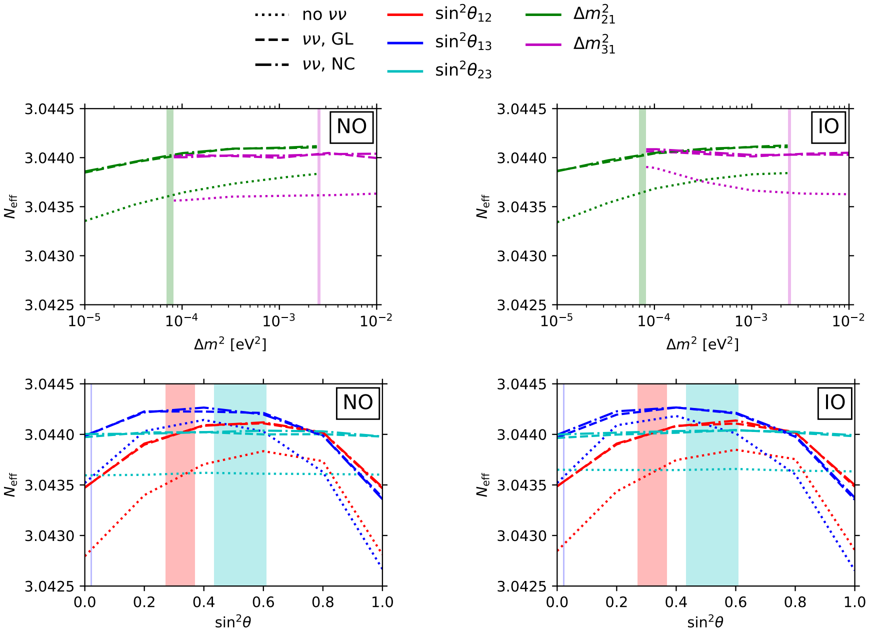
Neutrino momentum distribution and N_{eff} [Bennett, SG+, JCAP 2021]

$$N_{\mathrm{eff}}^{\text{any time}} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{1}{\rho_\gamma}\sum_i g_i \int \frac{d^3p}{(2\pi)^3} E(p)\, f_{\nu,i}(p)$$



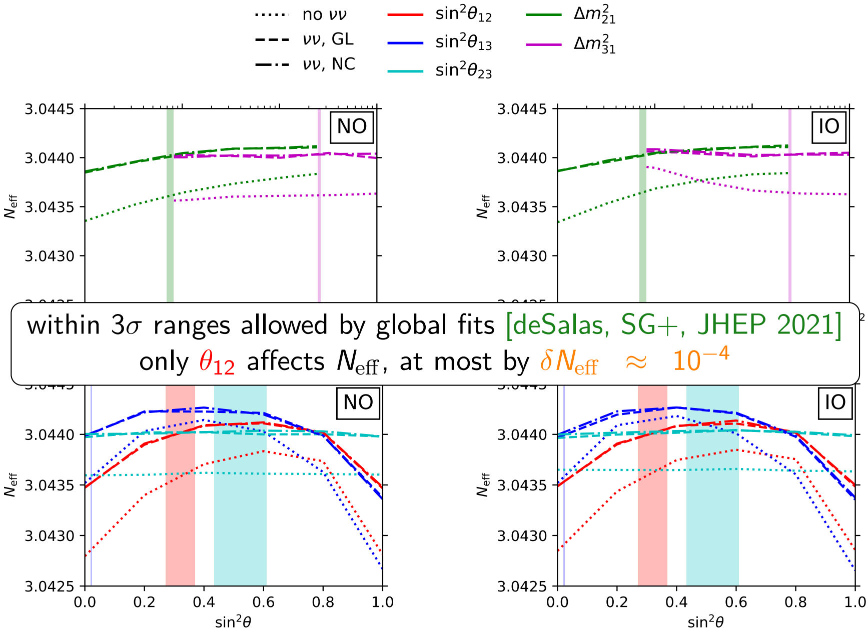
Effect of neutrino oscillations

[Bennett, SG+, JCAP 2021]



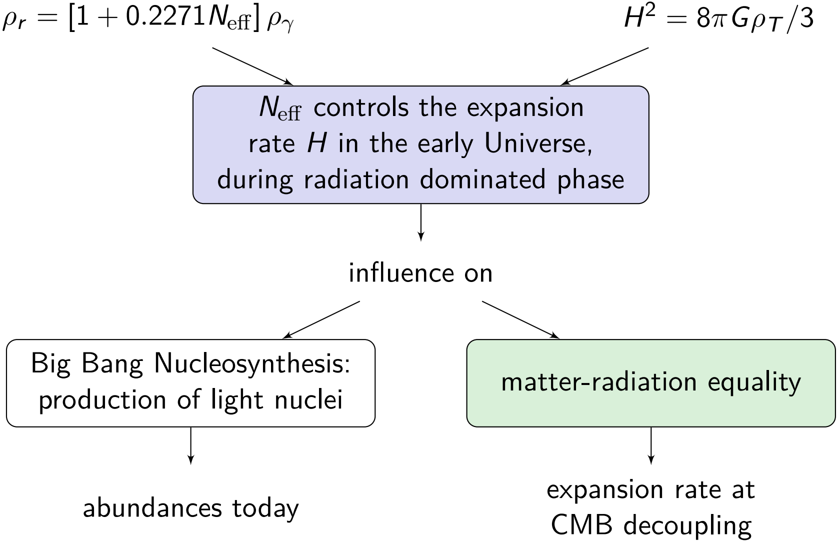
Effect of neutrino oscillations

[Bennett, SG+, JCAP 2021]



within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]
only θ_{12} affects $N_{\rm eff}$, at most by $\delta N_{\rm eff} \approx 10^{-4}$

Additional Radiation in the Early Universe



$\rho_r = [1 + 0.2271 N_{\rm eff}]\, \rho_\gamma$

$H^2 = 8\pi G \rho_T / 3$

$N_{\rm eff}$ controls the expansion rate H in the early Universe, during radiation dominated phase

influence on

Big Bang Nucleosynthesis: production of light nuclei

abundances today

matter-radiation equality

expansion rate at CMB decoupling

$N_{\rm eff}$ and BBN

BBN: production of light nuclei at $t \sim 1$s to $t \sim \mathcal{O}(10^2)$s

temperature $T_{fr} \simeq 1$ MeV
from nucleon freeze-out:

$$\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5 \;=\; H \sim \sqrt{g_\star G_N}\, T^2$$

$$\downarrow$$

$$\boxed{T_{fr} \simeq (g_\star G_N/G_F^4)^{1/6}}$$

enters

$$n/p = \exp(-Q/T_{fr})$$

which controls element abundances

$$\boxed{g_\star \text{ depends on } N_{\rm eff}}$$

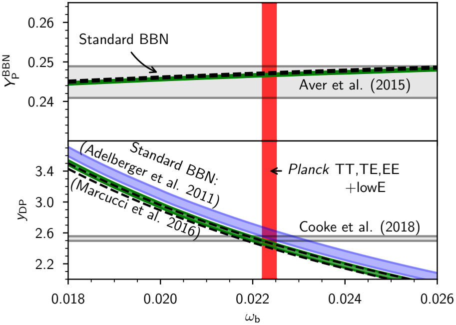
$$\downarrow$$

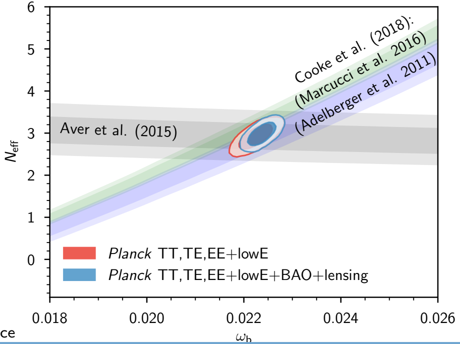
abundances depend on $N_{\rm eff}$

G_F Fermi constant
G_N Newton constant

n, p: neutron, proton density number
$Q = 1.293$ MeV neutron–proton mass difference

[Planck Collaboration, 2018]





$N_{\rm eff}$ and BBN

BBN: production of light nuclei
at $t \sim 1$s to $t \sim \mathcal{O}(10^2)$s

temperature $T_{fr} \simeq 1$ MeV
from nucleon freeze-out:

$$\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5 \ = \ H \sim \sqrt{g_\star G_N}\, T^2$$

$$\downarrow$$

$$\boxed{T_{fr} \simeq (g_\star G_N / G_F^4)^{1/6}}$$

enters

$$n/p = \exp(-Q/T_{fr})$$

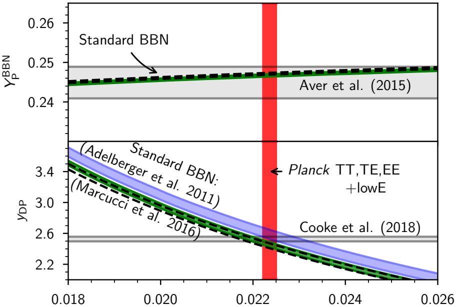
which controls element abundances

$$\boxed{g_\star \text{ depends on } N_{\rm eff}}$$

$$\downarrow$$

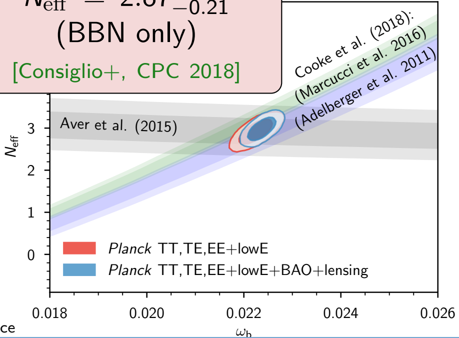
abundances depend on $N_{\rm eff}$

[Planck Collaboration, 2018]



$$N_{\rm eff} \ = \ 2.87^{+0.24}_{-0.21}$$
(BBN only)

[Consiglio+, CPC 2018]



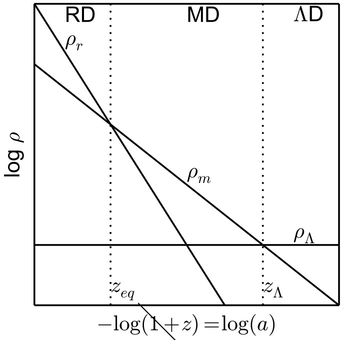
G_F Fermi constant
G_N Newton constant

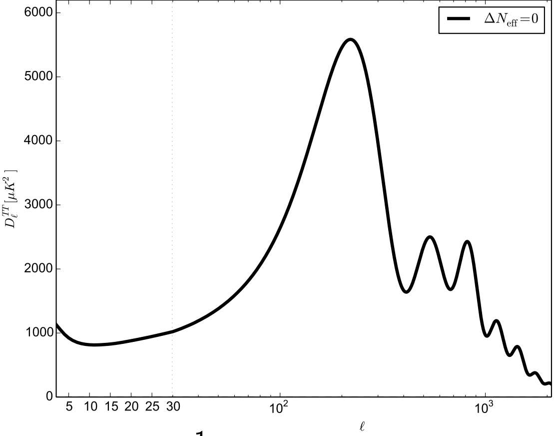
n, p: neutron, proton density number
$Q = 1.293$ MeV neutron–proton mass difference

Additional Radiation: Effects on the CMB

Starting configuration:

RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$



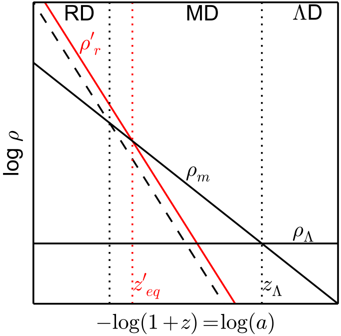


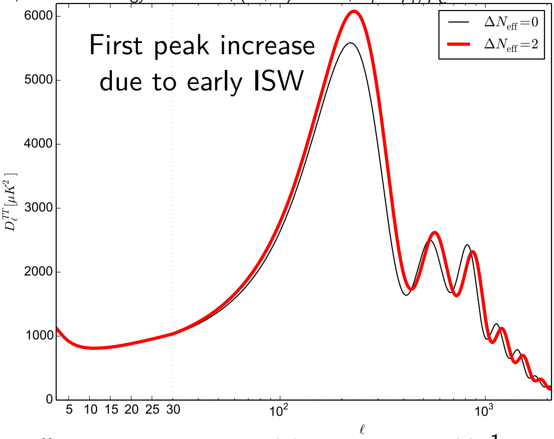
$$1 + z_{\rm eq} = \frac{\omega_m}{\omega_r} = \frac{\omega_m}{\omega_\gamma}\,\frac{1}{1 + 0.2271\,N_{\rm eff}}$$

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, all the other parameters fixed:

RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$





First peak increase due to early ISW

At z_{CMB}: higher $H \propto \rho_r \Rightarrow$ smaller comoving sound horizon $r_s \propto H^{-1}$

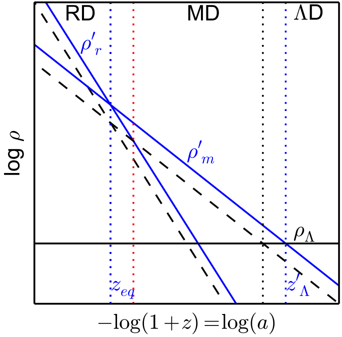
$\Rightarrow$ decrease of the angular scale of the acoustic peaks $\theta_s = r_s/D_A$

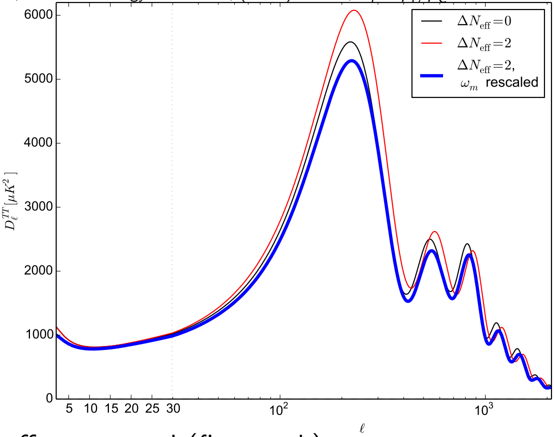
$\Rightarrow$ shift of the peaks at higher ℓ

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, plus ω_m to fix $z_{\rm eq}$:

RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$



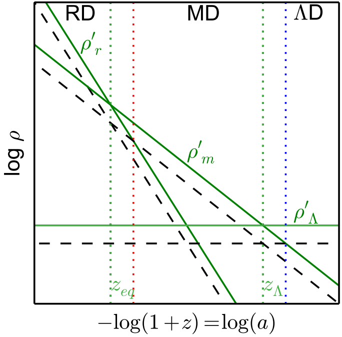


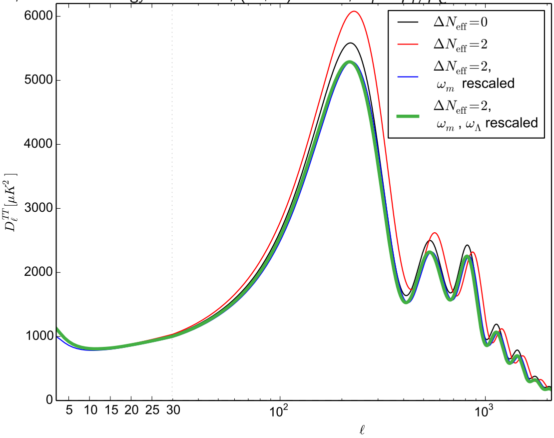
- Contribution from early ISW effect restored (first peak)
- different slope of the Sachs-Wolfe plateau, peak positions, envelope of high-ℓ peaks ⇒ due to later z_Λ

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, plus ω_m, ω_Λ to fix $z_{\rm eq}$, z_Λ:

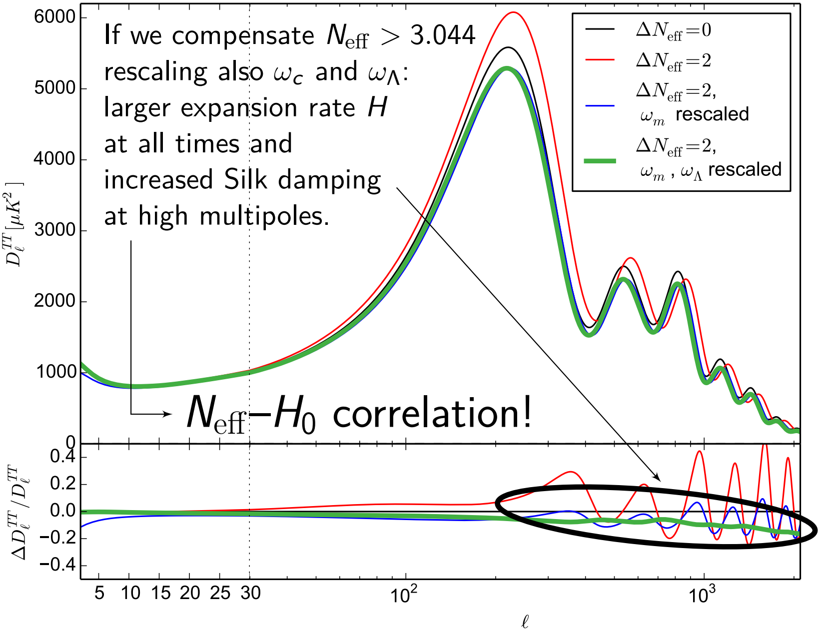
RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z)=a^{-1}$; $\omega_i=\rho_i/\rho_C$





- peak positions recovered;
- slope of the Sachs-Wolfe plateau recovered;
- peak amplitude not recovered!

Additional Radiation: Effects on the CMB

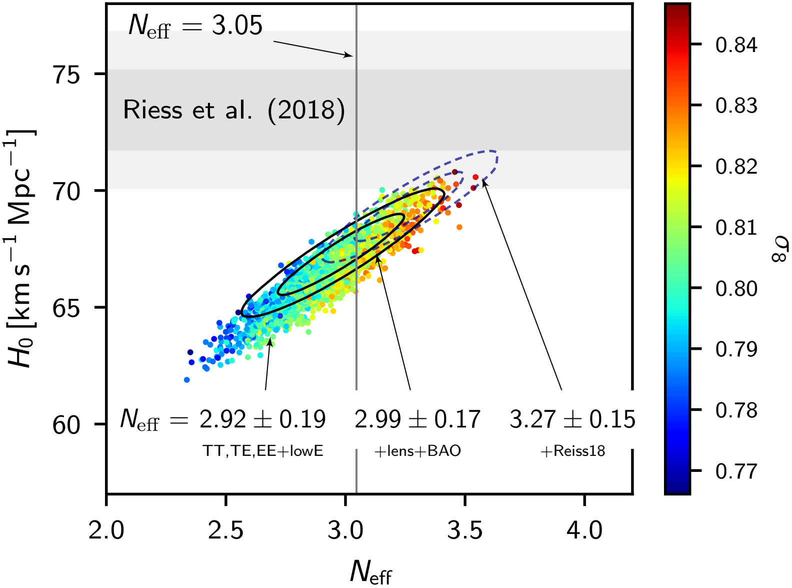


If we compensate $N_{\rm eff} > 3.044$ rescaling also ω_c and ω_Λ: larger expansion rate H at all times and increased Silk damping at high multipoles.

$N_{\rm eff}$–H_0 correlation!

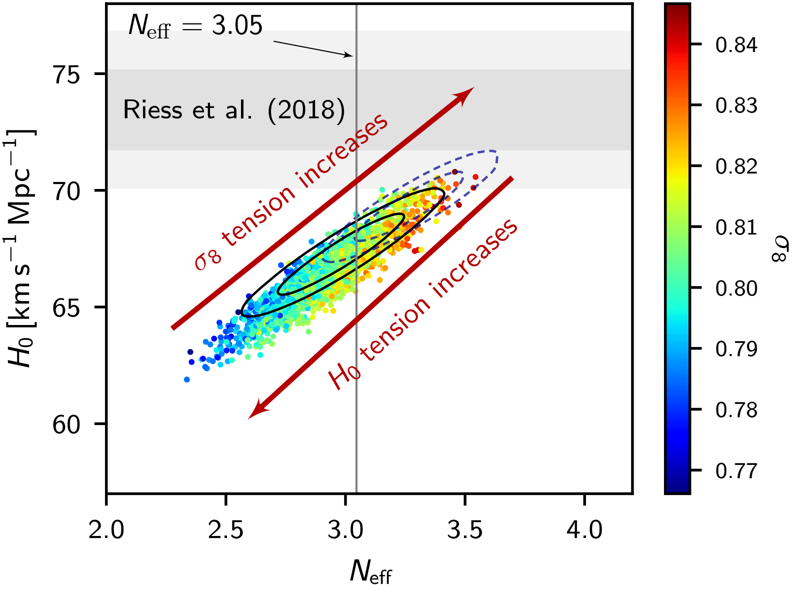
N_{eff} and the local tensions

[Planck Collaboration, 2018]



$N_{\rm eff}$ and the local tensions

[Planck Collaboration, 2018]



Neutrino masses from CMB

ω_i energy density of species i,
$i \in$ ($r$adiation, $m$atter, $b$aryons, $c$old dark matter, ν)
$z_{\rm eq}$ matter-radiation equality redshift

$$1 + z_{\rm eq} = (\omega_b + \omega_c)/\omega_r$$

independent of m_ν

$$\omega_m^0 = \omega_b^0 + \omega_c^0 + \omega_\nu^0 \text{ today}$$

mass of species relativistic at recombination
affects late time evolution only

small effects on the SW plateau
(cosmic variance, degeneracies...)

Effects on the early ISW effect

$$\frac{\Delta C_\ell}{C_\ell} \simeq -\left(\frac{\sum m_\nu}{0.1 \text{ eV}}\right)\%$$

effects on the position of peaks

$$\theta_s = r_s(\eta_{LS})/D_A(\eta_{LS})$$

$$D_A = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

(this effect can be compensated reducing H_0)

correlation m_ν–H_0

[Lesgourgues+, Neutrino Cosmology]

Neutrino masses from CMB

ω_i energy density of species i,
$i \in$ (radiation, matter, baryons, cold dark matter, ν)
$z_{\rm eq}$ matter-radiation equality redshift

$$1 + z_{\rm eq} = (\omega_b + \omega_c)/\omega_r$$

independent of m_ν

$$\omega_m^0 = \omega_b^0 + \omega_c^0 + \omega_\nu^0 \text{ today}$$

mass of species relativistic at recombination
affects late time evolution only

small effects on the SW plateau
(cosmic variance, degeneracies...)

Effects on the early ISW effect

$$\frac{\Delta C_\ell}{C_\ell} \simeq -\left(\frac{\sum m_\nu}{0.1 \text{ eV}}\right)\%$$

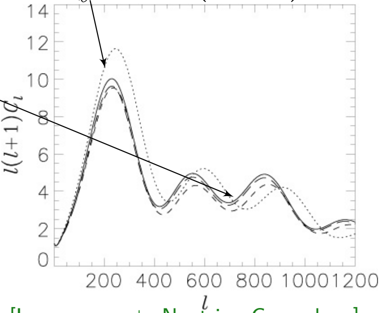
effects on the position of peaks

$$\theta_s = r_s(\eta_{LS})/D_A(\eta_{LS})$$

$$D_A = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

(this effect can be compensated reducing H_0)

correlation m_ν–H_0



[Lesgourgues+, Neutrino Cosmology]

Free-streaming - I

Non-cold relics $\Longrightarrow$ damping in the perturbations due to free-streaming

Growth equation: $\ddot{\delta} + \underbrace{\boxed{2H\dot{\delta}}}_{\text{Hubble drag}} + \underbrace{\boxed{c_s^2 k^2 \frac{\delta}{a^2}}}_{\text{pressure}} = \underbrace{\boxed{4\pi G_N \rho\, \delta}}_{\text{gravity}}$

Jeans scale: pressure=gravity

$$k_J \equiv \sqrt{\frac{4\pi G_N \rho}{c_s^2 (1+z)^2}}$$

$k < k_J$
growth of density perturbations

$k > k_J$
no growth can occur

neutrino free-streaming scale

$$\boxed{k_{fs}(z) \equiv \sqrt{\frac{3}{2}\frac{H(z)}{(1+z)\sigma_{v,\nu}(z)}} \simeq 0.7 \left(\frac{m_\nu}{1\ \text{eV}}\right) \sqrt{\frac{\Omega_M}{1+z}} h/\text{Mpc}}$$

ρ energy density of a given fluid
$\delta = \delta\rho/\rho$ perturbation (single fluid)
c_s sound speed of the fluid

$\sigma_{v,\nu}(z)$ ν velocity dispersion
$H = H(z)$ Hubble factor at redshift z
h reduced Hubble factor today

Free-streaming - II

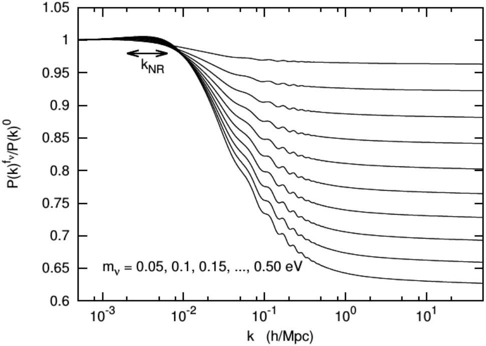
Damping occurs for all $k \gtrsim k_{\rm nr}$

k_{nr}: corresponding
to ν non-relativistic transition

Plot: $\dfrac{P_{m_\nu>0}(k)}{P_{m_\nu=0}(k)}$

- top to bottom: $m_\nu = 0.05$ eV
  to $m_\nu = 0.5$ eV
- $\dfrac{\Delta P}{P} \simeq -\dfrac{8\Omega_\nu}{\Omega_M} \simeq -\dfrac{\sum m_\nu}{0.01 \text{ eV}}\%$

[Lesgourgues+, Neutrino Cosmology]
(fixed h, ω_m, ω_b, ω_Λ)

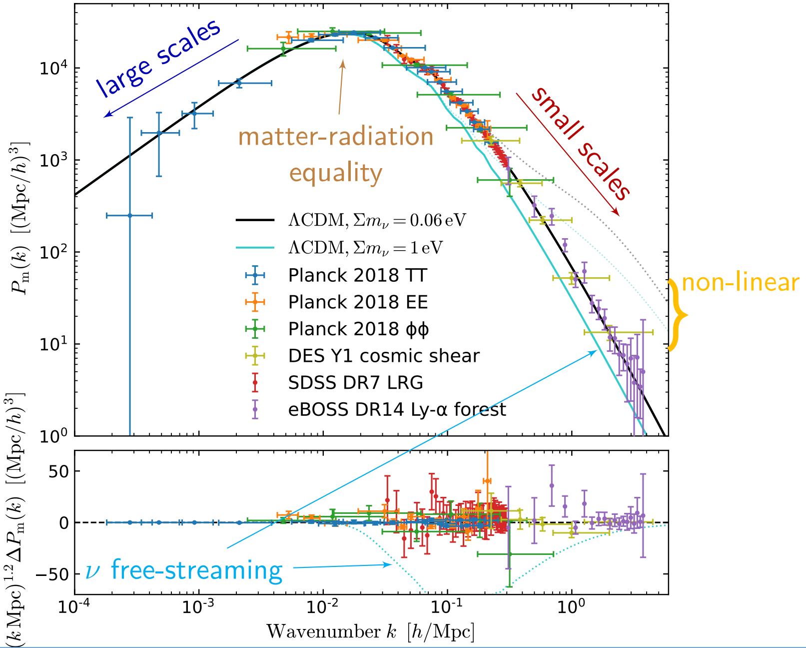


Expected constraints from future surveys:

- Planck CMB + DES: $\sigma(m_\nu) \simeq 0.04$–$0.06$ eV [Font-Ribera+, 2014]
- Planck CMB + Euclid: $\sigma(m_\nu) \simeq 0.03$ eV [Audren+, 2013]

(Linear) matter power spectrum with νs

[Chabanier+, 2019]



large scales

matter-radiation equality

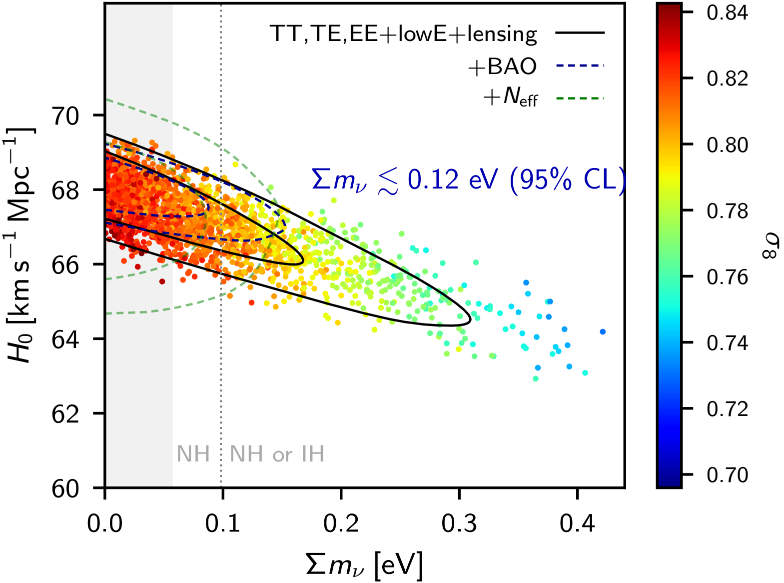
small scales

non-linear

ν free-streaming

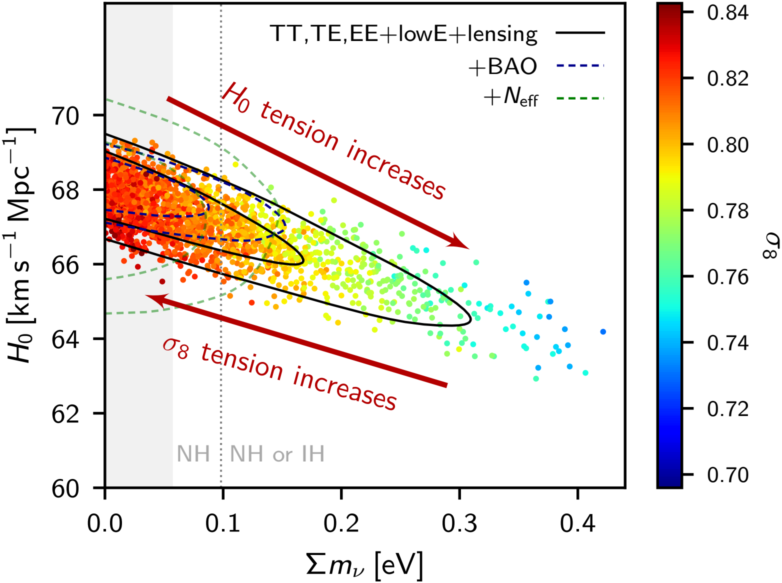
Σm_ν and the local tensions - I

[Planck Collaboration, 2018]



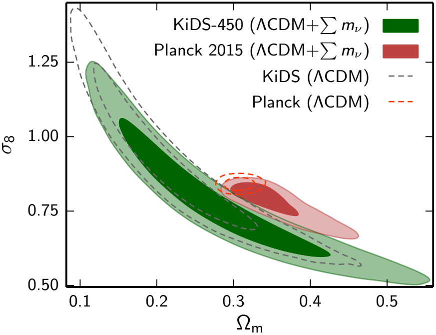
Σm_ν and the local tensions - I

[Planck Collaboration, 2018]

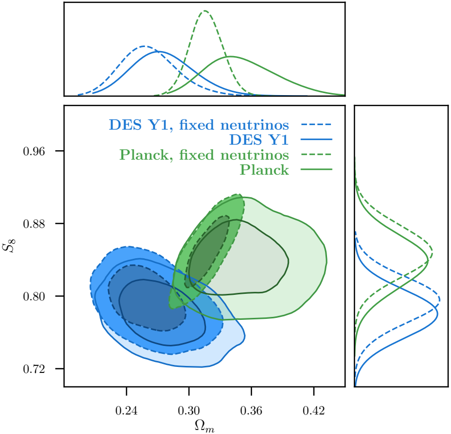


$\sum m_\nu$ and the local tensions - II

[KiDS collaboration, MNRAS 471 (2017) 1259]



[DES collaboration, arxiv:1708.01530]



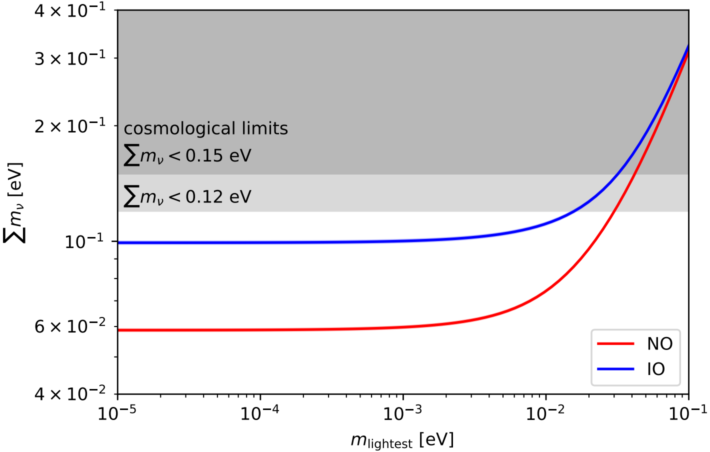
Overlapping of regions does not improve so much with massive neutrinos

From cosmology...

[de Salas+, Frontiers 5 (2018) 36]

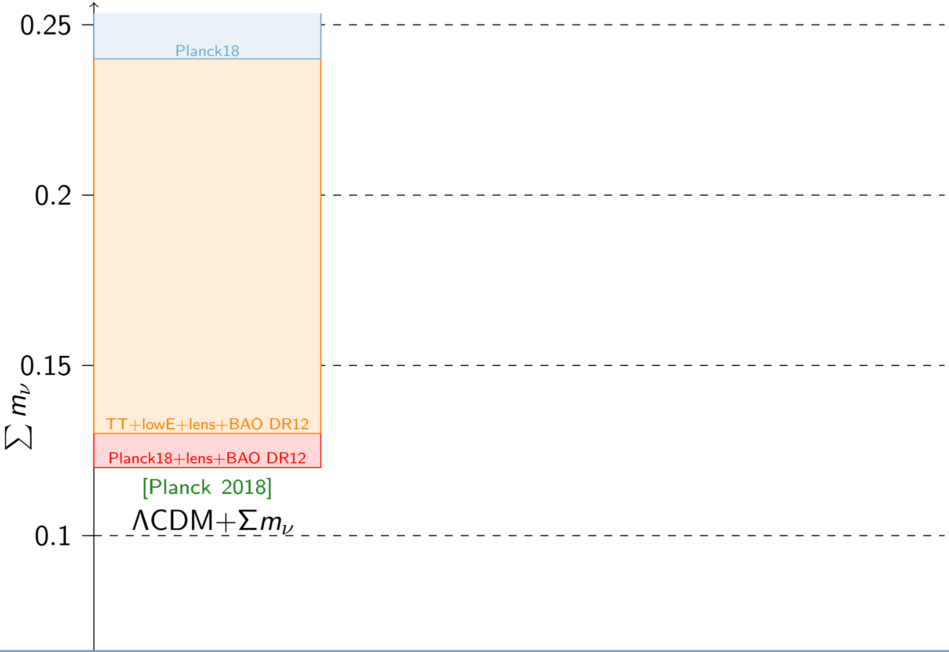
Warning: model dependent content!

How the limit change when considering extensions of the ΛCDM model?

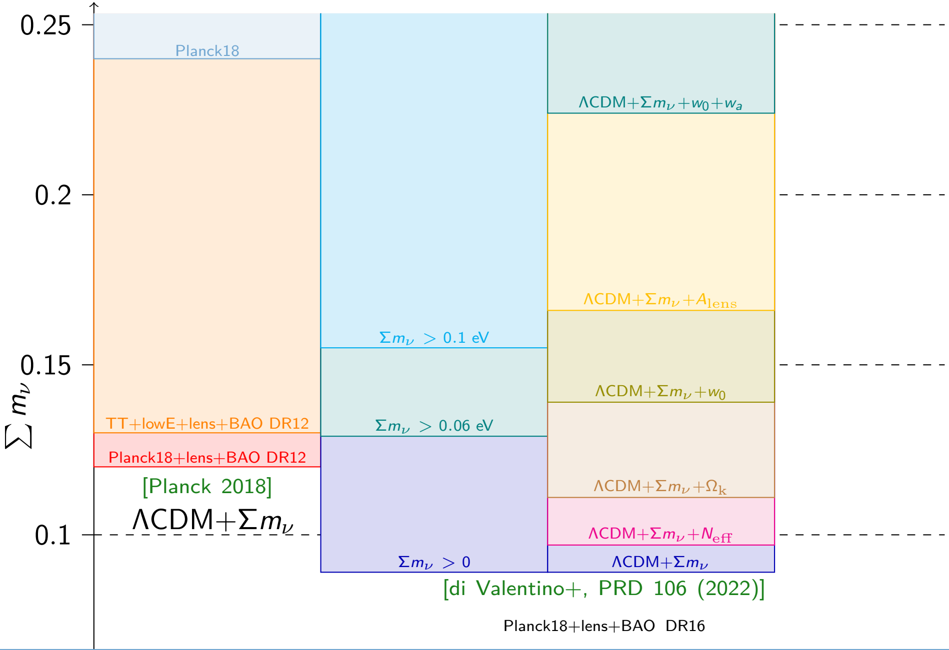


Warning: $\sum m_\nu \lesssim 0.1$ eV at 95% CL
does not mean IO disfavored at 95% CL!

Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)

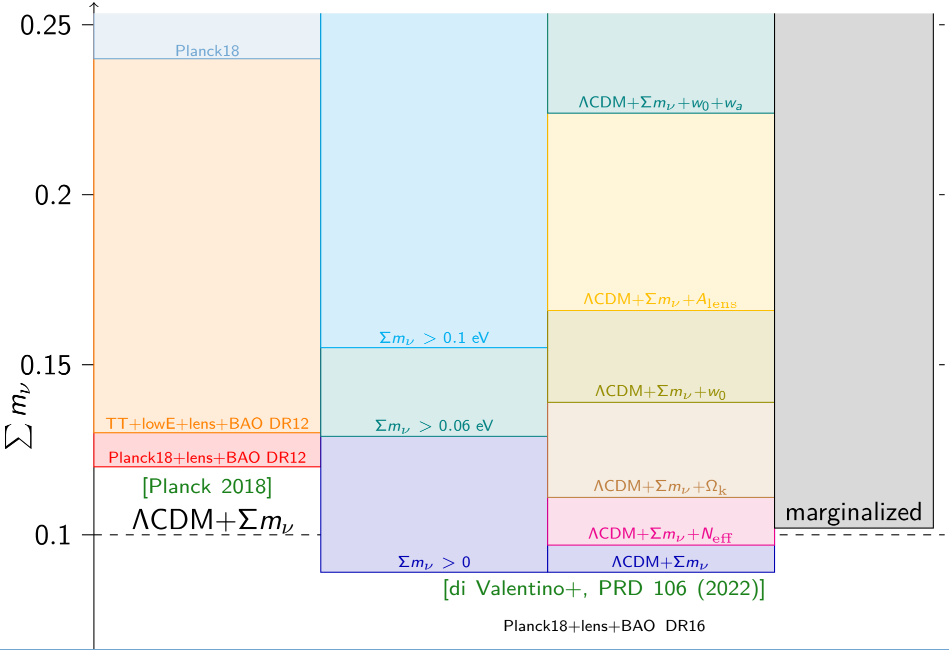


[Planck 2018]

[di Valentino+, PRD 106 (2022)]

Planck18+lens+BAO DR16

Cosmological neutrino mass bounds (95% CL)



Mass ordering results

Bayes theorem for models:

$$\boxed{p(\mathcal{M}|d) \propto Z_{\mathcal{M}}\pi(\mathcal{M})}$$

Bayesian evidence:

$$\boxed{Z_{\mathcal{M}} = \int_{\Omega_{\mathcal{M}}} \mathcal{L}(\theta)\,\pi(\theta)\,d\theta}$$

Bayes factor NO vs IO:

$$\boxed{B_{\mathrm{NO,IO}} = Z_{\mathrm{NO}}/Z_{\mathrm{IO}}}$$

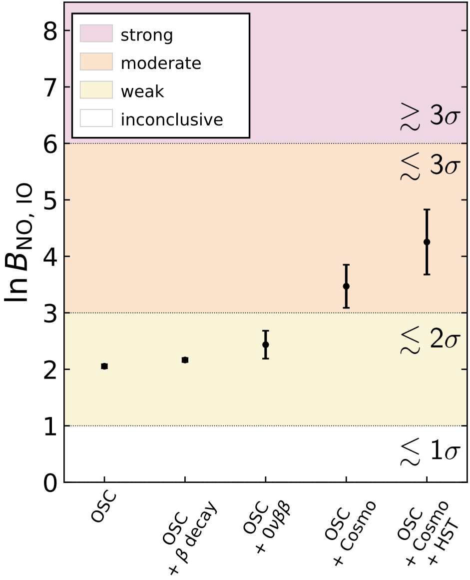
Posterior probability:

$$\boxed{\begin{aligned} P_{\mathrm{NO}} &= B_{\mathrm{NO,IO}}/(B_{\mathrm{NO,IO}}+1) \\ P_{\mathrm{IO}} &= 1/(B_{\mathrm{NO,IO}}+1) \end{aligned}}$$

$$N\sigma \text{ from } P_{\mathrm{NO}} = \mathrm{erf}(N/\sqrt{2})$$

[JHEP 02 (2021)]

http://globalfit.astroparticles.es/



$\pi(\mathcal{M})$ model prior

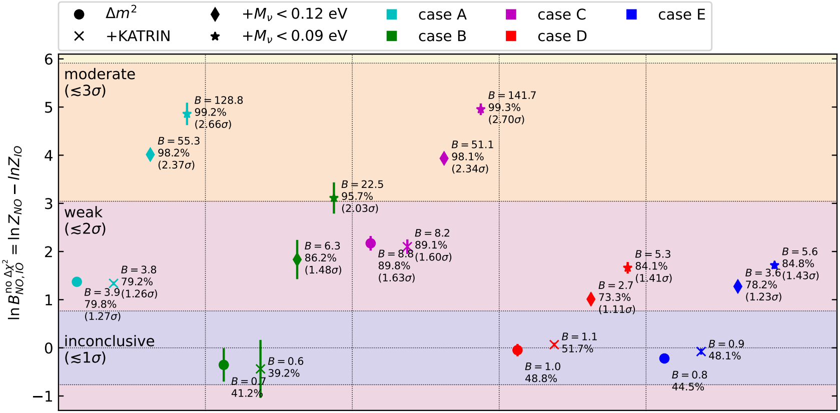
$p(\mathcal{M}|d)$ model posterior

$\mathcal{L}(\theta)$ likelihood

$\Omega_{\mathcal{M}}$ parameter space, for parameters θ

Mass ordering and Bayesian analyses

→ oscillation Δm^2 alone should not generate a difference



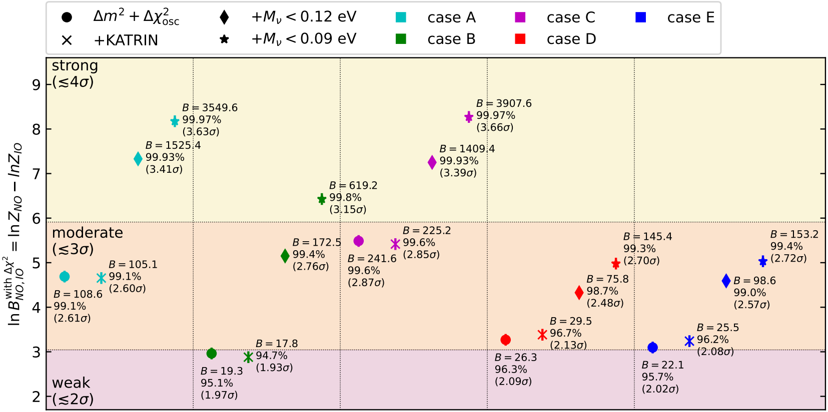
A, B, C:
Gauss. prior on
ln m_1, ln m_2, ln m_3,
different prior ranges or sampling

D, E:
linear prior on
Δm^2_{21}, $|\Delta m^2_{31}|$, $m_{\text{lightest}}/\sum m_\nu$

Mass ordering and Bayesian analyses

[JCAP 10 (2022)]

→ oscillation $\Delta\chi^2$ DOES prefer NO over IO at $\sim 2\sigma$



A, B, C:

Gauss. prior on

$\ln m_1$, $\ln m_2$, $\ln m_3$,

different prior ranges or sampling

D, E:

linear prior on

Δm^2_{21}, $|\Delta m^2_{31}|$, $m_{\rm lightest}/\sum m_\nu$

Can a cosmological limit on Σm_ν disfavor IO?

[PDU (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu/(94.12\text{ eV})$

NO: $\Sigma m_\nu \gtrsim 0.06$ eV

Current: $\Sigma m_\nu \lesssim 0.1$ eV (95%)

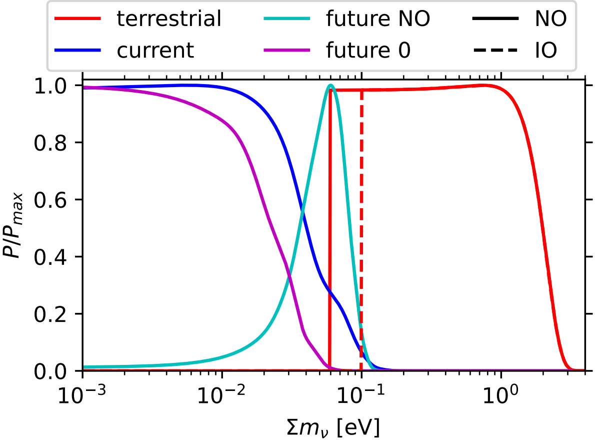
IO: $\Sigma m_\nu \gtrsim 0.1$ eV

Future sensitivity: $\sigma(\Sigma m_\nu) \simeq 0.02$ eV

Still preferring $\Sigma m_\nu = 0$?

Will measure e.g. $\Sigma m_\nu = 0.06$ eV?

tension ever with NO!



confirm NO, disfavor IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU (2023)]

standard factor

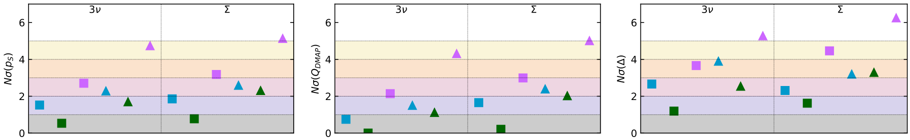
Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu/(94.12 \text{ eV})$

Is there a tension between cosmology and oscillations?

or will there be a tension?

several possible tests can be considered, similar results

$\Sigma m_\nu \lesssim 0.1$ eV (95%)	●	current	■	NO
$\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ)	●	future NO	▲	IO
$\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ)	●	future 0		



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

future 0 can be at $\sim 2-3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU (2023)]

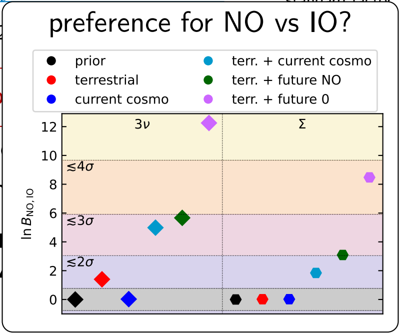
standard factor

Cosmology measures $\omega_\nu \;=\; \Omega_\nu h^2$

Is there a tension between cosmo

or will there be a t

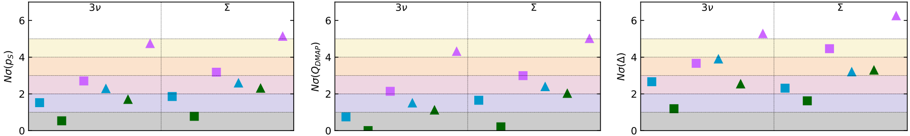
several possible tests can be con


preference for NO vs IO?

$\Sigma m_\nu \lesssim 0.1$ eV (95%) ● current ■

$\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ) ● future NO ▲

$\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ) ● future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

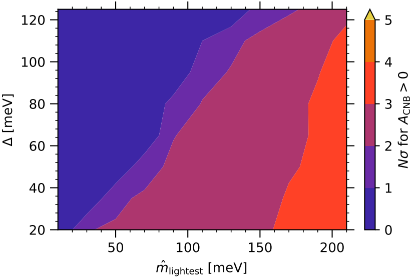
future 0 can be at $\sim 2-3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

D Direct detection of relic neutrinos

Proposed methods and their pros/cons

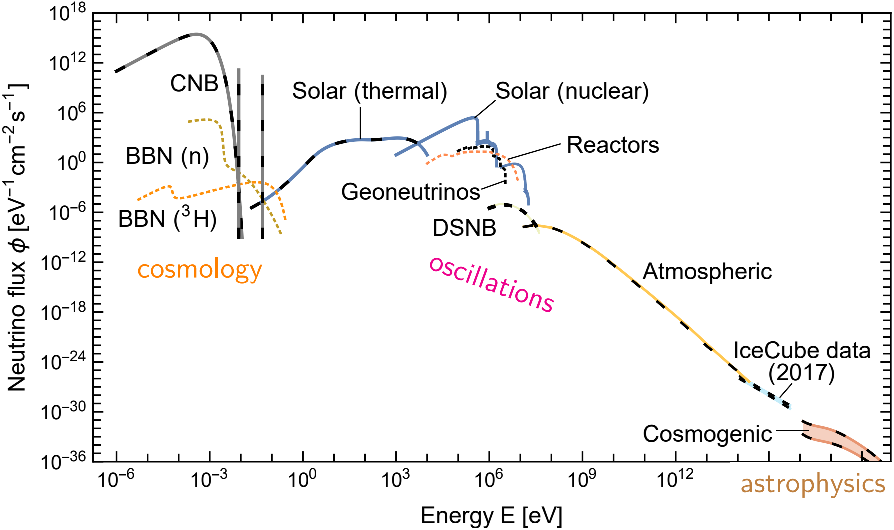
Based on:

- Cocco+,
  JCAP 06 (2007) 015
- Long+,
  JCAP 08 (2014) 038
- JCAP 09 (2017) 034
- JCAP 01 (2020) 015



Neutrino spectrum

[Vitagliano+, RMP 92 (2020)]

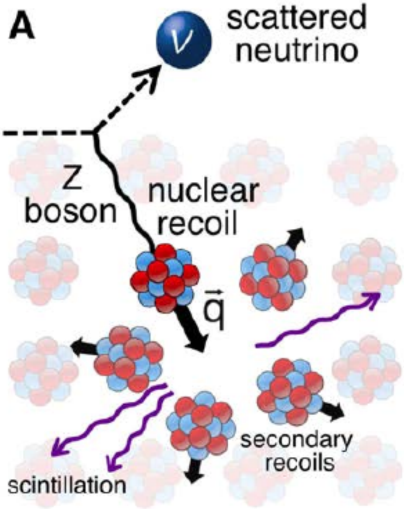


CNB neutrinos have extremely small energy!

CEνNS?

First of all: what's Coherent Elastic ν-Nucleous Scattering?

elastic scattering where ν interacts with nucleous "as a whole"



Predicted for $|\vec{q}|R \lesssim 1$
by [Freedman, PRD 1974]

small recoil energies! $\lesssim$ 10 keV...
difficult to measure

$$\frac{d\sigma}{dT}(E_\nu, T) \sim \frac{G_F^2 M}{4\pi} N^2$$

[Drukier, Stodolsky, PRD 1984]

enhancement N^2 because
ν interacts
coherently with all nucleons

may give huge cross
section enhancement

CEνNS?

[Shergold, arxiv:2109.07482]

First of all: what's Coherent Elastic ν-Nucleous Scattering?

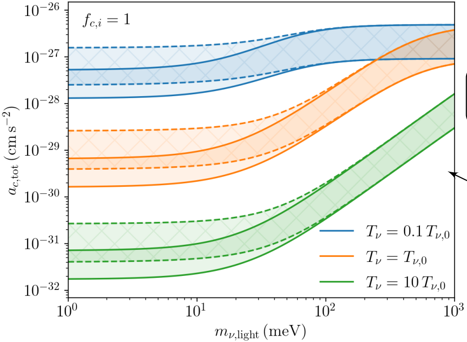
elastic scattering where ν interacts with nucleous "as a whole"

Can we detect relic neutrinos with CEνNS?

relic neutrinos have de Broglie length $\lambda \sim 2\pi/p_\nu$

↓

enhancement in interactions due to coherence with nuclei in volume λ^3



Acceleration induced by CEνNS of relic ν on test mass M:

$$a^N \propto ((A-Z)/A)^2 E_\nu/p_\nu^2 \Delta p_\nu n_\nu \rho$$

A, Z mass, atomic numbers

p_ν, E_ν neutrino momentum and energy

Δp_ν net momentum transfer

n_ν neutrino number density

ρ target mass density

unclustered relic νs, $n_\nu = n_0$

a^N of atoms in silicon target

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

Stodolsky effect

[Stodolsky, 1974][Duda et al., 2001]

(only if there is lepton asymmetry)

energy splitting of e^- spin states due to coherent scattering with relic neutrinos

↓

torque on e^- in lab rest frame

↓

use a ferromagnet to build detector

↓

measure torque with a torsion balance

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

Stodolsky effect

[Stodolsky, 1974][Duda et al., 2001]

(only if there is lepton asymmetry)

energy splitting of e^- spin states due to coherent scattering with relic neutrinos

↓

torque on e^- in lab rest frame

↓

use a ferromagnet to build detector

↓

measure torque with a torsion balance

expected $a_\nu \simeq \mathcal{O}(10^{-26})$ cm/s^2 ⟷ $a_{\rm exp} \simeq \mathcal{O}(10^{-12})$ cm/s^2

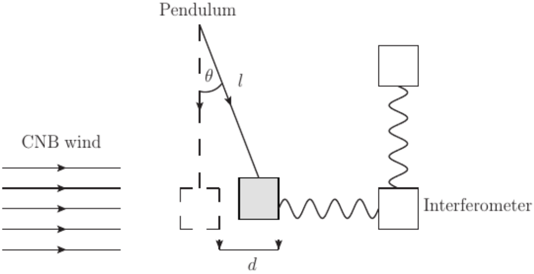
At interferometers?

How to directly detect non-relativistic neutrinos?

At interferometers

[Domcke et al., 2017]

coherent scattering of relic ν on a pendulum $\longrightarrow$ measure oscillations at interferometers



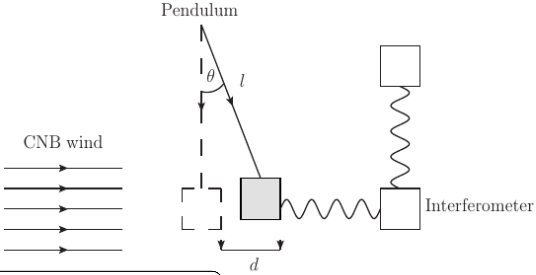
At interferometers?

How to directly detect non-relativistic neutrinos?

At interferometers

[Domcke et al., 2017]

coherent scattering of relic ν on a pendulum $\longrightarrow$ measure oscillations at interferometers



expected $10^{-33} \lesssim a_\nu/(\mathrm{cm/s^2}) \lesssim 10^{-27}$ $\longleftrightarrow$ $a_{\mathrm{LIGO/Virgo}} \simeq 10^{-16}\ \mathrm{cm/s^2}$

Neutrino capture? (I)

How to directly detect non-relativistic neutrinos?

Remember that
$\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today $\longrightarrow$ a process without energy threshold is necessary

(anti)neutrino capture on
electron-capture-decaying nuclei

[Cocco et al., 2009]

electron capture (EC): $e^- + A^+ \to \nu_e + B^*$
(e^- from inner level)

$$\boxed{\bar{\nu}_e + A \to B^- + e^+}$$

must have very specific Q value in order to avoid EC background and have no threshold

$$\boxed{\bar{\nu}_e + e^- + A^+ \to B}$$

specific energy conditions required

but Q value depends on ionization fraction!

Neutrino capture? (I)

How to directly detect non-relativistic neutrinos?

Remember that $\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today → a process without energy threshold is necessary

(anti)neutrino capture on electron-capture-decaying nuclei

[Cocco et al., 2009]

electron capture (EC): $e^- + A^+ \to \nu_e + B^*$

(e^- from inner level)

$$\boxed{\bar\nu_e + A \to B^- + e^+}$$

must have very specific Q value in order to avoid EC background and have no threshold

$$\boxed{\bar\nu_e + e^- + A^+ \to B}$$

specific energy conditions required

but

Q value depends on ionization fraction!

process useful only “if specific conditions on the Q-value are met or significant improvements on ion storage rings are achieved”

Neutrino capture (II) - a viable method

[Long+, JCAP 08 (2014) 038]

How to directly detect non-relativistic neutrinos?

Remember that $\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today → a process without energy threshold is necessary

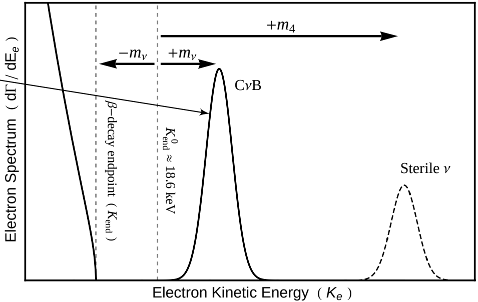
[Weinberg, 1962]: neutrino capture in β–decaying nuclei $\nu + n \to p + e^-$

Main background: β decay $n \to p + e^- + \bar{\nu}$!

signal is a peak at $2m_\nu$ above β–decay endpoint

only with a lot of material

need a very good energy resolution



What material?

[Cocco+, JCAP 06 (2007) 015]

best element has highest $\sigma_{\rm NCB}(v_\nu/c) \cdot t_{1/2}$

to minimize contamination from β decay background

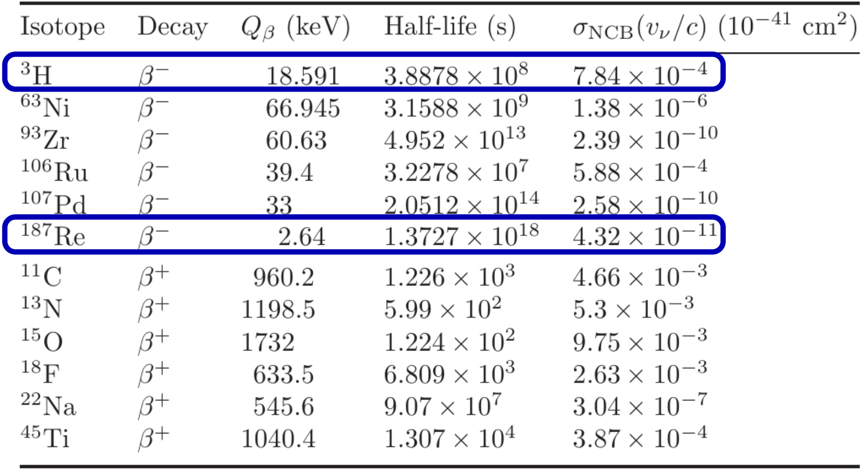
Isotope	Decay	Q_β (keV)	Half-life (s)	$\sigma_{\rm NCB}(v_\nu/c)$ (10^{-41} cm^2)
^{3}H	β^-	18.591	3.8878×10^{8}	7.84×10^{-4}
^{63}Ni	β^-	66.945	3.1588×10^{9}	1.38×10^{-6}
^{93}Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
^{106}Ru	β^-	39.4	3.2278×10^{7}	5.88×10^{-4}
^{107}Pd	β^-	33	2.0512×10^{14}	2.58×10^{-10}
^{187}Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
^{11}C	β^+	960.2	1.226×10^{3}	4.66×10^{-3}
^{13}N	β^+	1198.5	5.99×10^{2}	5.3×10^{-3}
^{15}O	β^+	1732	1.224×10^{2}	9.75×10^{-3}
^{18}F	β^+	633.5	6.809×10^{3}	2.63×10^{-3}
^{22}Na	β^+	545.6	9.07×10^{7}	3.04×10^{-7}
^{45}Ti	β^+	1040.4	1.307×10^{4}	3.87×10^{-4}

What material?

[Cocco+, JCAP 06 (2007) 015]

best element has highest $\sigma_{\rm NCB}(v_\nu/c)\cdot t_{1/2}$

to minimize contamination from β decay background



Isotope	Decay	Q_β (keV)	Half-life (s)	$\sigma_{\rm NCB}(v_\nu/c)$ (10^{-41} cm^2)
^{3}H	β^-	18.591	3.8878×10^8	7.84×10^{-4}
^{63}Ni	β^-	66.945	3.1588×10^9	1.38×10^{-6}
^{93}Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
^{106}Ru	β^-	39.4	3.2278×10^7	5.88×10^{-4}
^{107}Pd	β^-	33	2.0512×10^{14}	2.58×10^{-10}
^{187}Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
^{11}C	β^+	960.2	1.226×10^3	4.66×10^{-3}
^{13}N	β^+	1198.5	5.99×10^2	5.3×10^{-3}
^{15}O	β^+	1732	1.224×10^2	9.75×10^{-3}
^{18}F	β^+	633.5	6.809×10^3	2.63×10^{-3}
^{22}Na	β^+	545.6	9.07×10^7	3.04×10^{-7}
^{45}Ti	β^+	1040.4	1.307×10^4	3.87×10^{-4}

What material?

[Cocco+, JCAP 06 (2007) 015]

best element has highest $\sigma_{\rm NCB}(v_\nu/c) \cdot t_{1/2}$

to minimize contamination from β decay background

Isotope	Decay	Q_β (keV)	Half-life (s)	$\sigma_{\rm NCB}(v_\nu/c)$ (10^{-41} cm^2)
^{3}H	β^-	18.591	3.8878×10^{8}	7.84×10^{-4}
^{63}Ni	β^-	66.945	3.1588×10^{9}	1.38×10^{-6}
^{93}Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
^{106}Ru	β^-	39.4	3.2278×10^{7}	5.88×10^{-4}
^{107}Pd	β^-	33	2.0512×10^{14}	2.58×10^{-10}
^{187}Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
^{11}C	β^+	960.2	1.226×10^{3}	4.66×10^{-3}
^{13}N	β^+	1198.5	5.99×10^{2}	5.3×10^{-3}
^{15}O	β^+	1732	1.224×10^{2}	9.75×10^{-3}
^{18}F	β^+	633.5	6.809×10^{3}	2.63×10^{-3}
^{22}Na	β^+	545.6	9.07×10^{7}	3.04×10^{-7}
^{45}Ti	β^+	1040.4	1.307×10^{4}	3.87×10^{-4}

^{3}H better because the cross section (→ event rate) is higher

PTOLEMY

[PTOLEMY LoI, arxiv:1808.01892]

Pontecorvo Tritium Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1$ eV? 0.05 eV?

built mainly for CNB

can probe $m_\nu \simeq 1.4\Delta \simeq 0.1$ eV

$M_T = 100$ g of atomic ^{3}H

$$\boxed{\Gamma_{\text{CNB}} = \sum_{i=1}^{3} |U_{ei}|^2 [n_i(\nu_{h_R}) + n_i(\nu_{h_L})]\, N_T\, \bar{\sigma}} \quad \sim \mathcal{O}(10)\ \text{yr}^{-1}$$

N_T number of ^{3}H nuclei in a sample of mass M_T $\bar{\sigma} \simeq 3.834 \times 10^{-45}$ cm^2 n_i number density of neutrino i

(without clustering)

PTOLEMY

[PTOLEMY LoI, arxiv:1808.01892]

Pontecorvo Tritium Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1$ eV?
0.05 eV?

built mainly for CNB

can probe $m_\nu \simeq 1.4\Delta \simeq 0.1$ eV

$M_T = 100$ g of atomic ^{3}H

ehnancement from
ν clustering in the galaxy?

ehnancement from
other effects?

$$\Gamma_{\rm CNB} = \sum_{i=1}^{3} |U_{ei}|^2 [n_i(\nu_{h_R}) + n_i(\nu_{h_L})]\, N_T\, \bar\sigma \qquad \sim \mathcal{O}(10)\ {\rm yr}^{-1}$$

N_T number of ^{3}H nuclei in a sample of mass M_T $\bar\sigma \simeq 3.834 \times 10^{-45}$ cm^2 n_i number density of neutrino i

(without clustering)

Time variations of ν capture rates

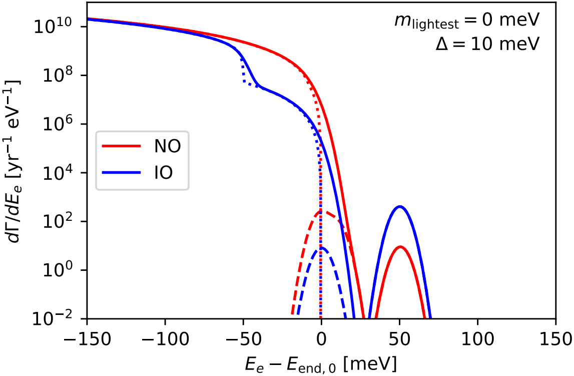
[Akhmedov, JCAP 2019]

What if the lightest neutrino is massless and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β-decay)!

$$\frac{\nu \text{ capture rate}}{\beta \text{ decay rate}} = \frac{\Gamma_{\rm NC}}{\Gamma_\beta} \simeq \frac{n_\nu}{56\ \text{cm}^{-3}} \frac{2.54\times 10^{-11}}{(\Delta/\text{eV})^3}$$

rates in the bin Δ on the endpoint



Time variations of ν capture rates

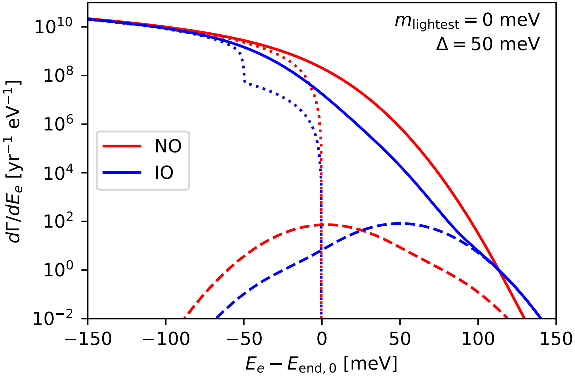
[Akhmedov, JCAP 2019]

What if the lightest neutrino is massless
and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β-decay)!

$$\frac{\nu \text{ capture rate}}{\beta \text{ decay rate}} = \frac{\Gamma_{\rm NC}}{\Gamma_\beta} \simeq \frac{n_\nu}{56\ \text{cm}^{-3}} \frac{2.54 \times 10^{-11}}{(\Delta/\text{eV})^3}$$

rates in the bin Δ
on the endpoint



Time variations of ν capture rates

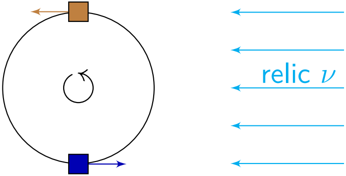
[Akhmedov, JCAP 2019]

What if the lightest neutrino is massless
and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β-decay)!

$$\frac{\nu\text{ capture rate}}{\beta\text{ decay rate}} = \frac{\Gamma_{\mathrm{NC}}}{\Gamma_\beta} \simeq \frac{n_\nu}{56\text{ cm}^{-3}} \frac{2.54\times 10^{-11}}{(\Delta/\mathrm{eV})^3}$$

rates in the bin Δ
on the endpoint



can be daily or annual modulation!

only for ν capture (no β-decay)

Time variations of ν capture rates

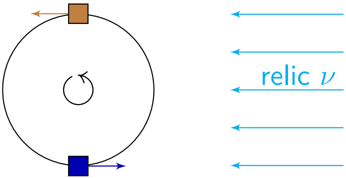
[Akhmedov, JCAP 2019]

What if the lightest neutrino is massless and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β-decay)!

$$\frac{\nu\ \text{capture rate}}{\beta\ \text{decay rate}} = \frac{\Gamma_{\rm NC}}{\Gamma_\beta} \simeq \frac{n_\nu}{56\ \text{cm}^{-3}} \frac{2.54 \times 10^{-11}}{(\Delta/\text{eV})^3}$$

rates in the bin Δ on the endpoint



can be daily or annual modulation!

only for ν capture (no β-decay)

Problem:

Expected daily modulation is $\sim$ 1% of the signal!!

Must use powerful technique for signal/noise separation

Fourier analysis and frequency filtering may be sufficient

no m_ν information in this way!

ν clustering with N-one-body simulations

[JCAP 09 (2017) 034]

Milky Way (MW) matter attracts neutrinos!

clustering →

$$\Gamma_{\rm CNB} = \sum_{i=1}^{3} |U_{ei}|^2 \, f_c(m_i) \, [n_{i,0}(\nu_{h_R}) + n_{i,0}(\nu_{h_L})] \, N_T \, \bar{\sigma}$$

$f_c(m_i) = n_i / n_{i,0}$ clustering factor ⟶ How to compute it?

Idea from [Ringwald & Wong, 2004] ⟶ N-one-body= N × single ν simulations

→ each ν evolved from initial conditions at $z = 3$

→ spherical symmetry, coordinates $(r,\ \theta,\ p_r,\ l)$

→ need $\rho_{\rm matter}(z) = \rho_{\rm DM}(z) + \rho_{\rm baryon}(z)$

Assumptions:

- νs are independent
- only gravitational interactions
- νs do not influence matter evolution ($\rho_\nu \ll \rho_{\rm DM}$)

how many νs is "N"?

→ must sample all possible r, p_r, l

→ must include all possible νs that reach the MW (fastest ones may come from several (up to $\mathcal{O}(100)$) Mpc!)

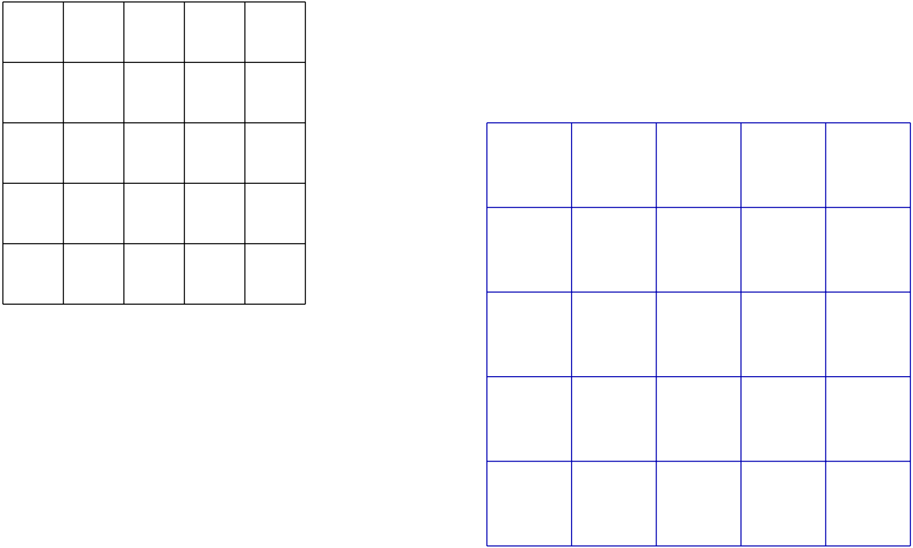
given N ν:

→ weigh each neutrinos

→ reconstruct final density profile with kernel method from [Merritt&Tremblay, 1994]

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution

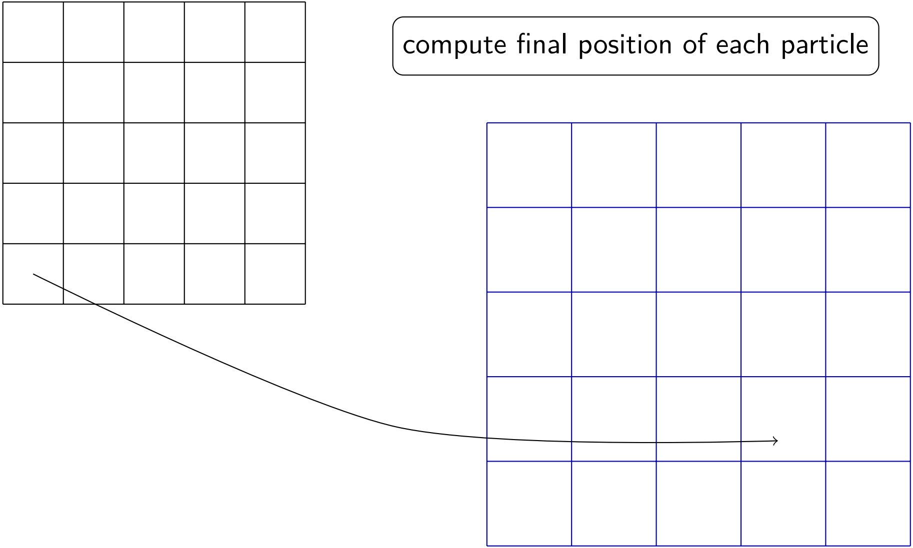


final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution

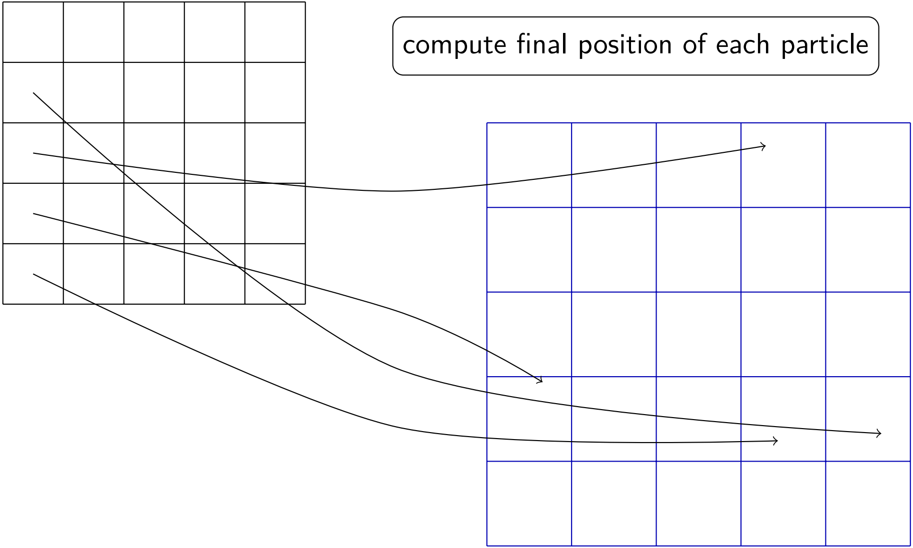
compute final position of each particle



final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution

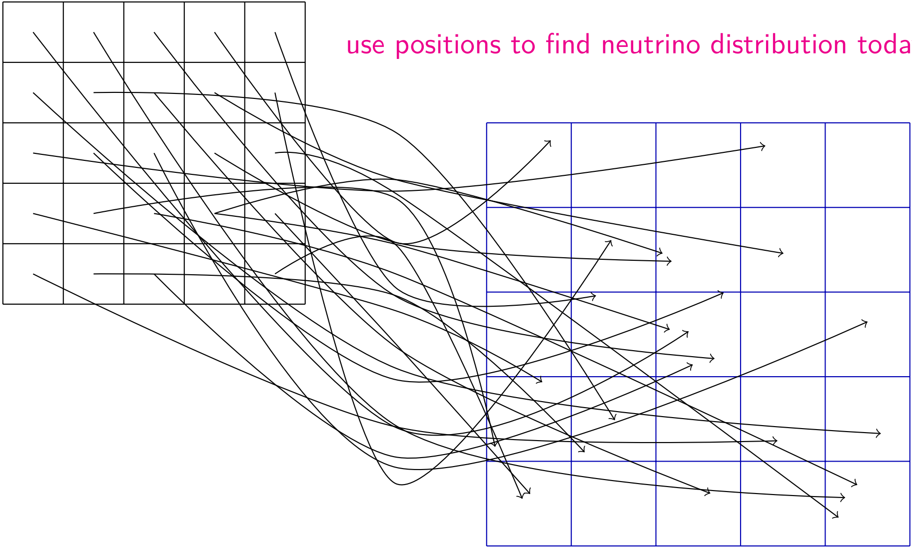


compute final position of each particle

final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution



use positions to find neutrino distribution today

final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution

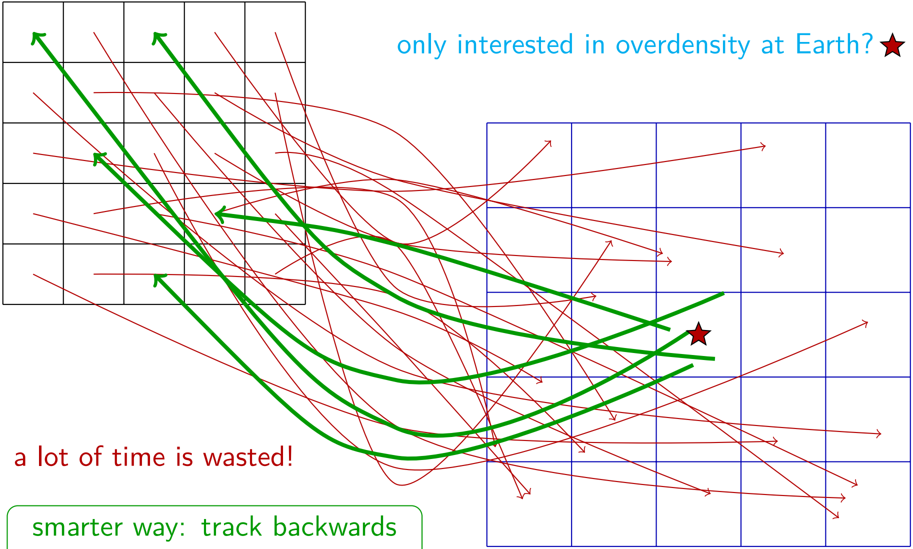
only interested in overdensity at Earth? ★

a lot of time is wasted!

final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4 \longrightarrow$ homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★

a lot of time is wasted!

smarter way: track backwards
only interesting particles!

final phase space, $z = 0$

Advantages of tracking back

[JCAP 01 (2020) 015]

First advantage is in computational terms: much less points to compute

Advantages of tracking back

[JCAP 01 (2020) 015]

First advantage is in computational terms: much less points to compute

Second advantage: no need to use spherical symmetry!

Forward-tracking

initial conditions need to sample 1D for position + 2D for momentum when using spherical symmetry

with full grid would require 3+3 dimensions!

Impossible to relax spherical symmetry!

Back-tracking

“Initial” conditions only described by 3D in momentum

(position is fixed, apart for checks)

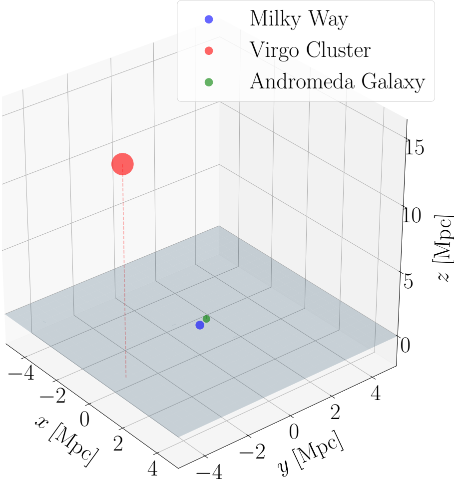
can do the calculation with any astrophysical setup

Advantages of tracking back

[JCAP 01 (2020) 015]

First advantage is in computational terms: much less points to compute

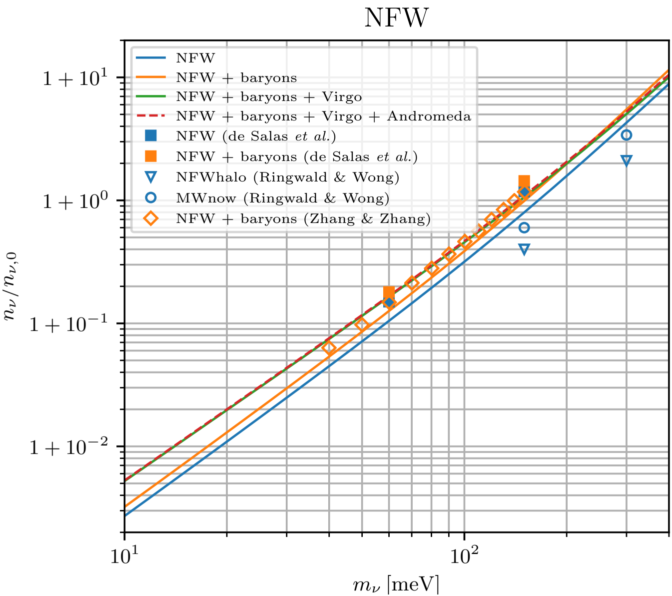
Second advantage: no need to use spherical symmetry!



Clustering results with back-tracking

[JCAP 01 (2020) 015]

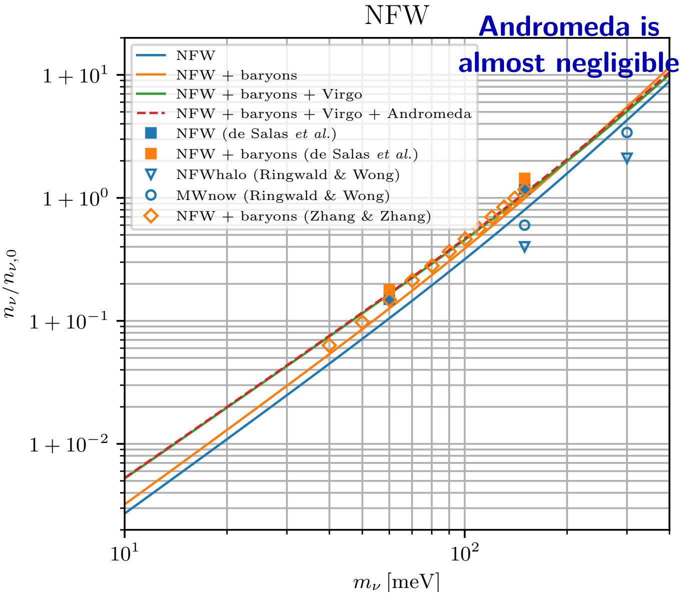
In comparison with previous results:



Clustering results with back-tracking

[JCAP 01 (2020) 015]

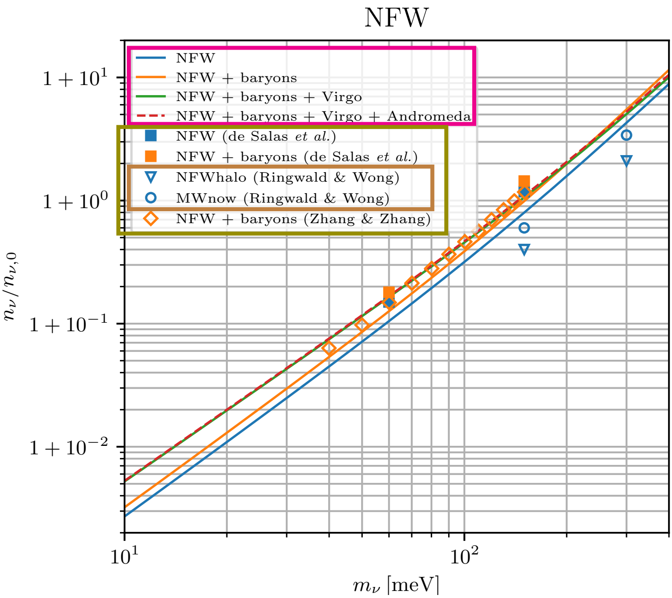
In comparison with previous results:



Andromeda is almost negligible

Clustering results with back-tracking

In comparison with previous results:



Warning: NFW is not the same for all the cases!

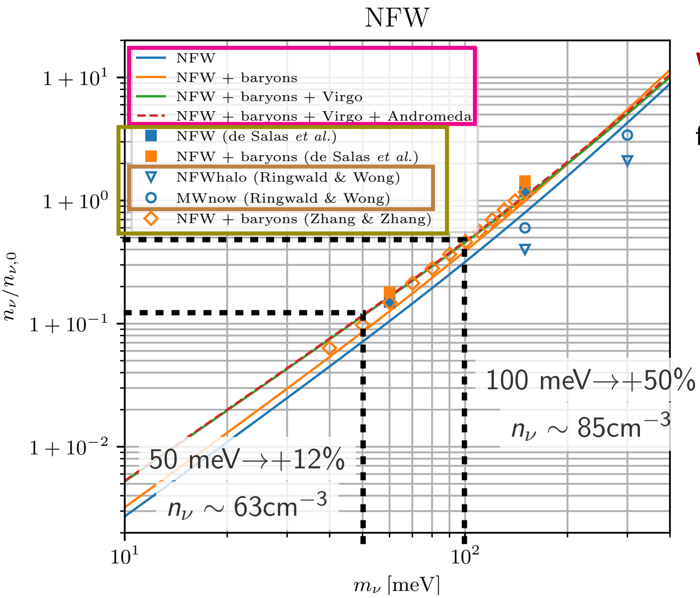
[de Salas+, 2017] and [Zhang^2, 2018] use $\gamma \neq 1$, now we have $\gamma = 1$

[Ringwald&Wong, 2004] uses old parameters

Clustering results with back-tracking

[JCAP 01 (2020) 015]

In comparison with previous results:



100 meV→+50%

$n_\nu \sim 85\text{cm}^{-3}$

50 meV→+12%

$n_\nu \sim 63\text{cm}^{-3}$

Warning: NFW is not the same for all the cases!

[de Salas+, 2017]
and
[Zhang^2, 2018]
use $\gamma \neq 1$,
now we have
$\gamma = 1$

[Ringwald&Wong, 2004] uses old parameters

Detection of the relic neutrinos

[PTOLEMY, JCAP 07 (2019) 047]

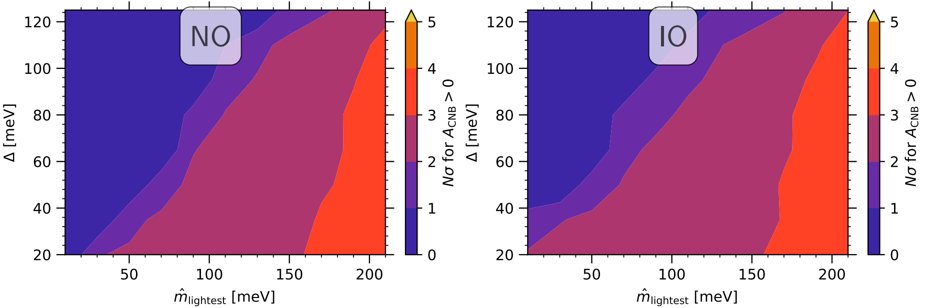
using the definition:

$$\boxed{N^i_{\rm th}(\boldsymbol{\theta}) = A_\beta N^i_\beta(\hat{E}_{end} + \Delta E_{end}, m_i, U) + \boldsymbol{A}_{\bf CNB} N^i_{\rm CNB}(\hat{E}_{end} + \Delta E_{end}, m_i, U) + N_b}$$

if $\boldsymbol{A}_{\bf CNB} > 0$ at $N\sigma$, direct detection of CNB accomplished at $N\sigma$

statistical only!

significance on $A_{\rm CNB} > 0$
as a function of $\hat{m}_{\rm lightest}$, Δ



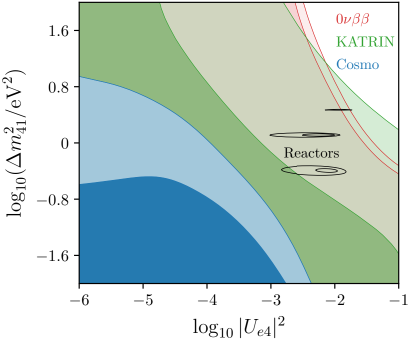
S

Light sterile neutrinos

Assuming they exist. . .

Based on:

- JCAP 07 (2019)
- PRD 104 (2021)



Sterile neutrino in the early universe

[SG+, JCAP 07 (2019) 014]

Four neutrinos $\longrightarrow$ new oscillations in the early Universe

sterile $\Longrightarrow$ no weak/em interactions in the thermal plasma

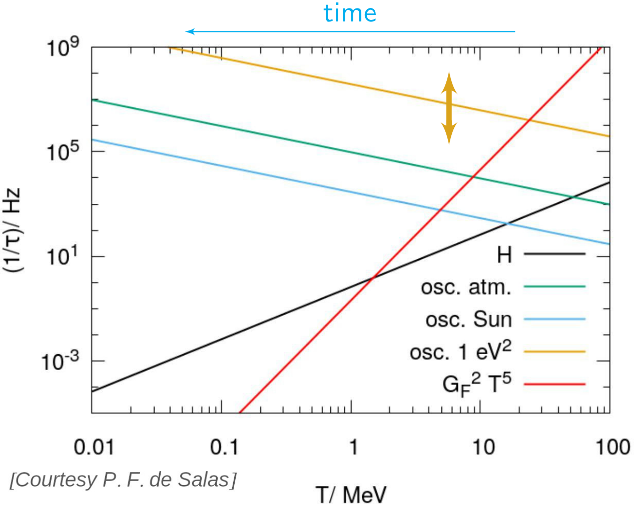
Sterile neutrino in the early universe

[SG+, JCAP 07 (2019) 014]

Four neutrinos $\longrightarrow$ new oscillations in the early Universe

sterile $\Longrightarrow$ no weak/em interactions in the thermal plasma

need to produce it through oscillations, but matter effects may block them



[Courtesy P. F. de Salas]

beginning of
oscillations
depends on Δm^2_{41}

later oscillations

$\Downarrow$

less time before
ν decoupling!

Sterile neutrino in the early universe

[SG+, JCAP 07 (2019) 014]

Four neutrinos ⟶ new oscillations in the early Universe

sterile $\Longrightarrow$ no weak/em interactions in the thermal plasma

need to produce it through oscillations, but matter effects may block them

when are they enough to allow full equilibrium of active-sterile states?

$$0 \longleftarrow \Delta N_{\rm eff} = N_{\rm eff}^{4\nu} - N_{\rm eff}^{3\nu} \longrightarrow \simeq 1$$

no sterile production

active&sterile in equilibrium

$$\frac{\Delta m^2_{as}}{\text{eV}^2} \sin^4\left(2\vartheta_{as}\right) \simeq 10^{-5} \ln^2\left(1 - \Delta N_{\rm eff}\right) \qquad \text{(1+1 approx.)}$$

[Dolgov&Villante, 2004]

e.g.: $\Delta m^2_{as} = 1$ eV^2, $\sin^2\left(2\vartheta_{as}\right) \simeq 10^{-3} \Longrightarrow \Delta N_{\rm eff} \simeq 1$

$N_{\rm eff}^{3\nu} = 3.044$ [JCAP 2021]

Sterile neutrino in the early universe

[SG+, JCAP 07 (2019) 014]

Four neutrinos ⟶ new oscillations in the early Universe

sterile ⟹ no weak/em interactions in the thermal plasma

need to produce it through oscillations, but matter effects may block them

when are they enough to allow full equilibrium of active-sterile states?

$$0 \longleftarrow \Delta N_{\rm eff} = N_{\rm eff}^{4\nu} - N_{\rm eff}^{3\nu} \longrightarrow \simeq 1$$

no sterile production — active&sterile in equilibrium

$$\frac{\Delta m^2_{as}}{\mathrm{eV}^2} \sin^4\left(2\vartheta_{as}\right) \simeq 10^{-5} \ln^2\left(1 - \Delta N_{\rm eff}\right) \qquad \text{(1+1 approx.)}$$

[Dolgov&Villante, 2004]

e.g.: $\Delta m^2_{as} = 1\ \mathrm{eV}^2$, $\sin^2\left(2\vartheta_{as}\right) \simeq 10^{-3} \Longrightarrow \Delta N_{\rm eff} \simeq 1$

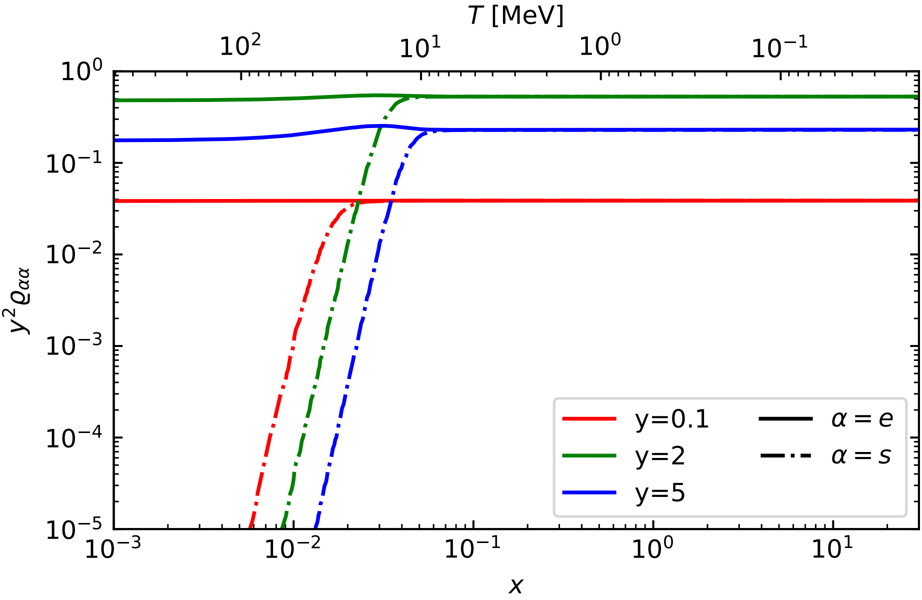
Full calculation: use numerical code!

`FORTran-Evolved PrimordIAl Neutrino Oscillations`
`(FortEPiaNO)`
`https://bitbucket.org/ahep_cosmo/fortepiano_public`

Momentum distributions

[SG+, JCAP 07 (2019) 014]

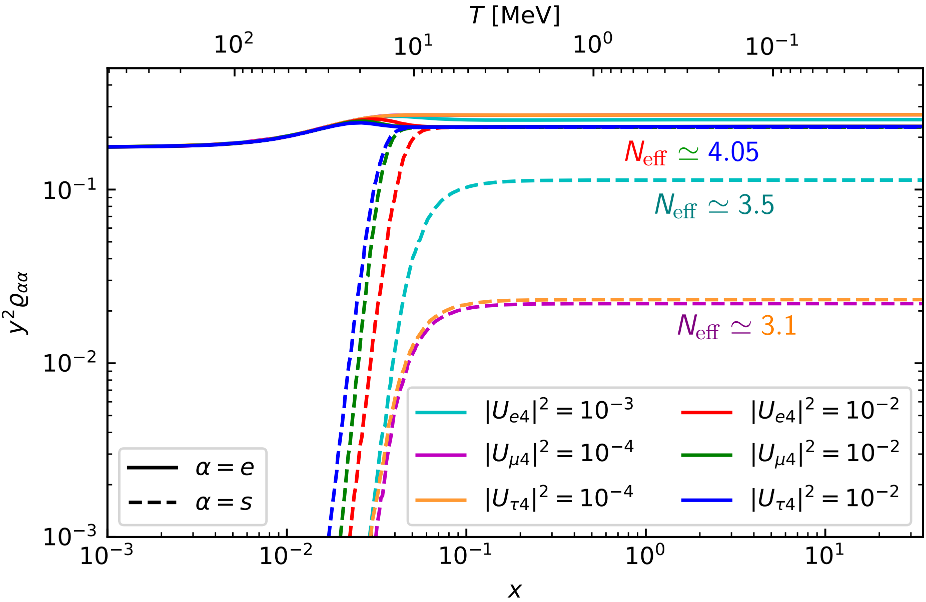
$\Delta m^2_{41} = 1.29\ \text{eV}^2,\ |U_{e4}|^2 = 10^{-2},\ |U_{\mu4}|^2 = |U_{\tau4}|^2 = 0,\ N_\text{eff} \simeq 4.05$



Momentum distributions

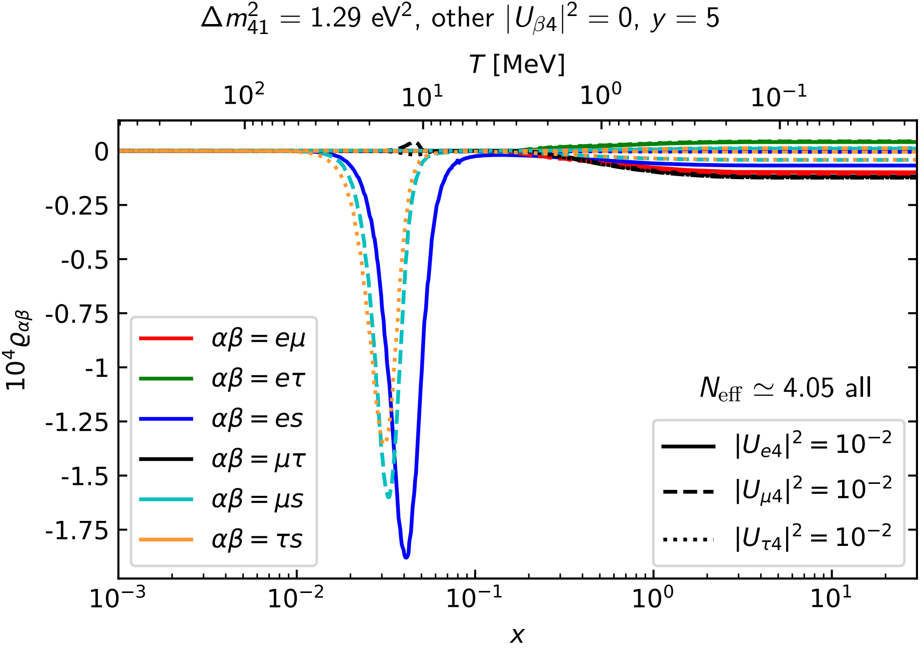
[SG+, JCAP 07 (2019) 014]

$\Delta m^2_{41} = 1.29\ \text{eV}^2$, other $|U_{\beta 4}|^2 = 0$, $y = 5$



Momentum distributions

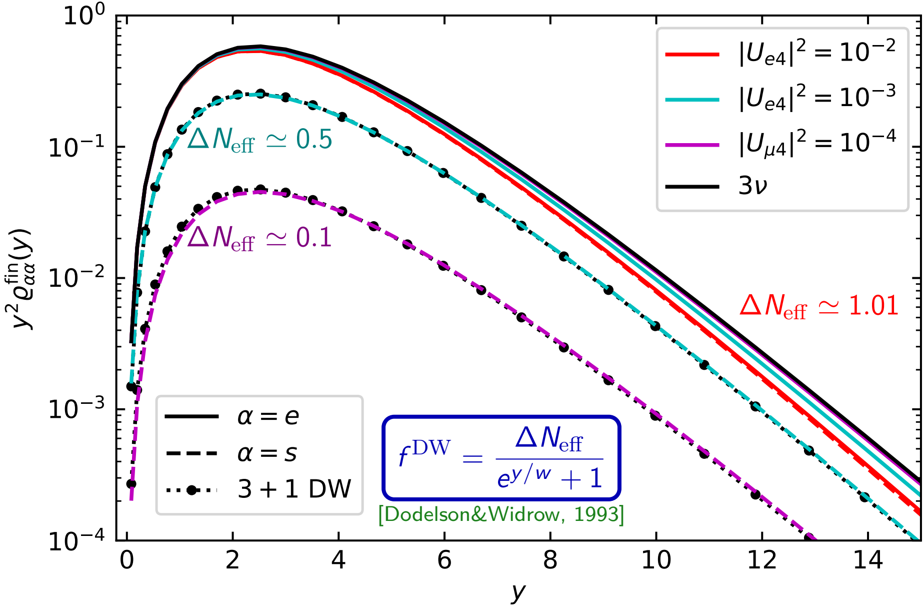
[SG+, JCAP 07 (2019) 014]



Momentum distributions

[SG+, JCAP 07 (2019) 014]

$\Delta m^2_{41} = 1.29$ eV^2, other $|U_{\beta 4}|^2 = 0$, $\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm active}$



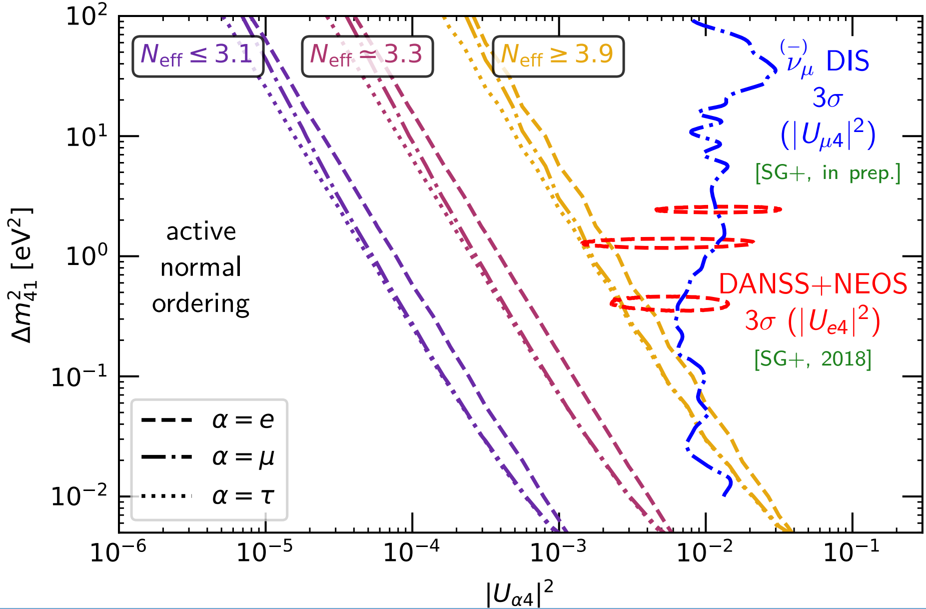
$$f^{\rm DW} = \frac{\Delta N_{\rm eff}}{e^{y/w}+1}$$

[Dodelson&Widrow, 1993]

N_{eff} and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

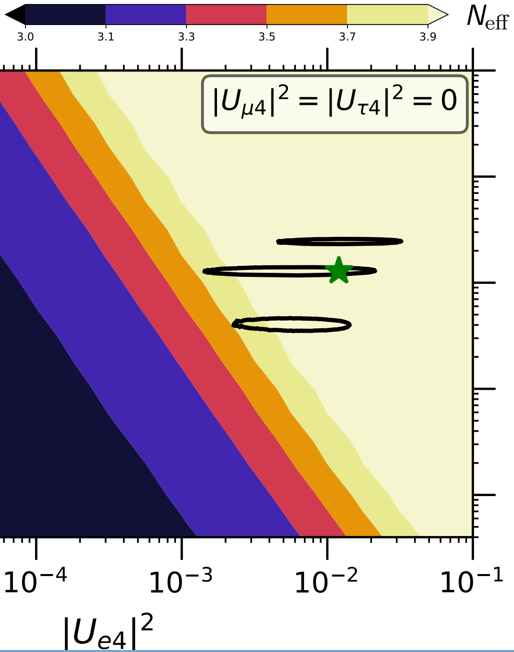
Only vary one angle and fix two to zero: do they have the same effect?



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

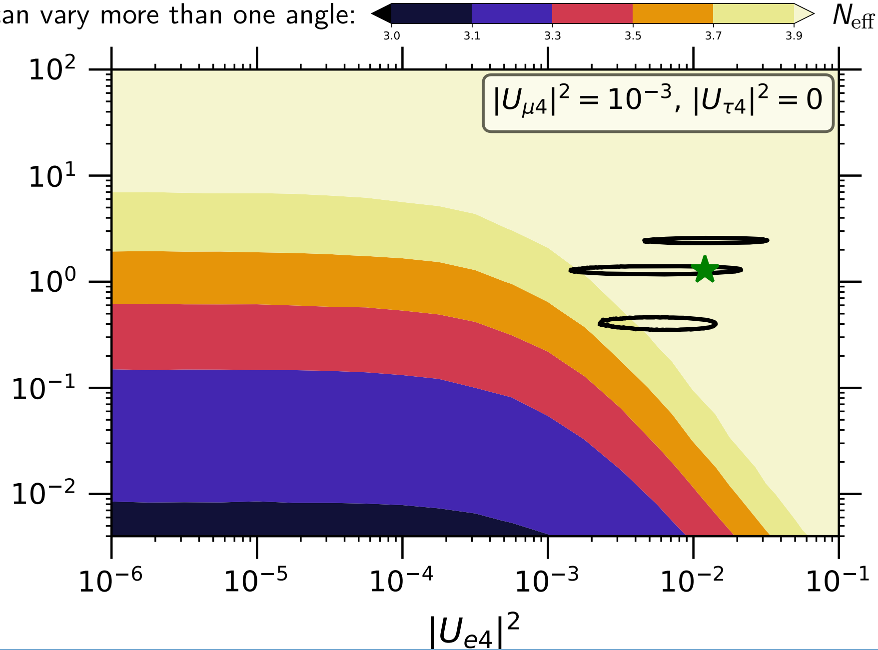
We can vary more than one angle:



N_{eff} and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

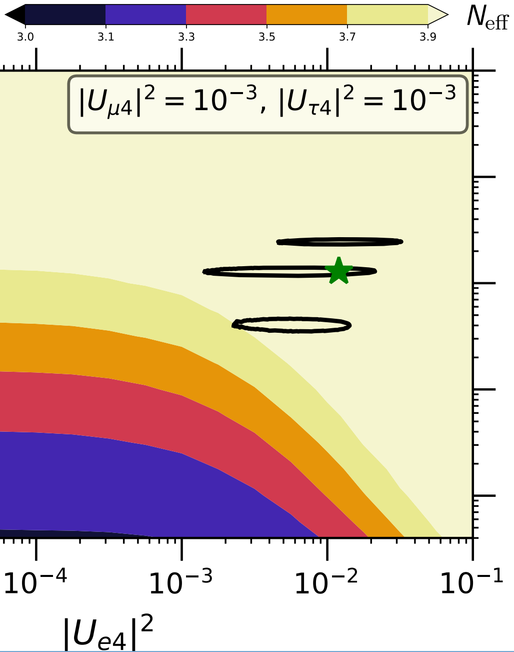
We can vary more than one angle:



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

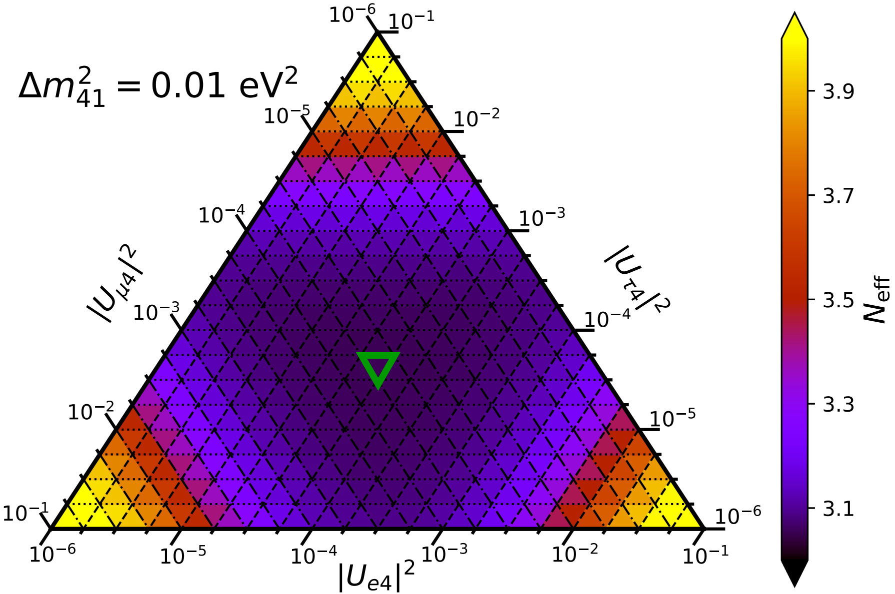
We can vary more than one angle:



N_{eff} and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

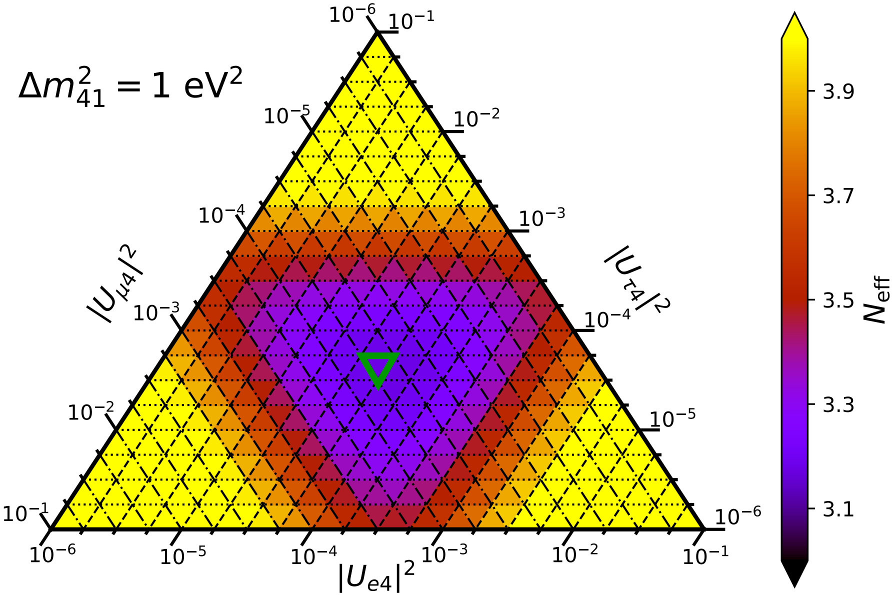
Sort of ternary plot (sum of $|U_{\alpha 4}|^2$ does not add up to 1!):



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

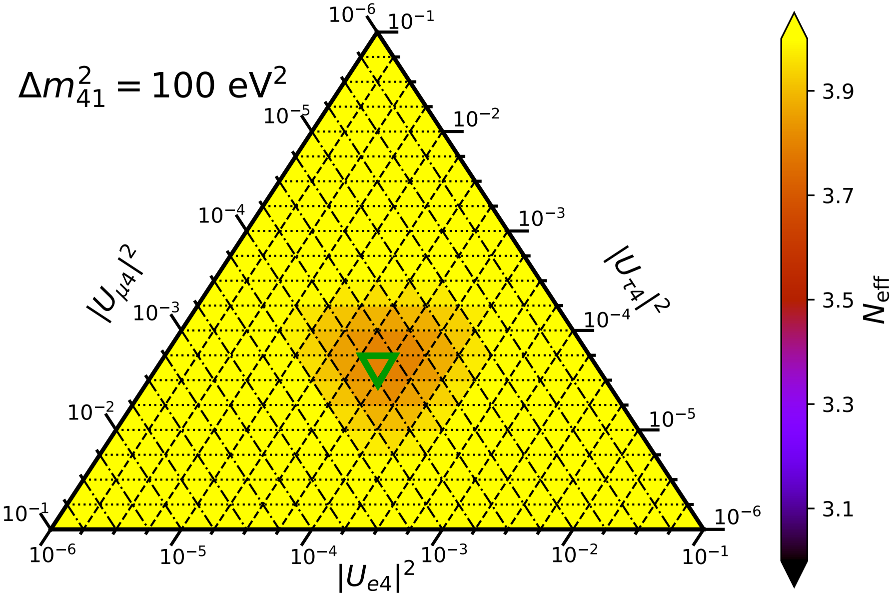
Sort of ternary plot (sum of $|U_{\alpha4}|^2$ does not add up to 1!):



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

Sort of ternary plot (sum of $|U_{\alpha 4}|^2$ does not add up to 1!):



LSν mass in cosmology: $m_s^{\rm eff}$ and m_s

most neutrinos are non-relativistic today

light sterile $m_s \simeq 1$ eV is non-relativistic already at CMB decoupling

Non-relativistic neutrinos: $\omega_\nu = \dfrac{\rho_\nu}{\rho_c} h^2 = \dfrac{\sum m_\nu}{94.12 \text{ eV}}$

$$\omega_s = \Omega_s h^2 = \frac{\rho_s}{\rho_c} h^2 = \frac{h^2}{\rho_c} \frac{m_s}{\pi^2} \int dp\, p^2 f_s(p)$$ [Acero+, PRD 2009]

ρ_s energy density of non-relativistic LSν, ρ_c critical density and h reduced Hubble parameter

Dodelson-Widrow distribution function: $f_s \approx \Delta N_{\rm eff}\, f_a$

$$\boxed{m_s^{\rm eff} = \Delta N_{\rm eff}\, m_s}$$

so that

- $\omega_s = m_s^{\rm eff}/(94.12 \text{ eV})$
- $m_s^{\rm eff} \simeq m_s$ for thermalized LSν ($\Delta N_{\rm eff} \simeq 1$)
- if $\Delta N_{\rm eff} \simeq 0$, $m_s^{\rm eff} \simeq 0 \Rightarrow$ cannot constrain m_s

LSν mass in cosmology: m_s^{eff} and m_s

most neutrinos are non-relativistic today

light sterile $m_s \simeq 1$ eV is non-relativistic already at CMB decoupling

Non-relativistic neutrinos: $\omega_\nu = \frac{\rho_\nu}{\rho_c} h^2 = \frac{\Sigma m_\nu}{94.12 \text{ eV}}$

$$\omega_s = \Omega_s h^2 = \frac{\rho_s}{\rho_c} h^2 = \frac{h^2}{\rho_c} \frac{m_s}{\pi^2} \int dp\, p^2 f_s(p)$$ [Acero+, PRD 2009]

ρ_s energy density of non-relativistic LSν, ρ_c critical density and h reduced Hubble parameter

alternative production mechanism, it may appear in the literature:

thermal distribution function $f_s(p) = \frac{1}{e^{p/T_s} + 1}$

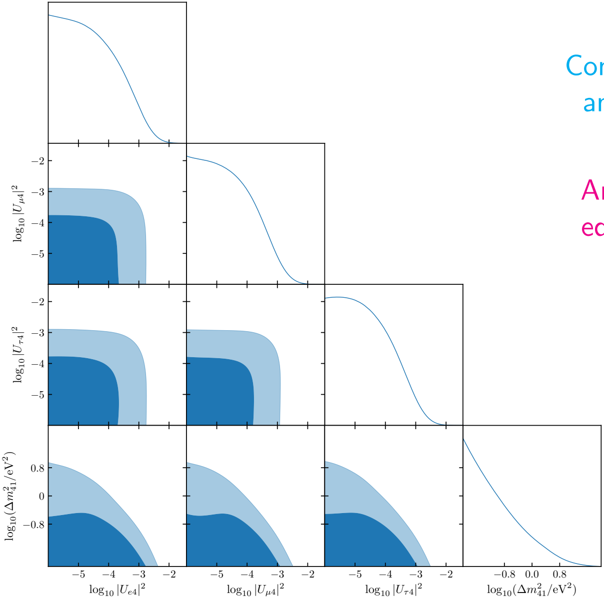
$$T_s = \Delta N_{\mathrm{eff}}^{1/4} T_a \implies \boxed{m_s^{\mathrm{eff}} = \Delta N_{\mathrm{eff}}^{3/4} m_s}$$

similar behavior as DW case, different dependence on ΔN_{eff}

Cosmological constraints on $|U_{\alpha 4}|^2$

[PRD 104 (2021) 123524]

Use multi-angle results from `FortEPiaNO` to derive constraints on $|U_{\alpha 4}|^2$:



Constraints come from $N_{\rm eff}$ and late-time density Ω_s

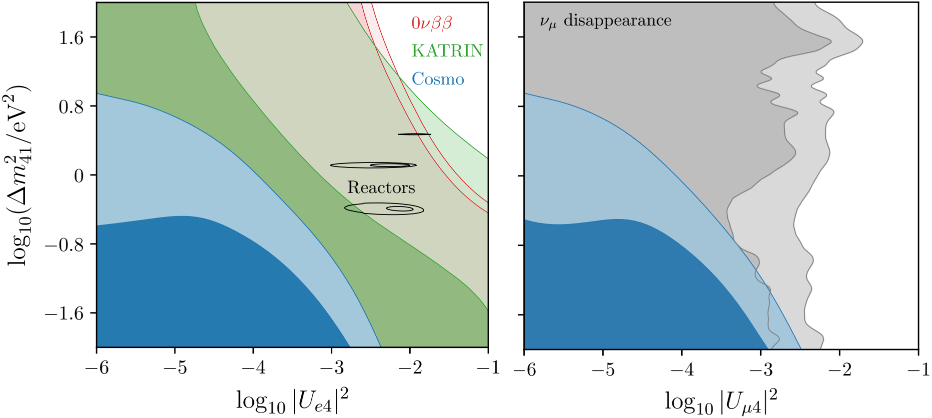
Angles $|U_{\alpha 4}|^2$ are almost equivalent for cosmology

Comparing constraints

[PRD 104 (2021) 123524]

Cosmological constraints are stronger than most other probes

But much more model dependent (as all the cosmological constraints)!

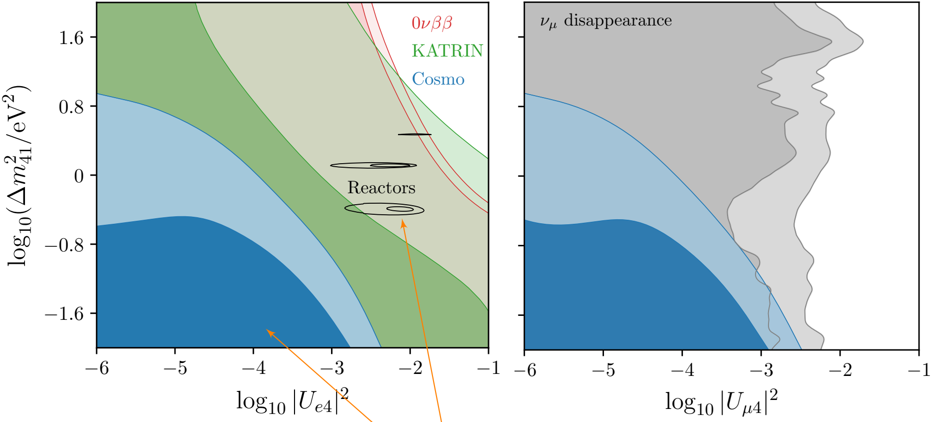


Comparing constraints

[PRD 104 (2021) 123524]

Cosmological constraints are stronger than most other probes

But much more model dependent (as all the cosmological constraints)!



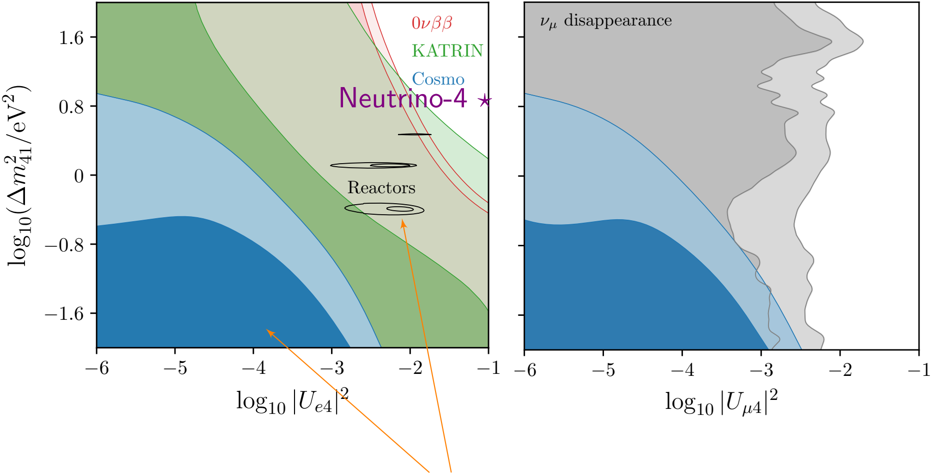
Warning: tension between reactor experiments and CMB bounds!

Comparing constraints

[PRD 104 (2021) 123524]

Cosmological constraints are stronger than most other probes

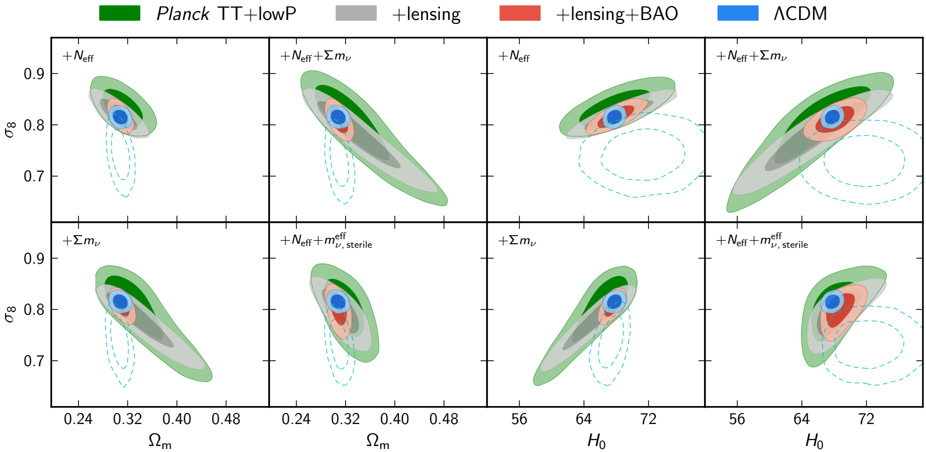
But much more model dependent (as all the cosmological constraints)!



Warning: tension between reactor experiments and CMB bounds!

Solving both σ_8 and H_0 Tension?

[Planck Collaboration, 2015]



dashed: local measurements – ΛCDM model, ΛCDM + $\nu_{a,s}$ models: full cosmological dataset

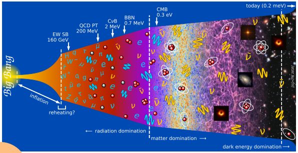
H_0 increases ⇒ σ_8 increases (and viceversa)!
The correlations do not help.

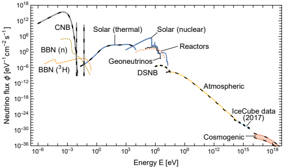
S Summary

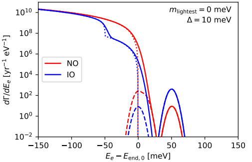
What did we learn about neutrinos and cosmology?

U

Neutrinos influenced the Universe evolution at most times!

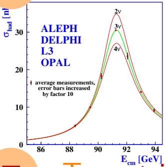


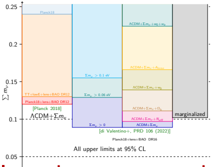


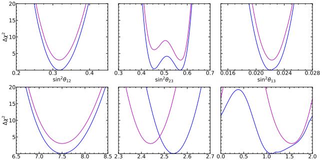


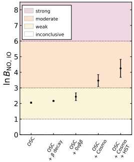
C

Neutrino physics is reasonably well Constrained



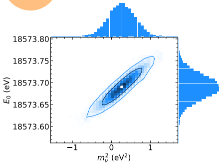


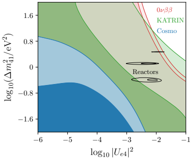


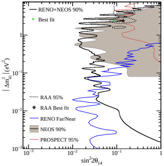


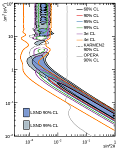
T

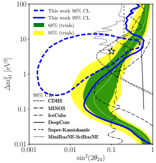
Terrestrial probes are our best short-term hope to learn more











What did we learn about neutrinos and cosmology?

Neutrino physics is like the UCT:
beautiful, public (open to anybody),
it takes some efforts

Thanks for your attention!