Heavy Higgs Lineshape

Many Questions – Few Answers

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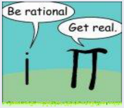
One might wonder why

considering an heavy SM Higgs boson. There are classic constraints on the Higgs boson mass coming from

- unitarity
- triviality



- vacuum stability
- precision electroweak data
- absence of fine-tuning

However, the search for a SM Higgs boson over a mass range from 80 *GeV* to 1 *TeV* is clearly indicated as a priority in many experimental papers (confirmed by Higgs conveners).

Preludio

From matrix elements to pseudo-observables

- Most of the people just want to use some well-defined recipe without having to dig any deeper;
- however, there is no alternative to a complete description of LHC processes which has to include the **complete matrix elements** for all relevant processes;
- **splitting** the whole S-matrix element into **components** is just conventional wisdom.
- However, the precise tone and degree of formality must be dictated by **gauge invariance**.

Heavy or light, standard or not

the Higgs boson is an unstable particle; as such it is described by a **complex pole on the second Riemann sheet**

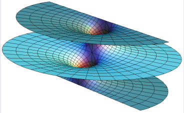
$$s_{\rm H} - M^2_{\rm H} + S_{\rm HH}\left(s_{\rm H}, M^2_{\rm t}, M^2_{\rm H}, M^2_{\rm W}, M^2_{\rm Z}\right) = 0$$

To lowest order accuracy the Higgs propagator can be rewritten as

$$\Delta^{-1}_{\rm H} = s - s_{\rm H}$$

The complex pole describing an unstable particle is conventionally parametrized as

$$s_i = \boxed{\mu_i^2 - i\,\mu_i\,\gamma_i}$$



Example of a nonexistent object

is it contradictory (Hume) or logically ill-formed (Kant) or wholeheartedly embraceable (Leibniz)?
A (gauge) meaningful definition of a nonexistent object. Take

$$\mathrm{H}(P) \quad \rightarrow \quad \mathrm{Z}_\mu(p_1) + \mathrm{Z}_\nu(p_2)$$

Work in the R_ξ -gauge; for any quantity $f(\xi)$ write

$$f(\xi) \quad = \quad f(1) + \Delta f(\xi), \qquad \Delta f(1) = 0$$

and look for ξ -independence

Step 1

Given the Higgs self-energy,

$$S_{\mathrm{HH}}(s) \;=\; S^{(1)}_{\mathrm{HH}} + \mathcal{O}\left(g^4\right) = \frac{g^2}{16\,\pi^2}\,\Sigma_{\mathrm{HH}}(s) + \mathcal{O}\left(g^4\right)$$

Let M_{H} be the renormalized Higgs mass, we obtain

$$\Delta\Sigma^{(1)}_{\mathrm{HH}}(\xi, s, M^2_{\mathrm{H}}) \;=\; \left(s - M^2_{\mathrm{H}}\right)\,\sigma^{(1)}_{\mathrm{HH}}(\xi, s, M^2_{\mathrm{H}})$$

The main equation is the one for the Higgs complex pole,

$$s_{\mathrm{H}} - M^2_{\mathrm{H}} + S^{(1)}_{\mathrm{HH}}\left(\xi\,,\,s_{\mathrm{H}}\,,\,M^2_{\mathrm{H}}\right) \;=\; 0$$

from which we derive $M^2_{\mathrm{H}} = s_{\mathrm{H}} + \mathcal{O}\left(g^2\right)$

Step 2

Easy to prove:

$$\frac{\partial}{\partial \xi} S^{(1)}_{\mathrm{HH}}\left(\xi\,,\,s_{\mathrm{H}}\,,\,s_{\mathrm{H}}\right) \;=\; 0$$

Next consider the one-loop vertices contributing to $\mathrm{H} \to \mathrm{ZZ}$

and obtain an S-matrix element

$$A^{(1)}_{\mathrm{V}} \;=\; \left(V^{(1)}_{d}\,\delta_{\mu\nu} + V^{(1)}_{p}\,p_{2\mu}\,p_{1\nu}\right) e^{\mu}\left(p_1,\lambda_1\right)\, e^{\nu}\left(p_2,\lambda_2\right)$$

Step 3

Using the decomposition

$$V_{d,p}^{(1)}\left(\xi\,,\,s\,,\,M_{\mathrm{H}}^{2}\right) \;=\; V_{d,p}^{(1)}\left(1\,,\,s\,,\,M_{\mathrm{H}}^{2}\right)+\Delta V_{d,p}^{(1)}\left(\xi\,,\,s\,,\,M_{\mathrm{H}}^{2}\right)$$

we obtain the following results:

1. after reduction to scalar form-factors there are no scalar vertices remaining in $\Delta V_{d}^{(1)}\left(\xi\,,\,s_{\mathrm{H}}\,,\,s_{\mathrm{H}}\right)$.
2. Furthermore, $\Delta V_{p}^{(1)}\left(\xi\,,\,s_{\mathrm{H}}\,,\,s_{\mathrm{H}}\right)=0$.

Step 4

Compute renormalization Z -factors for the external legs

$$s - M_{\rm H}^2 + S_{\rm HH}^{(1)}\left(\xi\,,\,s\,,\,M_{\rm H}^2\right) =$$

$$\left(s - s_{\rm H}\right)\,\left[1 + \frac{S_{\rm HH}^{(1)}\left(\xi\,,\,s\,,\,s_{\rm H}\right) - S_{\rm HH}^{(1)}\left(\xi\,,\,s_{\rm H}\,,\,s_{\rm H}\right)}{s - s_{\rm H}}\right] =$$

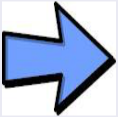
$$\left(1 + {\rm Z}_{\rm H}\right)\,\left(s - s_{\rm H}\right) + \mathcal{O}\left(\left(s - s_{\rm H}\right)^2\right)$$

Step 5

The main result follows:

$$\Delta V_d^{(1)}\left(\xi\,,\, s_{\rm H}\,,\, s_{\rm H}\right) - \left[\frac{1}{2}\,\Delta Z_{\rm H}(\xi) + \Delta Z_{\rm Z}(\xi)\right] A^{(0)} \;=\; 0$$

which gives to key to deal with processes where unstable particles play a role:

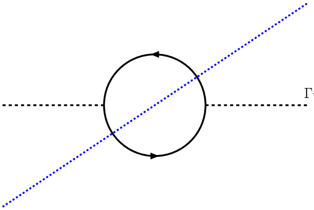


- **Define them at the complex pole**

The Unbearable Heaviness of Higgsing

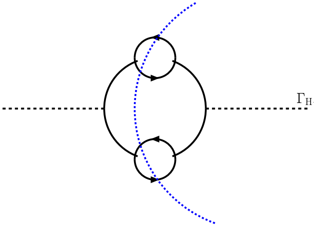
Table: The Higgs boson complex pole at fixed values of the W, t complex poles compared with the complete solution for $s_{\mathrm{H}}, s_{\mathrm{W}}$ and s_{t}

μ_{H} [GeV]	γ_{W} [GeV] f	γ_{t} [GeV] f	γ_{H} [GeV] d
200	2.088	1.481	1.355
250			3.865
300			8.137
350			14.886
400			26.598
μ_{H} [GeV]	γ_{W} [GeV] d	γ_{t} [GeV] d	γ_{H} [GeV] derived
200	2.130	1.085	1.356
250	2.119	0.962	3.823
300	2.193	0.836	8.139
350	2.607	0.711	14.653
400	3.922	0.566	25.498



$\Gamma_{\overline{b}b} = 2.55 \times 10^{-3}$ GeV

$M_H = 140$ GeV



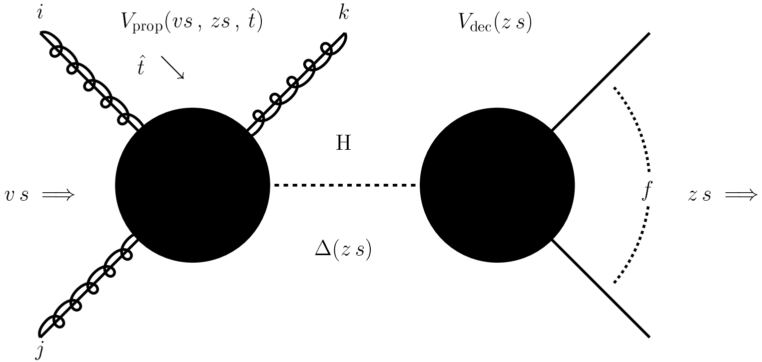
$\Gamma_{H \to 4f} = 4.64 \times 10^{-3}$ GeV

Lineshape Master Formulas

The final result is written in terms of **pseudo-observables**

$$\begin{aligned}
\sigma_{ij\to\mathrm{H}\to\mathrm{F}}(s) &= \frac{1}{\pi} \\
\Rightarrow &\times \sigma_{ij\to\mathrm{H}} \\
\Rightarrow &\times \frac{s^2}{\left|s - s_{\mathrm{H}}\right|^2} \\
\Rightarrow &\times \frac{\Gamma_{\mathrm{H}\to\mathrm{F}}}{\sqrt{s}}
\end{aligned}$$

$$\begin{aligned}
\sigma_{ij\to\mathrm{H}\to\mathrm{F}}(s) &= \dots \\
\Rightarrow &\times \frac{\Gamma^{\mathrm{tot}}_{\mathrm{H}}}{\sqrt{s}} \mathrm{BR}\left(\mathrm{H}\to\mathrm{F}\right)
\end{aligned}$$



$$\Longrightarrow\quad \sigma_{ij\to \mathrm{H}+k}(vs\,,\,\hat{t}\,,\,zs)\,\frac{vzs^2}{\left|zs - s_{\mathrm{H}}\right|^2}\,\frac{\Gamma_{\mathrm{H}\to f}(zs)}{(zs)^{1/2}} + \mathrm{NR}$$

$$= \quad \sigma_{ij\to \mathrm{H}+k}(vs\,,\,\hat{t}\,,\,s_{\mathrm{H}})\,\frac{vs\,\left|s_{\mathrm{H}}\right|^{1/2}}{\left|zs - s_{\mathrm{H}}\right|^2}\,\Gamma_{\mathrm{H}\to f}(s_{\mathrm{H}}) + \mathrm{NR}'$$

It is worth noting that

the introduction of complex poles does not imply complex kinematics. Only the residue of the propagator at the complex pole becomes complex, not any element of the phase-space integral (**details are non trivial** aspects of $L(C)$ and analytic continuation).

About on-off gauge variance

If the Higgs boson is off shell,

- in LO and NLO QCD in most cases the matrix element still respects gauge invariance,
- but in NLO EW gauge invariance is lost, unless the right scheme is used.

Technically speaking, we have a matrix element

$$\Gamma\left(\mathrm{H} \to \mathrm{F}\right) = \boxed{\boldsymbol{f}\left(\boldsymbol{s}\,,\;\mu_{\mathrm{H}}^{2}\right)}$$

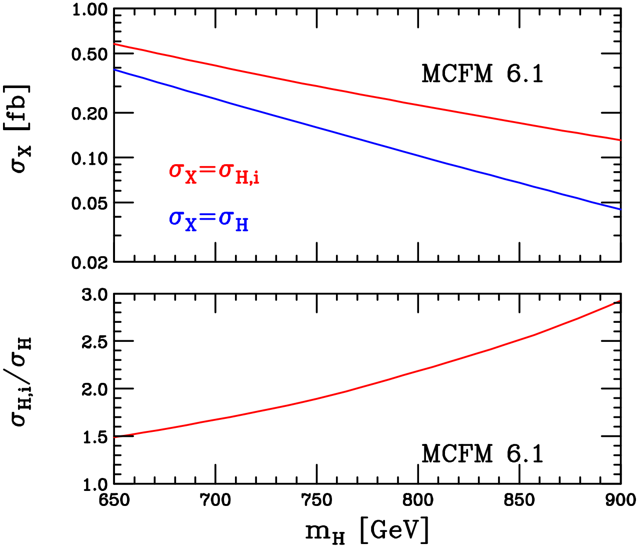
where s is the virtuality of the external Higgs boson, μ_{H} is the mass of internal Higgs lines and Higgs wave-function renormalization has been included.

The following happens:

- $f(s_{\rm H}\,,\, s_{\rm H})$ is gauge-parameter **independent** to all orders while
- $f(\mu_{\rm H}^2\,,\, \mu_{\rm H}^2)$ is gauge-parameter **independent** at one-loop but not beyond,
- $f(s\,,\, \mu_{\rm H}^2)$ is not.

In order to account for the off-shellness of the Higgs boson we can use (at one loop level) $f(s\,,\, s)$, i.e.,

- we intuitively replace the on-shell decay of the Higgs boson of mass $\mu_{\rm H}$ with the *on-shell* decay of an Higgs boson of mass $\sqrt{s}$ and
- not with the off-shell decay of an Higgs boson of mass $\mu_{\rm H}$.


1.00
0.50
0.20
0.10
0.05
0.02
σ_X [fb]
MCFM 6.1
$\sigma_X=\sigma_{H,i}$
$\sigma_X=\sigma_H$
3.0
2.5
2.0
1.5
1.0
$\sigma_{H,i}/\sigma_H$
MCFM 6.1
650
700
750
800
850
900
m_H [GeV]

Simulating interference?

Of course, the behavior for $s \to \infty$ is known and any correct treatment of PT (no mixing of different orders) will respect unitarity cancellations. The Higgs decays almost completely into longitudinals Zs, thus for $s \to \infty$

$$
\begin{aligned}
A_{\mathrm{H}} &\sim \frac{s m_q^2}{2 M_{\mathrm{Z}}^2}\, \Delta_{\mathrm{H}}\, \ln^2 \frac{s}{m_q^2} \\
A_{\mathrm{B}} &\sim -\frac{m_q^2}{2 M_{\mathrm{Z}}^2}\, \ln^2 \frac{s}{m_q^2}
\end{aligned}
$$

- but the behavior for $s \to \infty$ (unitarity) **should not/cannot** be used to **simulate** the interference for $s < M_{\mathrm{H}}^2$.
- The only relevant message is: unitarity requires the interference to be destructive at large s.

Schemes beyond ZWA

- **OFFBW**

$$S\,(\zeta,\,\dots) \;=\; V_{\rm prod}\,(\zeta,\,\dots)\;\Delta_{\rm BW}(\zeta)\,V_{\rm dec}(\zeta)$$

- **OFFP**

$$S\,(\zeta,\,\dots) \;=\; V_{\rm prod}\,(\zeta,\,\dots)\;\Delta_{\rm prop}(\zeta)\,V_{\rm dec}(\zeta)$$

- **CPP**

$$S\,(\zeta,\,\dots) \;=\; V_{\rm prod}\,(s_{\rm H},\,\dots)\;\Delta_{\rm prop}(\zeta)\,V_{\rm dec}(s_{\rm H})$$

The proof that CPP-scheme

satisfies **gauge-parameter independence** can be sketched as follows:

$$S(s) = \frac{V_{\text{prod}}(s_{\rm H})\, V_{\text{dec}}(s_{\rm H})}{\left[1 - S'_{\rm HH}(s_{\rm H})\right] (s - s_{\rm H})},$$

$$\frac{\partial}{\partial \xi} V_{\text{prod,dec}}(s_{\rm H}) \left[1 - S'_{\rm HH}(s_{\rm H})\right]^{-1/2} = 0$$

where ξ is an arbitrary gauge parameter and $S'_{\rm HH}(s_{\rm H})$ is the derivative of $S_{\rm HH}(s)$ computed at $s = s_{\rm H}$. Note that this equation follows from the use of Nielsen identities.

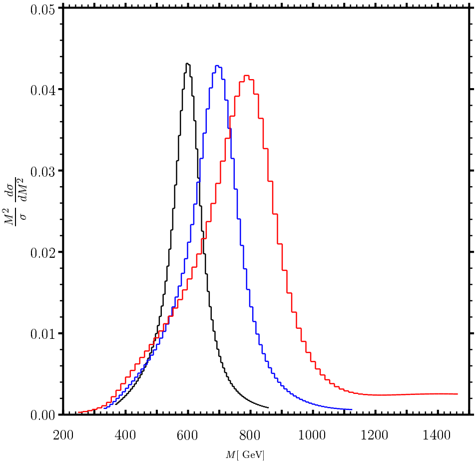


Figure: The normalized invariant mass distribution in the OFFP-scheme with running QCD scales for 600 *GeV* (black), 700 *GeV* (blue), 800 *GeV* (red).

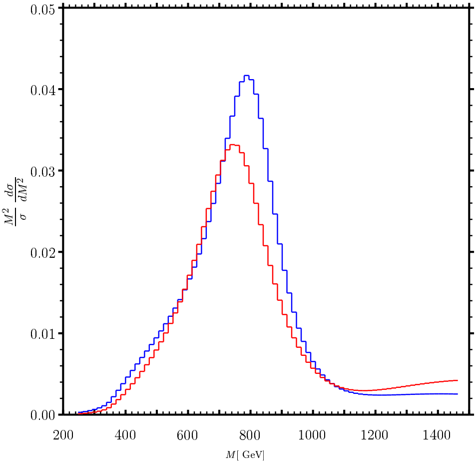


Figure: The normalized invariant mass distribution in the OFFP-scheme (blue) and OFFBW-scheme (red) with running QCD scales at 800 *GeV*.

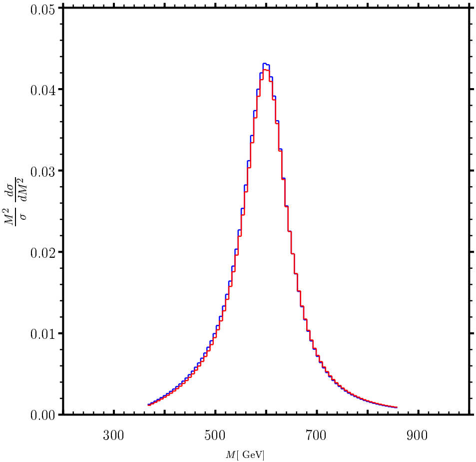


Figure: The normalized invariant mass distribution in the OFFP-scheme with running QCD scales for 600 GeV. The blue line refers to 8 TeV, the red one to 7 TeV.

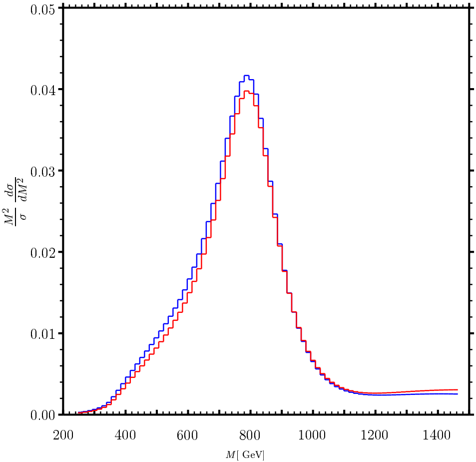


Figure: The normalized invariant mass distribution in the OFFP-scheme with running QCD scales for 800 *GeV*. The blue line refers to 8 *TeV*, the red one to 7 *TeV*.

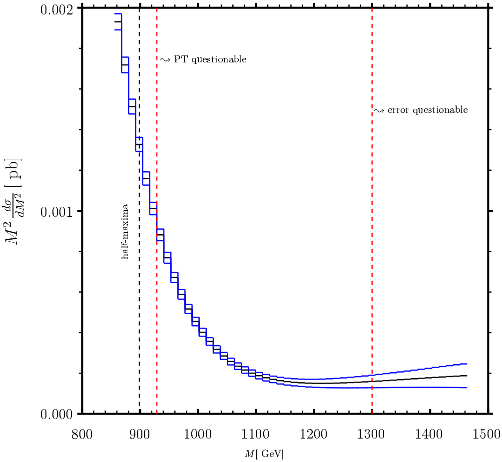


Figure: The invariant mass distribution in the OFFP-scheme for $\mu_{\rm H} = 800$ *GeV* with THU introduced by $\Gamma_{\rm H}^{\rm tot}(\zeta)$.

Estimate of THU is also needed

It would we desirable to include two- and three-loop contributions as well in γ_{H} and for some of these contributions only on-shell results have been computed so far.

- Use the Higgs-Goldstone Lagrangian of the SM

$$S_{\mathrm{HH}}(s) = A\,M_{\mathrm{H}}^2 + B\,(s - M_{\mathrm{H}}^2) + \frac{C}{M_{\mathrm{H}}^2}\,(s - M_{\mathrm{H}}^2)^2 + \dots$$

$$A = \sum_{n=0}^{\infty} a_n \left(\frac{G_{\mathrm{F}}\,M_{\mathrm{H}}^2}{2\sqrt{2}\pi^2}\right)^n$$

Derive

$$\gamma_{\rm H} = \frac{G_{\rm F}\, M_{\rm H}^3}{2\sqrt{2}\pi^2}\, a_1 \left(1 + \frac{G_{\rm F}\, M_{\rm H}^2}{2\sqrt{2}\pi^2}\, \frac{a_2}{a_1} + \dots \right)$$

- The ratio a_2/a_1 can be used to **estimate that the first correction** to $\gamma_{\rm H}$ is roughly given by

$$\delta_{\rm H} = 0.350119\, \frac{G_{\rm F}\, \mu_{\rm H}^2}{2\sqrt{2}\pi^2}$$

- No large variations up to 1*TeV* with a **breakdown** of the perturbative expansion around 1.74 *TeV*.

in the Higgs-Goldstone model one has

$$\frac{\gamma_{\rm H}}{\mu_{\rm H}} = 1.1781\, g_{\rm H} + 0.4125\, g^2_{\rm H} + 1.1445\, g^3_{\rm H}$$

$$g_{\rm H} = \frac{G_{\rm F}\mu^2_{\rm H}}{2\sqrt{2}\pi^2}$$

- $\gamma_{\rm H} = 168.84\; GeV$ (LO), $180.94\; GeV$ (NLO), $186.59\; GeV$ (NNLO) for $\mu_{\rm H} = 700\; GeV$.
- Using the three known terms in the series we estimate a 68% **credible interval** of $\gamma_{\rm H} = 186.59 \pm 1.93\; GeV$.
- The difference NNLO−LO is 17.8 GeV and in the full SM our estimate is $\gamma_{\rm H} = 163.26 \pm 11.75\; GeV$.

It is better to quantify the uncertainty

at the level of those quantities that characterize the resonance.

σ^{prod}	total production cross-section
$\Sigma = \dfrac{d\sigma^{\text{prod}}}{d\zeta}$	differential distribution
$\{\zeta_{\text{max}}\,,\,\Sigma_{\text{max}}\}$	the maximum of the lineshape
$\{\zeta_{\pm}\,,\,\frac{1}{2}\,\Sigma_{\text{max}}\}$	the half-maxima of the lineshape
$\mathrm{A} = \int_{\zeta_-}^{\zeta_+} d\zeta\,\Sigma$	the area between half-maxima

where ζ is the Higgs virtuality.

Table: Theoretical uncertainty on the production cross-section, the height of the maximum, the position of the half-maxima and the area of the resonance.

μ_H[GeV]	δ_H[%]	$\delta\sigma^{\text{prod}}$[%]	$\delta\Sigma_{max}$[%]	$\Delta\zeta_-$, $\Delta\zeta_+$[GeV]	δA[%]
600	5.3	-6.0 $+6.3$	-10.8 $+11.4$	$(-2.5, +2.5)$ $(+2.5, -2.5)$	-4.8 $+6.0$
700	7.2	-8.0 $+8.6$	-14.8 $+16.0$	$(-9.0, +8.0)$ $(+4.0, -4.0)$	-7.0 $+11.8$
800	9.4	-9.7 $+10.6$	-19.3 $+21.5$	$(-18.2, +18.2)$ $(+6.1, -6.1)$	-8.7 $+9.5$

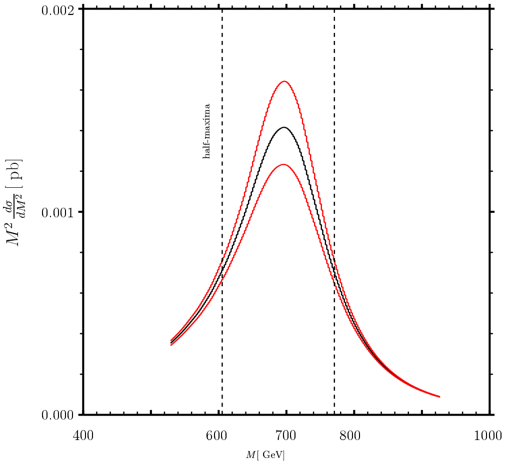


Figure: The invariant mass distribution in the OFFP-scheme with running QCD scales for $\mu_{\rm H} = 700\ GeV$. The red lines give the associated theoretical uncertainty.

The *right* factor $\Gamma_{\mathrm{H}}^{\mathrm{tot}}(\zeta)$

in the master formulas represents the "on-shell" decay of an Higgs boson of mass $\sqrt{\zeta}$ and we have to quantify the corresponding uncertainty.

$$\frac{\Gamma_{\mathrm{H}}}{\sqrt{\zeta}}\Big|_{HG} = \sum_{n=1}^{3} a_n\,\lambda^n = X_{HG},\ \lambda = \frac{G_{\mathrm{F}}\zeta}{2\sqrt{2}\pi^2}$$

Let $\Gamma_p = X_p\,\sqrt{\zeta}$ the width computed by PROPHECY4F, we redefine the total width as

$$\frac{\Gamma_{\mathrm{tot}}}{\sqrt{\zeta}} = (X_p - X_{HG}) + X_{HG} = \sum_{n=0}^{3} a_n\,\lambda^n$$

where now $a_0 = X_p - X_{HG}$.

As long as λ is not too large

we can define a $p\% < 80\%$ **credible interval** as

$$\begin{aligned}\Gamma_{\rm tot}(\zeta) &= \Gamma_p(\zeta) \pm \Delta\Gamma = \Gamma_p(\zeta)\,(1 \pm \delta\Gamma) \\ \Delta\Gamma &= \frac{5}{4}\,\mathrm{max}\{\mid a_0 \mid, a_1\}\, p\%\, \lambda^4\, \sqrt{\zeta}\end{aligned}$$

It is easily seen that

- for $\sqrt{\zeta} = 929\ GeV$ the **two-loop corrections are of the same size of the one-loop corrections**
- for $\sqrt{\zeta} = 2.6\ TeV$ **one-loop and Born become of the same size**

Table: Theoretical uncertainty on the total decay width, $\Gamma_{\rm H}^{\rm tot}$. Γ_p is the total width computed by PROPHECY4F and $\Delta\Gamma$ gives the credible intervals.

$\sqrt{\zeta}$[GeV]	Γ_p[GeV]	$\delta\Gamma$[68%]	$\delta\Gamma$[95%]
600	123	0.25 [%]	0.42 [%]
700	199	0.62 [%]	1.03 [%]
800	304	1.35 [%]	2.24 [%]
900	449	2.63 [%]	4.38 [%]
1000	647	4.72 [%]	7.85 [%]
1200	1205	13.1 [%]	21.7 [%]
1500	3380	34.7 [%]	57.8 [%]
2000	15800	98.9 [%]	165 [%]

Table: Total theoretical uncertainty on the production cross-section, the height of the maximum, the position of the half-maxima and the area of the resonance. The total is obtained by considering the THU on $\gamma_{\rm H}$ and on $\Gamma_{\rm tot}$ with a cut $\sqrt{\zeta} < 1.5\ TeV$.

$\mu_{\rm H}$[GeV]	$\delta\sigma^{\rm prod}$[%]	
600	-5.5	$+5.9$
700	-7.0	$+7.5$
800	-7.7	$+8.8$
900	-7.0	$+8.9$

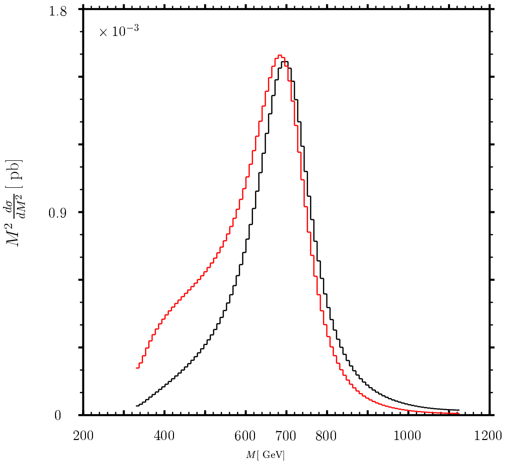


Figure: The invariant mass distribution in the OFFP-scheme (black) and in the CPP-scheme (red) for $\mu_{\rm H} = 700$ *GeV* for the process $gg \to H \to Z^c Z^c$.

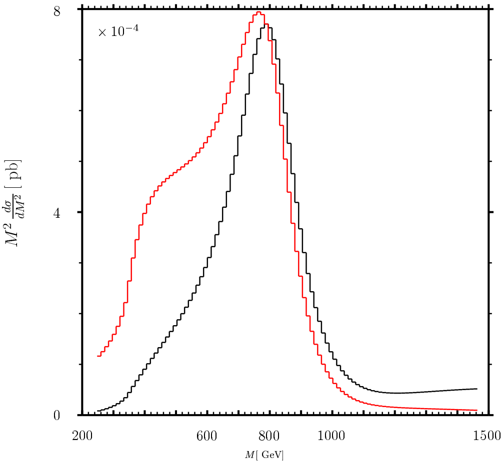


Figure: The invariant mass distribution in the OFFP-scheme (black) and in the CPP-scheme (red) for $\mu_{\mathrm{H}} = 800$ *GeV* for the process $\mathrm{gg} \to \mathrm{H} \to \mathrm{Z}^c\mathrm{Z}^c$.

Off-shell Pandora box

$\star\ \Gamma_{\mathrm{H}}\ \star\ M_{\mathrm{H}}\ \star\ \xi\ \star\ \sigma_{\text{prod}}$

Conclusions

- A conclusion is the place where you get tired of thinking (A. Bloch)
- Many questions, few answers . . . but
- aking the right questions takes as much skill as giving the right answers

- Higgs signal is in a good shape

- Interference is not in a good shape

Define the Signal: Step 1

General structure of any process containing a Higgs boson intermediate state:

$$A(s) = \frac{f(s)}{s - s_{\mathrm{H}}} + N(s),$$

Signal (S) and background (B) are defined as follows:

$$\begin{aligned} A(s) &= S(s) + B(s) \\ S(s) &= \frac{f(s_{\mathrm{H}})}{s - s_{\mathrm{H}}} \\ B(s) &= \frac{f(s) - f(s_{\mathrm{H}})}{s - s_{\mathrm{H}}} + N(s) \end{aligned}$$

Step 2

Consider the process $ij \to \mathrm{H} \to \mathrm{F}$ where $i, j \in$ partons and F is a generic final state; the complete cross-section will be written as follows:

$$
\begin{aligned}
\sigma_{ij\to \mathrm{H} \to \mathrm{F}}(s) &= \\
\Rightarrow & \quad \frac{1}{2s} \int d\Phi_{ij\to \mathrm{F}} \\
\Rightarrow & \times \quad \Big[\sum_{s,c} \big| A_{ij\to \mathrm{H}} \big|^2 \Big] \\
\Rightarrow & \times \quad \frac{1}{\big| s - s_{\mathrm{H}} \big|^2} \\
\Rightarrow & \times \quad \Big[\sum_{s,c} \big| A_{\mathrm{H}\to \mathrm{F}} \big|^2 \Big]
\end{aligned}
$$

Step 3

Strictly speaking and for reasons of gauge invariance, one should consider only

- the **residue** of the Higgs-resonant amplitude at the complex pole

If we decide to keep the Higgs boson off-shell also in the resonant part of the amplitude (interference signal/background remains unaddressed) then we can write

$$\int d\Phi_{ij\to \mathrm{H}} \sum_{s,c} \left| A_{ij\to \mathrm{H}} \right|^2 = s\,\overline{A}_{ij}(s).$$

Step 4

$$\Gamma_{\mathrm{H}\to\mathrm{F}}(s) = \frac{1}{2\sqrt{s}} \int d\Phi_{\mathrm{H}\to\mathrm{F}} \sum_{s,c} \left| A_{\mathrm{H}\to\mathrm{F}} \right|^2,$$

which gives the partial decay width of a Higgs boson of virtuality s into a final state F.

$$\sigma_{ij\to\mathrm{H}} = \frac{\overline{A}_{ij}(s)}{s},$$

which gives the production cross-section of a Higgs boson of virtuality s.

Step 5

We can write the final result in terms of **pseudo-observables**

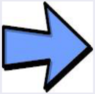
$$
\begin{aligned}
\sigma_{ij\to\mathrm{H}\to\mathrm{F}}(s) &= \frac{1}{\pi} \\
\Rightarrow &\times \sigma_{ij\to\mathrm{H}} \\
\Rightarrow &\times \frac{s^2}{\left|s - s_{\mathrm{H}}\right|^2} \\
\Rightarrow &\times \frac{\Gamma_{\mathrm{H}\to\mathrm{F}}}{\sqrt{s}}.
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{ij\to\mathrm{H}\to\mathrm{F}}(s) &= \dots \\
\Rightarrow &\times \frac{\Gamma^{\mathrm{tot}}_{\mathrm{H}}}{\sqrt{s}} \mathrm{BR}\left(\mathrm{H}\to\mathrm{F}\right)
\end{aligned}
$$

The complex-mass scheme

can be translated into a more familiar language by introducing the Bar – scheme.

$$\overline{M}_{\rm H}^2 = \mu_{\rm H}^2 + \gamma_{\rm H}^2 \qquad \mu_{\rm H}\,\overline{\Gamma}_{\rm H} = \overline{M}_{\rm H}\,\gamma_{\rm H}$$



It follows a remarkable identity:

$$\frac{1}{s - s_{\rm H}} = \left(1 + i\,\frac{\overline{\Gamma}_{\rm H}}{\overline{M}_{\rm H}}\right)\left(s - \overline{M}_{\rm H}^2 + i\,\frac{\overline{\Gamma}_{\rm H}}{\overline{M}_{\rm H}}\,s\right)^{-1},$$

showing that the Bar-scheme is equivalent to introducing a running width in the propagator with parameters that are not the on-shell ones.

It is important to realize however, that

$$\mathrm{Im}\Pi_{VV}(s) = 0, \quad s < 0,$$

since a space-like pair cannot appear as on-shell lines in a bubble.

To translate (4) to the full electroweak theory, we rewrite it

$$\frac{s}{s - m_H^2 + i\Gamma_H s/m_H} = \frac{s^2/m_H(1 + i\Gamma_H/m_H)}{s - m_H^2 + i\Gamma_H s/m_H} - \frac{s}{m_H^2}. \tag{5}$$

The apparently higher order term in the numerator is essential for the high energy limit, and cannot be neglected. Equation (5) provides a calculational implementation of (4) that is equally valid in the full electroweak theory. Namely that one makes the replacement

$$\frac{i}{s - m_H^2} \to \frac{i(1 + i\Gamma_H/m_H)}{s - m_H^2 + i\Gamma_H s/m_H}$$

for the s-channel Higgs boson propagator, leaving all other amplitudes unchanged. It would be extremely simple to make this substitution in computer programs that calculate the *amplitude* for $qq \to qqVV$ such as [6] and, with slightly more effort, in those that directly calculate the differential cross-section.

Unitarity requires that each partial wave of definite angular momentum and isospin, a_J^I, obeys

$$|a_J^I| \le 1.$$

Since the condition applies to the exact amplitude, one expects small violations at any given order in perturbation theory, owing to the truncation of the series. However, gross violations should be taken as an indication of the failure of the perturbation series. The $I = 0$ case is obtained by scattering the state $(2WW + ZZ)/\sqrt{6}$ to itself, and $J = 0$ from the integral

From Seymour paper

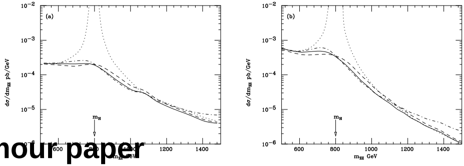
it's Bar - scheme !!!

$$a_0^I = \frac{1}{16\pi}\int_{-s}^{0}\frac{dt}{s}\mathcal{A}_I.$$

For a_0^0, the only partial wave to which the Higgs resonance contributes, we obtain

$$a_0^0 = -\frac{\Gamma_H}{m_H}\frac{s}{s - m_H^2 + i\Gamma_H s/m_H} - \frac{2}{3}\frac{\Gamma_H}{m_H}\left(1 - \frac{m_H^2}{s}\log\left(1 + \frac{s}{m_H^2}\right)\right).$$

This is shown in Fig. 4, in comparison with various amplitudes that have been used in the past. Note that only the full amplitude satisfies unitarity both in the resonance region and well above it. Note also that it peaks very close to m_H, unlike the other cases.



From Seymour paper
no need to approximate

Figure 7: The ZZ invariant mass spectrum at the LHC from (a) $qq \to qqZZ$ and (b) $gg \to ZZ$. We set $m_t = 175$ GeV, $m_Z = 91.2$ GeV, $\alpha = 1/128$, $\sin^2\theta_w = 0.23$, $m_W = m_Z \cos\theta_w$ and $\alpha_s(m_Z) = 0.120$, and use the MRS D–′ parton distribution functions. Curves are as in Fig. 4.

model the Higgs boson 'signal', and not the $\mathcal{O}(g^2)$ 'background' we compare it with the full result after subtraction of this background. As usual[9], we define the background to be the full result in the limit $m_H \to 0$, as this gives the lowest rate one could expect. It is clear from (1) that this background is zero in the effective theory. The comparison is shown in Fig. 8, where it can be seen that the improved s-channel approximation performs much better than the naïve one.

To conclude, the principal result of this paper is shown in Fig. 3 and Eq. (4). It is that it is possible to resum the sum of resonant and non-resonant diagrams to all orders, and the result smoothly extrapolates the well-known correct behaviour below, above and on the resonance. As a calculational prescription, it is possible to represent the result as a modification of the Higgs boson propagator,

$$\frac{i}{s - m_H^2} \to \frac{i(1 + i\Gamma_H/m_H)}{s - m_H^2 + i\Gamma_H s/m_H},$$

although it should be stressed that it includes effects that are not strictly associated with the propagation of a Higgs boson, namely the interference with non-resonant diagrams. In calculations that use the s-channel approximation, a better modification is

$$\frac{i}{s - m_H^2} \to \frac{i m_H^2/s}{s - m_H^2 + i\Gamma_H s/m_H}.$$

We have shown that the impact on the Higgs boson lineshape, and hence on the whole phenomenology of high energy vector boson pair production, is significant.

8

nothing to do with interference!!!

Modified Seymour scheme?

$$\frac{m_{\rm H}^2}{s}\left[s - m_{\rm H}^2 + i\,\frac{\Gamma_{\rm H}}{m_{\rm H}}\,s\right]^{-1}$$

- not normalizable in $[0\,,\,\infty]$
- not derivable from first principles, not justifiable, not simulating interference

The Higgs propagator doesn't know about real and imaginary parts of boxes, e.g., $\mathrm{gg} \to \mathrm{ZZ}$.

$$\begin{aligned} I &= 2\,\mathrm{Re}\,\Delta_{\rm H}\,\mathrm{Re}\left(\overline{A}_{\rm H}\,A_{\rm B}^{\dagger}\right) \\ &+ 2\,\mathrm{Im}\,\Delta_{\rm H}\,\mathrm{Im}\left(\overline{A}_{\rm H}\,A_{\rm B}^{\dagger}\right) \end{aligned}$$

$\overline{A}_{\rm H} = (s - s_{\rm H})\,A_{\rm H}$.

Modified Seymour scheme?

Of course, the behavior for $s \to \infty$ is known and any correct treatment of PT (no mixing of different orders) will respect unitarity cancellations. The Higgs decays almost completely into longitudinals Zs, thus for $s \to \infty$

$$A_{\mathrm{H}} \sim \frac{s m_q^2}{2 M_{\mathrm{Z}}^2} \Delta_{\mathrm{H}} \ln^2 \frac{s}{m_q^2}$$

$$A_{\mathrm{B}} \sim -\frac{m_q^2}{2 M_{\mathrm{Z}}^2} \ln^2 \frac{s}{m_q^2}$$

- but the behavior for $s \to \infty$ (unitarity) **should not/cannot** be used to **simulate** the interference for $s < M_{\mathrm{H}}^2$.
- The only relevant message is: unitarity requires the interference to be destructive at large s.

Leading K -factor for the decay width $\mathrm{H} \to \mathrm{VV}$:

$$
\begin{aligned}
\text{OS} \qquad & K \sim 1 + a_1 \frac{G_{\mathrm{F}} M_{\mathrm{H}}^2}{16\sqrt{2}\pi^2} + a_2 \left(\frac{G_{\mathrm{F}} M_{\mathrm{H}}^2}{16\sqrt{2}\pi^2} \right)^2 \\
\text{CPP} \qquad & K \sim 1 + a_1 \frac{G_{\mathrm{F}} s_{\mathrm{H}}}{16\sqrt{2}\pi^2} \\
+ \quad & (a_2 + 3i\pi\, a_1) \left(\frac{G_{\mathrm{F}} s_{\mathrm{H}}}{16\sqrt{2}\pi^2} \right)^2 ,
\end{aligned}
$$

$$
a_1 \;=\; 1.40 - 11.35\, i \qquad a_2 = -34.41 - 21.00\, i
$$

Above 1 TeV the NNLO term dominates the K -factor $\sim 0.17(M_{\mathrm{H}}/1\ TeV)^4$.

More on interference

- Consider $\mathrm{gg} \to 4\,\mathrm{f}$, one would like to have the best prediction for signal, i.e., $\Gamma(\mathrm{H} \to 4\,\mathrm{f})$ at NLO+NNLO (NNLO dominates for large masses).
- Therefore the Signal is (at least) at two-loop level and is not gauge invariant for off-shell Higgs.
- One-loop (complete) Background(+ Interference) is under construction, two-loop Background seems out of reach for the foreseeable future.
- Dura lex sed lex . . .

Warning

- It is clear that it **does not make much sense** to have an error estimate **beyond** 1.3 *TeV* and, therefore, all results for the Higgs lineshape that have a sizable fraction of events in this high-mass region should not be taken too seriously. Here, once again, the only viable alternative to define the Higgs signal is the CPP-scheme.
- **above** 0.93 *TeV* perturbation theory becomes questionable since the two-loop corrections start be become larger than the one-loop ones
- **above** 1.3 *TeV* the error estimate also becomes questionable since the expansion parameter is $\lambda = 0.7$ and the 95% credible interval (after inclusion of the leading two-loop effects) is 32.2%.