 "la Caixa" Foundation
Junior Leader
Fellowship
LCF/BQ/PI23/11970034

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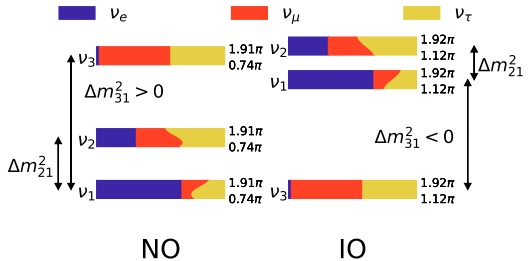
`stefano.gariazzo@ift.csic.es`

Relic neutrinos: decoupling and direct detection perspectives

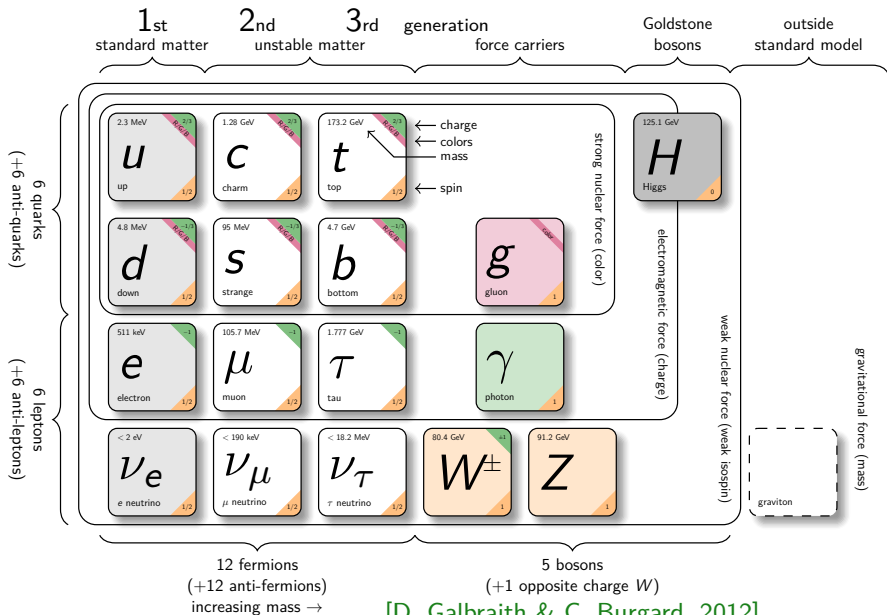
N Neutrinos

Based on:

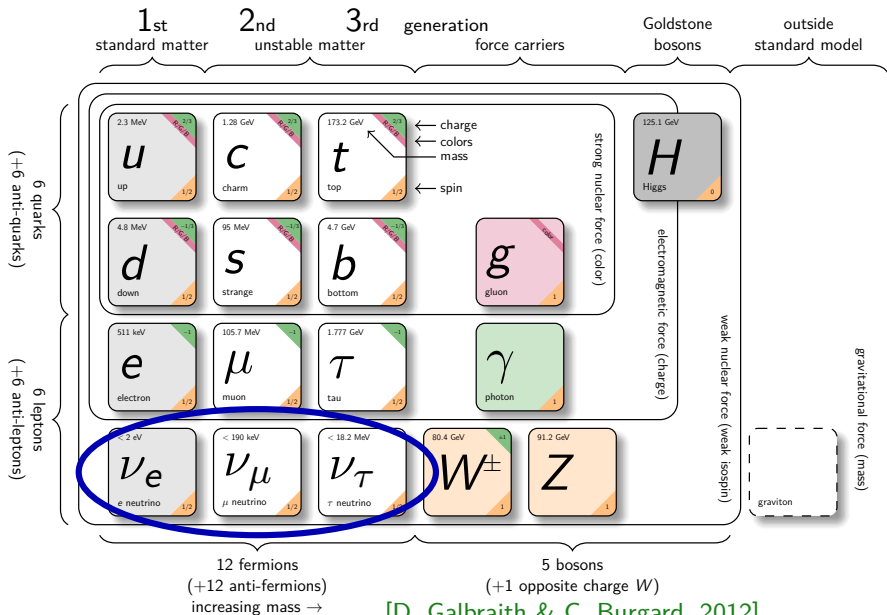
- JHEP 02 (2021) 071 and update

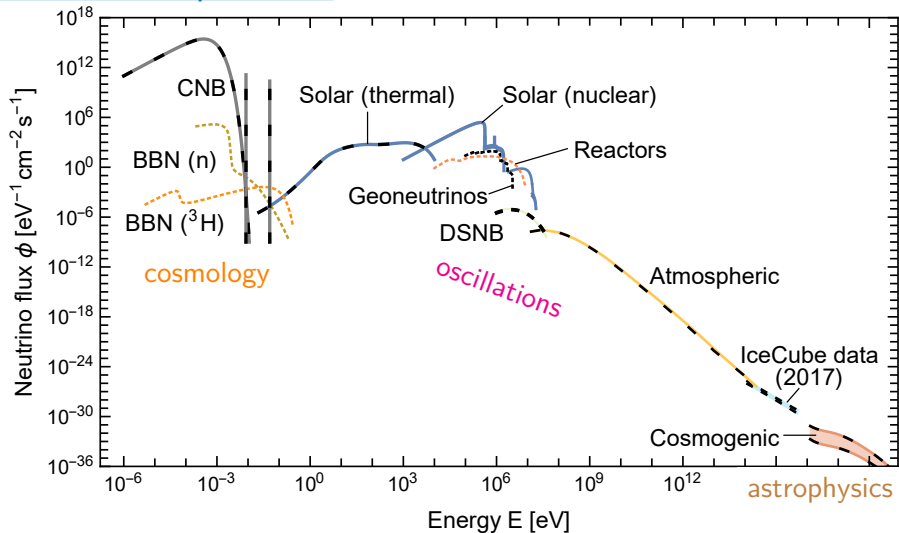


The Standard Model of Particle Physics



The Standard Model of Particle Physics



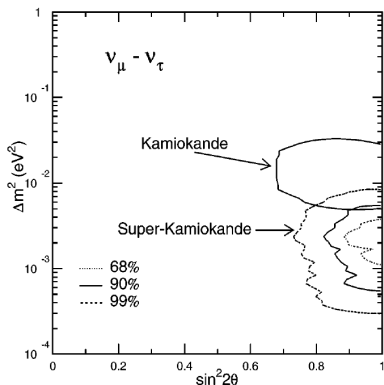


neutrinos at all energies provide valuable information!

Neutrino oscillations

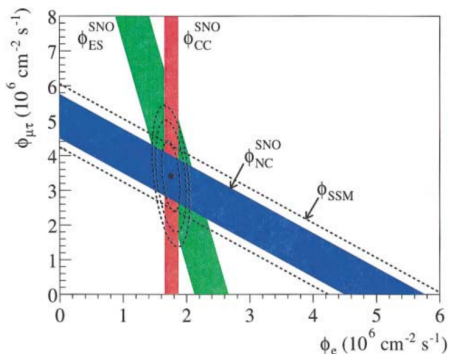
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \rightarrow \nu_\tau$
oscillations from atmospheric ν

[SNO, 2001-2002]



first discovery of $\nu_e \rightarrow \nu_\mu, \nu_\tau$
oscillations from solar ν

Nobel prize in 2015

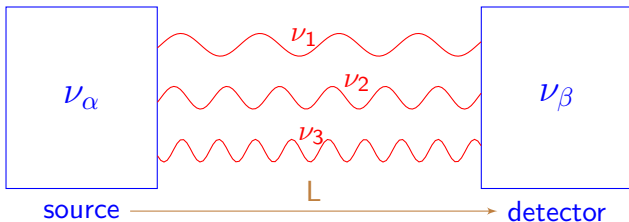
Two neutrino bases

flavor neutrinos ν_α

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1} |\nu_1\rangle + U_{\alpha 2} |\nu_2\rangle + U_{\alpha 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = |\nu_\beta\rangle = U_{\alpha 1} e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2} e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \longleftarrow \text{define} \longrightarrow t = L$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

The mixing matrix

U can be parameterized using 3 angles (θ_{12} , θ_{13} , θ_{23}) and max 3 (1 Dirac δ , 2 Majorana [\exists only for Majorana ν]) phases

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{mainly atmospheric} \\ \text{and LBL} \\ \text{accelerator} \\ \text{disappearance}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\substack{\text{mainly LBL reactors and} \\ \text{LBL accelerator} \\ \text{appearance}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{mainly solar and} \\ \text{VLBL reactors}}} M$$

Majorana phases irrelevant for oscillation experiments ←

Relevant for example in neutrinoless double-beta decay

$$s_{ij} \equiv \sin \theta_{ij}; \quad c_{ij} \equiv \cos \theta_{ij}$$

LBL = long baseline; VLBL = very long baseline;

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12} , θ_{13} , θ_{23} and one CP phase δ

Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

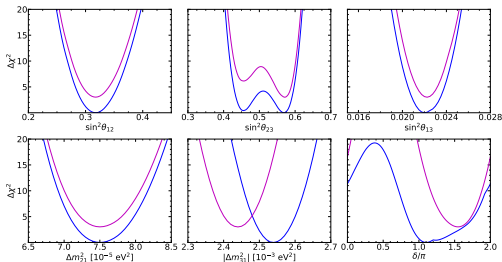
NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$

IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned} \Delta m_{21}^2 &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5} \text{ eV}^2 \\ |\Delta m_{31}^2| &= (2.54 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (NO)} \\ &= (2.44 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (IO)} \end{aligned}$$

$$\begin{aligned} 10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\ 10^2 \sin^2(\theta_{13}) &= 2.200^{+0.069}_{-0.062} \text{ (NO)} \\ &= 2.225^{+0.064}_{-0.070} \text{ (IO)} \\ 10 \sin^2(\theta_{23}) &= 4.55 \pm 0.13 \text{ (NO)} \\ &= 5.71^{+0.14}_{-0.17} \text{ (IO)} \end{aligned}$$

$$\begin{aligned} \delta/\pi &= 1.10^{+0.27}_{-0.12} \text{ (NO)} \\ &= 1.54 \pm 0.14 \text{ (IO)} \end{aligned}$$



mass ordering
still unknown

δ still unknown

see also: <http://globalfit.astroparticles.es>

Normal ordering (NO)

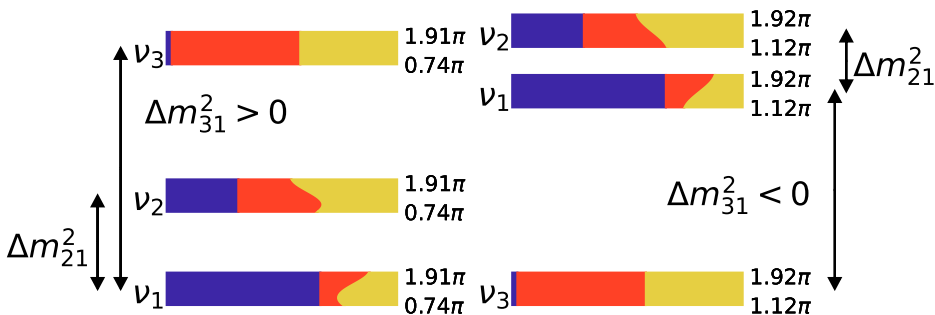
$$m_1 < m_2 < m_3$$

$$\sum m_k \gtrsim 0.06 \text{ eV}$$

Inverted ordering (IO)

$$m_3 < m_1 < m_2$$

$$\sum m_k \gtrsim 0.1 \text{ eV}$$



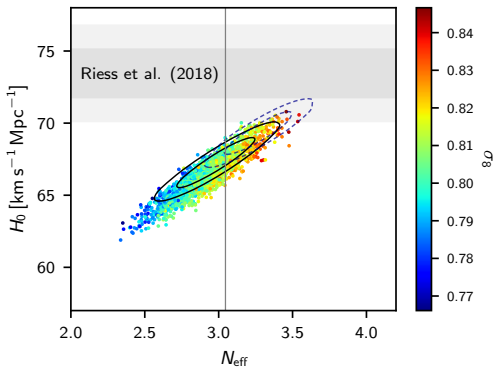
Absolute scale unknown!

Can we constrain the mass ordering using bounds on $\sum m_\nu$?

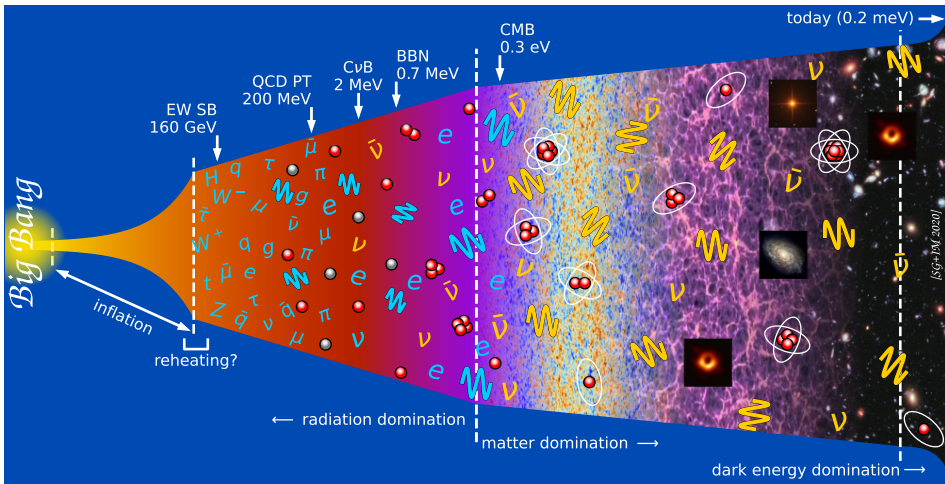
E Neutrinos in the Early Universe

Based on:

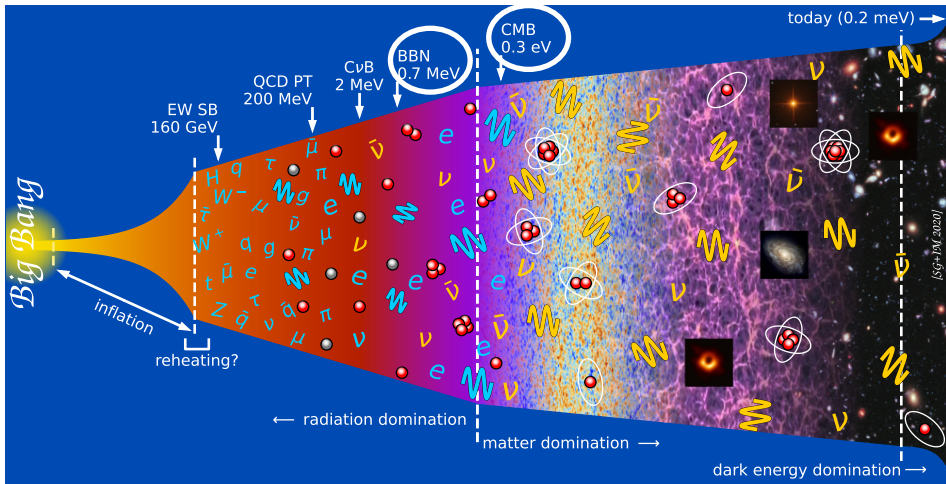
- Planck 2018
- JCAP 04 (2021) 073
- PRD 106 (2022) 043540



History of the universe



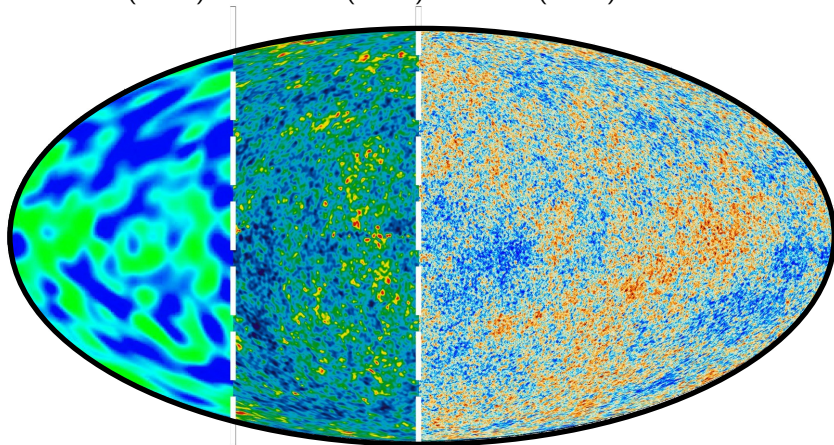
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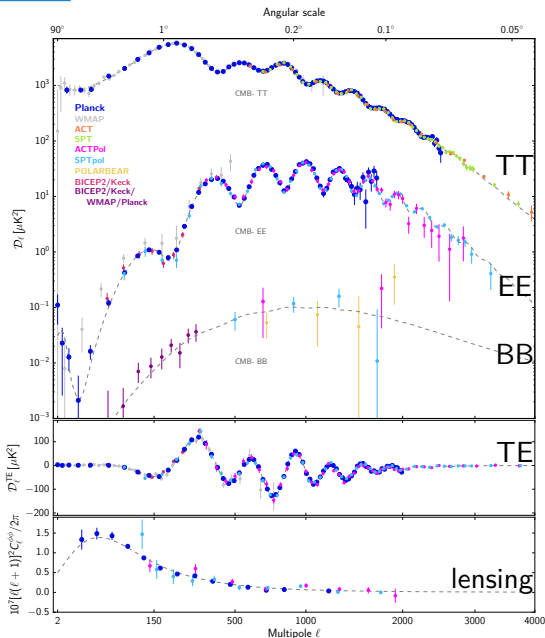
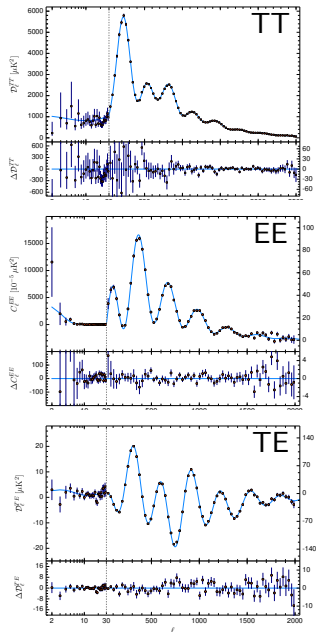


The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

COBE (1992) WMAP (2003) Planck (2013)





Big Bang Nucleosynthesis (BBN)

BBN: production of light nuclei at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

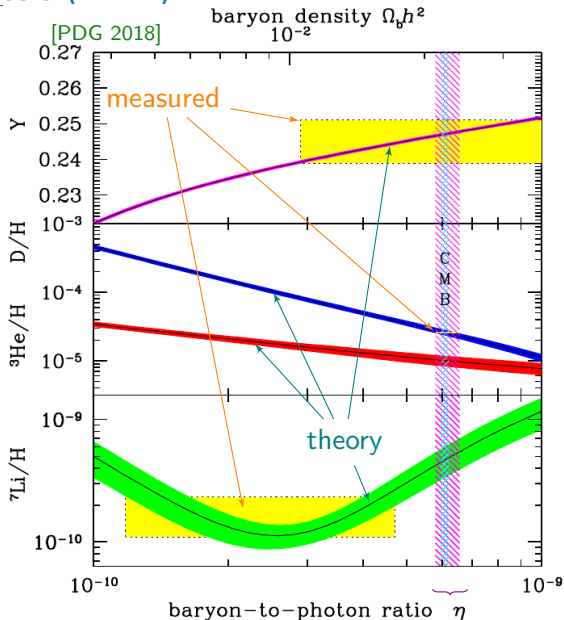
temperature $T_{fr} \simeq 1\text{ MeV}$
from nucleon freeze-out

much earlier than CMB!

strong probe for physics
before the CMB

e.g. neutrinos!

ν affect
universe expansion
and
reaction rates ($\nu_e/\bar{\nu}_e$)
at BBN time...



BBN concordance

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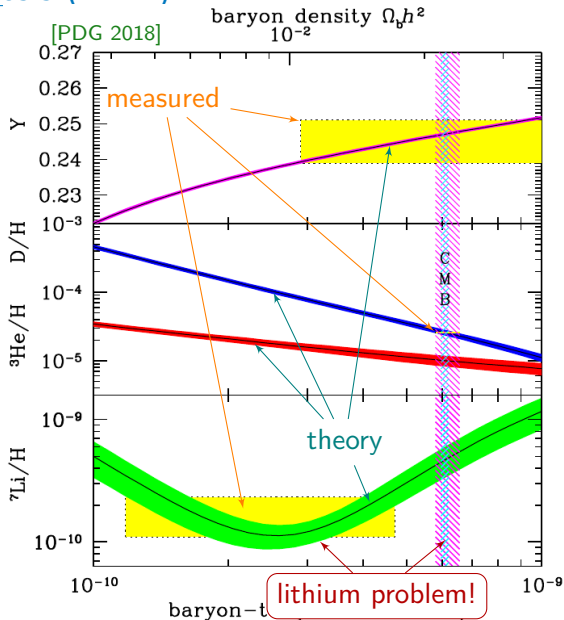
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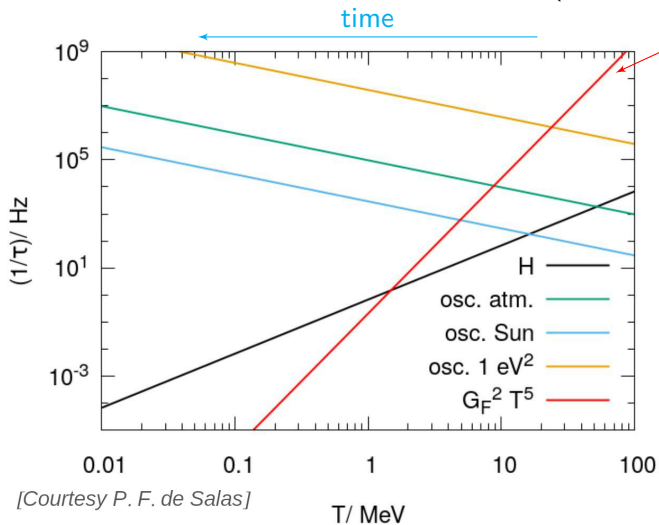
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BBN concordance

Neutrinos in the early Universe

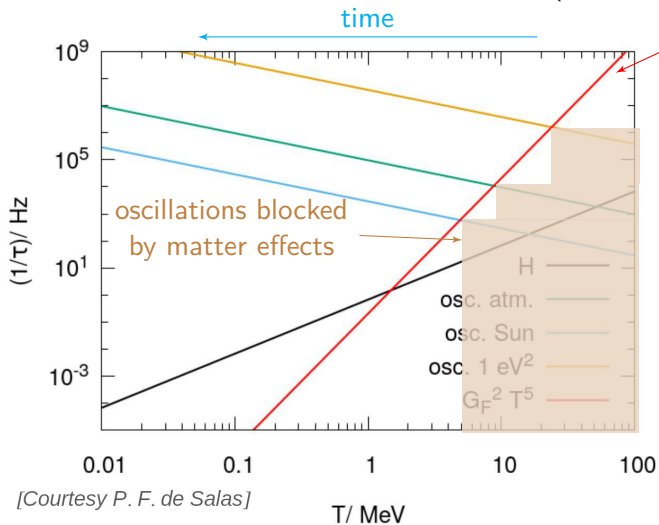
before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

Neutrinos in the early Universe

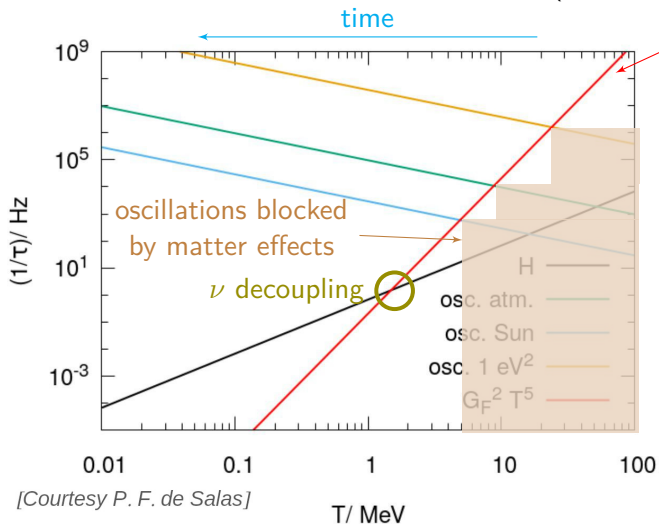
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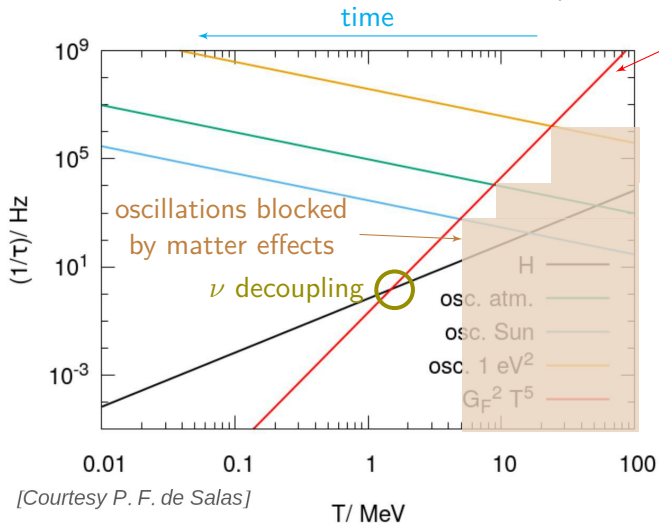
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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

$T_\nu \simeq (4/11)^{1/3} T_\gamma$
after $e^+ e^- \rightarrow \gamma\gamma$

f_ν : frozen Fermi-Dirac distribution

Today:

$T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV}$

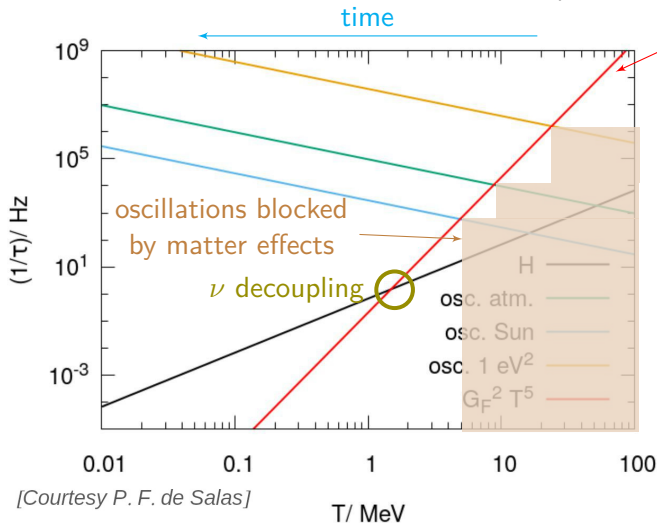
$\langle E_\nu \rangle \simeq 3.1 T_{\nu,0} \simeq 5 \times 10^{-4} \text{ eV}$

$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56 \text{ cm}^{-3}$ per family

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Neutrinos in the early Universe

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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!
 actually, the decoupling T is momentum dependent!

distortions to equilibrium f_ν !

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]
[Sigl, Raffelt, 1993]

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

density matrix: $\varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$
 $\propto \langle a_j^\dagger(p, t) a_i(p, t) \rangle$
off-diagonals to take into account coherency in the neutrino system

$$\varrho \text{ evolution from } x \text{ to } y: \quad x H \frac{d\varrho(y, x)}{dx} = -i a [\mathcal{H}_{\text{eff}}, \varrho] + b \mathcal{I}$$

H Hubble factor \rightarrow expansion (depends on universe content)

effective Hamiltonian $\mathcal{H}_{\text{eff}} = \frac{M_F}{2y} - \frac{2\sqrt{2}G_F y m_e^6}{x^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4}{3} \frac{E_\nu}{m_Z^2} \right)$

vacuum oscillations \longleftarrow \longrightarrow matter effects

\mathcal{I} collision integrals

take into account ν -e scattering and pair annihilation, ν - ν interactions

2D integrals over momentum, take most of the computation time

$$\text{solve together with } z \text{ evolution, from } x \frac{d\rho(x)}{dx} = \rho - 3P$$

ρ, P total energy density and pressure, also take into account FTQED corrections

ν oscillations in the early universe

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$$\varrho \text{ evolution from } xH \frac{d\varrho(y, x)}{dx} = -ia[\mathcal{H}_{\text{eff}}, \varrho] + b\mathcal{I}$$

FORTRAN-Evolved Primordial Neutrino Oscillations
(FortEPiano)

https://bitbucket.org/ahep_cosmo/fortepiano_public

vacuum oscillations



matter effects

\mathcal{I} collision integrals

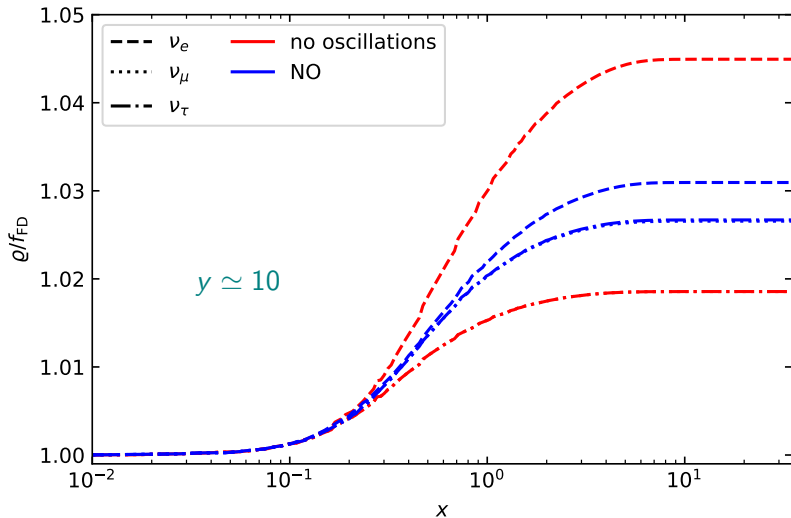
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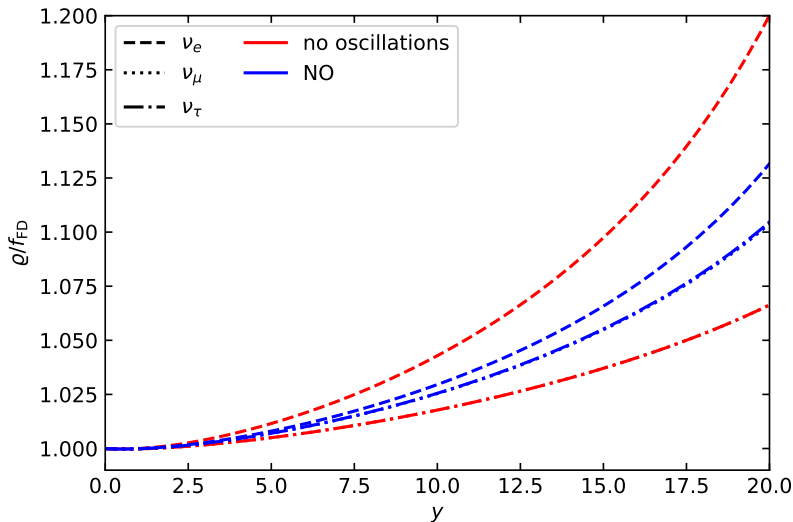
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ρ, P total energy density and pressure, also take into account FTQED corrections

Distortion of the momentum distribution (f_{FD} : Fermi-Dirac at equilibrium)

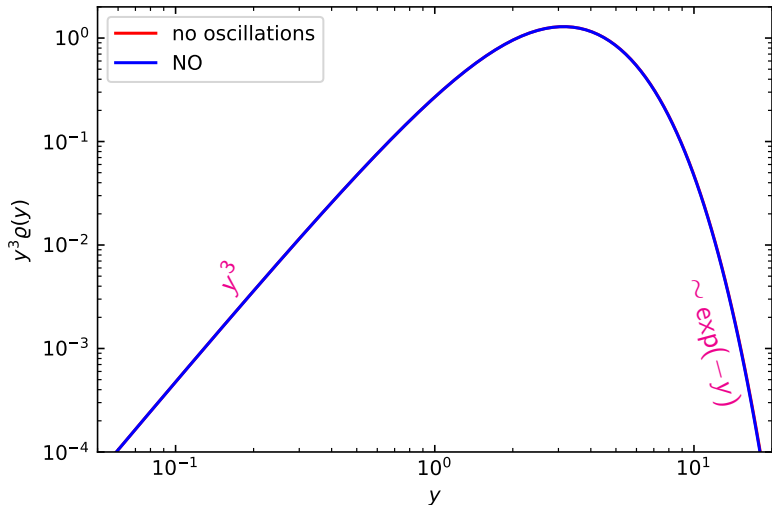


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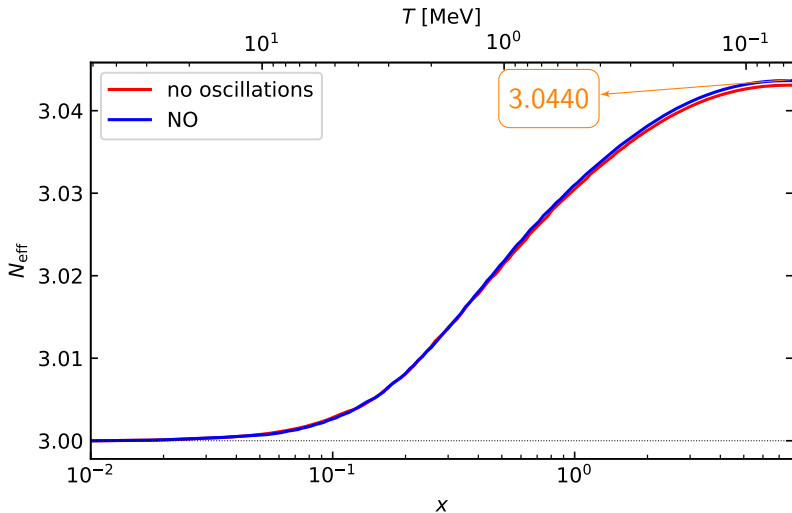


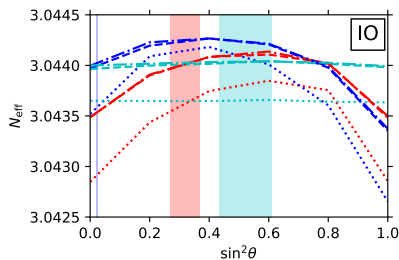
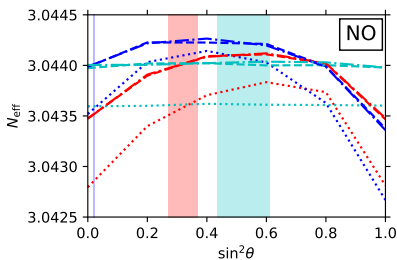
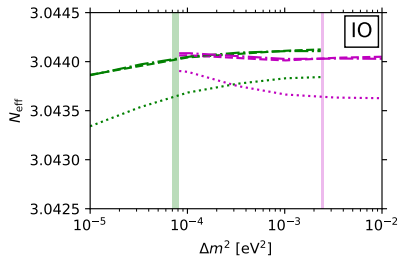
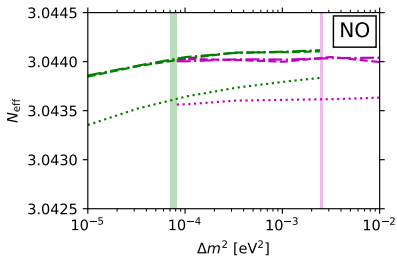
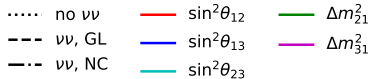
$$N_{\text{eff}}^{\text{final}} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$

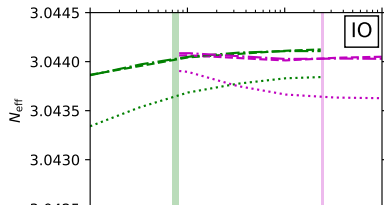
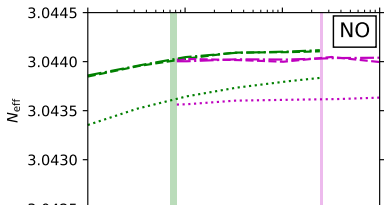
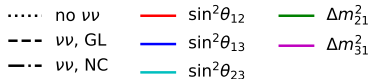
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\text{fin}}$
 $\hookrightarrow \propto y^3 g_{ii}(y)$



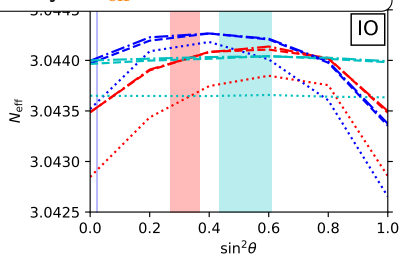
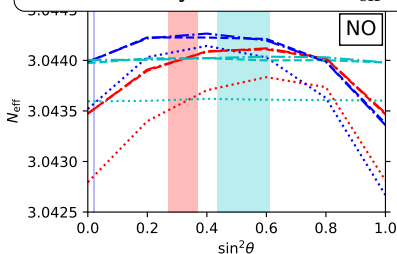
$$N_{\text{eff}}^{\text{any time}} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$





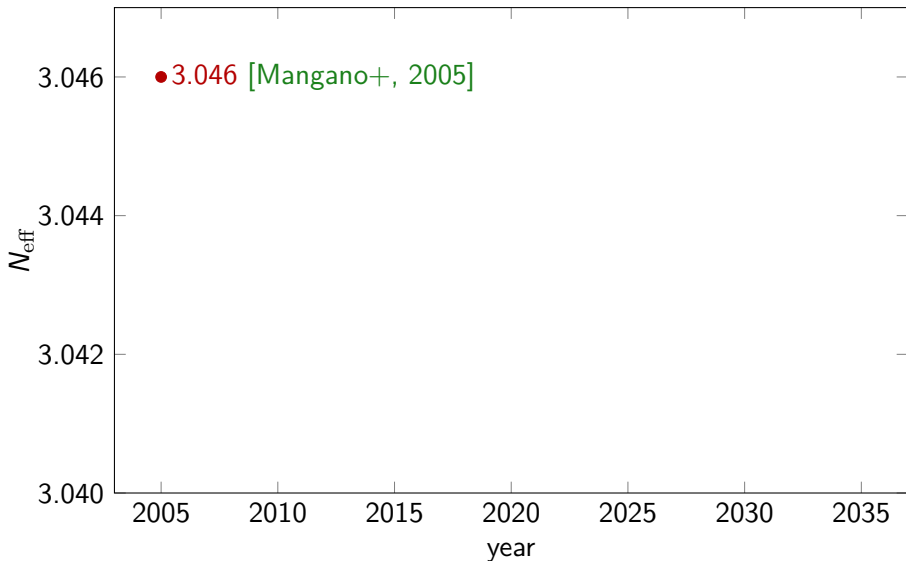


within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]
 only θ_{12} affects N_{eff} , at most by $\delta N_{\text{eff}} \approx 10^{-4}$



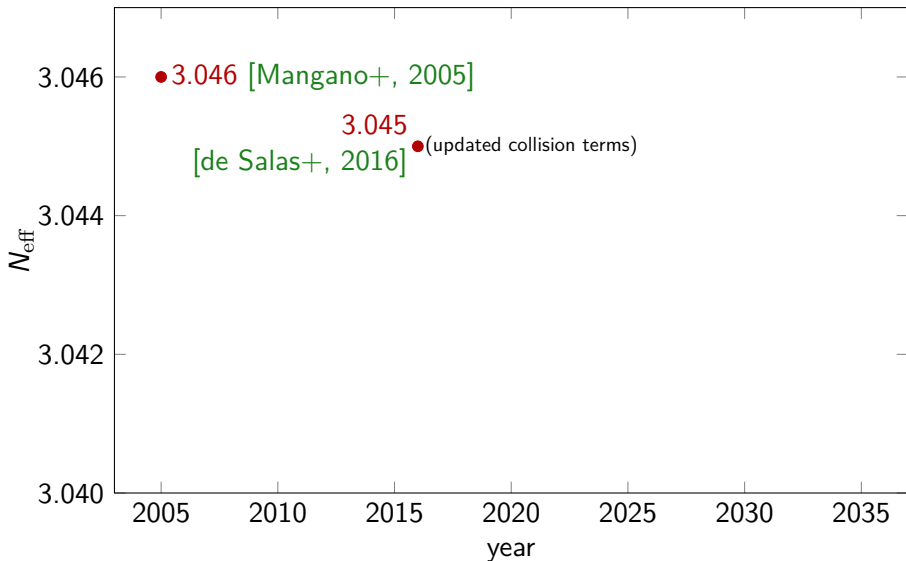
How precise is $N_{\text{eff}} = 3.04\dots?$

Full 3ν mixing results:



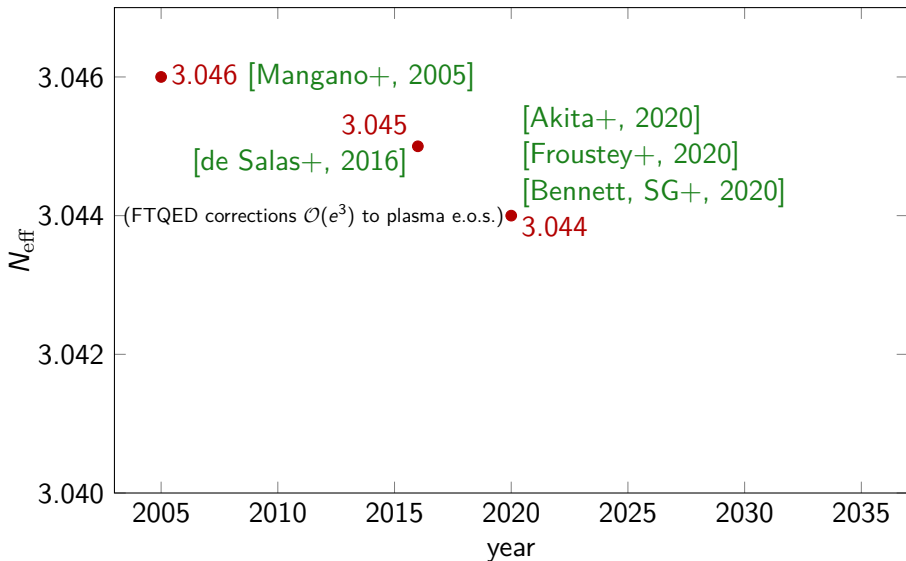
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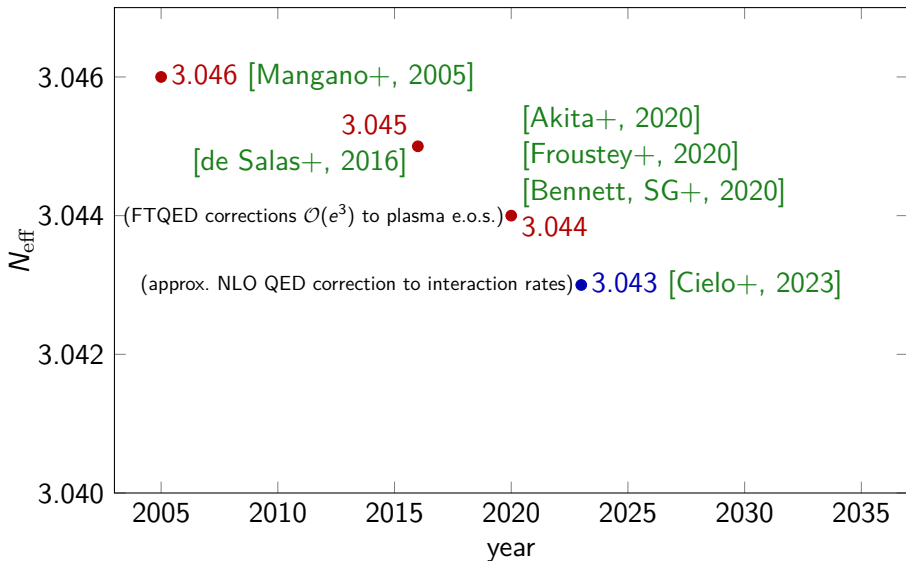
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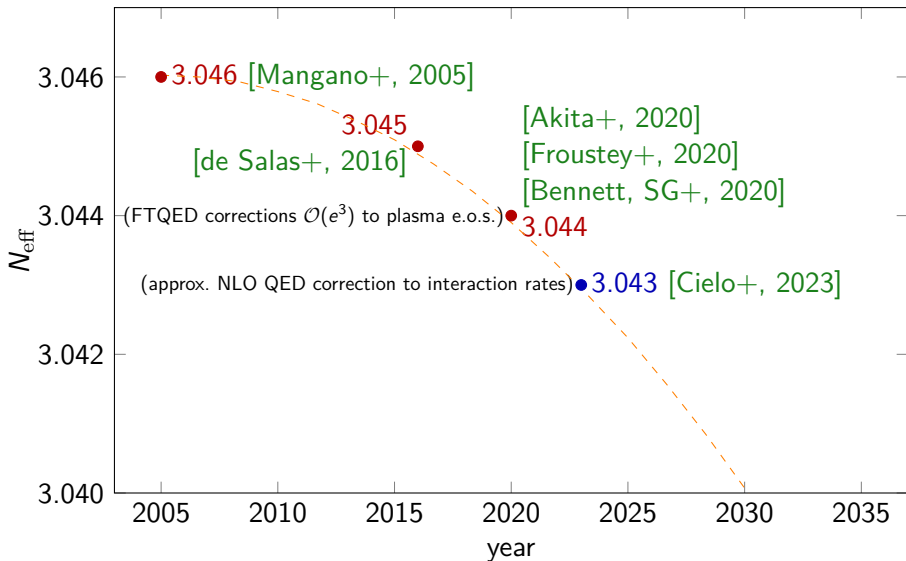
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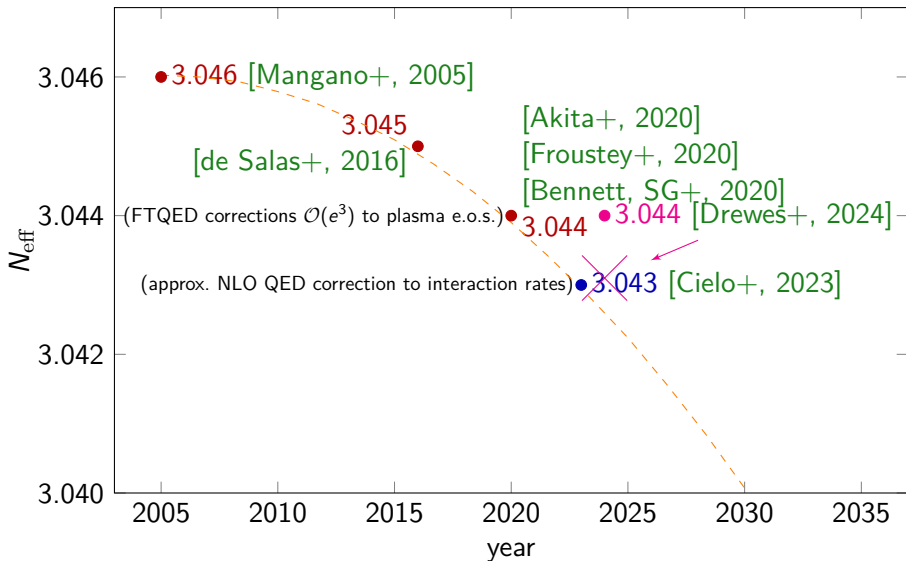
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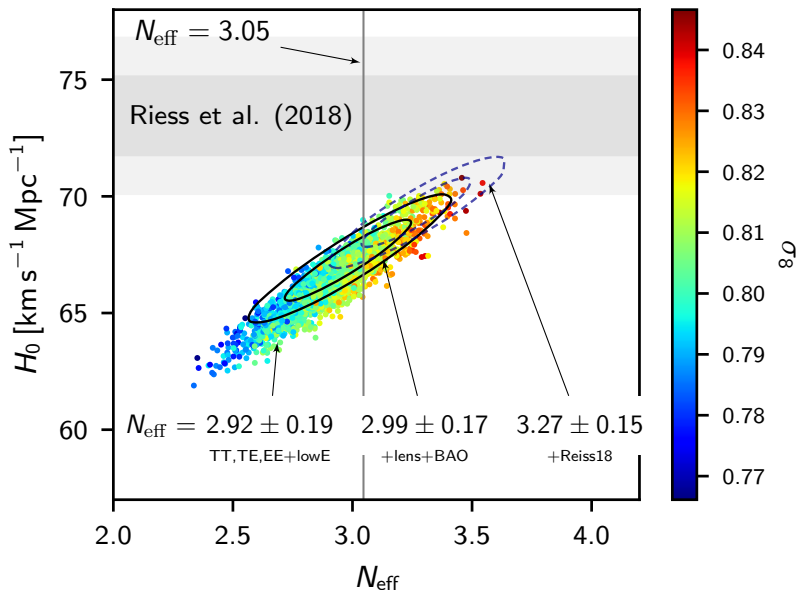
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N_{eff} and BBN

BBN: production of light nuclei
at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

temperature $T_{\text{fr}} \simeq 1\text{ MeV}$
from nucleon freeze-out:

$$\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5 = H \sim \sqrt{g_* G_N T^2}$$

$$T_{\text{fr}} \simeq (g_* G_N / G_F^4)^{1/6}$$

enters

$$n/p = \exp(-Q/T_{\text{fr}})$$

which controls element abundances

$$g_* \text{ depends on } N_{\text{eff}}$$

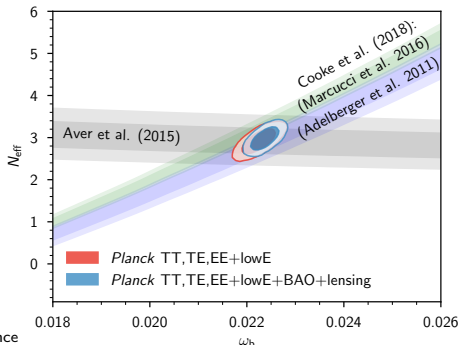
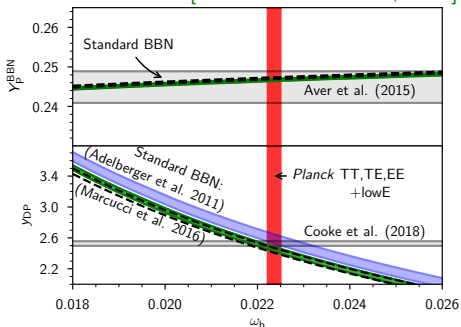
abundances depend on N_{eff}

G_F Fermi constant n, p : neutron, proton density number
 G_N Newton constant $Q = 1.293\text{ MeV}$ neutron-proton mass difference

S. Gariazzo

"Relic neutrinos: decoupling and direct detection perspectives"

[Planck Collaboration, 2018]



U. of Sheffield, 08/03/2024

18/38

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from nucleon freeze-out:

$$\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5 = H \sim \sqrt{g_* G_N T^2}$$

$$T_{\text{fr}} \simeq (g_* G_N / G_F^4)^{1/6}$$

enters

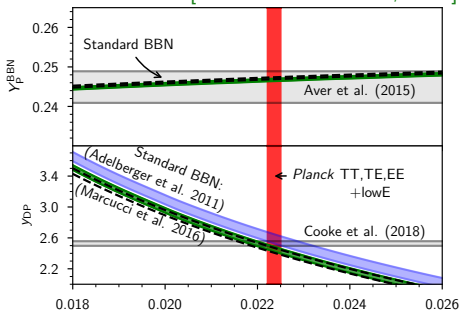
$$n/p = \exp(-Q/T_{\text{fr}})$$

which controls element abundances

$$g_* \text{ depends on } N_{\text{eff}}$$

abundances depend on N_{eff}

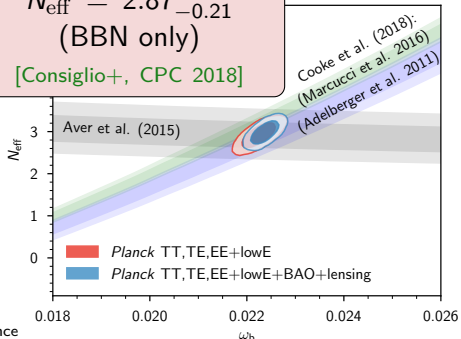
G_F Fermi constant n, p : neutron, proton density number
 G_N Newton constant $Q = 1.293\text{ MeV}$ neutron-proton mass difference

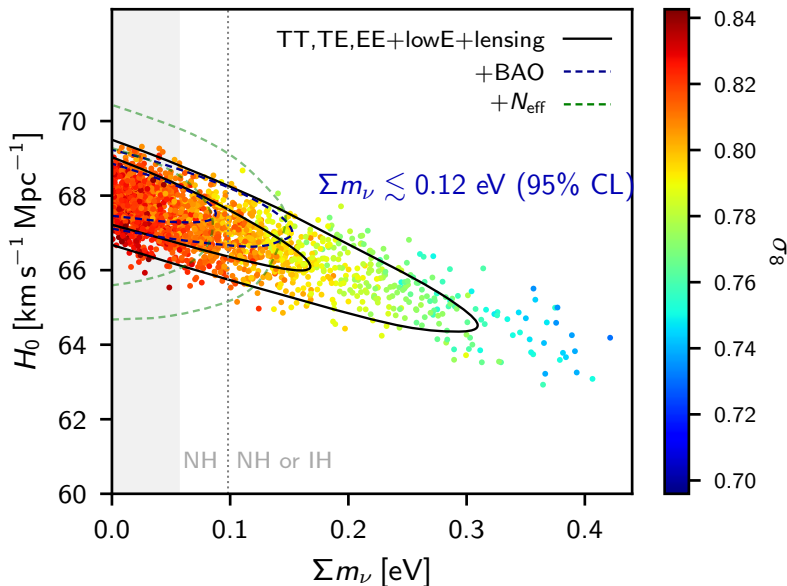


$$N_{\text{eff}} = 2.87^{+0.24}_{-0.21}$$

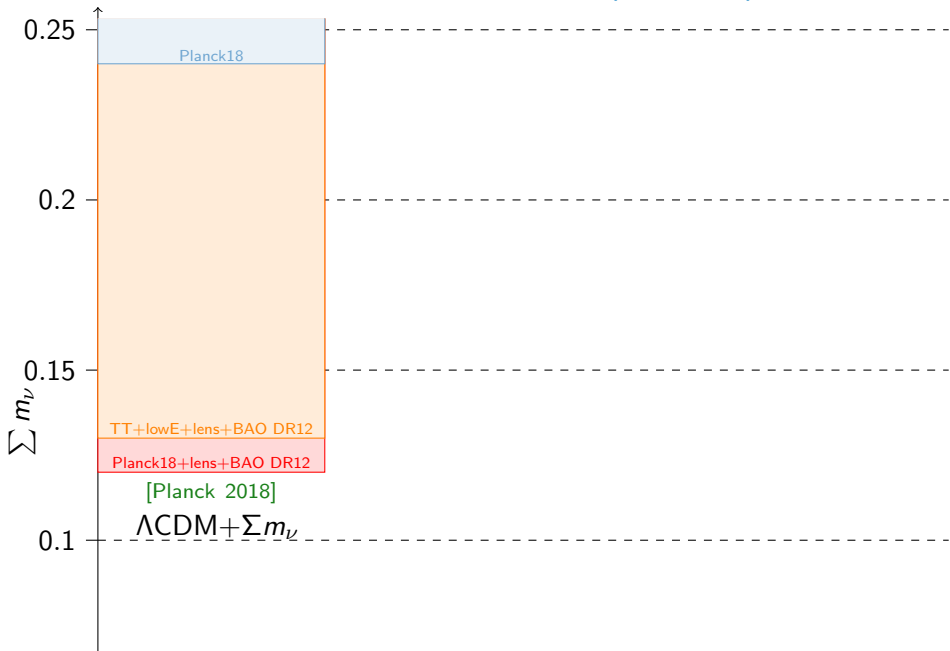
(BBN only)

[Consiglio+, CPC 2018]

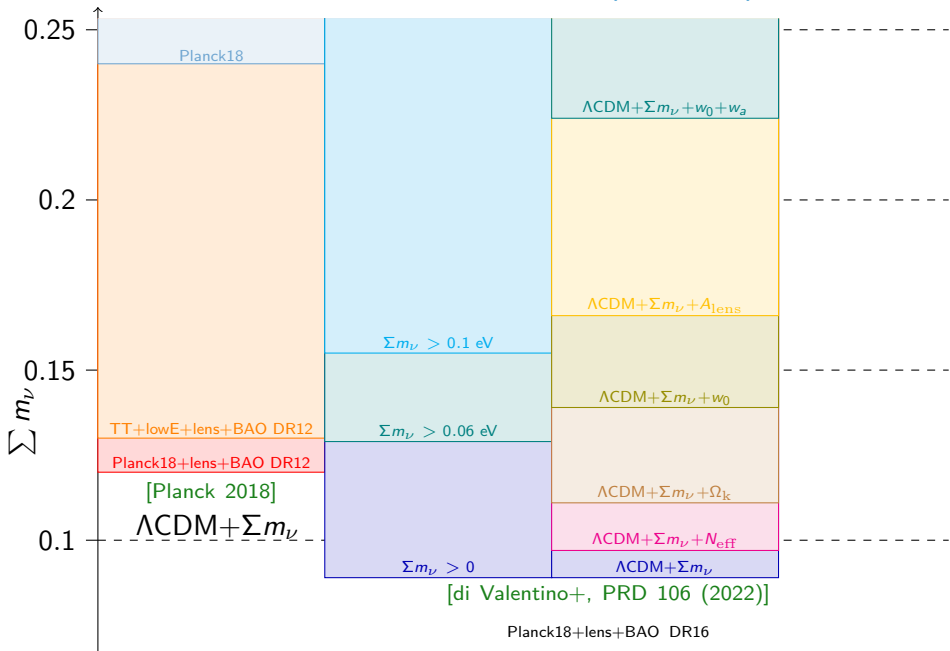




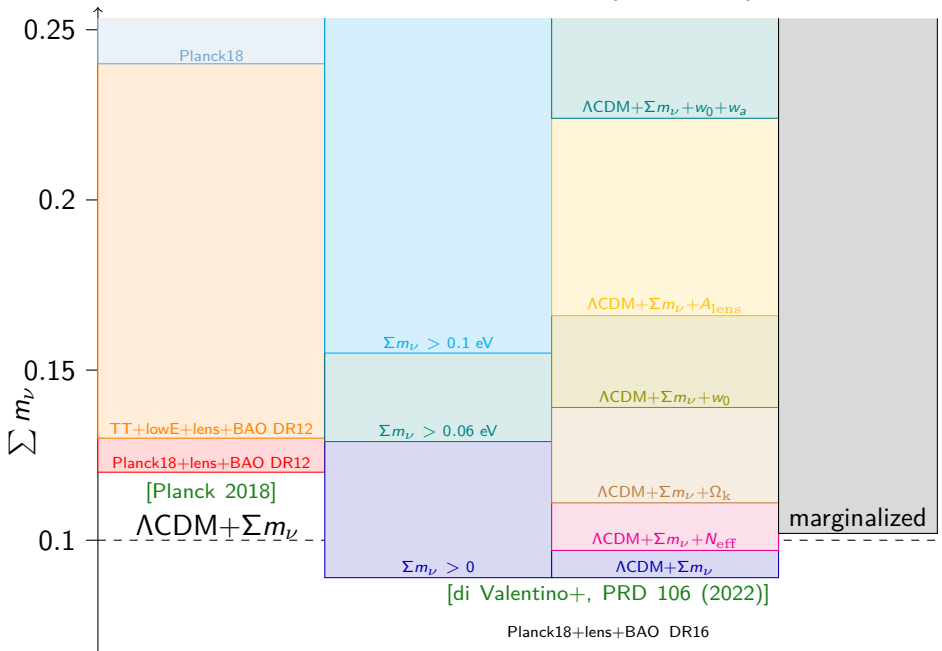
Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu / (94.12 \text{ eV})$

NO: $\Sigma m_\nu \gtrsim 0.06 \text{ eV}$

Current: $\Sigma m_\nu \lesssim 0.1 \text{ eV}$ (95%)

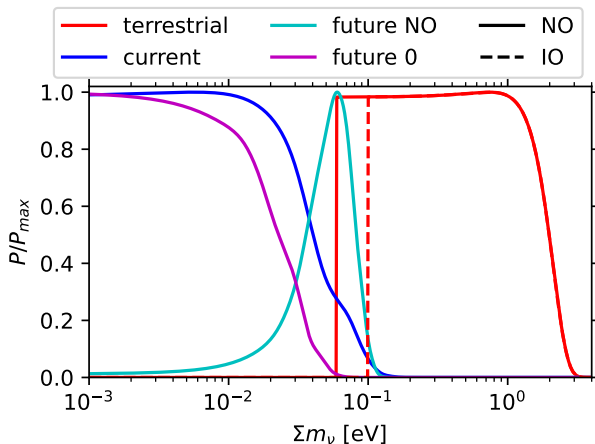
IO: $\Sigma m_\nu \gtrsim 0.1 \text{ eV}$

Future sensitivity: $\sigma(\Sigma m_\nu) \simeq 0.02 \text{ eV}$

Still preferring $\Sigma m_\nu = 0$?

Will measure e.g. $\Sigma m_\nu = 0.06 \text{ eV}$?

tension even
with NO!



confirm NO,
disfavor IO

Can a cosmological limit on Σm_ν disfavor IO? [PDU 40 (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu / (94.12 \text{ eV})$

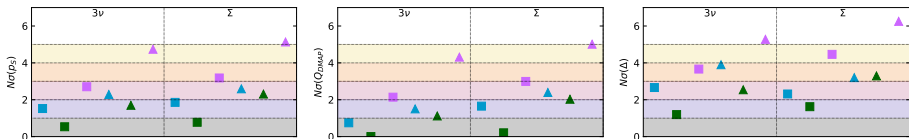
Is there a tension between cosmology and oscillations?

or will there be a tension?

several possible tests can be considered, similar results

$\Sigma m_\nu \lesssim 0.1 \text{ eV}$ (95%)
 $\Sigma m_\nu = 0.06 \pm 0.02 \text{ eV}$ (1σ)
 $\Sigma m_\nu = 0.00 \pm 0.02 \text{ eV}$ (1σ)

● current ■ NO
● future NO ▲ IO
● future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

future 0 can be at $\sim 2 - 3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

standard factor

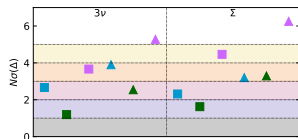
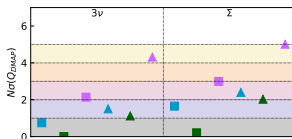
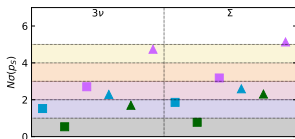
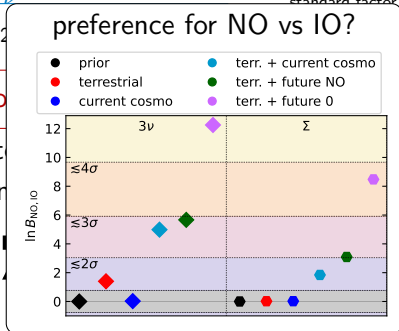
Cosmology measures $\omega_\nu = \Omega_\nu h^2$

Is there a tension between cosmo

or will there be a t

several possible tests can be con

- $\Sigma m_\nu \lesssim 0.1$ eV (95%) ● current
- $\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ) ● future NO
- $\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ) ● future 0



currently only mild tension between cosmology and oscillations

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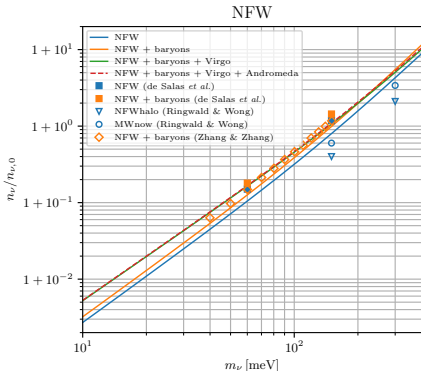
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C

Clustering in the local Universe

Based on:

- JCAP 09 (2017) 034
- JCAP 01 (2020) 015



ν clustering with N-one-body simulations

Relic neutrinos are **slow!** [$c_\nu \sim 160(1+z)(1 \text{ eV}/m_\nu) \text{ km s}^{-1}$]

Can be trapped in the gravitational potential of the Milky Way and neighbours

$f_c(m_i) = n_i/n_{i,0}$ clustering factor \rightarrow How to compute it?

Idea from [Ringwald & Wong, 2004] \rightarrow **N-one-body** = $N \times$ single ν simulations

Assumptions:

- ν s are independent
- only gravitational interactions
- ν s do not influence matter evolution
- ($\rho_\nu \ll \rho_{\text{DM}}$)

\rightarrow each ν evolved from initial conditions at $z = 3$

\rightarrow spherical symmetry, coordinates (r, θ, p_r, l)

\rightarrow need $\rho_{\text{matter}}(z) = \rho_{\text{DM}}(z) + \rho_{\text{baryon}}(z)$

how many ν s is "N"?

\rightarrow must sample all possible r, p_r, l

\rightarrow must include all possible ν s that reach the MW

(fastest ones may come from
several (up to $\mathcal{O}(100)$) Mpc!)

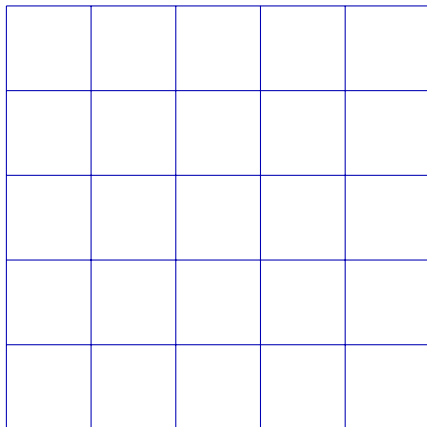
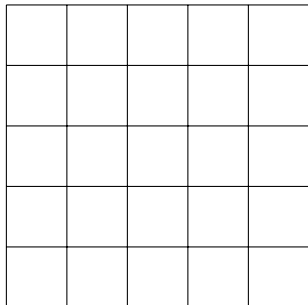
given $N \nu$:

\rightarrow weigh each neutrinos

\rightarrow reconstruct final density profile with kernel method from [Merritt & Tremblay, 1994]

Forward-tracking and back-tracking

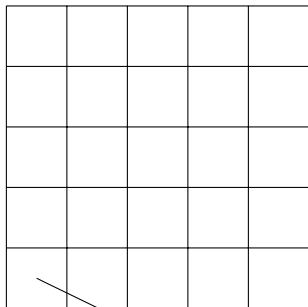
initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



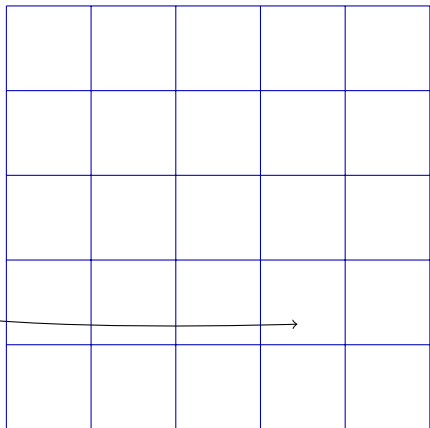
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



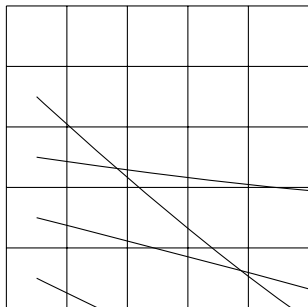
compute final position of each particle



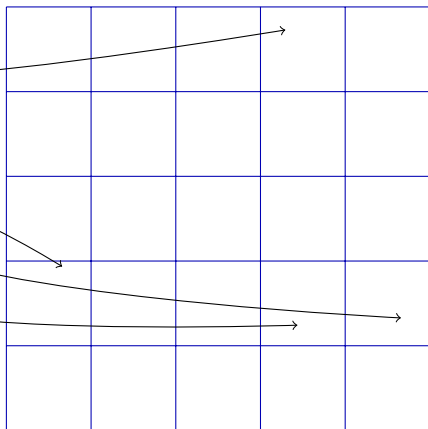
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Forward-tracking and back-tracking

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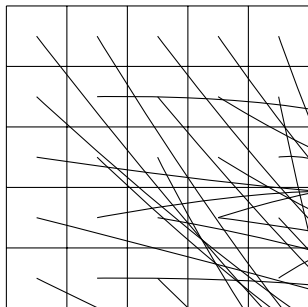
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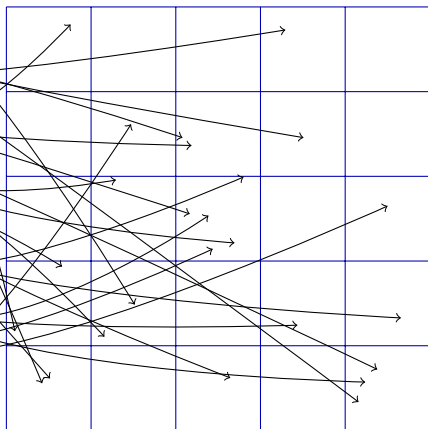
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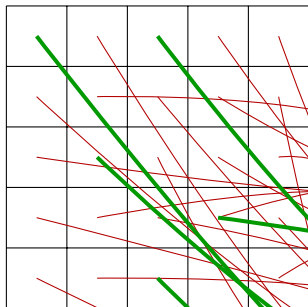
use positions to find neutrino distribution today



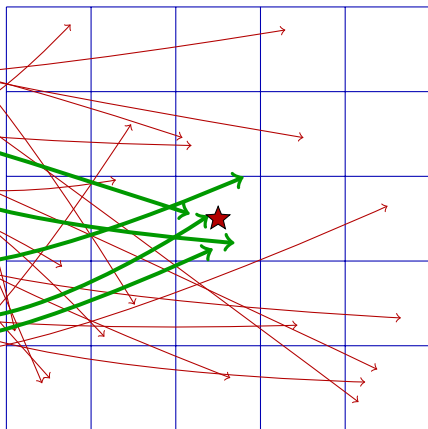
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Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★

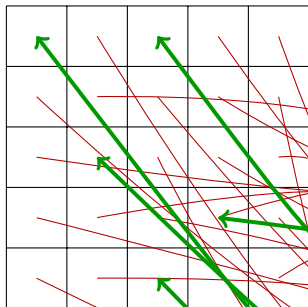


a lot of time is wasted!

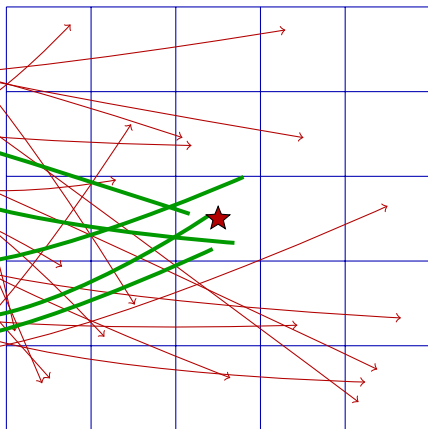
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★



a lot of time is wasted!

smarter way: track backwards
only interesting particles!

final phase space, $z = 0$

Advantages of tracking back

First advantage is in computational terms: much less points to compute

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Second advantage: no need to use spherical symmetry!

Forward-tracking

initial conditions need to sample
1D for position + 2D for momentum
when using spherical symmetry

with full grid would re-
quire 3+3 dimensions!

Impossible to relax
spherical symmetry!

Back-tracking

“Initial” conditions only described
by 3D in momentum

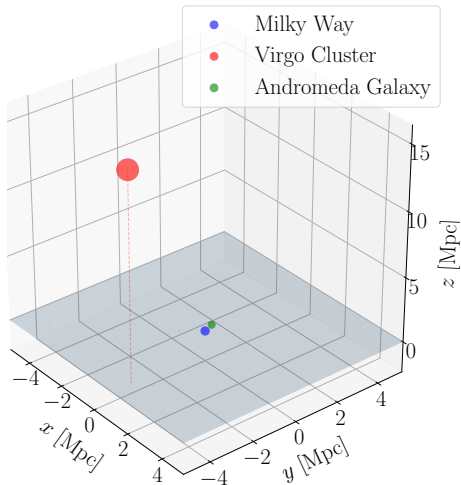
(position is fixed, apart for checks)

can do the calculation with
any astrophysical setup

Advantages of tracking back

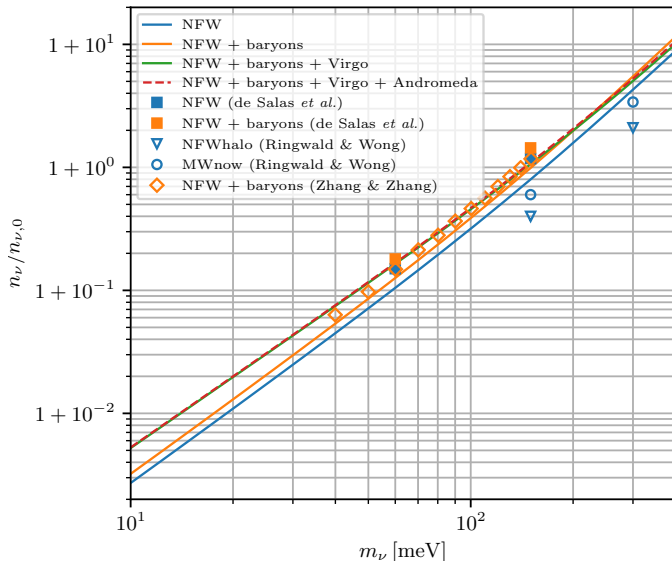
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Second advantage: no need to use spherical symmetry!



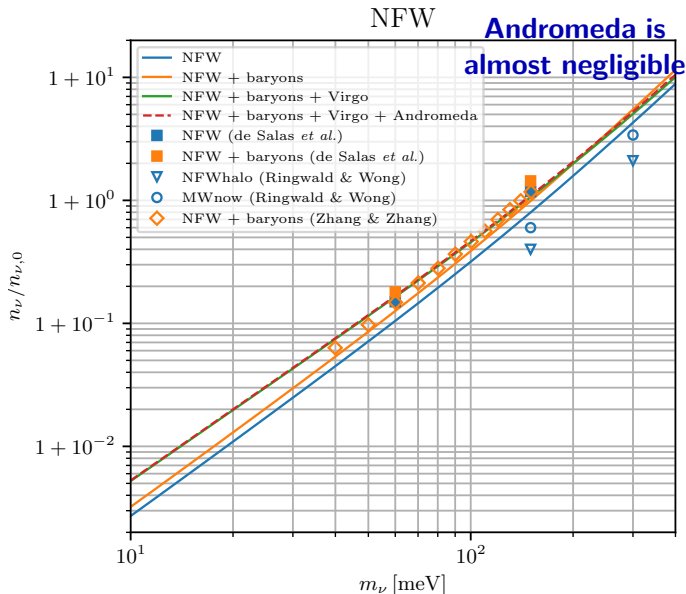
In comparison with previous results:

NFW



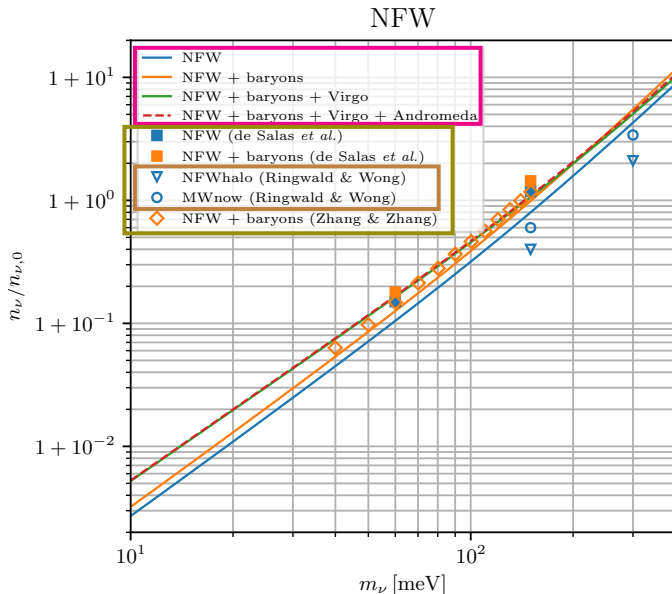
Clustering results with back-tracking

In comparison with previous results:



Clustering results with back-tracking

In comparison with previous results:



Warning: NFW
is not the same
for all the cases!

[de Salas+, 2017]

and

[Zhang², 2018]

use $\gamma \neq 1$,
now we have

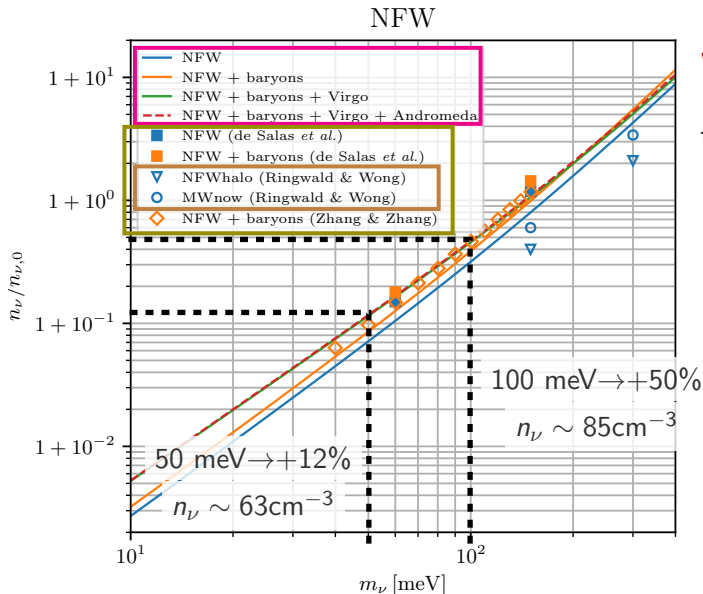
$$\gamma = 1$$

[Ringwald&Wong,

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Clustering results with back-tracking

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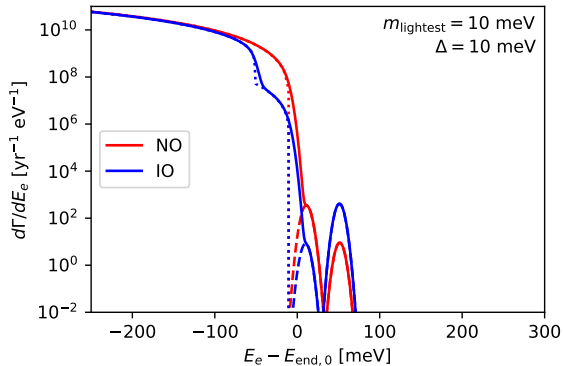
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D Direct detection of relic neutrinos

Based on:

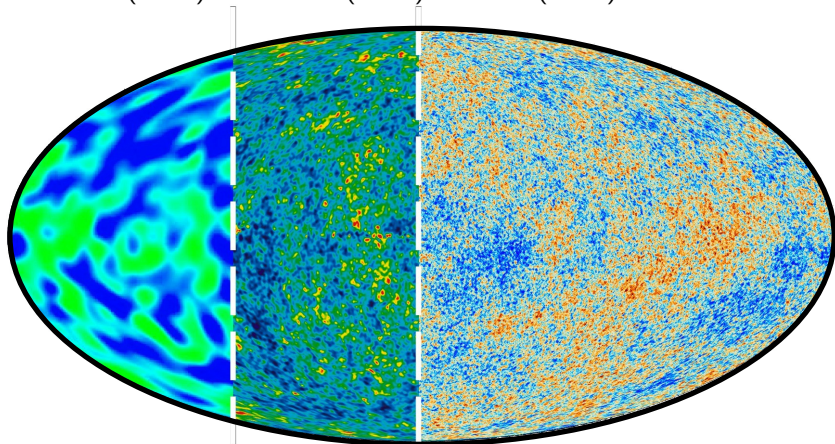
- JCAP 01 (2023) 003
- JCAP 08 (2014) 038
- JCAP 07 (2019) 047



The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

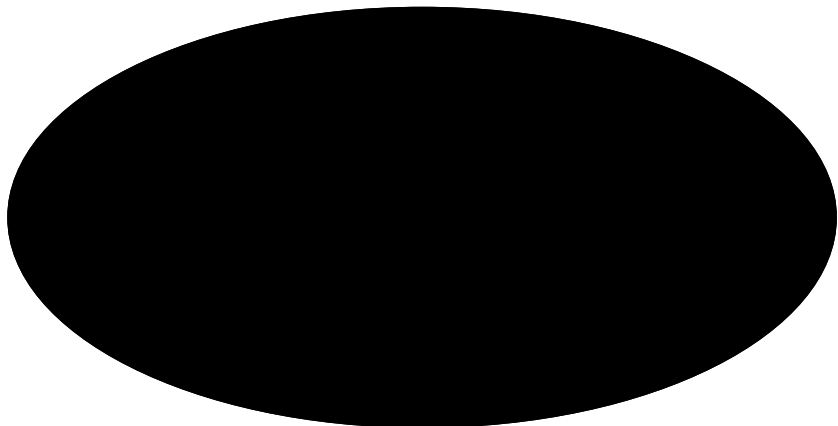
COBE (1992) WMAP (2003) Planck (2013)



The oldest picture of the Universe

The Cosmic Neutrino Background, generated at $t \simeq 1$ s

... → 2024 → ...



$$T_\nu \sim 10^{-4} \text{ eV}, E_\nu \sim 5 \times 10^{-4} \text{ eV today!}$$

We need **thresholdless detection process...** **How do we get them?**

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

Stodolsky effect

[Stodolsky, 1974][Duda+, 2001]

(only if there is
lepton asymmetry)

energy splitting of e^- spin states due to
coherent scattering with relic neutrinos



torque on e^- in lab rest frame



use a ferromagnet to build detector



measure torque with a torsion balance

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

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[Stodolsky, 1974][Duda+, 2001]

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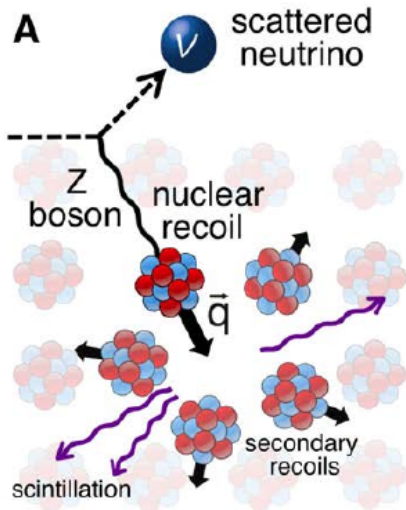
expected $a_\nu \simeq \mathcal{O}(10^{-26}) \text{ cm/s}^2$



$a_{\text{exp}} \simeq \mathcal{O}(10^{-12}) \text{ cm/s}^2$

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

elastic scattering where ν interacts with **nucleous** "as a whole"



Predicted for $|\vec{q}|R \lesssim 1$
by [Freedman, PRD 1974]

small recoil energies! $\lesssim 10$ keV...
difficult to measure

$$\frac{d\sigma}{dT}(E_\nu, T) \sim \frac{G_F^2 M}{4\pi} N^2$$

[Drukier, Stodolsky, PRD 1984]

enhancement N^2 because
 ν interacts
coherently with all nucleons

may give huge cross
section enhancement

CE ν NS?

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

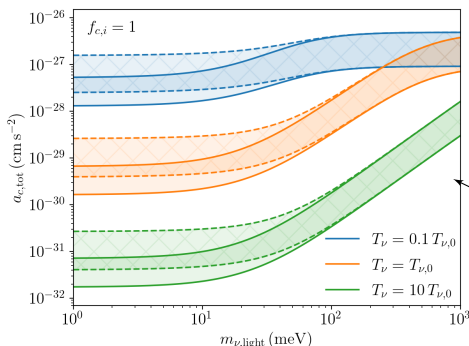
elastic scattering where ν interacts with **nucleous "as a whole"**

Can we detect relic neutrinos with CE ν NS?

relic neutrinos have **de Broglie length** $\lambda \sim 2\pi/p_\nu$



enhancement in interactions due to **coherence** with nuclei in volume λ^3



Acceleration induced by CE ν NS
of relic ν on test mass M :

$$a^N \propto ((A - Z)/A)^2 E_\nu / p_\nu^2 \Delta p_\nu n_\nu \rho$$

A, Z mass, atomic numbers
 p_ν, E_ν neutrino momentum and energy
 Δp_ν net momentum transfer
 n_ν neutrino number density
 ρ target mass density

unclustered relic ν s, $n_\nu = n_0$
 a^N of atoms in silicon target

CE ν NS?

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

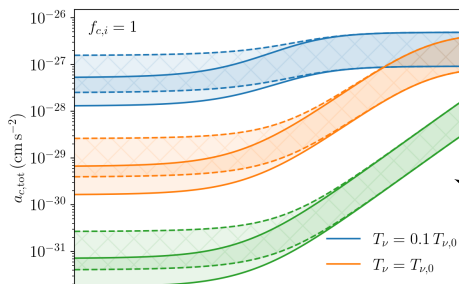
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unclustered relic ν s. $n_\nu = n_0$

proposed torsion balances can most optimistically reach $a \sim 10^{-23} \text{ cm s}^{-2}$

At interferometers?

How to directly detect non-relativistic neutrinos?

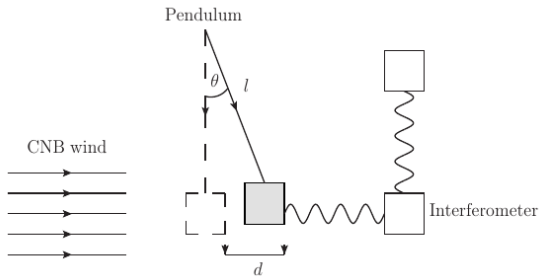
At interferometers

[Domcke+, 2017] [Shergold, 2021]

coherent scattering of relic ν on a pendulum



measure oscillations at interferometers



At interferometers?

How to directly detect non-relativistic neutrinos?

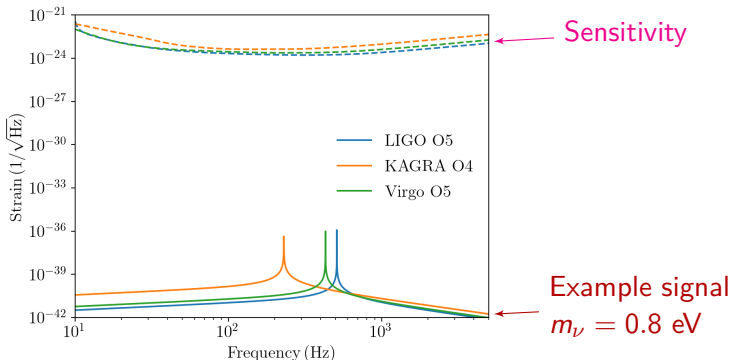
At interferometers

[Domcke+, 2017] [Shergold, 2021]

coherent scattering of relic ν on a pendulum



measure oscillations at interferometers



How to directly detect non-relativistic neutrinos?

Remember that
 $\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today



a process without energy
 threshold is necessary

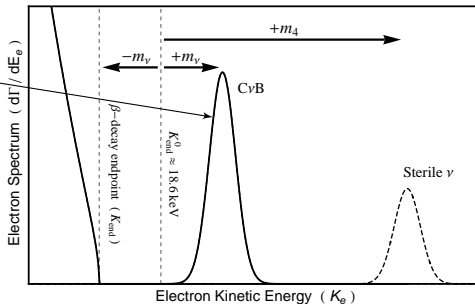
[Weinberg, 1962]: neutrino capture in β -decaying nuclei $\nu + n \rightarrow p + e^-$

Main background: β decay $n \rightarrow p + e^- + \bar{\nu}$!

signal is a peak at $2m_\nu$
 above β -decay endpoint

only with a lot of material

need a very good energy resolution



best element has highest $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$

to minimize contamination from β decay background

Isotope	Decay	Q_β (keV)	Half-life (s)	$\sigma_{\text{NCB}}(v_\nu/c)$ (10^{-41} cm ²)
³ H	β^-	18.591	3.8878×10^8	7.84×10^{-4}
⁶³ Ni	β^-	66.945	3.1588×10^9	1.38×10^{-6}
⁹³ Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
¹⁰⁶ Ru	β^-	39.4	3.2278×10^7	5.88×10^{-4}
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¹⁸⁷ Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
¹¹ C	β^+	960.2	1.226×10^3	4.66×10^{-3}
¹³ N	β^+	1198.5	5.99×10^2	5.3×10^{-3}
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²² Na	β^+	545.6	9.07×10^7	3.04×10^{-7}
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³H better because the cross section (\rightarrow event rate) is higher

At accelerator facilities?

What if we consider **accelerated tritium ions**, ${}^3\text{H}^+ + \nu_e \rightarrow {}^3\text{He}^{++} + e^-$?

Large background due to tritium beta decay...

Inverse process ${}^3\text{He}^{++} + \bar{\nu}_e \rightarrow {}^3\text{H}^+ + e^+$ would require **energy threshold**

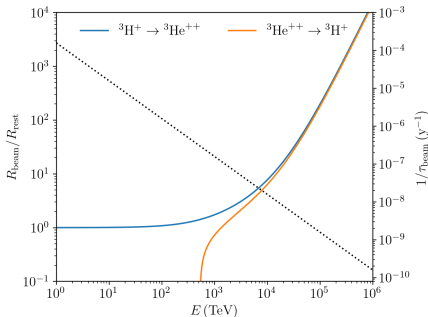
Match threshold in **beam rest frame**: $\tilde{E}_\nu = \frac{m_\nu}{M} E \geq Q$
with M , E ion mass, energy in lab frame

Even better:
resonant

${}^3\text{He}^{++} + \bar{\nu}_e + e^-$ (bound) $\rightarrow {}^3\text{H}^+$

can have many orders of magnitude larger cross-section
(which still scales with G_F^2)

but also **large background**...need
huge E to overcome it



All mentioned cross sections scale with G_F^2

resonant bound beta decay (RB β): ${}^A_Z P + \nu_e \rightarrow {}^A_{Z+1} D + e^-$ (bound)

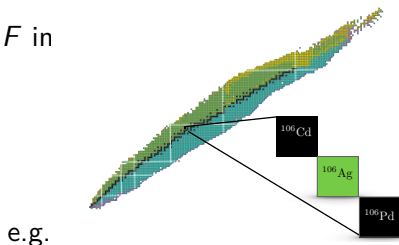
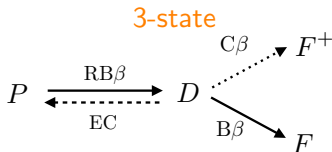
at resonance, G_F^2 suppression is lost in favor of Q^2 suppression!

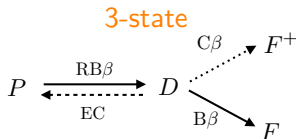
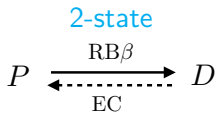
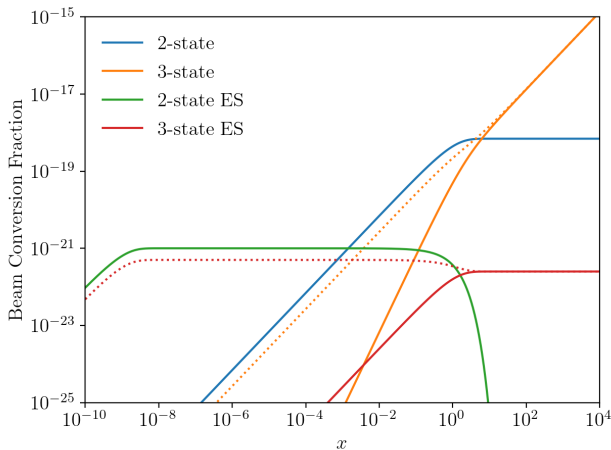
problem: final state D is converted back to P through electron capture (EC)!



Max event rate at equilibrium limits
information on RB β rate when running a long experiment

better: try to measure final stable state F in





excited states
also possible

$$\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2\sigma^2}}$$

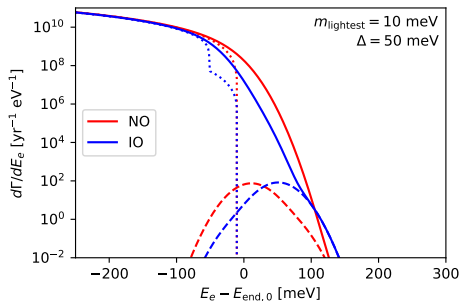
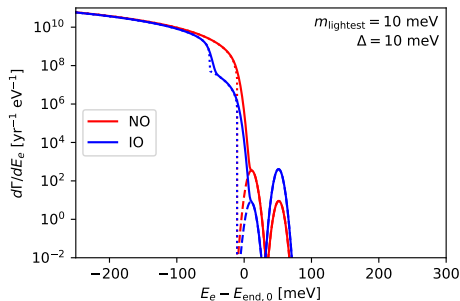
$$\frac{d\Gamma_\beta}{dE_e} = \frac{\bar{\sigma}}{\pi^2} N_T \sum_{i=1}^{N_\nu} |U_{ei}|^2 H(E_e, m_i)$$

$$\frac{d\tilde{\Gamma}_\beta}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} dx \frac{d\Gamma_\beta}{dE_e}(x) \exp\left[-\frac{(E_e - x)^2}{2\sigma^2}\right]$$

$$\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2\sigma^2}}$$

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Pontecorvo Tritium Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1 \text{ eV?}$
 0.05 eV?

can probe $m_\nu \simeq 1.4\Delta \simeq 0.1 \text{ eV}$

built mainly for CNB

$M_T = 100 \text{ g}$ of atomic ${}^3\text{H}$

$$\Gamma_{\text{CNB}} = \sum_{i=1}^3 |U_{ei}|^2 [n_i(\nu_{hR}) + n_i(\nu_{hL})] N_T \bar{\sigma} \sim \mathcal{O}(10) \text{ yr}^{-1}$$

N_T number of ${}^3\text{H}$ nuclei in a sample of mass M_T $\bar{\sigma} \simeq 3.834 \times 10^{-45} \text{ cm}^2$ n_i number density of neutrino i

(without clustering)

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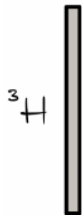
enhancement from
 ν clustering in the galaxy?

enhancement from
 other effects?

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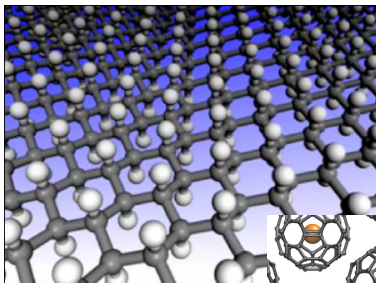
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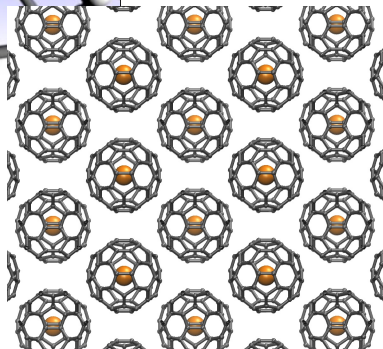
[Courtesy A. Esposito]

3
T



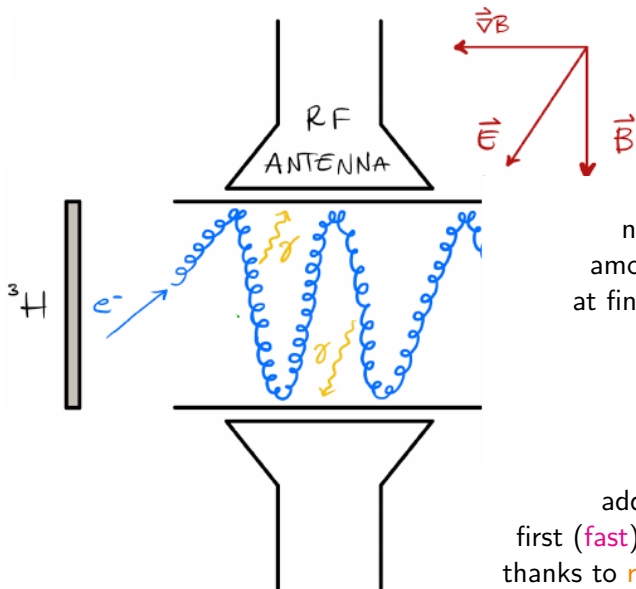
tritium on
graphene?

tritium on
fullerene?



[Courtesy A. Esposito]

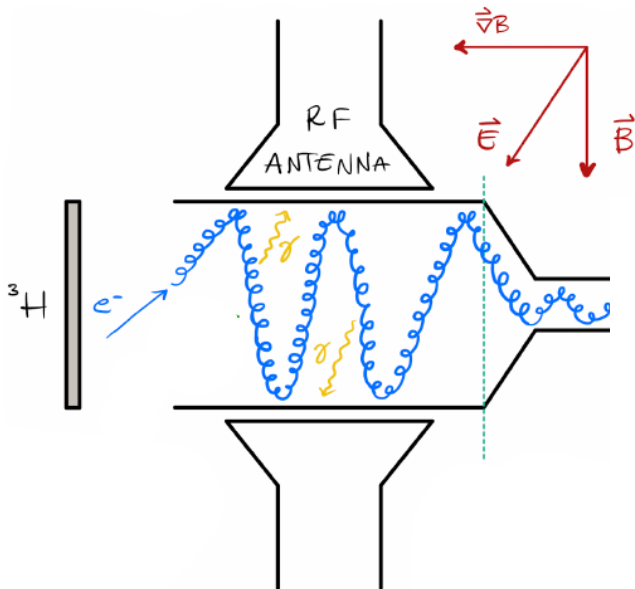
[Courtesy V. Tozzini]



need to reduce
amount of **electrons**
at final energy sensors:
EM filter

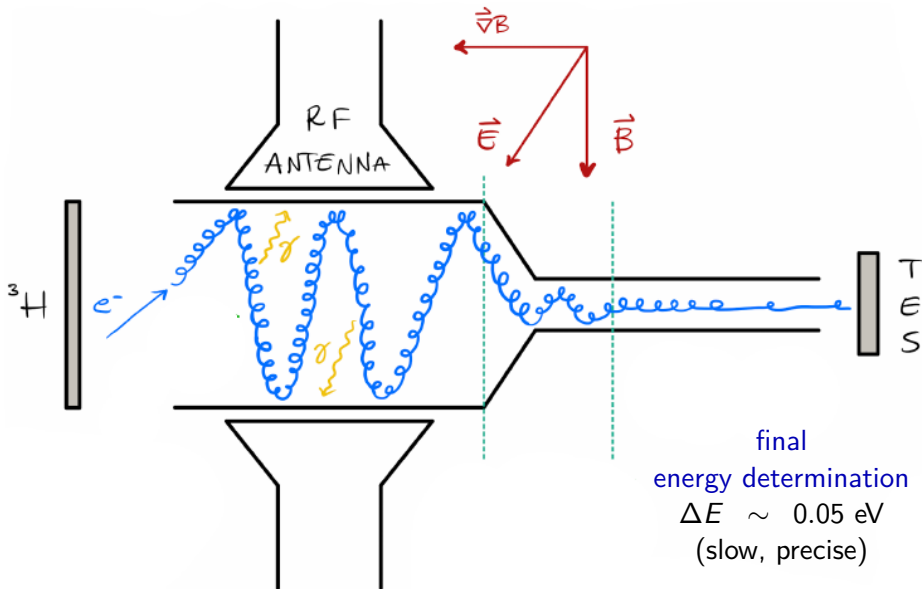
additional benefit:
first (**fast**) energy determination
thanks to **radio-frequency antenna**

[Courtesy A. Esposito]



filter only events
close to endpoint
($E \gtrsim E_0 - 10$ eV)

[Courtesy A. Esposito]



[Courtesy A. Esposito]

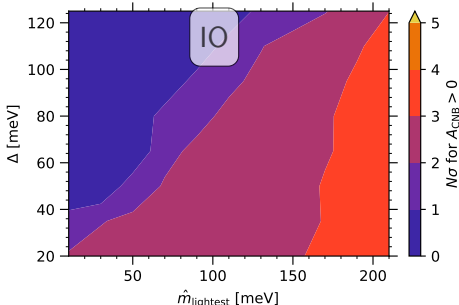
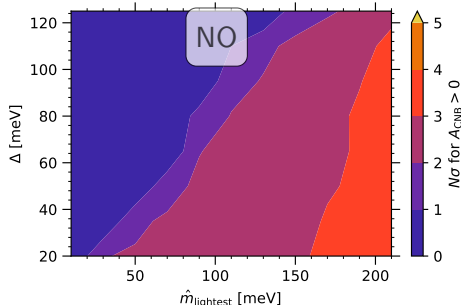
using the definition:

$$N_{\text{th}}^i(\theta) = A_\beta N_\beta^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + \mathbf{A}_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + N_b$$

if $\mathbf{A}_{\text{CNB}} > 0$ at $N\sigma$, direct detection of CNB accomplished at $N\sigma$

statistical only!

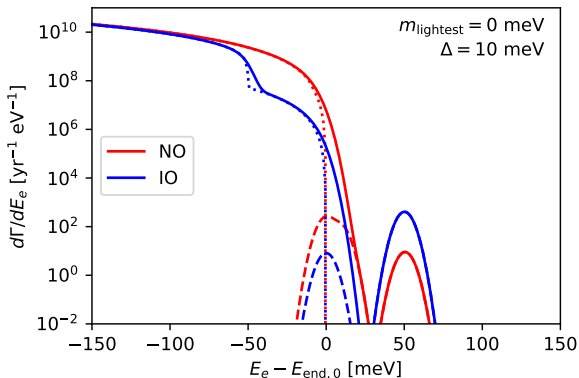
significance on $A_{\text{CNB}} > 0$
as a function of $\hat{m}_{\text{lightest}}$, Δ



What if the lightest neutrino is massless
and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β -decay)!

$$\frac{\nu \text{ capture rate}}{\beta \text{ decay rate}} = \frac{\Gamma_{\text{NC}}}{\Gamma_{\beta}} \approx \frac{n_{\nu}}{56 \text{ cm}^{-3}} \frac{2.54 \times 10^{-11}}{(\Delta/\text{eV})^3} \quad \text{rates in the bin } \Delta \text{ on the endpoint}$$

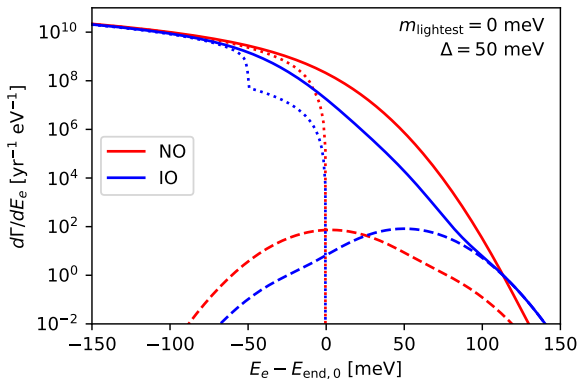


Time variations of ν capture rates

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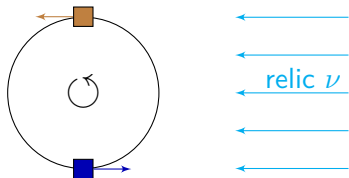
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rates in the bin Δ
on the endpoint



can be **daily** or annual modulation!

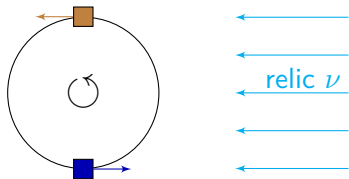
only for ν capture (no β -decay)

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can be **daily** or annual modulation!

only for ν capture (no β -decay)

Problem:

Expected **daily modulation**
is $\sim 1\%$ of the signal!!

Must use powerful technique
for signal/noise separation

**Fourier analysis and frequency
filtering may be sufficient**

no m_{ν} information in this way!



Z

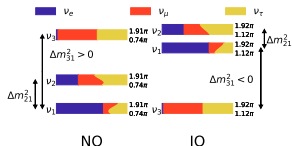
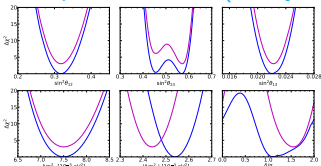
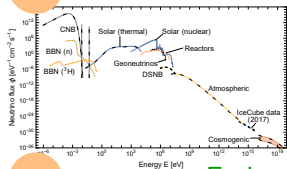
Conclusions

almost there!

What do we learn from relic neutrinos?

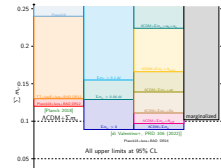
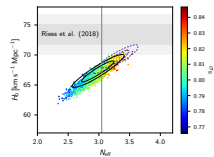
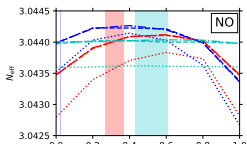
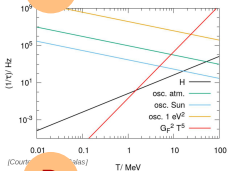
N

Neutrinos: precision era (many sources)



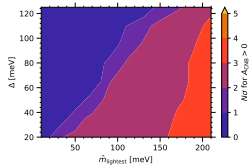
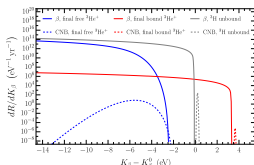
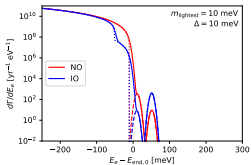
E

Early universe effects: indirect indications



D

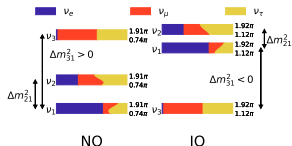
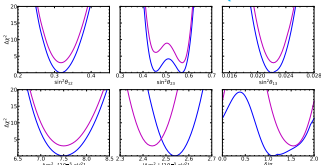
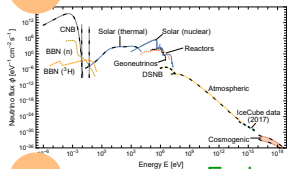
Direct detection: still long to go



What do we learn from relic neutrinos?

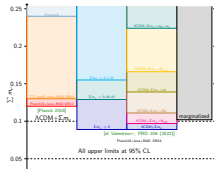
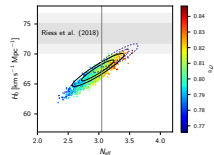
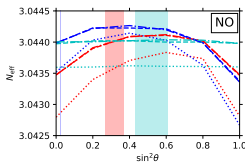
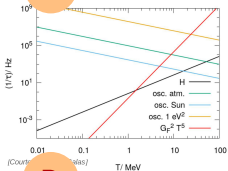
N

Neutrinos: precision era (many sources)



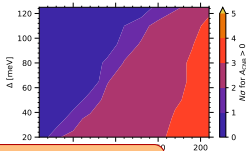
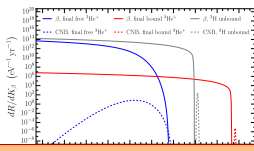
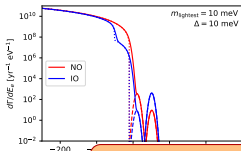
E

Early universe effects: indirect indications



D

Direct detection: still long to go



Thanks for your attention!