TWO-LOOP QFT in the Making

Stefano ACTIS[‡], Giampiero PASSARINO[†],
Sandro UCCIRATI[†]

‡Deutsches Electronen - Synchrotron, Zeuthen, Germany
†Dipartimento di Fisica Teorica, Università di Torino, Italy
INFN, Sezione di Torino, Italy

Outline of Part I

1. **The Project**
   - Numerical evaluation
   - Status:
   - Back to renormalization
   - Dressed propagators
   - Loops with dressed propagators
   - New problems with complex poles
   - Change of strategy

Outline of Part II

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2. **The running of α**
   - Is there an $\alpha(s)$?
   - Ingredients for $\alpha_{\overline{MS}}$
   - Results for $\alpha_{\overline{MS}}$
   - $\alpha(s), \qquad \xi = 1$
3. **Infrared at two - loops**
   - Examples
   - IR numerica
4. **Complex poles: *numerica***
   - Input: on-shell masses
   - Input: complex poles
   - Complex poles handling: more details
5. **R - equations: *numerica***
   - numbers & renormalization I
   - numbers & renormalization II
6. **Conclusions**

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Part I

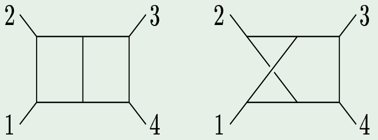
Goals and perspectives

the project

Problem

HO perturbative QFT is a rather challenging field requiring:
clever ideas and new algorithms

Example



Solutions

- Develop portable graph generators

- import new ideas from functional analysis into EW physics to confront the *practical difficulties* there,

- especially as concerns *massive Feynman diagrams*.

The road map for an NNLO process

(1, 2, 3)

1. *A variety of important processes will benefit from NLO(NNLO) computations*
2. *two-loop accuracy in conjuction with resummation*
3. *Ideally, one would like a fast and reliable (general) NNLO program*

Complexity: $n\,!$ growth

Different diagrams interfere

(a, b, c)

1. *tree level (obvious)*
2. *$1\,L$ with finite $1\,L$ renormalization*
3. *$2\,L$, but beware:*

Example

$2\,L$ renormalization is (much) more than 

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Deliverable now

Generation

Diagrams are generated & manipulated by Graphshot (Form 3.1)

Evaluation

Observables are computed by LoopBack

No *external* Black Box

Solutions

- A Fortran 95 code has been written (LoopBack)

- with huge gains in CPU time

- array handling, assignment overloading, vector/recursive functions, etc.

two-loop R

(Awramik .... Weiglein)

Counter-terms

Not needed, but useful for dealing with overlapping divergencies

Skeleton expansion

The relevant objects beyond 1 L are *dressed propagators*

Dogma

- $\{p_R\}$ are REAL

- finite R $\in$ consistent solution of R -equations

- complex poles $\longleftrightarrow$ dressing, but cutting equations must be verified

FCC

Cutting equations & D propagators

Problem

Use *dressed* propagators,

Example

$$\bar{\Delta}_V = \frac{\Delta_V}{1 - i\,\Delta_V\,\Sigma_{VV}},$$

Theorem

cutting-equations and unitarity of the S-matrix can be proven

Solutions

- $2L$ $\bar{\Delta}$ in tree diagrams,
- $1L$ $\bar{\Delta}$ in $1L$ diagrams,
- tree in $2L$ diagrams.

- $\bar{\Delta}$ satisfy the Källen - Lehmann representation.
- only skeleton diagrams are included

Proof: Veltman

Proof.

$$\bar{\Delta}^{+}_{V}(p^2) \;=\; \theta(p_0)\left[\bar{\Delta}_V(p^2)\right]^2 2\,i\,\mathrm{Re}\,\Sigma_{VV}(p^2),$$

while, for a stable particle, the pole term shows up as

$$\bar{\Delta}^{+}_{V}(p^2) \;=\; \theta(p_0)\left[\bar{\Delta}_V(p^2)\right]^2 2\,i\,\mathrm{Re}\,\Sigma_{VV}(p^2) + 2\,i\,\pi\,\delta(p^2 + m_V^2).$$

$\mathrm{Re}\,\Sigma_{VV} \;\to\;$ cut self-energy / repeat ad libidum

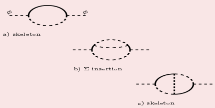
$\hookrightarrow$ contributions from cut lines $\in$ stable particles only □

The consistent way with unstable particles

Problem

Toy model with Φ unstable:

$$L = \frac{g}{2}\,\Phi(x)\,\phi^2(x).$$



skpf1

Example

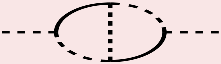
$$\overline{\Delta}_\Phi = \frac{\Delta_\Phi}{1 - \Delta_\Phi\,\Sigma_{\Phi\Phi}},$$
$$\overline{\Delta}_\phi = \frac{\Delta_\phi}{1 - \Delta_\phi\,\Sigma_{\phi\phi}},$$

Solutions

- $\mathrm{Im}\,\Sigma_{\phi\phi} \neq 0 \longleftarrow$ 3 p cut b)
- With D propagators only a) and c) are retained, but a) $\longleftarrow \overline{\Delta}_\Phi$, $\mathcal{O}(1\,L)$

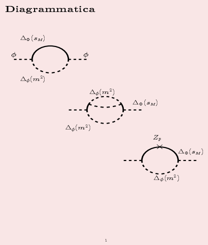
a) skeleton

b) Σ insertion



c) skeleton

The consistent way with unstable particles



skpf2

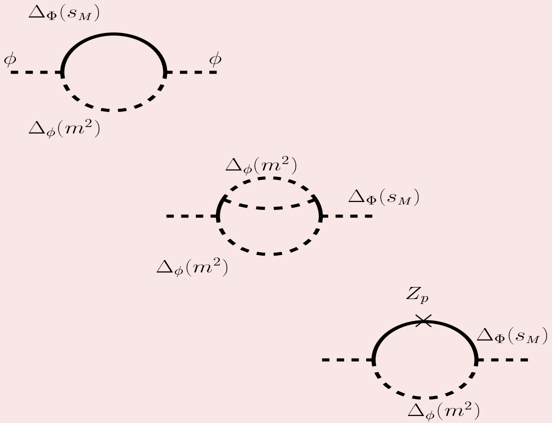
Theorem

$1\,L$ FD with $1\,L$ $\overline{\Delta}_\Phi$
$\mathcal{O}\left(g^4\right) \equiv$
3 FD whith $\Delta_\Phi(s_M)$

with

$$Z_p = \frac{g^2}{16\,\pi^2} B_0\left(-s_M\,;\,m\,,\,m\right).$$

Diagrammatica



Complex poles

(see also Denner Dittmaier @ 1 L)

Problem

R - equations need $M_{\rm exp}$?
OS PO are derived by fitting
lineshapes $\longleftrightarrow$ experiments

Solutions

define pole PO

$$
\begin{aligned}
M_P &= M_{OS}\,\cos\psi, \\
\Gamma_P &= \Gamma_{OS}\,\sin\psi, \\
\psi &= \arctan\frac{\Gamma_{OS}}{M_{OS}},
\end{aligned}
$$

@ 1 L we can use M_{OS}. Beyond 1 L GI $\hookrightarrow$

$$
s_V = \mu_V^2 - i\,\gamma_V\,\mu_V
$$

complex poles beyond $1L$

(Jegerlehner .... Veretin)

Problem

@ $2L$ R - equations change their structure.

Example

change of perspective: @ $1L$ one considers M_{OS} as IP independent of s_P and *derive* s_P. @ $2L$ R - equations are written for real p_R and solved in terms of (among other things) experimental s_P

Solutions

consistently with an order-by-order R, $M_R \hookrightarrow$ real solutions of truncated R - equations,

Theorem

there is no problem with cutting-equations and unitarity.

Part II

Numbers, nothing more than numbers . . .

What's the running of α?

Problem

the role played by the running of α has been crucial in the development of precision tests of the SM.

popular wisdom

universal corrections are the important ingredient, non-universal ones should be made as small as possible

once again, problems

- UC should be linked to a set of PO's and data should be presented in the language of PO's
- this language $\longleftrightarrow$ resummation, against GI
- $\approx M_z$ it is easy to perform a discrimination relevant vs. irrelevant terms, paying a very little price to GI

is it useful?

Why not?

One (fuzzy) idea is to import from QCD the concept of $\overline{MS}$ couplings

Example

express th. predictions through $\overline{MS}$ couplings. Open for criticism.

- The $\overline{MS}$ parameter seems unambiguos,
- however, it will violate decoupling

another solution

- do the calculation in $R_{|xi}$,
- select a ξ - independent part of S,
- perform resummation while leaving the rest to ensure independence when combined with V & B
- the obvious criticism: it violates uniqueness; then what?

what we need for $\alpha_{\overline{MS}}$

fermion $\supset$ 3 lepton generations, a perturbative quark contribution, top or diagrams where light quarks are coupled internally to vector bosons

non-perturbative $\supset$ diagrams where a light quark couple to a photon, is related to $\Delta\alpha^{5}_{\rm had}(M_z^2)$

QED and QCD contributions to the light-quark part is always subtracted

main equation

$$\Pi_{QQ}(0) \quad = \quad \Pi^{\rm bos}_{QQ}(0) + \Pi^{\rm lep}_{QQ}(0) + \Pi^{\rm per}_{QQ}(0) + \Pi^{\rm had}_{QQ}(0).$$

Numerical results

(see also Degrassi et al)

Definition

$$\alpha^{-1}_{MSB}(s) \;=\; \alpha^{-1} - \frac{1}{4\,\pi}\,\Pi^{MSB}_{QQ}(0)\Big|_{\mu^2=s}$$

$m_t = 174.3\,\mathrm{GeV}$	$M_H = 150\,\mathrm{GeV}$				
$\sqrt{s}$ [GeV]	M_Z	120	160	200	500
one-loop	128.105	127.974	127.839	127.734	127.305
two-loop	128.042	127.967	127.891	127.831	127.586
%					0.22
$m_t = 179.3\,\mathrm{GeV}$	$M_H = 150\,\mathrm{GeV}$				
one-loop	128.113	127.982	127.847	127.742	127.313
two-loop	128.048	127.980	127.911	127.857	127.636
%					0.25
$m_t = 174.3\,\mathrm{GeV}$	$M_H = 300\,\mathrm{GeV}$				
$\sqrt{s}$ [GeV]	M_Z	120	160	200	500
one-loop	128.105	127.974	127.839	127.734	127.305
two-loop	128.041	127.914	127.784	127.683	127.266
%					0.03

Fine points: LQ basis more complex

Definition

$$\alpha^{-1}(s) \;=\; \alpha^{-1} - \frac{1}{4\,\pi}\,\Pi^{\rm ren}_{QQ\,;\,{\rm ext}}(s)$$

$$
\begin{aligned}
D_{AA} &= s^2\,\Pi_{QQ\,;\,{\rm ext}}\,p^2 = s^2 \sum_{n=1}^{\infty}\left(\frac{g^2}{16\,\pi^2}\right)^n \Pi^{(n)}_{QQ\,;\,{\rm ext}}\,p^2,\\
D_{AZ} &= \frac{s}{c}\,\Sigma_{AZ\,;\,{\rm ext}} = \frac{s}{c}\sum_{n=1}^{\infty}\left(\frac{g^2}{16\,\pi^2}\right)^n \Sigma^{(n)}_{AZ\,;\,{\rm ext}},\\
D_{ZZ} &= \frac{1}{c^2}\,\Sigma_{ZZ\,;\,{\rm ext}} = \frac{1}{c^2}\sum_{n=1}^{\infty}\left(\frac{g^2}{16\,\pi^2}\right)^n \Sigma^{(n)}_{ZZ\,;\,{\rm ext}},\\
\Sigma^{(n)}_{AZ\,;\,{\rm ext}} &= \Sigma^{(n)}_{3Q\,;\,{\rm ext}} - s^2\,\Pi^{(n)}_{QQ\,;\,{\rm ext}}\,p^2,\\
\Sigma^{(n)}_{ZZ\,;\,{\rm ext}} &= \Sigma^{(n)}_{33\,;\,{\rm ext}} - 2\,s^2\,\Sigma^{(n)}_{3Q\,;\,{\rm ext}} + s^4\,\Pi^{(n)}_{QQ\,;\,{\rm ext}}\,p^2.
\end{aligned}
$$

Anatomy at 200 GeV

Definition

$$\frac{\alpha}{\alpha(s)} = 1 + \Delta\alpha(s)$$

$\Delta\alpha$	value at $\sqrt{(s)} = 200\,\mathrm{GeV}$
Re EW	$-0.003578(8)$
Im EW	$+0.002156(8)$
Re p QCD	$-0.0005522(4)$
Im p QCD	$+0.0001178(3)$
fin ren	$-0.0000977 - 0.0000998\,i$
Re $\alpha(s)$	$0.0078782(2)$
Re $\alpha^{-1}(s)$	$126.933(4)$

Vertices: enough for a long talk

New

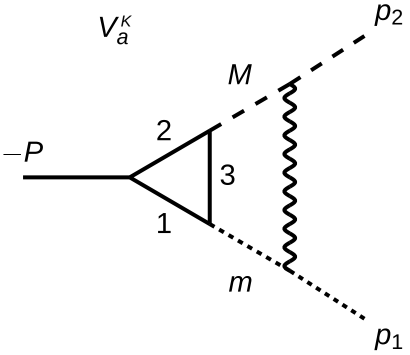
FD $\equiv$ integral representations

Theorem

$$\int dC_k(\{x\})\frac{1}{A}\ln\left(1+\frac{A}{B}\right) \quad \text{or} \quad \int dC_k(\{x\})\frac{1}{A}\,\mathrm{Li}_n\left(\frac{A}{B}\right)$$

where A, B are multivariate polynomials in the Feynman parameters. Two - loop diagrams are always reducible to combinations of integrals of this type where the usual monomials that appear in the integral representation of Nielsen - Goncharov generalized polylogarithms are replaced by multivariate polynomials of arbitrary degree.

Examples



Solutions

- IR conf. classified
- $\hookrightarrow$ IR residues and finite part computed
- suitable also for coll. regions
- fully multi-scale

BST funct. rel. $\Rightarrow$ h.o. transcendental functions

Results: just a sample

	$\sqrt{s}$ [GeV]	Re $V_{0;K}$ [GeV^{-4}]	Im $V_{0;K}$ [GeV^{-4}]
Our	400	$5.1343(1) \times 10^{-8}$	$1.94009(8) \times 10^{-8}$
DK		5.13445×10^{-8}	1.94008×10^{-8}
Our	300	5.68801×10^{-8}	-1.61218×10^{-8}
DK		5.68801×10^{-8}	-1.61218×10^{-8}
Our	200	9.36340×10^{-8}	-2.84232×10^{-8}
DK		9.36340×10^{-8}	-2.84232×10^{-8}
Our	100	2.94726×10^{-7}	-9.74218×10^{-8}
DK		2.94726×10^{-7}	-9.74218×10^{-8}
	$\sqrt{-t}$ [GeV]	Re $V_{0;K}$ [GeV^{-4}]	Im $V_{0;K}$ [GeV^{-4}]
Our	100	-2.85709×10^{-7}	0
DK		-2.85709×10^{-7}	0
Our	200	-7.61695×10^{-8}	0
DK		-7.61695×10^{-8}	0
Our	300	-3.29938×10^{-8}	0
DK		-3.29938×10^{-8}	0
Our	400	-1.74228×10^{-8}	0
DK		-1.74228×10^{-8}	0

Table: Comparison with the results of Davydychev - Kalmykov Only the infrared finite part is shown

Old fashioned one-loop

Example

$$s_H = \mu_H^2 - i\,\mu_H\,\gamma_H$$

M_H^{OS} [GeV]	120	150	300
μ_H	119.96 GeV	149.91 GeV	299.74 GeV
γ_H	5.62 MeV	7.00 MeV	7.90 GeV

τ and b-quark

input

On-Shell Masses

The current fashion

Example

$$s_H^{\rm exp} = \mu_H^2 - i\,\mu_H\,\gamma_H$$
$$s_H^{\rm th} = M_H^2 - i\,M_H\,\Gamma_H$$

μ_H	γ_H	M_H	Γ_H
300	4	299.96	8.374
300	12	299.87	8.376
500	40	500.17	63.37
500	80	500.42	63.34

Note:

it's the imaginary part that matters finally beyond $m_H^{MSB}(s_H)$

Skip details

Complex poles: details

$$s_V - m^2 + \Pi\left(s_V\,,\, m^2\,,\,\dots\right) \;\to\; s_V = m^2 - \Pi^{(1)}\left(m^2\,,\, m^2\,\dots\right) + \dots$$

Example

You get complex pole (renorm. mass) an $\overline{MS}$ concept

$$\textbf{Improve}: \quad s_V - m^2 + \Pi\left(s_V\,,\, m^2\,,\,\{p\}\,,\,\dots\right)$$

Solution

m^2 and $\{p\}$ from R - equations $\qquad m^2\,,\,\{p\} = \mathrm{Re}\, f\left(s_{V_1}\,,\, s_{V_2}\,,\,\dots\right)$

No expansion for *exp* - dependent quantities $\hookrightarrow$ 2 L on second R-sheet (try it!)

R - equations $\hookrightarrow$ Born in 2 L , 1 L in 1 L

(in principle) *masses to complex poles* in propagators

$\hookrightarrow$ **prediction** if $V \not\in \{V_1\,,\,V_2\,,\,\dots\}$

$\hookrightarrow$ **consistency of QC** if $V \in \{V_1\,,\,V_2\,,\dots\}$ $\qquad s_V$ -expansion

Return

R - equations: details

Definition

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\,M^2}\,(1+\Delta g)$$

$$\Delta g = \delta_G + \Delta g^S$$

Solutions

$$g^2 = 8\,G\,\mu_W^2\,\left[1 + C_g^{(1)}\,\frac{G}{\pi^2} + \dots\right]$$

$$C_g^{(1)} = \frac{1}{2}\,\left[\mathrm{Re}\,\Sigma_{WW}^{(1)}(s_W) - \Sigma_{WW}^{(1)}(0)\right]$$

$G_F \to G$ $\delta_G^{(1)}$ finite - $\delta_G^{(2)}$ finite after $1\,L$ Ren.

$$G = G_F\left\{1 - \delta_G^{(1)}\,\frac{G_F\,\mu_W^2}{2\,\pi^2} + \left[2\,(\delta_G^{(1)})^2 - \frac{2}{\mu_W^2}\,\delta_G^{(1)}\,C_g^{(1)} - \delta_G^{(2)}\right]\left(\frac{G_F\,\mu_W^2}{2\,\pi^2}\right)^2\right\}$$

The UV, IR finite remainder for G_F

M_H^{OS} [GeV]	150	300	500
$\frac{G_F\,\mu_W^2}{2\,\pi^2}\,\frac{\delta_G^{(2)}}{\delta_G^{(1)}}$	18.29 %	8.89 %	-24.62%

Playing with numbers in R - equations

Definition

$$X = x\,(1 + a_1\,x + a_2\,x^2)$$

$$X = \frac{G_F\,\mu_W^2}{2\,\pi^2}, \quad x = \frac{g^2}{16\,\pi^2}$$

$$a_1 = \delta_G^{(1)} + S^{(1)}$$

$$a_2 = S^{(1)}\left[\delta_G^{(1)} + S^{(1)}\right] + \delta_G^{(2)} + S^{(2)}$$

PT solution

$$x = X + X^2\,(b_1 + b_2\,X)$$

$$S^{(n)} = \frac{1}{\mu_W^2}\,\Sigma_{WW}^{(n)}(0)$$

PT questionable $\hookrightarrow$

M_H^{OS} [GeV]	150	200	250	300	350
$b_1\,X$ (%)	$+3.31$	$+0.13$	-2.30	-4.84	-7.85
b_1	$+12.28$	$+0.47$	-8.51	-17.95	-29.07
$b_2\,X$	$+0.25$	-1.31	-1.38	-2.58	-9.26
$b_2/b_1\,X$ (%)	$+2.06$	-277.31	$+16.16$	$+14.35$	$+31.85$

accidental $1\,L$ cancellation

Return

The road map for an NNLO calculation

(1, 2, 3, 4, 5)

1. *We have created an independent integrated system which*
   - *uses FORM to generate*
   - *uses FORTRAN 95 to compute*
2. *Has a built-in Renormalization procedure*
3. *Can deal with multi-scale diagrams (also IR and coll.)*
4. *Is fully operative at two-loop level,*
   - *expanding & improving PO (two-leg) results*
   - *classifying & computing three-leg diagrams (d-by-d)*
5. *Is evolving towards PO / O (three-leg) (already implanted in other projects)*
   - *Yes, I'm slow;* no hurry, no worry, I'm going my way

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