Fellowship for Innovation at INFN

H2020 MSCA COFUND
G.A. 754496

Stefano Gariazzo

INFN, Turin section
Turin (IT)

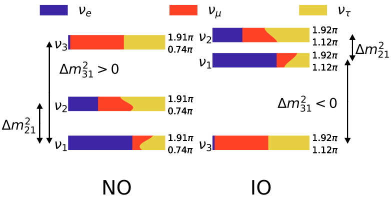
Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

gariazzo@to.infn.it

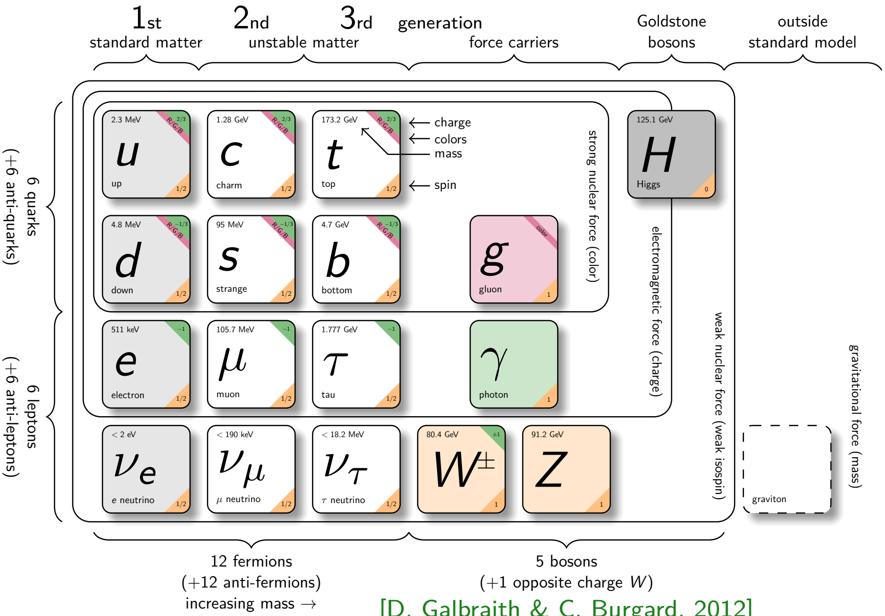
http://personalpages.to.infn.it/~gariazzo/

Neutrino masses in cosmology

21th Lomonosov Conference on Elementary Particle Physics, Moscow State University / online, 24/08/2023

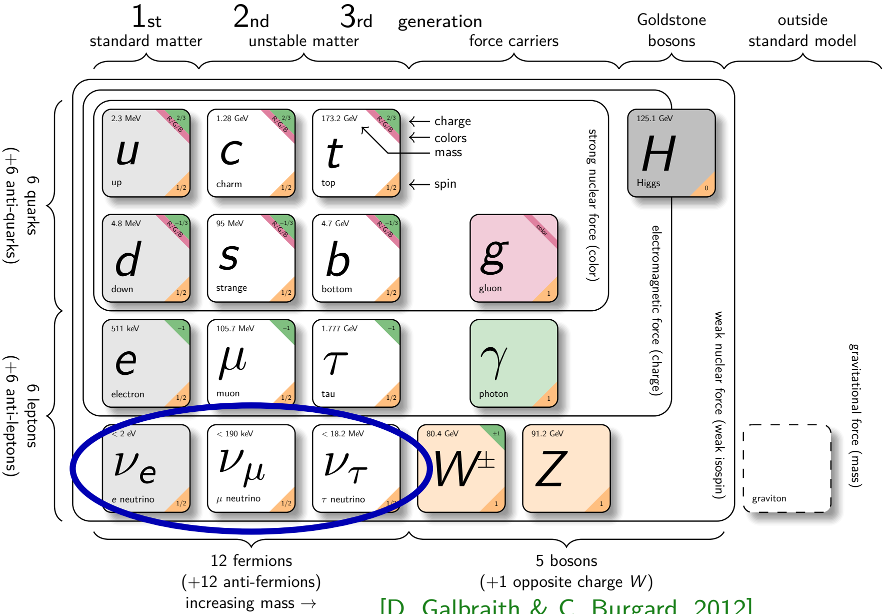

ν_e
ν_μ
ν_τ
ν_3
ν_2
ν_1
$\Delta m^2_{31} > 0$
Δm^2_{21}
1.91π
0.74π
$\Delta m^2_{31} < 0$
1.92π
1.12π
NO
IO

The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

The Standard Model of Particle Physics

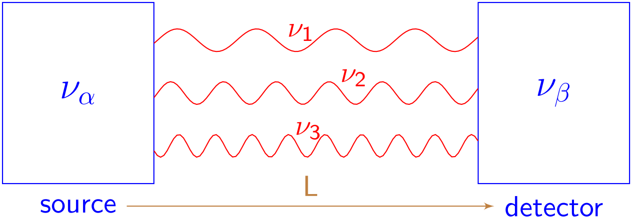


[D. Galbraith & C. Burgard, 2012]

Two neutrino bases

flavor neutrinos ν_α | $|\nu_\alpha\rangle = \sum_k U_{\alpha k}|\nu_k\rangle$ | massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}|\nu_1\rangle + U_{\alpha 2}|\nu_2\rangle + U_{\alpha 3}|\nu_3\rangle$$



$$|\nu(t>0)\rangle = |\nu_\beta\rangle = U_{\alpha 1}\, e^{-iE_1 t}|\nu_1\rangle + U_{\alpha 2}\, e^{-iE_2 t}|\nu_2\rangle + U_{\alpha 3}\, e^{-iE_3 t}|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$E_k^2 = p^2 + m_k^2$ ← define → $t = L$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\alpha | \nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k} U^*_{\alpha k} U^*_{\beta j} U_{\alpha j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

$$\Delta m^2_{ij} = m_i^2 - m_j^2$$

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12}, θ_{13}, θ_{23} and one CP phase δ

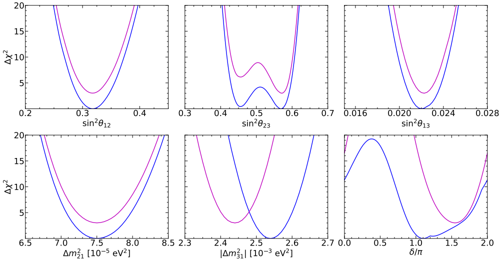
Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$ IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\Delta m^2_{21} = (7.50^{+0.22}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$$
$$|\Delta m^2_{31}| = (2.54 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{NO})$$
$$= (2.44 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{IO})$$

$$10 \sin^2(\theta_{12}) = 3.18 \pm 0.16$$
$$10^2 \sin^2(\theta_{13}) = 2.200^{+0.069}_{-0.062}\ (\mathrm{NO})$$
$$= 2.225^{+0.064}_{-0.070}\ (\mathrm{IO})$$
$$10 \sin^2(\theta_{23}) = 4.55 \pm 0.13 U 5.71 \pm 0.12\ (\mathrm{NO})$$
$$= 5.71^{+0.14}_{-0.17}\ (\mathrm{IO})$$

$$\delta/\pi = 1.10^{+0.27}_{-0.12}\ (\mathrm{NO})$$
$$= 1.54 \pm 0.14\ (\mathrm{IO})$$



mass ordering still unknown

δ still unknown

see also: `http://globalfit.astroparticles.es`

Neutrinos and their masses

[de Salas+, Frontiers 5 (2018) 36]

Normal ordering (NO)

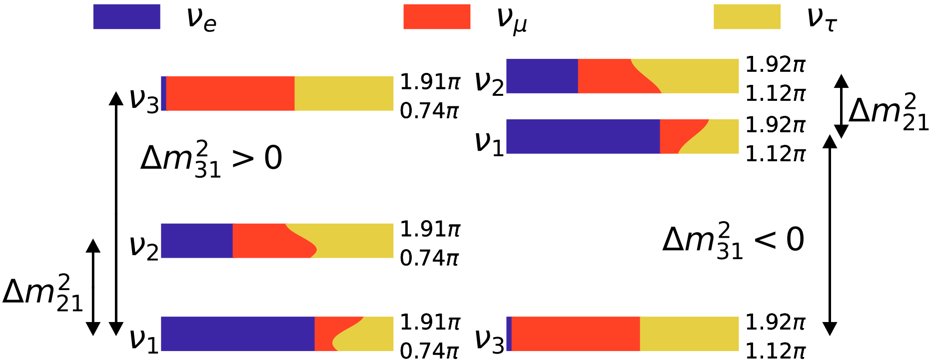
$m_1 < m_2 < m_3$

$\sum m_k \gtrsim 0.06$ eV

Inverted ordering (IO)

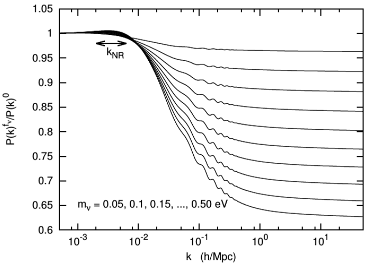
$m_3 < m_1 < m_2$

$\sum m_k \gtrsim 0.1$ eV



Absolute scale unknown!

Can we constrain the mass ordering using bounds on $\sum m_\nu$?


1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
P(k)^fν/P(k)^0
k_NR
m_ν = 0.05, 0.1, 0.15, ..., 0.50 eV
10^-3
10^-2
10^-1
10^0
10^1
k (h/Mpc)

Neutrino masses from CMB

ω_i energy density of species i,
$i \in$ (radiation, matter, baryons, cold dark matter, ν)
$z_{\rm eq}$ matter-radiation equality redshift

$$1 + z_{\rm eq} = (\omega_b + \omega_c)/\omega_r$$

independent of m_ν

$$\omega_m^0 = \omega_b^0 + \omega_c^0 + \omega_\nu^0 \text{ today}$$

mass of species relativistic at recombination
affects late time evolution only

small effects on the SW plateau
(cosmic variance, degeneracies...)

Effects on the early ISW effect

$$\frac{\Delta C_\ell}{C_\ell} \simeq -\left(\frac{\sum m_\nu}{0.1\text{ eV}}\right)\%$$

effects on the position of peaks

$$\theta_s = r_s(\eta_{LS})/D_A(\eta_{LS})$$

$$D_A = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

(this effect can be compensated reducing H_0)

correlation m_ν–H_0

[Lesgourgues+, Neutrino Cosmology]

Neutrino masses from CMB

ω_i energy density of species i,
$i \in$ (radiation, matter, baryons, cold dark matter, ν)
$z_{\rm eq}$ matter-radiation equality redshift

$$1 + z_{\rm eq} = (\omega_b + \omega_c)/\omega_r$$

independent of m_ν

$$\omega_m^0 = \omega_b^0 + \omega_c^0 + \omega_\nu^0 \text{ today}$$

mass of species relativistic at recombination affects late time evolution only

small effects on the SW plateau (cosmic variance, degeneracies...)

Effects on the early ISW effect

$$\frac{\Delta C_\ell}{C_\ell} \simeq -\left(\frac{\sum m_\nu}{0.1\ \text{eV}}\right)\%$$

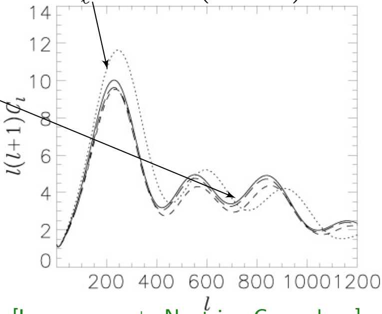
effects on the position of peaks

$$\theta_s = r_s(\eta_{LS})/D_A(\eta_{LS})$$

$$D_A = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

(this effect can be compensated reducing H_0)

correlation m_ν–H_0



[Lesgourgues+, Neutrino Cosmology]

Free-streaming - I

Non-cold relics ⟹ damping in the perturbations due to free-streaming

Growth equation: $\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{Hubble drag}} + \underbrace{c_s^2 k^2 \frac{\delta}{a^2}}_{\text{pressure}} = \underbrace{4\pi G_N \rho\, \delta}_{\text{gravity}}$

Jeans scale: pressure=gravity

$$k_J \equiv \sqrt{\frac{4\pi G_N \rho}{c_s^2 (1+z)^2}}$$

$k < k_J$
growth of density perturbations

$k > k_J$
no growth can occur

neutrino free-streaming scale

$$k_{fs}(z) \equiv \sqrt{\frac{3}{2} \frac{H(z)}{(1+z)\sigma_{v,\nu}(z)}} \simeq 0.7 \left(\frac{m_\nu}{1\ \text{eV}}\right) \sqrt{\frac{\Omega_M}{1+z}}\, h/\text{Mpc}$$

ρ energy density of a given fluid
$\delta = \delta\rho/\rho$ perturbation (single fluid)
c_s sound speed of the fluid

$\sigma_{v,\nu}(z)$ ν velocity dispersion
$H = H(z)$ Hubble factor at redshift z
h reduced Hubble factor today

Free-streaming - II

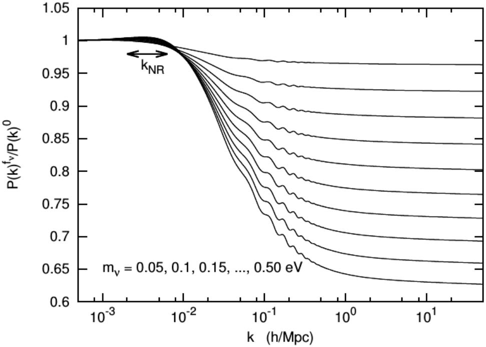
Damping occurs for all $k \gtrsim k_{nr}$

k_{nr}: corresponding
to ν non-relativistic transition

Plot: $\frac{P_{m_\nu>0}(k)}{P_{m_\nu=0}(k)}$

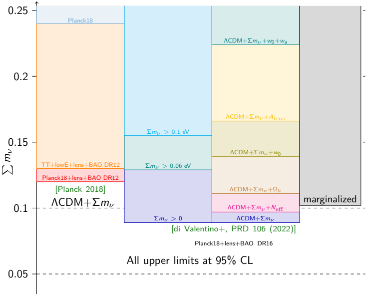
- top to bottom: $m_\nu = 0.05$ eV
  to $m_\nu = 0.5$ eV
- $\frac{\Delta P}{P} \simeq -\frac{8\Omega_\nu}{\Omega_M} \simeq -\frac{\sum m_\nu}{0.01 \text{ eV}}\%$

[Lesgourgues+, Neutrino Cosmology]
(fixed h, ω_m, ω_b, ω_Λ)



Expected constraints from future surveys:

- Planck CMB + DES: $\sigma(m_\nu) \simeq 0.04$–$0.06$ eV [Font-Ribera+, 2014]
- Planck CMB + Euclid: $\sigma(m_\nu) \simeq 0.03$ eV [Audren+, 2013]



$\sum m_\nu$
0.25
0.2
0.15
0.1
0.05
Planck18
TT+lowE+lens+BAO DR12
Planck18+lens+BAO DR12
[Planck 2018]
ΛCDM+Σm_ν
Σm_ν > 0.1 eV
Σm_ν > 0.06 eV
Σm_ν > 0
ΛCDM+Σm_ν+w_0+w_a
ΛCDM+Σm_ν+$A_{\rm lens}$
ΛCDM+Σm_ν+w_0
ΛCDM+Σm_ν+Ω_k
ΛCDM+Σm_ν+$N_{\rm eff}$
ΛCDM+Σm_ν
[di Valentino+, PRD 106 (2022)]
Planck18+lens+BAO DR16
marginalized
All upper limits at 95% CL

Playing with priors

Bayes theorem:

$$p(\theta|d,\mathcal{M}) = \mathcal{L}(\theta)\frac{\pi(\theta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

parameters θ, model $\mathcal{M}$, data d $\pi(\theta|\mathcal{M})$ prior $p(\theta|d,\mathcal{M})$ posterior $\mathcal{L}(\theta)$ likelihood $Z_{\mathcal{M}}$ Bayesian evidence

Playing with priors

Bayes theorem:

$$p(\theta|d,\mathcal{M}) = \mathcal{L}(\theta)\frac{\pi(\theta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

[Planck 2018]: prior
$0 < \Sigma m_\nu < \mathcal{O}(1)$ eV

strongest upper limit (95%):
$\Sigma m_\nu < 113$ meV
(CMB+lens+BAO+SN)

corresponding to
$\Sigma m_\nu < 53.6$ meV (68%)

below minimum for NO!
does it make sense?

parameters θ, model $\mathcal{M}$, data d $\pi(\theta|\mathcal{M})$ prior $p(\theta|d,\mathcal{M})$ posterior $\mathcal{L}(\theta)$ likelihood $Z_{\mathcal{M}}$ Bayesian evidence

Playing with priors

Bayes theorem:

$$p(\theta|d,\mathcal{M}) = \mathcal{L}(\theta)\frac{\pi(\theta|\mathcal{M})}{Z_{\mathcal{M}}}$$

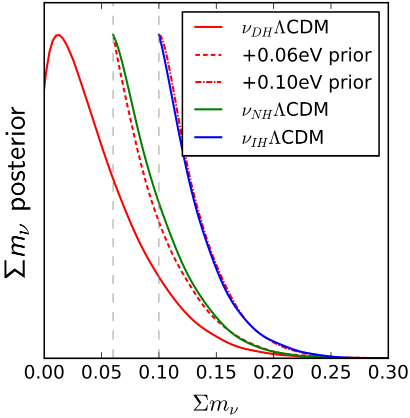
posterior depends on prior!

[Wang+, 2017]

degenerate (DH)
vs normal (NH)
vs inverted (IH) hierarchy

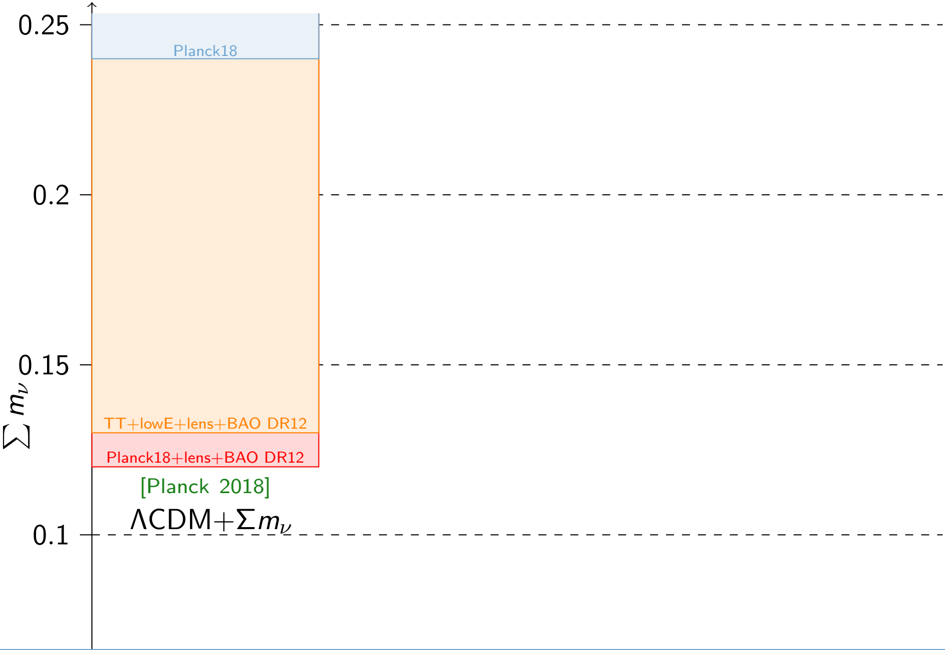
(i.e. change the prior lower bound)

Different limits if you consider simply $\Sigma m_\nu > 0$ or you take into account oscillation results...

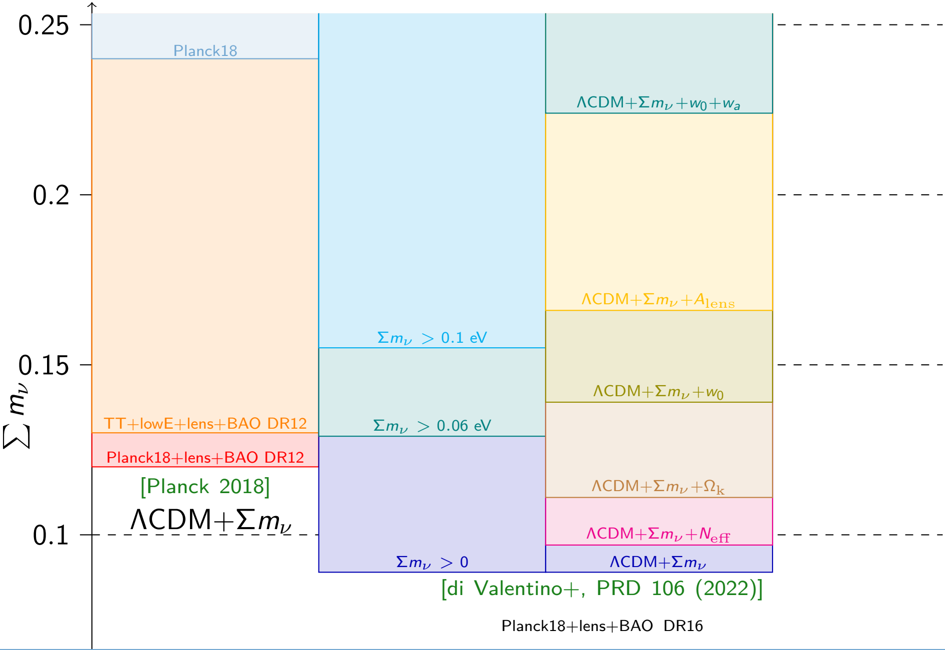


parameters θ, model $\mathcal{M}$, data d $\pi(\theta|\mathcal{M})$ prior $p(\theta|d,\mathcal{M})$ posterior $\mathcal{L}(\theta)$ likelihood $Z_{\mathcal{M}}$ Bayesian evidence

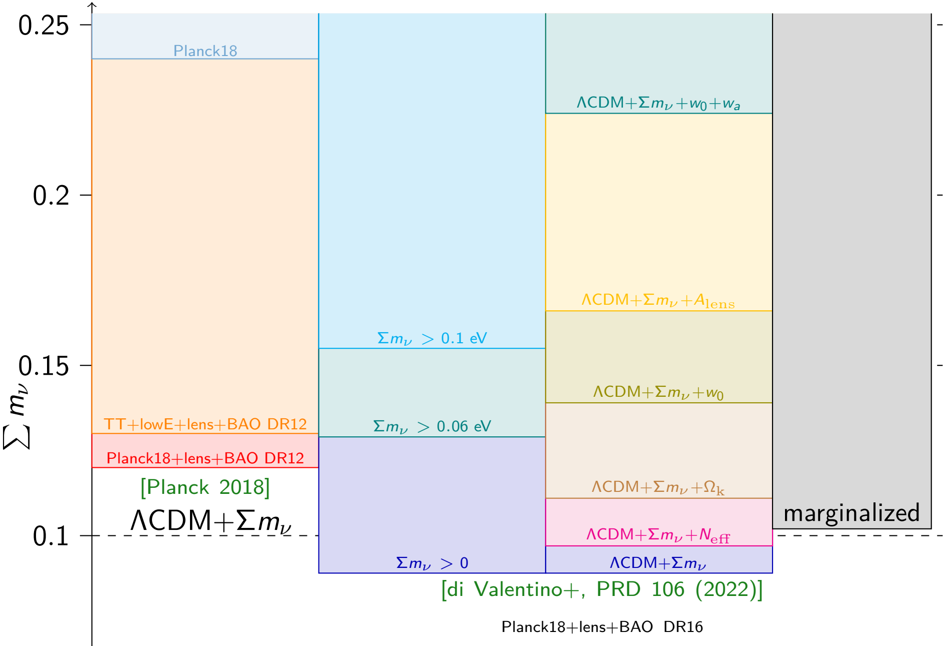
Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Remove prior dependency?

[SG, EPJC 80 (2020)]

relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

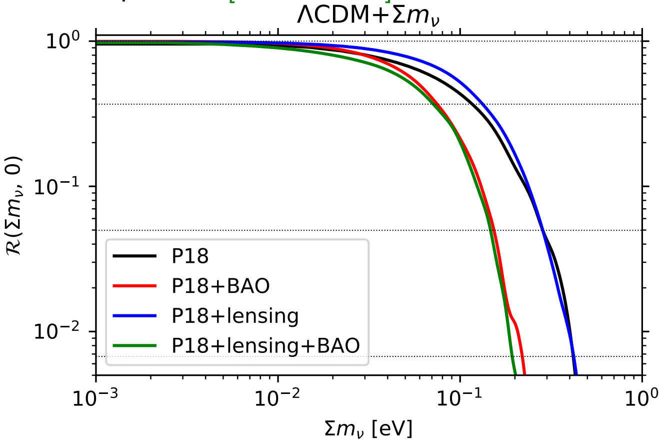
Remove prior dependency?

relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

Example with [Planck 2018] chains from PLA



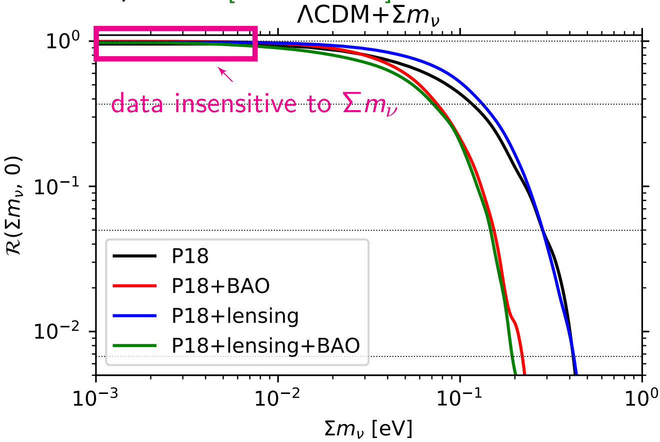
Remove prior dependency?

relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

Example with [Planck 2018] chains from PLA



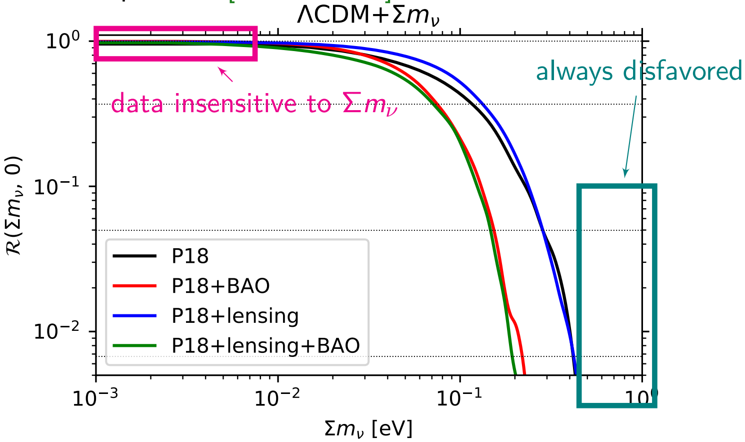
Remove prior dependency?

relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

Example with [Planck 2018] chains from PLA



Remove prior dependency?

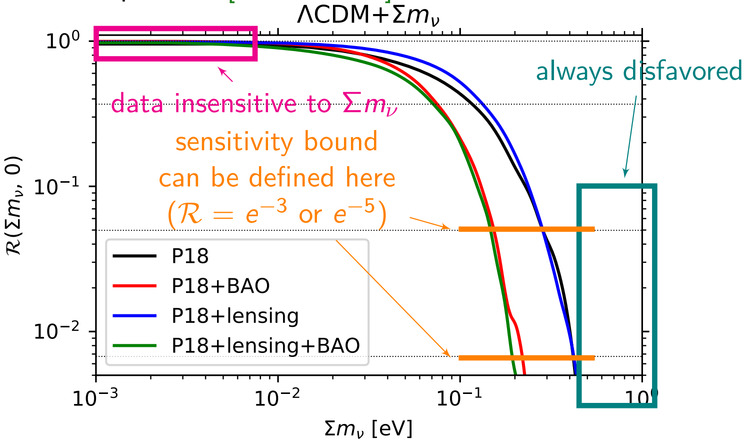
[SG, EPJC 80 (2020)]

relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Numerically easy to compute: fix $\pi(x)$, get $p(x|d)$ normally and divide

Example with [Planck 2018] chains from PLA



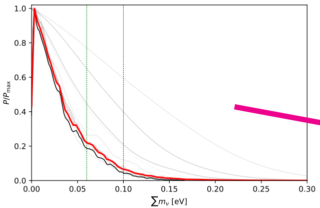
A model-marginalized example

[PRD 106 (2022)]

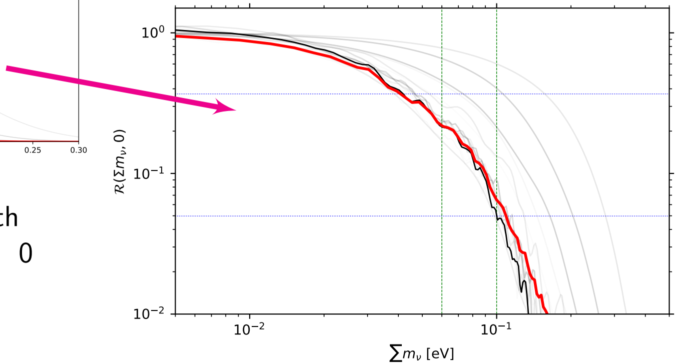
relative belief updating ratio

$$\mathcal{R}(x, x_0|d) \equiv \frac{p(x|d)/\pi(x)}{p(x_0|d)/\pi(x_0)}$$

Example with [Planck 2018] data, ΛCDM+$\sum m_\nu$ and extensions:

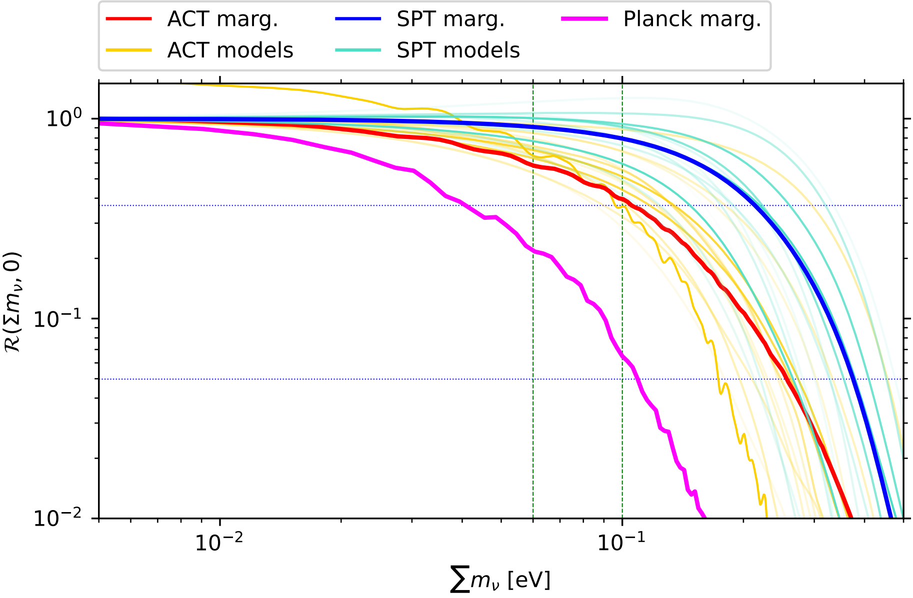


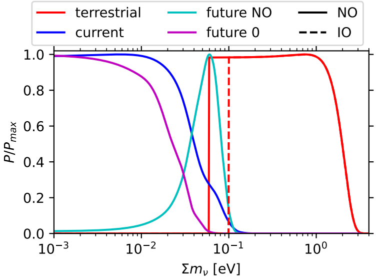
Posterior, with prior $\sum m_\nu > 0$



Constraints on the sum of neutrino masses

[arxiv:2305.12989]





terrestrial
current
future NO
future 0
NO
IO
P/P_max
Σm_ν [eV]

Can current data tell us the neutrino mass ordering?

1. [Hannestad, Schwetz, 2016]: extremely weak (2:1, 3:2) preference for NO (cosmology + [Bergstrom et al., 2015] neutrino oscillation fit) Bayesian approach;
2. [Gerbino et al, 2016]: extremely weak (up to 3:2) preference for NO (cosmology only), Bayesian approach;
3. [Simpson et al., 2017]: strong preference for NO (cosmological limits on $\sum m_\nu$ + constraints on Δm^2_{21} and $|\Delta m^2_{31}|$) Bayesian approach;
4. [Schwetz et al., 2017], "Comment on ..."[Simpson et al., 2017]: effect of prior?
5. [Capozzi et al., 2017]: 2σ preference for NO (cosmology + [Capozzi et al., 2016, updated 2017] neutrino oscillation fit) frequentist approach;
6. [Caldwell et al., 2017] very mild indication for NO (cosmology + neutrinoless double-beta decay + [Esteban et al., 2016] readapted oscillation results) Bayesian approach;
7. [Wang, Xia, 2017]: Bayes factor NO vs IO is not informative (cosmology only).
8. ...

Can current data tell us the neutrino mass ordering?

1. [Hannestad, Schwetz, 2016]: extremely weak (2:1, 3:2) preference for NO
(cosmology + [Bergstrom et al., 2015] neutrino oscillation fit)
Bayesian approach;
2. [Gerbino et al, 2016]: extremely weak (up to 3:2) preference for NO
(cosmology only), Bayesian approach;
3. [Simpson et al., 2017]: strong preference for NO
(cosmological limits on $\sum m_\nu$ + constraints on Δm^2_{21} and $|\Delta m^2_{31}|$)
Bayesian approach;
4. [Schwetz et al., 2017], "Comment on ..."[Simpson et al., 2017]: effect of prior?
5. [Capozzi et al., 2017]: 2σ preference for NO
(cosmology + [Capozzi et al., 2016, updated 2017] neutrino oscillation fit)
frequentist approach;
6. [Caldwell et al., 2017] very mild indication for NO
(cosmology + neutrinoless double-beta decay + [Esteban et al., 2016]
readapted oscillation results)
Bayesian approach;
7. [Wang, Xia, 2017]: Bayes factor NO vs IO is not informative
(cosmology only).
8. ...

Parameterizing neutrino masses

[SG+, JCAP 03 (2018) 11]

[Simpson et al, 2017]

use $m_1,\ m_2,\ m_3$ (3M)

[Caldwell et al, 2017]

use $m_{\rm lightest},\ \Delta m^2_{21},\ |\Delta m^2_{31}|$ ($M\Delta$)

Alternatively:

$\sum m_\nu,\ \Delta m^2_{21},\ |\Delta m^2_{31}|$ ($\Sigma\Delta$)

intuition says: $\Sigma\Delta$ is closer to observable quantities! Better than 3M?

Cosmology measures $\sum m_\nu$, oscillations $\Delta m^2_{21},\ |\Delta m^2_{31}|$

$M\Delta$ should be equivalent to $\Sigma\Delta$

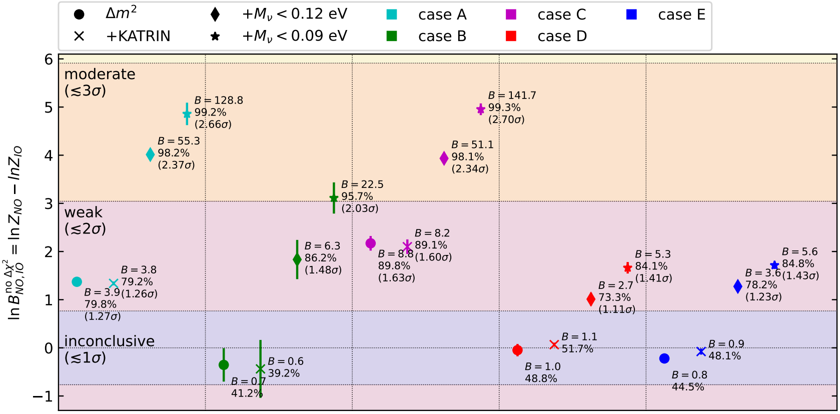
Should we use linear or logarithmic priors on m_k ($m_{\rm lightest}$)?

Can data help to select 3M or $M\Delta/\Sigma\Delta$, linear or log?

Mass ordering and Bayesian analyses

[JCAP 10 (2022)]

→ oscillation Δm^2 alone should not generate a difference

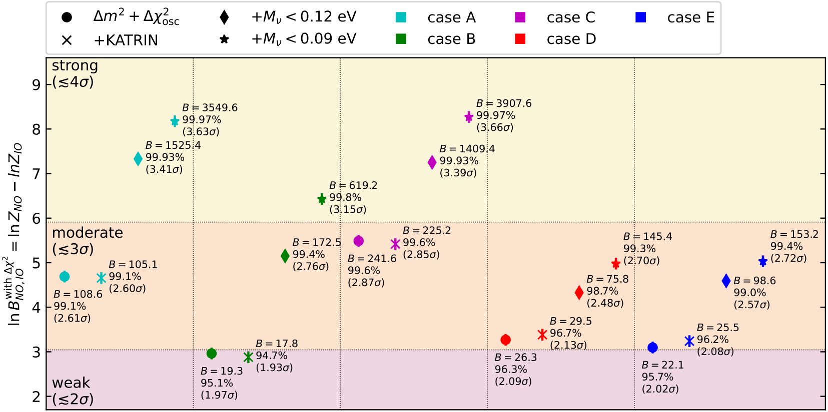


A, B, C:
Gauss. prior on
$\ln m_1$, $\ln m_2$, $\ln m_3$,
different prior ranges or sampling

D, E:
linear prior on
Δm^2_{21}, $|\Delta m^2_{31}|$, $m_{\rm lightest}/\sum m_\nu$

Mass ordering and Bayesian analyses

→ oscillation $\Delta\chi^2$ DOES prefer NO over IO at $\sim 2\sigma$



A, B, C:
Gauss. prior on
$\ln m_1$, $\ln m_2$, $\ln m_3$,
different prior ranges or sampling

D, E:
linear prior on
Δm^2_{21}, $|\Delta m^2_{31}|$, $m_{\text{lightest}}/\sum m_\nu$

Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

standard factor

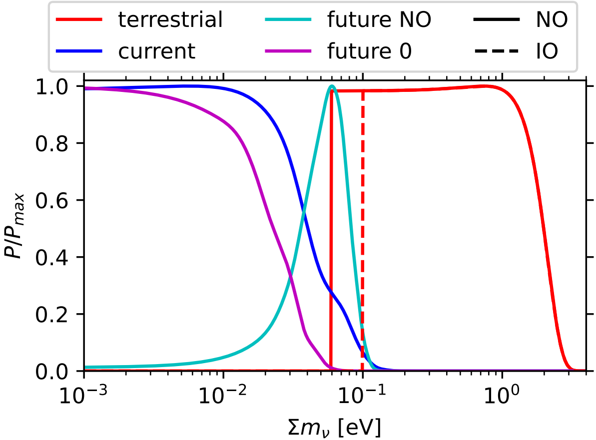
Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu/(94.12\text{ eV})$

NO: $\Sigma m_\nu \gtrsim 0.06$ eV   Current: $\Sigma m_\nu \lesssim 0.1$ eV (95%)   IO: $\Sigma m_\nu \gtrsim 0.1$ eV

Future sensitivity: $\sigma(\Sigma m_\nu) \simeq 0.02$ eV

Still preferring $\Sigma m_\nu = 0$?   Will measure e.g. $\Sigma m_\nu = 0.06$ eV?

tension ever with NO!



confirm NO, disfavor IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu / (94.12\ \text{eV})$

Is there a tension between cosmology and oscillations?

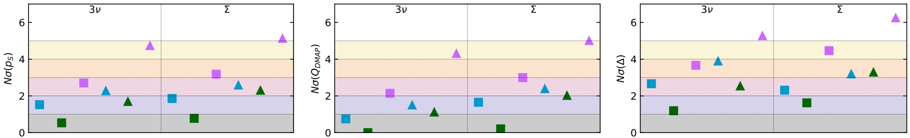
or will there be a tension?

several possible tests can be considered, similar results

$\Sigma m_\nu \lesssim 0.1$ eV (95%) ● current ■ NO

$\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ) ● future NO ▲ IO

$\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ) ● future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

future 0 can be at $\sim 2 - 3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

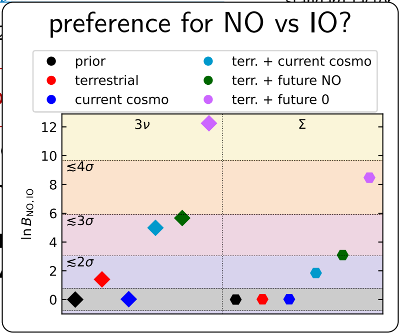
standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2$

Is there a tension between cosmo

or will there be a t

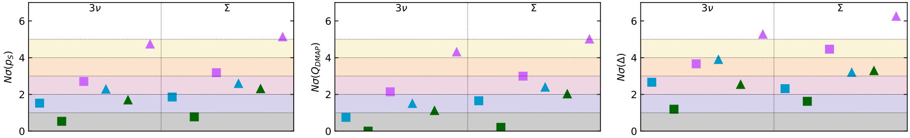
several possible tests can be con



$\Sigma m_\nu \lesssim 0.1$ eV (95%) — current

$\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ) — future NO

$\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ) — future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

future 0 can be at $\sim 2-3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

Mass ordering results

[JHEP 02 (2021)]
http://globalfit.astroparticles.es/

Bayes theorem for models:

$$p(\mathcal{M}|d) \propto Z_{\mathcal{M}}\pi(\mathcal{M})$$

Bayesian evidence:

$$Z_{\mathcal{M}} = \int_{\Omega_{\mathcal{M}}} \mathcal{L}(\theta)\,\pi(\theta)\,d\theta$$

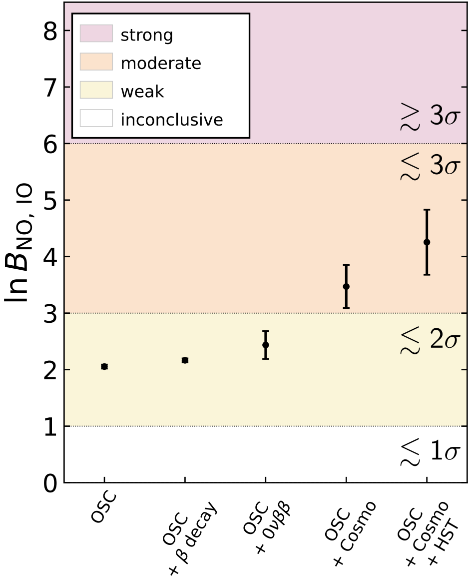
Bayes factor NO vs IO:

$$B_{\mathrm{NO,IO}} = Z_{\mathrm{NO}}/Z_{\mathrm{IO}}$$

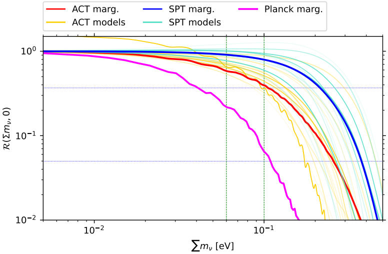
Posterior probability:

$$P_{\mathrm{NO}} = B_{\mathrm{NO,IO}}/(B_{\mathrm{NO,IO}}+1)$$
$$P_{\mathrm{IO}} = 1/(B_{\mathrm{NO,IO}}+1)$$

$N\sigma$ from $P_{\mathrm{NO}} = \mathrm{erf}(N/\sqrt{2})$



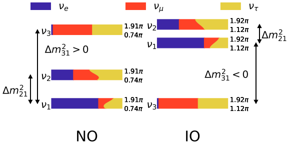
$\pi(\mathcal{M})$ model prior
$p(\mathcal{M}|d)$ model posterior
$\mathcal{L}(\theta)$ likelihood
$\Omega_{\mathcal{M}}$ parameter space, for parameters θ

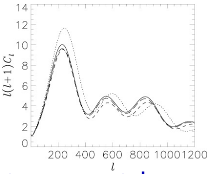

ACT marg.
ACT models
SPT marg.
SPT models
Planck marg.
$\mathcal{R}(\Sigma m_\nu, 0)$
10^0
10^{-1}
10^{-2}
10^{-2}
10^{-1}
$\sum m_\nu$ [eV]

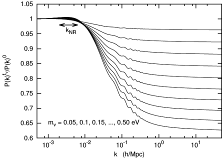
What do we learn on neutrino masses from cosmology?

C

Effects of neutrino masses in cosmology

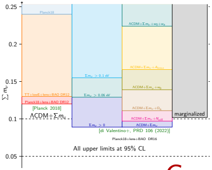


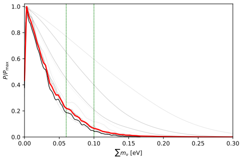


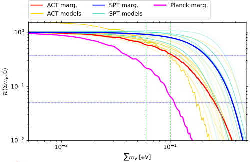


M

Constraints on neutrino masses

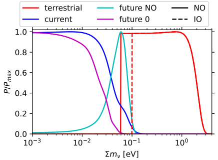


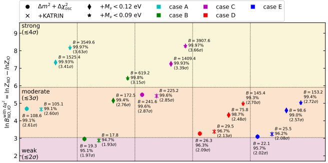


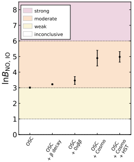


O

Constraints on the mass ordering



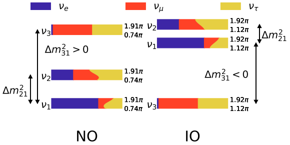


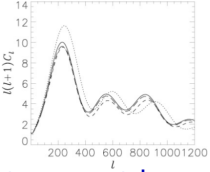


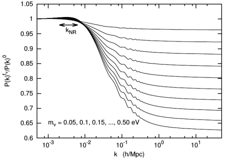
What do we learn on neutrino masses from cosmology?

C

Effects of neutrino masses in cosmology

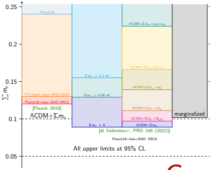


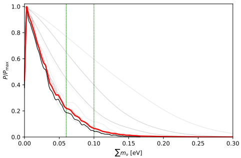


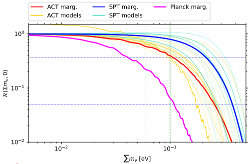


M

Constraints on neutrino masses

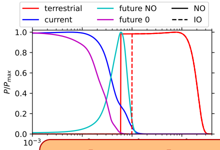


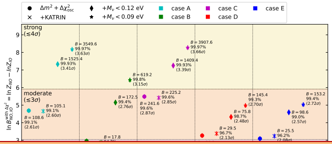


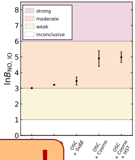


O

Constraints on the mass ordering







Thanks for your attention!