

SMEFT

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Loops and Legs, 24–29 April 2016, Leipzig

Synopsis

Practices that define hep at this point in time

A set of constructs, definitions, and propositions that present
a systematic view of SMEFT¹

... while attempting to provide a consistency proof²
of quasi-renormalization in SMEFT³

Theory deals with the well founded theoretical results obtained from first principles, while phenomenology deals with not so well founded effective models with a smaller domain of application⁴.

¹ how the influence of higher energy processes is localizable in a few structural properties which can be captured by a handful of Wilson coefficients

² Not only power counting, but a proof that proves that there are enough Wilson coefficients

³  A theory without loss of calculability

⁴ For a definition see Hartmann (Studies in History and Philosophy of Modern Physics)

One-loop divergencies in the theory of gravitation

par

G. 't HOOFT (*) and M. VELTMAN (*)

C E R N , Geneva

ABSTRACT. — All one-loop divergencies of pure gravity and all those of gravitation interacting with a scalar particle are calculated. In the case of pure gravity, no physically relevant divergencies remain; they can all be absorbed in a field renormalization. In case of gravitation interacting with scalar particles, divergencies in physical quantities remain, even when employing the so-called improved energy-momentum tensor.

1. INTRODUCTION

The recent advances in the understanding of gauge theories make a fresh approach to the quantum theory of gravitation possible. First, we now know precisely how to obtain Feynman rules for a gauge theory [1]; secondly, the dimensional regularization scheme provides a powerful tool to handle divergencies [2]. In fact, several authors have already published work using these methods [3], [4].

One may ask why one would be interested in quantum gravity. The foremost reason is that gravitation undeniably exists; but in addition we may hope that study of this gauge theory, apparently realized in nature, gives insight that can be useful in other areas of field theory. Of course, one may entertain all kinds of speculative ideas about the role of gravitation in elementary particle physics, and several authors have amused themselves imagining elementary particles as little black holes etc. It may well be true that gravitation functions as a cut-off for other interactions; in view of the fact that it seems possible to formulate all known

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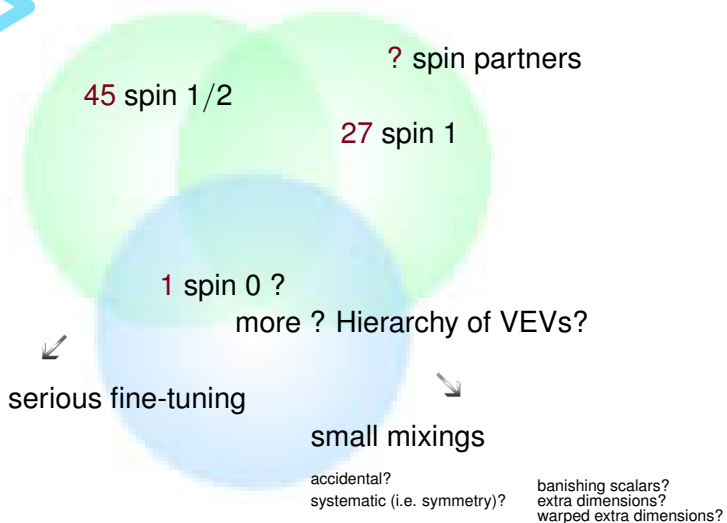


methodological antireductionism It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically, it may even be possible that there is no end, simply more and more scales (Georgi).

This prompts the important question whether there is a last fundamental theory in this tower of EFTs which supersede each other with rising energies. Some people conjecture that this deeper theory could be a string theory, i.e. a theory which is not a field theory any more.

epistemological antifoundationalism Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain? (Hartmann, Castellani)

Or ... one should not resort to arguments involving gravity: let us banish further thoughts about gravity and the damage it could do to the weak scale
(J. D. Wells)



Thinking UV ...



Back to the **more and more scales** scenario. Let's undergo revision (SMEFT) but it is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended.

The very effort for rigor forces us to find out simpler methods of proof

D. Hilbert



Executive summary (so far) Need for a consistent theoretical framework in which deviations from the SM (or NSM) predictions can be calculated⁶. Such a framework should be applicable to comprehensively describe measurements in all sectors of particle physics: LHC Higgs measurements, past EWPD, etc.

⁶Every 20 bogus hypotheses you test, one of them will give you a p of < 0.05

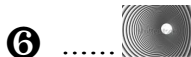
- 1 SM augmented with the inclusion of higher dimensional operators (\mathbf{T}_1); not strictly renormalizable. Although workable to all orders, \mathbf{T}_1 fails above a certain scale, Λ_1 .
- 2 Consider any BSM model that is strictly renormalizable and respects unitarity (\mathbf{T}_2); its parameters can be fixed by comparison with data, while masses of heavy states are presently unknown. $\mathbf{T}_1 \neq \mathbf{T}_2$ in the UV but must have the same IR behavior.
- 3 Consider now the whole set of data below Λ_1 .

\mathbf{T}_1 should be able to explain them by fitting Wilson coefficients,

\mathbf{T}_2 adjusting the masses of heavy states (as SM did with the Higgs mass at LEP) should be able to explain the data.

Goodness of both explanations are crucial in understanding how well they match and how reasonable is to use \mathbf{T}_1 instead of the full \mathbf{T}_2

- 4 Does \mathbf{T}_2 explain everything? Certainly not, but it should be able to explain something more than \mathbf{T}_1 .
- 5 We could now define \mathbf{T}_3 as \mathbf{T}_2 augmented with (its own) higher dimensional operators; it is valid up to a scale Λ_2 .





SMEFT rulebook

- 1 The construction of the SMEFT, to all orders, is not based on assumptions on the size of the Wilson coefficients of the higher dimensional operators
- 2 Restricting to a particular UV case is not an integral part of a general SMEFT treatment and various cases can be chosen once the general calculation is performed.
- 3 If the value of Wilson coefficients in broad UV scenarios could be inferred in general this would be of significant scientific value.



What can be said at all can be said clearly and whereof one cannot speak thereof one must be silent L. Wittgenstein



... constructing SMEFT

- Experiments occur at finite energy and measure $\mathbf{S}^{\text{eff}}(\Lambda)$
- Whatever QFT should give low energy $\mathbf{S}^{\text{eff}}(\Lambda)$, $\forall \Lambda < \infty$
- There is no fundamental scale above which $\mathbf{S}^{\text{eff}}(\Lambda)$ is not defined (K. Costello, Renormalization and EFT, AMS)
- $\mathbf{S}^{\text{eff}}(\Lambda)$ loses its predictive power if a process at $E = \Lambda$ requires ∞ renormalized parameters (J. Preskill, CALT-68-1493)

Based on [arXiv:1505.03706](https://arxiv.org/abs/1505.03706), [arXiv:1510.00414](https://arxiv.org/abs/1510.00414),



<https://cds.cern.ch/record/2138031> about to be rejected by HXSWG

Don't say it's



It is remarkable that when constructive proofs are provided, their simplicity always seems to detract from their originality

Don't expect an immediate translation into easy-to-use, pre-Moriond recipe

Not covered in this talk:

connection with PO-framework (<https://cds.cern.ch/record/2138023>),

connection with kappa-framework ([arXiv:1505.03706](https://arxiv.org/abs/1505.03706))

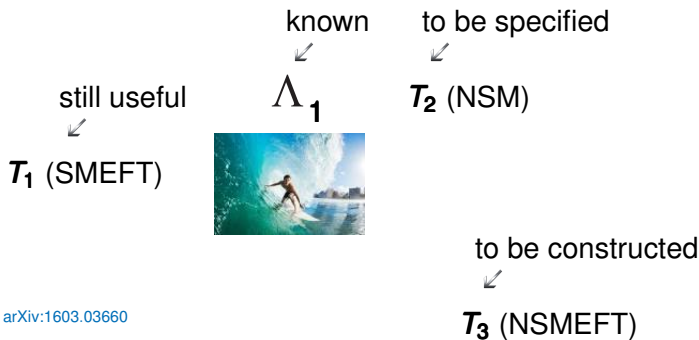
What about a $6M_H$ bump? First read

Two Comments on LEE with Two Data Sets

Bob Cousins
Univ. of California, Los Angeles

LL Statistics Workshop
13 February 2013

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$$M_{H^\pm}^2 = \Lambda^2 + \frac{1}{2} v^2 (\lambda_4 + \lambda_5) \quad M_{A^0}^2 = \Lambda^2 + v^2 \lambda_5 \quad M_H^2 = \Lambda^2 - \frac{1}{4} \left[v^2 (\lambda_1 - 2\bar{\lambda}) \right]$$

$$\sin \beta = 1 - \frac{1}{8} \frac{v^4}{\Lambda^4} + \mathcal{O} \left(\frac{v^6}{\Lambda^6} \right) \quad \cos \beta = \frac{1}{2} \frac{v^2}{\Lambda^2} + \mathcal{O} \left(\frac{v^4}{\Lambda^4} \right)$$

$$\sin(\alpha - \beta) = -1 + \mathcal{O} \left(\frac{v^6}{\Lambda^6} \right) \quad \cos(\alpha - \beta) = -\frac{1}{2} \left(M_h^2 + v^2 \bar{\lambda} \right) \frac{v^2}{\Lambda^4} + \mathcal{O} \left(\frac{v^6}{\Lambda^6} \right)$$



The UV connection



for off-peak

$$\mathcal{A} = \sum_{n=N}^{\infty} g^n \mathcal{A}_n^{(4)} + \sum_{n=N_6}^{\infty} \sum_{l=1}^n \sum_{k=1}^{\infty} g^n g^l \mathcal{A}_{4+2k}^{(4+2k)} \frac{1}{n/k}$$

where g is the $SU(2)$ coupling constant and $g_{4+2k} = 1/(\sqrt{2} G_F \Lambda^2)^k = g_6^k$, where G_F is the Fermi coupling constant and Λ is the scale around which new physics (NP) must be resolved. For each process N defines the dim = 4 LO (e.g. $N = 1$ for $H \rightarrow VV$ etc. but $N = 3$ for $H \rightarrow \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N - 2$ for loop initiated ones. Here we consider single insertions of dim = 6 operators, which defines NLO SMEFT.

Ex: HAA (tree) vertex generated by $\mathcal{O}_{\phi W}^{(6)} = (\Phi^\dagger \Phi) F^{a\mu\nu} F_{\mu\nu}^a$, by

$$\mathcal{O}_{\phi W}^{(8)} = \Phi^\dagger F^{a\mu\nu} F_{\mu\rho}^a D^\rho D_\nu \Phi \text{ etc.}$$

SMEFT ordertable for tree initiated 1 → 2 processes

$$\begin{array}{rcccl}
 g / \text{dim} & \longrightarrow & & & \\
 \downarrow & & g \mathcal{A}_1^{(4)} & + & g g_6 \mathcal{A}_{1,1,1}^{(6)} & + & g g_8 \mathcal{A}_{1,1,2}^{(8)} \\
 & & g^3 \mathcal{A}_3^{(4)} & + & g^3 g_6 \mathcal{A}_{3,1,1}^{(6)} & + & g^3 g_6^2 \mathcal{A}_{3,2,1}^{(6)} \\
 & & \dots & & \dots & & \dots
 \end{array}$$

- $g g_6 \mathcal{A}_{1,1,1}^{(6)}$ LO SMEFT. There is also RG-improved LO ([arXiv:1308.2627](https://arxiv.org/abs/1308.2627)) and MHOU for LO SMEFT ([arXiv:1508.05060](https://arxiv.org/abs/1508.05060))
- $g^3 g_6 \mathcal{A}_{3,1,1}^{(6)}$ ([arXiv:1505.03706](https://arxiv.org/abs/1505.03706)) NLO SMEFT
- $g g_8 \mathcal{A}_{1,1,2}^{(8)}$ ([arXiv:1510.00372](https://arxiv.org/abs/1510.00372)), $g^3 g_6^2 \mathcal{A}_{3,2,1}^{(6)}$ MHOU for NLO SMEFT

N.B. g_8 denotes a single $\mathcal{O}^{(8)}$ insertion, g_6^2 denotes two, distinct, $\mathcal{O}^{(6)}$ insertions

Compendium Records



CT_{4,6} + Mix

$$A = g^N A_{\text{LO}}^{(4)}(\{p\}) + g^N g_6 A_{\text{LO}}^{(6)}(\{p\}, \{a\}) + \frac{1}{16\pi^2} g^{N+2} A_{\text{NLO}}^{(4)}(\{p\}) + \frac{1}{16\pi^2} g^{N+2} g_6 A_{\text{NLO}}^{(6)}(\{p\}, \{a\})$$

CT₄

$\{p\} = \{g, \sin \theta_W, M, M_H, M_t\} \in \text{SM}$

$\{a\} = \text{Wilson coeff.} \in \text{Warsaw basis}$

$$\{p\}, \{a\} \longrightarrow \{p_{\text{ren}}\}, \{a_{\text{ren}}\} \longrightarrow \text{IPS}, \{a_{\text{ren}}(\mu_R)\}$$

$\underbrace{\hspace{10em}}_{G_F, M_W, M_Z, M_H}$



CT = counterterm

The role of $H \rightarrow \text{VEV}$



$$\mathcal{O} = \Lambda^{-n} M^l \partial^c \overbrace{\overbrace{\overbrace{\psi^a \psi^b}^{\text{codim}} (\Phi^\dagger)^d}^{\text{N}_F}}^{\text{dim}} \Phi^e A^f$$
$$\frac{3}{2}(a+b) + c + d + e + f + l + n = 4$$

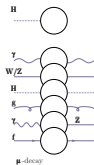
one loop renormalization is controlled by:

$$\boxed{\text{dim} = 6 \quad \text{codim} = 4 \quad \text{N}_F > 2 \quad (\text{Jargon: LO SMEFT})}$$

The hearth of the problem: a large number of operators implodes into a small number of coefficients

$$\boxed{92 \text{ SM vertices} \iff 28 \text{ CP even operators (1 flavor, } N_\psi = 0, 2)}$$

Self-energies



$$S_{HH} = \frac{g^2}{16\pi^2} \Sigma_{HH} = \frac{g^2}{16\pi^2} \left(\Sigma_{HH}^{(4)} + g_6 \Sigma_{HH}^{(6)} \right)$$

$$S_{AA}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{AA}^{\mu\nu} \quad \Sigma_{AA}^{\mu\nu} = \Pi_{AA} T^{\mu\nu}$$

$$S_{VV}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{VV}^{\mu\nu} \quad \Sigma_{VV}^{\mu\nu} = D_{VV} \delta^{\mu\nu} + P_{VV} p^\mu p^\nu$$

$$D_{VV} = D_{VV}^{(4)} + g_6 D_{VV}^{(6)} \quad P_{VV} = P_{VV}^{(4)} + g_6 P_{VV}^{(6)}$$

$$S_{ZA}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{ZA}^{\mu\nu} + g_6 T^{\mu\nu} a_{AZ} \quad \Sigma_{ZA}^{\mu\nu} = \Pi_{ZA} T^{\mu\nu} + P_{ZA} p^\mu p^\nu$$

$$S_f = \frac{g^2}{16\pi^2} \left[\Delta_f + (V_f - A_f \gamma^5) i\not{p} \right]$$

Counterterms



$$\Delta_{UV} = \frac{2}{4-n} - \gamma - \ln \pi - \ln \frac{\mu_R^2}{\mu^2}$$

n is space-time dimension
loop measure $\mu^{4-n} d^n q$

μ_R ren. scale

Warsaw basis

$$Z_i = 1 + \frac{g^2}{16\pi^2} \left(dZ_i^{(4)} + g_6 dZ_i^{(6)} \right) \Delta_{UV}$$

With field/parameter counterterms we can make

$S_{HH}, \Pi_{AA}, D_{VV}, \Pi_{ZA}, V_f, A_f$ and the corresponding Dyson resummed propagators UV finite at $\mathcal{O}(g^2 g_6)$ (Q.E.D.)

which is enough when working under the assumption that gauge bosons couple to conserved currents

A gauge-invariant description turns out to be mandatory



Field/parameter counterterms are not enough to make UV finite the Green's functions with more than two legs. A mixing matrix among Wilson coefficients is needed:

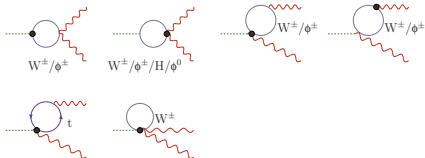
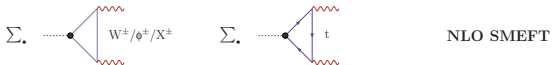
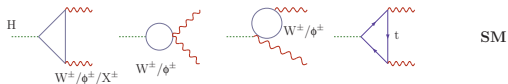
$$\mathbf{a}_i = \sum_j \mathbf{Z}_{ij}^{\mathbf{W}} \mathbf{a}_j^{\text{ren}} \quad \mathbf{Z}_{ij}^{\mathbf{W}} = \delta_{ij} + \frac{g^2}{16\pi^2} d\mathbf{Z}_{ij}^{\mathbf{W}} \Delta_{UV}$$



$$|g^N \mathcal{A}_N^{(4)} + g^K g_6 \mathcal{A}_{K,1,1}^{(6)}|^2 \rightsquigarrow |g^N \mathcal{A}_N^{(4)}|^2 + 2g^{N+K} g_6 \text{Re} \left[\mathcal{A}_N^{(4)} \right]^\dagger \mathcal{A}_{K,1,1}^{(6)}$$

Remark negative bin entries judge the validity of the dim = 6 “linear” approach ([arXiv:1511.05170](https://arxiv.org/abs/1511.05170))

Nihil novi: for a similar problem in EWPD see [The standard model in the making](#)



Diagrams contributing to the amplitude for $H \rightarrow \gamma\gamma$ in the R_ξ -gauge: SM (first row), LO SMEFT (second row), and NLO SMEFT. Black circles denote the insertion of one $\mathbf{dim} = 6$ operator. Σ_\bullet implies summing over all insertions in the diagram (vertex by vertex). For triangles with internal charge flow ($t, W^\pm, \phi^\pm, X^\pm$) only the clockwise orientation is shown. Non-equivalent diagrams obtained by the exchange of the two photon lines are not shown. Higgs and photon wave-function factors are not included. The Fadeev-Popov ghost fields are denoted by X .



1



Define the following combinations of Wilson coefficients (where $s_\theta(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle.

$$a_{ZZ} = s_\theta^2 a_{\phi_B} + c_\theta^2 a_{\phi_W} - s_\theta c_\theta a_{\phi_{WB}}$$

$$a_{AA} = c_\theta^2 a_{\phi_B} + s_\theta^2 a_{\phi_W} + s_\theta c_\theta a_{\phi_{WB}}$$

$$a_{AZ} = 2 c_\theta s_\theta (a_{\phi_W} - a_{\phi_B}) + (2 c_\theta^2 - 1) a_{\phi_{WB}}$$

and compute the (on-shell) decay $H(P) \rightarrow A_\mu(p_1) A_\nu(p_2)$ where the amplitude is

$$A_{HAA}^{\mu\nu} = \mathcal{T}_{HAA} T^{\mu\nu} \quad M_H^2 T^{\mu\nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}$$

Remark The amplitude is made UV finite by mixing a_{AA} with a_{AA}, a_{AZ}, a_{ZZ} and a_{QW} Q.E.D.



②



Compute the (on-shell) decay $H(P) \rightarrow A_\mu(p_1)Z_\nu(p_2)$. After adding 1PI and 1PR components we obtain

$$A_{\text{HAZ}}^{\mu\nu} = \mathcal{T}_{\text{HAZ}} T^{\mu\nu} \quad M_{\text{H}}^2 T^{\mu\nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}$$

Remark The amplitude is made UV finite by mixing \mathbf{a}_{AZ} with $\mathbf{a}_{\text{AA}}, \mathbf{a}_{\text{AZ}}, \mathbf{a}_{\text{ZZ}}$ and \mathbf{a}_{QW} Q.E.D.



③



Compute the (on-shell) decay $\mathbf{H}(P) \rightarrow Z_\mu(p_1)Z_\nu(p_2)$. The amplitude contains

- a \mathcal{D}_{HZZ} part proportional to $\delta^{\mu\nu}$ and
- a \mathcal{P}_{HZZ} part proportional to $p_2^\mu p_1^\nu$.

Remark Mixing of \mathbf{a}_{ZZ} with other Wilson coefficients makes \mathcal{P}_{HZZ} UV finite, while the mixing of $\mathbf{a}_{\phi\Box}$ makes \mathcal{D}_{HZZ} UV finite Q.E.D.



4



Compute the (on-shell) decay $\mathbf{H}(P) \rightarrow W^-_{\mu}(p_1)W^+_{\nu}(p_2)$. This process follows the same decomposition of $\mathbf{H} \rightarrow \mathbf{ZZ}$ and it is UV finite in the $\mathbf{dim} = 4$ part. However, for the $\mathbf{dim} = 6$ one, there are no Wilson coefficients left free in $\mathcal{P}_{\mathbf{HWW}}$ so that its UV finiteness follows from gauge cancellations
($\mathbf{H} \rightarrow \mathbf{AA}, \mathbf{AZ}, \mathbf{ZZ}, \mathbf{WW} = 6$ Lorentz structures controlled by 5 coefficients)

Proposition

This is the first part in proving closure of NLO SMEFT under renormalization Q.E.D.

Remark Mixing of $\mathbf{a}_{\phi D}$ makes $\mathcal{P}_{\mathbf{HWW}}$ UV finite Q.E.D.



5



Compute the (on-shell) decay $H(P) \rightarrow b(p_1)\bar{b}(p_2)$.

Remark

- It is **dim** = 4 UV finite and
- mixing of $a_{d\phi}$ makes it UV finite also at **dim** = 6 Q.E.D.



6



Compute the (on-shell) decay $Z(P) \rightarrow f(p_1)\bar{f}(p_2)$. It is $\mathbf{dim} = 4$
 UV finite and we introduce

$$\begin{aligned} a_{lW} &= S_\theta a_{lWB} + C_\theta a_{lBW} & a_{lB} &= S_\theta a_{lBW} - C_\theta a_{lWB} \\ a_{dW} &= S_\theta a_{dWB} + C_\theta a_{dBW} & a_{dB} &= S_\theta a_{dBW} - C_\theta a_{dWB} \\ a_{uW} &= S_\theta a_{uWB} + C_\theta a_{uBW} & a_{uB} &= C_\theta a_{uWB} - S_\theta a_{uBW} \end{aligned}$$

$$\begin{aligned} a_{\phi l}^{(3)} - a_{\phi l}^{(1)} &= \frac{1}{2} (a_{\phi lV} + a_{\phi lA}) & a_{\phi l} &= \frac{1}{2} (a_{\phi lA} - a_{\phi lV}) \\ a_{\phi uV} &= a_{\phi q}^{(3)} + a_{\phi u} + a_{\phi q}^{(1)} & a_{\phi uA} &= a_{\phi q}^{(3)} - a_{\phi u} + a_{\phi q}^{(1)} \\ a_{\phi dV} &= a_{\phi q}^{(3)} - a_{\phi d} - a_{\phi q}^{(1)} & a_{\phi dA} &= a_{\phi q}^{(3)} + a_{\phi d} - a_{\phi q}^{(1)} \end{aligned}$$

and obtain that (Q.E.D.)

$Z \rightarrow \bar{l}l$ requires **mixing** of a_{lBW} , $a_{\phi lA}$ and $a_{\phi lV}$ with other coefficients,
 $Z \rightarrow \bar{u}u$ requires **mixing** of a_{uBW} , $a_{\phi uA}$ and $a_{\phi uV}$ with other coefficients,
 $Z \rightarrow \bar{d}d$ requires **mixing** of a_{dBW} , $a_{\phi dA}$ and $a_{\phi dV}$ with other coefficients,
 $Z \rightarrow \bar{v}v$ requires **mixing** of $a_{\phi v} = 2(a_{\phi l}^{(1)} + a_{\phi l}^{(3)})$ with other coefficients.



7



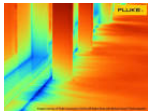
At this point we are left with the universality of the electric charge. In QED there is a Ward identity telling us that e is renormalized in terms of vacuum polarization and Ward-Slavnov-Taylor identities allow us to generalize the argument to the full SM.

We can give a quantitative meaning to the the previous statement by saying that the contribution from vertices (at zero momentum transfer) exactly cancel those from (fermion) wave function renormalization factors. Therefore,

Compute the vertex $A\bar{f}f$ (at $q^2 = 0$) and the f wave function factor in SMEFT, proving that the WST identity can be extended to $\text{dim} = 6$; this is non trivial since there are no free Wilson coefficients in these terms (after the previous steps); (non-trivial) finiteness of $e^+e^- \rightarrow \bar{f}f$ follows.

Proposition

This is the second part in proving closure of NLO SMEFT under renormalization Q.E.D.



The IR connection (e.g. $\mathbf{Z} \rightarrow \bar{\mathbb{I}}\mathbb{I}$)



$$= \rho_Z^f \gamma^\mu \left[\left(I_f^{(3)} + i a_L \right) \gamma_+ - 2 Q_f \kappa_Z^f \sin^2 \theta + i a_Q \right]$$

$$\mathcal{A}_\mu^{\text{tree}} = g \mathcal{A}_{1\mu}^{(4)} + g g_6 \mathcal{A}_{1\mu}^{(6)}$$

$$\mathcal{A}_{1\mu}^{(4)} = \frac{1}{4 c_\theta} \gamma_\mu \left(v_L + \gamma^5 \right) \quad \mathcal{A}_{1\mu}^{(6)} = \frac{1}{4} \gamma_\mu \left(V_1 + A_1 \gamma^5 \right)$$

$$V_1 = \frac{s_\theta^2}{c_\theta} \left(4 s_\theta^2 - 7 \right) a_{AA} + c_\theta \left(1 + 4 s_\theta^2 \right) a_{ZZ} + s_\theta \left(4 s_\theta^2 - 3 \right) a_{AZ}$$

$$+ \frac{1}{4 c_\theta} \left(7 - s_\theta^2 \right) a_{\phi D} + \frac{2}{c_\theta} a_{\phi 1V}$$

$$A_1 = \frac{s_\theta^2}{c_\theta} a_{AA} + c_\theta a_{ZZ} + s_\theta a_{AZ} - \frac{1}{4 c_\theta} a_{\phi D} + \frac{2}{c_\theta} a_{\phi LA}$$

After UV renormalization, i.e. after counterterms and mixing have been introduced, we perform analytic continuation in n (space-time dimension), $n = 4 + \varepsilon$ with ε positive.

Proposition

The infrared/collinear part of the one-loop virtual corrections shows double factorization.

$$\Gamma(Z \rightarrow \bar{1} + 1) |_{\text{div}} = -\frac{g^4}{384\pi^3} M_Z s_\theta^2 \mathcal{F}^{\text{virt}} \left[\Gamma_0^{(4)} (1 + g_6 \Delta\Gamma) + g_6 \Gamma_0^{(6)} \right]$$

Proposition

The infrared/collinear part of the real corrections shows double factorization.

$$\Gamma^{\text{app}}(Z \rightarrow \bar{1} + 1 + (\gamma)) = \frac{g^4}{384\pi^3} M_Z s_\theta^2 \mathcal{F}^{\text{real}} \left[\Gamma_0^{(4)} (1 + g_6 \Delta\Gamma) + g_6 \Gamma_0^{(6)} \right]$$

Proposition

The total = virtual + real is IR/collinear finite at $\mathcal{O}(g^4 g_6)$ (Q.E.D.).



Assembling everything gives

$$\Gamma_{\text{QED}}^1 = \frac{3}{4} \Gamma_0^1 \frac{\alpha}{\pi} \left(1 + g_6 \Delta_{\text{QED}}^{(6)} \right) \quad \Gamma_0^1 = \frac{G_F M_Z^3}{24 \sqrt{2} \pi} \left(v_1^2 + 1 \right)$$

$$\Delta_{\text{QED}}^{(6)} = 2 \left(2 - s_\theta^2 \right) a_{\text{AA}} + 2 s_\theta^2 a_{\text{ZZ}} + 2 \left(\frac{c_\theta^3}{s_\theta} + \frac{512}{26} \frac{v_L}{v_L^2 + 1} \right) a_{\text{AZ}}$$

$$- \frac{1}{2} \frac{c_\theta^2}{s_\theta^2} a_{\phi\text{D}} + \frac{1}{v_L^2 + 1} \delta_{\text{QED}}^{(6)}$$

$$\delta_{\text{QED}}^{(6)} = \left(1 - 6 v_1 - v_1^2 \right) \frac{1}{c_\theta^2} \left(s_\theta a_{\text{AA}} - \frac{1}{4} a_{\phi\text{D}} \right)$$

$$+ \left(1 + 2 v_1 - v_1^2 \right) \left(a_{\text{ZZ}} + \frac{s_\theta}{c_\theta} a_{\text{AZ}} \right)$$

$$+ \frac{2}{c_\theta^2} \left(a_{\phi\text{1A}} + v_1 a_{\phi\text{1V}} \right)$$



W-decay: solvable problems expected



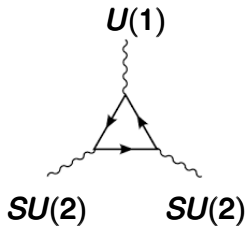
Triple/quadrupole gauge couplings,
last stop before renormalizability?

Gauge anomalies, anomaly cancellation; d'Hoker-Farhi
(Wess-Zumino) terms? Extra symmetry? Etc: severe
problems expected



Perhaps, a deeper understanding of SMEFT, a low-energy limit
of an underlying anomaly-free theory?

UPDATE



$$= \text{Diagram 1} + M_z \text{Diagram 2} = A J_\alpha J_\beta \epsilon^{\mu\nu\alpha\beta} p_{1\mu} p_{2\nu}$$

Proposition

X SMEFT anomalies are UV finite^a and local^b

It's another tiny step forward

^aIt's good for renormalizability, restoring gauge invariance order-by-order by adding finite counterterms, i.e. it is possible to quantize an anomalous theory in a manner that respects WST1 (Preskill)

^bIt's good for unitarity



implications for phenomenology

based on <https://cds.cern.ch/record/2138031>

- ✓ NLO results have already had an important impact on the SMEFT physics program. LEP constraints should not be interpreted to mean that effective SMEFT parameters should be set to zero in LHC analyses
- ✓ It is important to preserve the original data, not just the interpretation results, as the estimate of the missing higher order terms can change over time, modifying the lessons drawn from the data and projected into the SMEFT
- ✓ Considering projections for the precision to be reached in LHC RunII analyses, LO results for interpretations of the data in the SMEFT are challenged by consistency concerns and are not sufficient, if the cut off scale is in the few TeV range
- ✓ The assignment of a theoretical error for SMEFT analyses is always important



Ideas that require people to reorganize their picture of the world provoke hostility

To conclude, the journey to the next (and next-to-next) SM may require crossing narrow straits of precision physics. If that is what nature has in store for us, we must equip ourselves with both a range of concrete models as well as a general theories. Both will be indispensable tools in navigating an ocean of future experimental results.

Each paradigm will be shown to satisfy more or less the criteria that it dictates for itself and to fall short of a few of those dictated by its opponent

T. S. Kuhn

Backup Slides



- ☛ The naive version: for a theory or hypothesis to count as *scientific* it ought to be falsifiable in principle
 - ✓ SM is in. The reason is that SM has withstood *risky* tests that it could have easily failed
- ☛ The *non-empirical confirmation*, where the value of a theory is judged in conjunction with empirical confirmation elsewhere in the same field, assuming that a long term perspective of empirical confirmation exists for the given theory⁷

⁷A mature science, according to Kuhn, experiences alternating phases of normal science and revolutions. In normal science the key theories, instruments and values that comprise the disciplinary matrix are kept fixed, permitting the cumulative generation of puzzle-solutions, whereas in a scientific revolution the disciplinary matrix undergoes revision, in order to permit the solution of the more serious anomalous puzzles that disturbed the preceding period of normal science

$$\mathcal{A}^{\text{tree}, 1L} = \bar{u}_1 \mathcal{A}_\mu^{\text{tree}, 1L} v_2 e^\mu(\lambda, P)$$



$$\Gamma(Z \rightarrow \bar{l} + l) |_{\text{div}} = \frac{2}{3} \frac{1}{(2\pi)^2} \sum_{\text{spin}} \int d\Phi_{1 \rightarrow 2} \text{Re} \left[\mathcal{A}^{\text{tree}} \right]^\dagger \mathcal{A}^{1L} |_{\text{div}}$$

(ϵ, m_f) -scheme for (IR, collinear) singularities

$$\frac{1}{\hat{\epsilon}} = \frac{2}{\epsilon} + \bar{\gamma} - \ln \frac{M_W^2}{\mu^2} \quad L_{\text{cw}} = \ln \frac{m_f^2}{M_W^2} \quad L_{\text{cz}} = \ln \frac{m_f^2}{M_Z^2}$$

$$\bar{\gamma} = \gamma + \ln \pi \quad L = \ln \frac{M_Z^2}{M_W^2}$$

IR /collinear divergent factor

$$\begin{aligned}\mathcal{F}^{\text{virt}} &= -2 \left(\frac{1}{\hat{\epsilon}} + \bar{\gamma} \right) (1 + L_{\text{CZ}}) - L_{\text{CZ}}^2 - 4L_{\text{CZ}}L + 3L_{\text{CZ}} - 4L \\ &- 2 \ln \frac{M_{\text{W}}^2}{\mu^2} (1 + L_{\text{CZ}}) + 2 - 8\zeta(2)\end{aligned}$$

Sub-amplitudes

$$\Gamma_0^{(4)} = \frac{1}{2} (1 - 4s_\theta^2 + 8s_\theta^4) \frac{1}{c_\theta^2} = \frac{1}{4} (1 + v_1^2) \frac{1}{c_\theta^2}$$

$$\Gamma_{0A}^{(4)} = 2 (1 - 4s_\theta^2) \frac{s_\theta}{c_\theta} = 2v_1 \frac{s_\theta}{c_\theta}$$

$$\begin{aligned}\Gamma_0^{(6)} &= - (3 - 16s_\theta^2 + 8s_\theta^4) \frac{s_\theta^2}{c_\theta^2} a_{\text{AA}} + (1 - 8s_\theta^4) a_{\text{ZZ}} - (1 - 8s_\theta^2 + 8s_\theta^4) \frac{s_\theta}{c_\theta} a_{\text{AZ}} \\ &+ \frac{1}{4} (3 - 16s_\theta^2 + 8s_\theta^4) \frac{1}{c_\theta^2} a_{\phi\text{D}} + \frac{1}{c_\theta^2} a_{\phi\text{1A}} + (1 - 4s_\theta^2) \frac{1}{c_\theta^2} a_{\phi\text{1v}}\end{aligned}$$

Proposition

The infrared/collinear part of the one-loop virtual corrections shows double factorization.

$$\Gamma(Z \rightarrow \bar{1} + 1) |_{\text{div}} = -\frac{g^4}{384\pi^3} M_Z s_\theta^2 \mathcal{F}^{\text{virt}} \left[\Gamma_0^{(4)} (1 + g_6 \Delta\Gamma) + g_6 \Gamma_0^{(6)} \right]$$

$$\Delta\Gamma = 2 \left(2 - s_\theta^2 \right) a_{AA} + 2 s_\theta^2 a_{ZZ} + 2 \frac{c_\theta^3}{s_\theta} a_{AZ} - \frac{1}{2} \frac{1}{s_\theta^2 c_\theta^2} a_{\phi D}$$

Next we compute $Z(P) \rightarrow I(p_1) + \bar{I}(p_2) + \gamma(k)$, obtaining

$$\Gamma(Z \rightarrow \bar{I} + I + \gamma) = \frac{1}{3} \frac{1}{(2\pi)^5} \sum_{\text{spin}} \int d\Phi_{1 \rightarrow 3} |\mathcal{A}^{\text{real}}|^2$$

$$\mathcal{A}^{\text{real}} = \bar{u}_1 \mathcal{A}_{\mu\nu}^{\text{real}} v_2 e^\mu(\lambda, P) e^\nu(\sigma, k)$$

We split the total into

- “approximated”, $n \neq 4$, approximated phase-space, reproducing the exact structure of singularities
- “remainder”, $n = 4$, finite

After expanding in $\varepsilon = n - 4$ we obtain an overall infrared/collinear (real) factor

$$\begin{aligned} \mathcal{F}^{\text{real}} &= -2 \left(\frac{1}{\varepsilon} + \bar{\gamma} \right) (1 + L_{cZ}) - L_{cZ}^2 - 2L_{cZ}L + 3L_{cZ} - 2L \\ &- 2 \ln \frac{M_Z^2}{\mu^2} (1 + L_{cZ}) + 1 - 4 \zeta(2) \end{aligned}$$

and a partial width integrated over the whole photon phase space

$$\Gamma^{\text{app}}(Z \rightarrow \bar{l}l + (\gamma)) = \frac{g^4}{384 \pi^3} M_Z s_\theta^2 \mathcal{F}^{\text{real}} \left[\Gamma_0^{(4)} (1 + g_6 \Delta\Gamma) + g_6 \Gamma_0^{(6)} \right]$$

Proposition

The infrared/collinear part of the real corrections shows double factorization. The total = virtual + real is IR/collinear finite at $\mathcal{O}(g^4 g_6)$ (Q.E.D.).



NLO SMEFT for Higgs and EW precision data





No NP yet?

A study of SM-deviations: here the reference process is $gg \rightarrow H$
✓ κ -approach: write the amplitude as

$$A^{gg} = \sum_{q=t,b} \kappa_q^{gg} \mathcal{A}_q^{gg} + \kappa_C^{gg}$$

\mathcal{A}_t^{gg} being the SM t -loop etc. The **contact term** (which is the LO SMEFT) is given by κ_C^{gg} . Furthermore

$$\kappa_q^{gg} = 1 + \Delta \kappa_q^{gg}$$

Compute

$$\mathbf{R} = \sigma \left(\kappa_{\mathbf{q}}^{\text{gg}}, \kappa_{\mathbf{c}}^{\text{gg}} \right) / \sigma_{\text{SM}} - 1 \quad [\%]$$

- 1 In LO SMEFT $\kappa_{\mathbf{c}}$ is non-zero and $\kappa_{\mathbf{q}} = 1$.⁸ You measure a deviation and you get a value for $\kappa_{\mathbf{c}}$
- 2 However, at NLO $\Delta\kappa_{\mathbf{q}}$ is non zero and you get a degeneracy
- 3 The interpretation in terms of $\kappa_{\mathbf{c}}^{\text{LO}}$ or in terms of $\{\kappa_{\mathbf{c}}^{\text{NLO}}, \Delta\kappa_{\mathbf{q}}^{\text{NLO}}\}$ could be rather different.

⁸Certainly true in the linear realization

Going interpretational

$$\begin{aligned} A_{\text{SMEFT}}^{\text{gg}} &= \frac{g g_S^2}{\pi^2} \sum_{q=t,b} \kappa_q^{\text{gg}} \mathcal{A}_q^{\text{gg}} \\ &+ 2 g_S g_6 \frac{s}{M_W^2} a_{\phi g} + \frac{g g_S^2 g_6}{\pi^2} \sum_{q=t,b} \mathcal{A}_q^{\text{NF;gg}} a_{qg} \end{aligned}$$

Remark use [arXiv:1505.03706](https://arxiv.org/abs/1505.03706), adopt Warsaw basis ([arXiv:1008.4884](https://arxiv.org/abs/1008.4884)), eventually work in the Einhorn-Wudka PTG scenario ([arXiv:1307.0478](https://arxiv.org/abs/1307.0478))

- ① LO SMEFT: $\kappa_q = 1$ and $a_{\phi g}$ is scaled by $1/16 \pi^2$ being LG (blue color)
- ② NLO PTG-SMEFT: $\kappa_q \neq 1$ but only PTG operators inserted in loops (non-factorizable terms absent), $a_{\phi g}$ scaled as above
- ③ NLO full-SMEFT: $\kappa_q \neq 1$ LG/PTG operators inserted in loops (non-factorizable terms present), LG coefficients scaled as above

At NLO, $\Delta\kappa = g_6 \rho$

$$\begin{aligned}
 g_6^{-1} &= \sqrt{2} G_F \Lambda^2 \\
 4\pi\alpha_s &= g_S^2 \\
 \rho_t^{gg} &= a_{\phi W} + a_{t\phi} + 2a_{\phi\Box} - \frac{1}{2}a_{\phi D} \\
 \rho_b^{gg} &= a_{\phi W} - a_{b\phi} + 2a_{\phi\Box} - \frac{1}{2}a_{\phi D}
 \end{aligned}$$

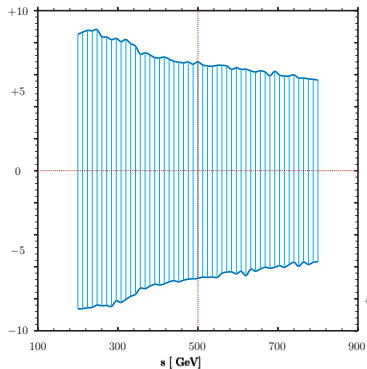
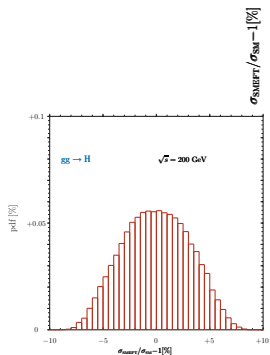
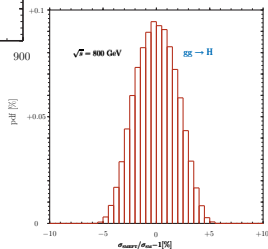
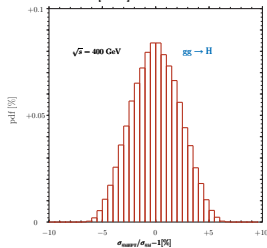


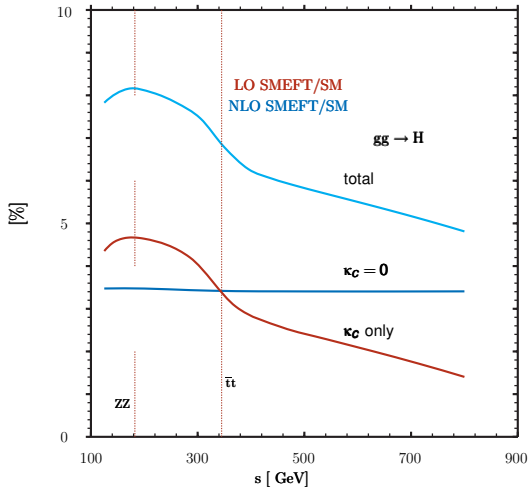
Relaxing the PTG assumption introduces

non-factorizable sub-amplitudes proportional to $\mathbf{a}_{tg}, \mathbf{a}_{bg}$ with a mixing among $\{\mathbf{a}_{\phi g}, \mathbf{a}_{tg}, \mathbf{a}_{bg}\}$. Meanwhile, renormalization has made one-loop SMEFT finite, e.g. in the G_F -scheme, with a residual μ_R -dependence.

What are POs? Experimenters collapse some “primordial quantities” (say number of observed events in some pre-defined set-up) into some “secondary quantities” which we feel closer to the theoretical description of the phenomena.

Residues of resonant poles, κ -parameters and Wilson coefficients are different layers of POs

$gg \rightarrow H$ off-shell
 $\text{unif}(-1, 1)$
 $\Lambda = 3$ TeV




Another reason to go NLO

The contact term is real ... $\kappa_C^{gg} \in \mathbb{R}$

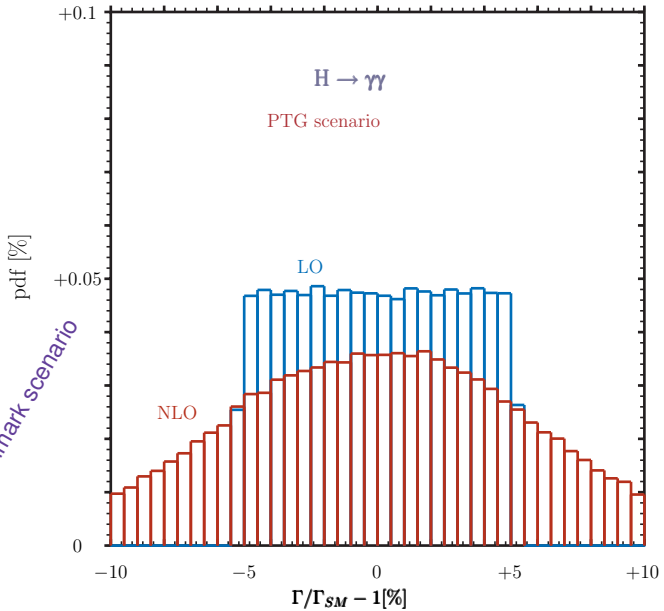
$$\frac{gg_S^2 g_6}{\pi^2} \sum_{q=t,b} \left[\Delta \kappa_q^{gg} \omega_q^{gg} + \omega_q^{NF:gg} a_{qg} \right] \in \mathbb{C}$$

$$2g_S g_6 \frac{s}{M_W^2} a_{\phi g} \in \mathbb{R}$$

$$a_i = 1, \forall i$$

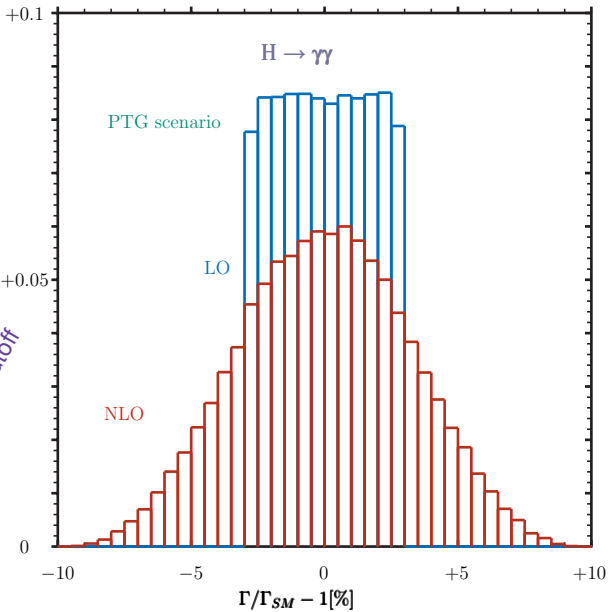
$$\Lambda = 3 \text{ TeV}$$

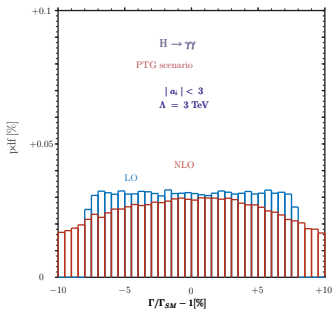
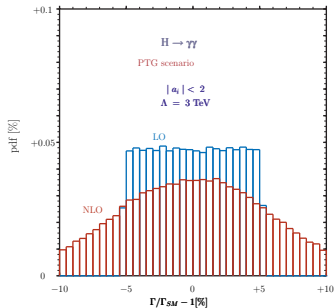
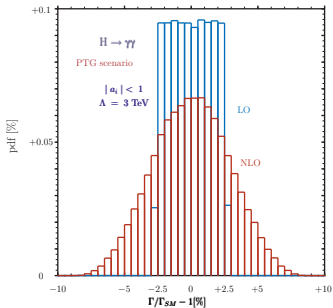
Benchmark scenario



$\Lambda = 4 \text{ TeV}$

Changing the cutoff





Changing the interval

Appendix C. Dimension-Six Basis Operators for the SM²².

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2 \varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^A G_\nu^{B\rho} G_\rho^C \mu$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^A G_\nu^{B\rho} G_\rho^C \mu$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$ (LG)		$\psi^2 X \varphi$ (LG)		$\psi^2 \varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

²²These tables are taken from [5], by permission of the authors.