Neutrino Mixing and Oscillations

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$\rightsquigarrow$ Part 1: Neutrino Masses and Mixing

$\rightsquigarrow$ Part 2: Neutrino Oscillations in Vacuum and in Matter

$\rightsquigarrow$ Part 3: Experimental Results and Theoretical Implications

Part 1: Neutrino Masses and Mixing

“Standard Model” $\Longleftrightarrow$ Massless Neutrinos

		I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet	$L_{\alpha L} = \begin{pmatrix} \nu_{\alpha L} \\ \ell_{\alpha L} \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet	$\ell_{\alpha R}$	0	0	-2	-1
quark doublet	$Q_{aL} = \begin{pmatrix} q^U_{aL} \\ q^D_{aL} \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$1/3$	$2/3$ $-1/3$
quark singlets	q^U_{aR} q^D_{aR}	0	0	$4/3$ $-2/3$	$2/3$ $-1/3$
Higgs doublet	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	1	1 0

$$\mathcal{L}_{H,\ell} = - \sum_{\alpha,\beta=e,\mu,\tau} y^{\ell}_{\alpha\beta} \, \overline{L_{\alpha L}} \, \Phi \, \ell_{\beta R} + \text{H.c.}$$

$$\mathcal{L}_{H,q} = - \sum_{a,b=d,s,b} y^{D}_{ab} \, \overline{Q_{aL}} \, \Phi \, q^D_{bR} - \sum_{a,b=d,s,b} y^{U}_{ab} \, \overline{Q_{aL}} \, \widetilde{\Phi} \, q^U_{bR} + \text{H.c.} \qquad (\widetilde{\Phi}=i\tau_2\Phi^*)$$

Spontaneous Symmetry Breaking $\Rightarrow$ Dirac Mass Terms of type $m\left(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L\right)$

“Standard Model” $\Longleftarrow$ Two-Component Theory of Massless Neutrinos

[Landau, Nucl. Phys. 3 (1957) 127; Lee and Yang, Phys. Rev. 105 (1957) 1671; Salam, Nuovo Cim. 5 (1957) 299]

$V - A$ coupling: $j_\mu = \overline{\nu}\,\gamma_\mu\,(1-\gamma_5)\;e = 2\,\overline{\nu_L}\,\gamma_\mu\,e_L$ $\qquad \nu_L \equiv \dfrac{1-\gamma_5}{2}\,\nu$ $\qquad \gamma_5\,\nu_L = -\nu_L$

$\Uparrow$

Chiral representation: $\gamma_5 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \Rightarrow \dfrac{1-\gamma_5}{2} = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ $\qquad$ Left-Handed Chirality

$$\underset{\substack{\uparrow \\ \text{four} \\ \text{components}}}{\nu = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}} \quad \Rightarrow \quad \nu_L = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \underset{\substack{\uparrow \\ \text{two} \\ \text{components}}}{\begin{pmatrix} 0 \\ \chi_L \end{pmatrix}}$$

Weak interactions involve only two of the four components of the Dirac neutrino field!

Four components: $\left\{ \begin{array}{c} \text{(particle+antiparticle)} \\ \otimes \\ \text{(negative+positive hel.)} \end{array} \right.$ $\qquad$ Two components: $\left\{ \left(\begin{array}{c} \text{particle} \\ \text{negative hel.} \end{array} \right) + \left(\begin{array}{c} \text{antiparticle} \\ \text{positive hel.} \end{array} \right) \right.$

Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\,\nu = 0 \quad \Rightarrow \quad (i\gamma^{0}\partial_{0} + i\,\underbrace{\gamma^{k}\partial_{k}}_{\vec{\gamma}\cdot\vec{\nabla}} - m)\,\nu = 0$

Chiral representation: $\gamma^{0}=\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \vec{\gamma}=\begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$ $\quad \begin{pmatrix} -m & i\,(-\partial_{0}+\vec{\sigma}\cdot\vec{\nabla}) \\ i\,(-\partial_{0}-\vec{\sigma}\cdot\vec{\nabla}) & -m \end{pmatrix}\begin{pmatrix} \chi_{R} \\ \chi_{L} \end{pmatrix}=0$

Two equations coupled by mass: $\begin{cases} i\,(\partial_{0} - \vec{\sigma}\cdot\vec{\nabla})\,\chi_{L} = m\,\chi_{R} \\ i\,(\partial_{0} + \vec{\sigma}\cdot\vec{\nabla})\,\chi_{R} = m\,\chi_{L} \end{cases}$

$m = 0 \Longrightarrow \chi_{R}$ (or χ_{L}) is not needed! $\Longrightarrow$ two components!

$\boxed{\left(\partial_{0} - \vec{\sigma}\cdot\vec{\nabla}\right)\chi_{L} = 0}$ Weyl Equation (1929) (Rejected by Pauli because parity violating!)

(two-component)

1947: $m_{\nu} \lesssim 500\,\mathrm{eV} \Longrightarrow$ neutrino may be massless (plausible because $m_{\nu} \ll m_{e}$)

Maximal Parity Violation + Massless Neutrino $\Longrightarrow$ Two-Component Theory

Chirality and Helicity

$$\left(\partial_0 - \vec{\Sigma}\cdot\vec{\nabla}\right)\nu_L(x) = 0$$

massless chiral field $\qquad \vec{\Sigma} \equiv \left(\begin{smallmatrix}\vec{\sigma} & 0 \\ 0 & \vec{\sigma}\end{smallmatrix}\right)$

(Weyl Equation in four-component)

Fourier expansion: $\nu_L(x) \propto \int \mathrm{d}^3p \sum_{h=\pm1}\left[b_p^{(h)} u_L^{(h)}(p) e^{-ip\cdot x} + d_p^{(h)\dagger} v_L^{(h)}(p) e^{ip\cdot x}\right]$

Wave function: $\nu_L^{(h)}(x,p) = \langle 0|\nu_L(x)|p,h\rangle \propto u_L^{(h)}(p)e^{-ip\cdot x} \leftarrow -iEt+i\vec{p}\cdot\vec{x}$

$$\left(\partial_0 - \vec{\Sigma}\cdot\vec{\nabla}\right)\nu_L^{(h)}(x,p) = 0 \Rightarrow \left(-iE - i\vec{\Sigma}\cdot\vec{p}\right)\nu_L^{(h)}(x,p) = 0$$

$$\underset{\text{massless}}{E = |\vec{p}|} \Longrightarrow \underset{\text{Helicity}}{\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}}\,\nu_L^{(h)}(x,p) = -\nu_L^{(h)}(x,p) \Rightarrow h = -1$$

Massless two-component neutrinos described by ν_L have negative helicity and antineutrinos have positive helicity!

$$\nu_L(x) \propto \int \mathrm{d}^3p \left[b_p^{(-)} u_L^{(-)}(p) e^{-ip\cdot x} + d_p^{(+)\dagger} v_L^{(+)}(p) e^{ip\cdot x}\right]$$

Massless fermion $\Rightarrow$ Chirality $=$ Helicity

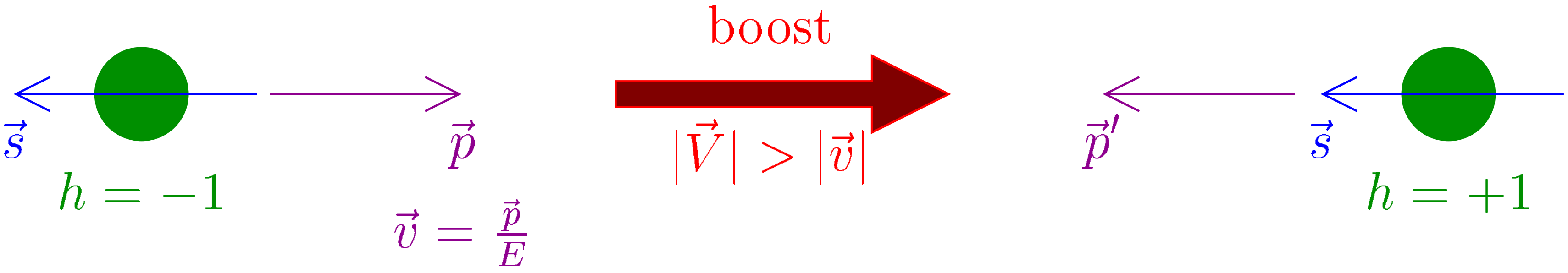
Massive fermion $\Rightarrow$ Chirality $\neq$ Helicity

Helicity in Different Frames

Helicity is conserved: $[\widehat{h}, \widehat{H}] = 0 \Longrightarrow$ Good quantum number for classification of states!

But in general not Lorentz invariant:

$$\left.\begin{array}{c} \vec{p}\,,\ \vec{s} \\ h = -1 \end{array}\right\} \xrightarrow[|\vec{V}| > |\vec{v}|]{\text{boost}} \left\{\begin{array}{c} -\vec{p}\,,\ \vec{s} \\ h = +1 \end{array}\right.$$



Massive fermion $\Longrightarrow$ both helicity states must exist: $f(h=-1) \xrightarrow[|\vec{V}| > |\vec{v}|]{\text{boost}} f(h=+1)$

Massless fermion $\Longrightarrow$ boost is impossible $\Longrightarrow$ Helicity is Lorentz invariant!

Neutrino can be exclusively left-handed only if massless!

Exotic Neutrino Properties

- ⇝ Dirac Mass
- ⇝ Majorana Mass
- ⇝ Magnetic Moment
- ⇝ Decay

Exotic = Beyond the Standard Model with Massless Neutrinos

what is exotic today may be standard tomorrow!

or

what was exotic yesterday may be standard today?

Original GWS Standard Model was different from the Standard Model of the 80's and 90's!

1967 - Weinberg - "A model of leptons". One generation (e).

1970 - Glashow-Iliopulos-Maiani - GIM Mechanism: c quark predicted.

1973 - Kobayashi-Maskawa - Three generation mixing.

1974 - BNL & SPEAR - c quark discovered ($J/\psi = c\bar{c}$).

1975 - SPEAR - τ lepton discovered.

1977 - FNAL - b quark discovered ($\Upsilon = b\bar{b}$).

1998 $\sim$ 2002 - SK, SNO, KamLAND, K2K - ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$?

Dirac neutrino mass terms generated with standard Higgs mechanism

But surprise: possible Majorana mass for ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$!

Majorana Neutrinos

1937: Majorana discovers the possibility of existence of truly neutral fermions

Charged Fermion (electron) + Electromagnetic Field [a]

$(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\,\psi = 0$ particle

$(i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu - m)\,\psi^c = 0$ antiparticle

$\psi^c = \psi$ forbidden

Neutral Fermion (neutrino) + Electromagnetic Field

$(i\gamma^\mu \partial_\mu - m)\,\nu = 0$ particle

$(i\gamma^\mu \partial_\mu - m)\,\nu^c = 0$ antiparticle

$\nu^c = \nu$ allowed

$\boxed{\nu^c = \nu}$ Majorana condition particle=antiparticle

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[a] $\psi^c = \mathcal{C}\,\overline{\psi}^T$, $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$, $\mathcal{C}^\dagger = \mathcal{C}^{-1}$, $\mathcal{C}^T = -\mathcal{C}$, $\mathcal{C}\,\gamma_5^T\,\mathcal{C}^{-1} = \gamma_5$

Chiral Representation: $\nu = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} , \quad \nu^c = \begin{pmatrix} -i\sigma^2\chi_L^* \\ i\sigma^2\chi_R^* \end{pmatrix}$ four independent components

Majorana Fermion $\nu^c = \nu \Longrightarrow \left\{ \begin{array}{l} \chi_R = -i\sigma^2\chi_L^* \\ \chi_L = i\sigma^2\chi_R^* \end{array} \right\}$ equivalent $\Longrightarrow$ two independent components

Dirac Fermion needs independent left and right chiral projections

$$\psi = \begin{pmatrix} \varphi_R \\ \varphi_L \end{pmatrix} = \begin{pmatrix} 0 \\ \varphi_L \end{pmatrix} + \begin{pmatrix} \varphi_R \\ 0 \end{pmatrix} = \psi_L + \psi_R$$

Majorana Fermion needs only one independent chiral projection

$$\nu = \begin{pmatrix} -i\sigma^2\chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} + \begin{pmatrix} -i\sigma^2\chi_L^* \\ 0 \end{pmatrix} = \nu_L + \nu_L^c$$

Two-component neutrino can have a Majorana mass!

$$\left.\begin{aligned} i\,(\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\,\chi_L &= m\,\chi_R \\ i\,(\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\,\chi_R &= m\,\chi_L \end{aligned}\right\} \underset{\nu^c = \nu}{\Longrightarrow} \left\{\begin{aligned} (\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\,\chi_L &= -m\,\sigma^2\,\chi_L^* \\ (\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\,\sigma^2\,\chi_L^* &= m\,\chi_L \end{aligned}\right.$$

Dirac equation

(chiral representation)

$\Updownarrow$

equivalent

$$\chi_R = -i\sigma^2\chi_L^*$$

Majorana condition

Majorana Equation: $$\underbrace{(\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\,\chi_L}_{\text{Weyl}} + m\,\sigma^2\,\chi_L^* = 0$$

Two-component neutrino with Majorana mass!

> Per quanto non sia forse ancora possibile chiedere all'esperienza una decisione tra questa nuova teoria e quella consistente nella semplice estensione delle equazioni di Dirac alle particelle neutre, va tenuto presente che la prima introduce, in questo campo ancora poco esplorato, un minor numero di entità ipotetiche. ... Il vantaggio di questo procedimento rispetto alla interpretazione elementare delle equazioni di Dirac è che non vi è più nessuna ragione di presumere l'esistenza di antineutroni o antineutrini.
>
> [E. Majorana, Nuovo Cimento 5 (1937) 171]

CPT Transformations of Dirac and Majorana Neutrinos

Parity (Space Inversion): $t \xrightarrow{\mathsf{P}} t \,, \quad \vec{x} \xrightarrow{\mathsf{P}} -\vec{x}$

$$\vec{p} \xrightarrow{\mathsf{P}} -\vec{p} \,, \qquad \vec{L} = \vec{x} \times \vec{p} \xrightarrow{\mathsf{P}} \vec{L} \quad \Rightarrow \quad \vec{s} \xrightarrow{\mathsf{P}} \vec{s} \,, \qquad \text{Helicity:} \quad h = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \xrightarrow{\mathsf{P}} -h$$

Time reversal: $t \xrightarrow{\mathsf{T}} -t \,, \quad \vec{x} \xrightarrow{\mathsf{T}} \vec{x}$

$$\vec{p} \xrightarrow{\mathsf{T}} -\vec{p} \,, \qquad \vec{L} = \vec{x} \times \vec{p} \xrightarrow{\mathsf{T}} -\vec{L} \quad \Rightarrow \quad \vec{s} \xrightarrow{\mathsf{T}} -\vec{s} \,, \qquad \text{Helicity:} \quad h = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \xrightarrow{\mathsf{T}} h$$

Space-Time Inversion: $t \xrightarrow{\mathsf{PT}} -t \,, \quad \vec{x} \xrightarrow{\mathsf{PT}} -\vec{x}$

$$\vec{p} \xrightarrow{\mathsf{PT}} \vec{p} \,, \qquad s \xrightarrow{\mathsf{PT}} -s \,, \qquad h \xrightarrow{\mathsf{PT}} -h \,, \qquad \nu(\vec{p}, h) \xrightarrow{\mathsf{PT}} \nu(\vec{p}, -h)$$

$$\text{CPT:} \quad \begin{cases} \nu(\vec{p}, h) \xrightarrow{\mathsf{CPT}} \bar{\nu}(\vec{p}, -h) & \text{Dirac} \\ \nu(\vec{p}, h) \xrightarrow{\mathsf{CPT}} \nu(\vec{p}, -h) & \text{Majorana} \end{cases}$$

Dirac and Majorana Degrees of Freedom

$$\begin{tikzcd}
\nu(\vec{p},h) \arrow[r, Leftrightarrow, "\text{CPT}"] \arrow[d, Leftrightarrow, "\text{Boost}"'] & \bar{\nu}(\vec{p},-h) \arrow[d, Leftrightarrow, "\text{Boost}"] \\
\nu(-\vec{p},-h) \arrow[r, Leftrightarrow, "\text{CPT}"'] & \bar{\nu}(-\vec{p},h)
\end{tikzcd}$$

$$\begin{tikzcd}
\nu(\vec{p},h) \arrow[r, Leftrightarrow, "\text{CPT}"] \arrow[d, Leftrightarrow, "\text{Boost}"'] \arrow[dr, Leftrightarrow, "180^\circ\ \text{Rotation}" description] & \nu(\vec{p},-h) \arrow[d, Leftrightarrow, "\text{Boost}"] \arrow[dl, Leftrightarrow] \\
\nu(-\vec{p},-h) \arrow[r, Leftrightarrow, "\text{CPT}"'] & \nu(-\vec{p},h)
\end{tikzcd}$$

$\nu(\vec{p},h)$ and $\bar{\nu}(-\vec{p},h)$

$\nu(-\vec{p},-h)$ and $\bar{\nu}(\vec{p},-h)$

have different interactions

$\Downarrow$

four degrees of freedom

$\nu(\vec{p},h)$ and $\nu(-\vec{p},h)$

$\nu(-\vec{p},-h)$ and $\nu(\vec{p},-h)$

have same interactions

$\Downarrow$

two degrees of freedom

Majorana Mass

Two-Component Majorana Equation:

$$\boxed{(\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\,\chi_L + m\,\sigma^2\,\chi_L^* = 0}$$

Four Components

(chiral representation)

$$\begin{pmatrix} 0 & i\,(\partial_0 - \vec{\sigma}\cdot\vec{\nabla}) \\ i\,(\partial_0 + \vec{\sigma}\cdot\vec{\nabla}) & 0 \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ \chi_L \end{pmatrix}}_{\nu_L} - \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \underbrace{\begin{pmatrix} -i\sigma^2\chi_L^* \\ 0 \end{pmatrix}}_{\nu_L^c} = 0$$

Four-Component Majorana Equation:

$$\boxed{i\gamma^\mu\partial_\mu\nu_L + m\nu_L^c = 0}$$

Lagrangian:

$$\mathcal{L}_L = \frac{1}{2}\left[-i\overline{\nu_L}\gamma^\mu(\partial_\mu\nu_L) + i(\partial_\mu\overline{\nu_L})\gamma^\mu\nu_L - m(\underbrace{\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c}_{-\nu_L^T\mathcal{C}^\dagger\nu_L \;\; \underbrace{+\overline{\nu_L}\mathcal{C}\overline{\nu_L}^T}_{-\overline{\nu_L}\mathcal{C}^T\overline{\nu_L}^T}})\right]$$

$$\boxed{\nu_L^c = \mathcal{C}\,\overline{\nu_L}^T,\;\overline{\nu_L^c} = -\nu_L^T\,\mathcal{C}^\dagger}$$

Euler-Lagrange

Equations

$$\partial_\mu\frac{\partial\mathcal{L}_L}{\partial(\partial_\mu\overline{\nu_L})} - \frac{\partial\mathcal{L}_L}{\partial\overline{\nu_L}} = 0 \Rightarrow \frac{1}{2}(i\gamma^\mu\partial_\mu\nu_L + i\gamma^\mu\partial_\mu\nu_L + m\mathcal{C}\overline{\nu_L}^T - m\,\underbrace{\mathcal{C}^T\overline{\nu_L}^T}_{-\mathcal{C}\overline{\nu_L}^T}) = 0$$

Majorana Mass Term:

$$\boxed{\mathcal{L}_L^{\mathrm{M}} = -\tfrac{1}{2}\,m\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)}$$

Majorana Neutrino $\Longleftrightarrow$ No Conserved Lepton Number

$$L_e\,,\;L_\mu\,,\;L_\tau\,,\;L=L_e+L_\mu+L_\tau$$

$$\xcancel{L=-1}\quad\longleftarrow\quad\boxed{\nu^c=\nu}\quad\longrightarrow\quad\xcancel{L=+1}$$

$$\text{Conserved Lepton Number}\quad\underset{\text{Theorem}}{\overset{\text{Noether}}{\begin{array}{c}\xrightarrow{\hspace{2cm}}\\ \xleftarrow{\hspace{2cm}}\end{array}}}\quad\text{Global Gauge Invariance}$$

Dirac mass term

$$\mathcal{L}^{\mathrm{D}}=-m_{\mathrm{D}}\left(\overline{\nu_L}\,\nu_R+\overline{\nu_R}\,\nu_L\right)$$

invariant under

$$\nu_L\to e^{i\Lambda}\nu_L \qquad \nu_R\to e^{i\Lambda}\nu_R$$

$$\overline{\nu_L}\to e^{-i\Lambda}\overline{\nu_L} \qquad \overline{\nu_R}\to e^{-i\Lambda}\overline{\nu_R}$$

Majorana mass term

$$\mathcal{L}^{\mathrm{M}}=-m_{\mathrm{M}}\left(\overline{\nu_L}\,\nu_L^c+\overline{\nu_L^c}\,\nu_L\right)$$

not invariant under

$$\nu_L\to e^{i\Lambda}\nu_L \qquad \nu_L^c\to e^{-i\Lambda}\nu_L^c$$

$$\overline{\nu_L}\to e^{-i\Lambda}\overline{\nu_L} \qquad \overline{\nu_L^c}\to e^{i\Lambda}\overline{\nu_L^c}$$

$$\boxed{\text{Majorana Neutrino}\quad=\quad\text{Truly Neutral Fermion}}$$

the chiral fields ν_L and ν_R (if it exists!)
are the building blocks of the neutrino Lagrangian

ONLY ν_L $\Longrightarrow$ Majorana Mass Term

$$\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2} m_L \, \overline{\nu} \, \nu = -\frac{1}{2} m_L \left(\overline{\nu_L} + \overline{\nu_L^c} \right) \left(\nu_L + \nu_L^c \right) = -\frac{1}{2} m_L \left(\overline{\nu_L^c} \, \nu_L + \overline{\nu_L} \, \nu_L^c \right)$$

$$= \frac{1}{2} m_L \left(\nu_L^T \, \mathcal{C}^\dagger \, \nu_L \underbrace{- \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T}_{\nu_L^\dagger \, \mathcal{C} \, \nu_L^*} \right)$$

$$\boxed{\nu_L^c = \mathcal{C} \, \overline{\nu_L}^T \, , \quad \overline{\nu_L^c} = -\nu_L^T \, \mathcal{C}^\dagger}$$

ν_L AND ν_R $\Longrightarrow$ Dirac Mass Term

$$\mathcal{L}^{\mathrm{D}} = -m_{\mathrm{D}} \, \overline{\nu} \, \nu = -m_{\mathrm{D}} \left(\overline{\nu_L} + \overline{\nu_R} \right) \left(\nu_L + \nu_R \right) = -m_{\mathrm{D}} \left(\overline{\nu_L} \, \nu_R + \overline{\nu_R} \, \nu_L \right)$$

SURPRISE!

ν_L AND ν_R $\Longrightarrow$ Dirac–Majorana Mass Term

$$\begin{aligned}\mathcal{L}^{\mathrm{D+M}} &= \mathcal{L}^{\mathrm{M}}_L + \mathcal{L}^{\mathrm{M}}_R + \mathcal{L}^{\mathrm{D}} \\ &= -\frac{1}{2}\begin{pmatrix}\overline{\nu^c_L} & \overline{\nu_R}\end{pmatrix}\begin{pmatrix}m_L & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu^c_R\end{pmatrix} + \mathrm{H.c.} \\ &= \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \mathrm{H.c.}\end{aligned}$$

$$M = \begin{pmatrix}m_L & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R\end{pmatrix}$$

$$N_L = \begin{pmatrix}\nu_L \\ \nu^c_R\end{pmatrix}$$

diagonalization
$\Downarrow$
fields with definite mass

$$N_L = U\, n_L\,, \quad n_L = \begin{pmatrix}\nu_{1L} \\ \nu_{2L}\end{pmatrix} \quad \Rightarrow \quad U^T\, M\, U = \begin{pmatrix}m_1 & 0 \\ 0 & m_2\end{pmatrix}$$

$$\mathcal{L}^{\mathrm{D+M}} = \frac{1}{2}\sum_{k=1,2} m_k \nu^T_{kL}\, \mathcal{C}^\dagger\, \nu_{kL} + \mathrm{h.c.} = -\frac{1}{2}\sum_{k=1,2} m_k \overline{\nu_k}\, \nu_k$$

$\nu_k = \nu_{kL} + \nu^c_{kL}$ Massive neutrinos are Majorana!

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{\nu_R}\end{pmatrix}\begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix} + \text{H.c.} = \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \text{H.c.}$$

m_L, m_R can be chosen real ≥ 0 by rephasing the fields ν_L, ν_R

simplest case: real m_{D} $\Longrightarrow$ $U = \mathcal{O}\,\rho$ (CP invariance)

$$M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}, \quad \mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}, \quad |\rho_k|^2 = 1\,, \quad U = \begin{pmatrix} \rho_1\cos\vartheta & \rho_2\sin\vartheta \\ -\rho_1\sin\vartheta & \rho_2\cos\vartheta \end{pmatrix}$$

$$\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \Longrightarrow \tan 2\vartheta = \frac{2m_{\text{D}}}{m_R - m_L}\,, \quad m_{2,1}' = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

m_1' negative if $m_{\text{D}}^2 > m_L m_R$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}\begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix}\begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} = \begin{pmatrix} \rho_1^2 m_1' & 0 \\ 0 & \rho_2^2 m_2' \end{pmatrix} \Longrightarrow m_k = \rho_k^2\, m_k' \quad \begin{pmatrix} \rho_1^2 = \pm 1 \\ \rho_2^2 = 1 \end{pmatrix}$$

$$\rho_1^2 = 1 \Longrightarrow U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \rho_1^2 = -1 \Longrightarrow U = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\nu_k(t,\vec{x}) \xrightarrow{\text{CP}} \eta_k\,\gamma^0\,\nu_k(t,-\vec{x})$$

$\eta_k = i\,\rho_k^2 = \pm i$ CP parity of ν_k

important in neutrinoless double-β decay

[Wolfenstein, Phys. Lett. B107 (1981) 77]
[Bilenky, Nedelcheva, Petcov, Nucl. Phys. B247 (1984) 61]
[Kayser, Phys. Rev. D30 (1984) 1023]

in general
$$\begin{cases} \nu_k(t,\vec{x}) \xrightarrow{\text{CP}} \eta_k\,\gamma^0\,\nu_k^c(t,-\vec{x}) \\ \nu_k^c(t,\vec{x}) \xrightarrow{\text{CP}} -\,\eta_k^*\,\gamma^0\,\nu_k(t,-\vec{x}) \end{cases}$$

the product of the CP parities of particle and antiparticle is -1

$$(|\eta_k|^2 = 1,\ \psi^c = \mathcal{C}\,\overline{\psi}^T)$$

Majorana Constraint $\nu_k^c = \nu_k \implies \eta_k = -\eta_k^* \implies \eta_k = \pm i$ imaginary CP parity!

CP transformation of $N_L = \left(\begin{smallmatrix} \nu_L \\ \nu_R^c \end{smallmatrix}\right)$ is determined by CP invariance of Lagrangian

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2}\,\overline{N_L^c}\,M\,N_L - \frac{1}{2}\,\overline{N_L}\,M^*\,N_L^c \qquad (M^T = M)$$

$$\left.\begin{array}{l} N_L \xrightarrow{\text{CP}} \xi\,\gamma^0\,N_L^c \\ N_L^c \xrightarrow{\text{CP}} -\xi^\dagger\,\gamma^0\,N_L \end{array}\right\} \Longrightarrow \mathcal{L}^{\text{D+M}} \xrightarrow{\text{CP}} \frac{1}{2}\,\overline{N_L}\,\xi\,M\,\xi\,N_L^c + \frac{1}{2}\,\overline{N_L^c}\,\xi^\dagger\,M^*\,\xi^\dagger\,N_L$$

M real $\Rightarrow$ CP invariance $\Leftrightarrow \xi\,M\,\xi = -M \Rightarrow \xi = \left(\begin{smallmatrix} i & 0 \\ 0 & i \end{smallmatrix}\right) = i\,I \Rightarrow \left\{\begin{array}{l} N_L \xrightarrow{\text{CP}} i\,\gamma^0\,N_L^c \\ N_L^c \xrightarrow{\text{CP}} i\,\gamma^0\,N_L \end{array}\right.$

$$N_L = U\,n_L \qquad n_L = U^\dagger\,N_L \qquad U = \mathcal{O}\,\rho \qquad \rho_{kj} = \rho_k\,\delta_{kj}$$

$$N_L^c = U^*\,n_L^c \qquad n_L^c = U^T\,N_L^c \qquad \mathcal{O}^T\,\mathcal{O} = I \qquad \rho_k^2 = \pm 1$$

$$n_L = U^\dagger\,N_L \xrightarrow{\text{CP}} i\,U^\dagger\,\gamma^0\,N_L^c = \underbrace{i\,U^\dagger\,U^*}_{\eta}\,\gamma^0\,n_L^c$$

$$\eta = i\,U^\dagger\,U^* = i\left(U^T\,U\right)^* = i\left(\rho\,\mathcal{O}^T\,\mathcal{O}\,\rho\right)^* = i\,\rho^2 \qquad \boxed{\eta_k = i\rho_k^2 = \pm i}$$

CP invariance of $\mathcal{L}_I^{\text{CC}} = -\frac{g}{\sqrt{2}} \overline{\nu_L} \gamma^\mu \ell_L W_\mu - \frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^\mu \nu_L W_\mu^\dagger$?

$$\nu_L \xrightarrow{\text{CP}} i \gamma^0 \mathcal{C} \overline{\nu_L}^T \qquad \ell_L \xrightarrow{\text{CP}} i \gamma^0 \mathcal{C} \overline{\ell_L}^T$$

$$\overline{\nu_L} \xrightarrow{\text{CP}} - i \nu_L^T \mathcal{C}^\dagger \gamma^0 \qquad \overline{\ell_L} \xrightarrow{\text{CP}} - i \ell_L^T \mathcal{C}^\dagger \gamma^0$$

$$W_\mu \xrightarrow{\text{CP}} - W^{\mu\dagger}$$

$$\mathcal{L}_I^{\text{CC}} \xrightarrow{\text{CP}} - \frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^{\mu\dagger} \nu_L W^{\mu\dagger} - \frac{g}{\sqrt{2}} \overline{\nu_L} \gamma^{\mu\dagger} \ell_L W^\mu$$

$$\gamma^{\mu\dagger} = \left(\gamma^{0\dagger}, \vec{\gamma}^\dagger\right) = (\gamma^0, -\vec{\gamma}) = \gamma_\mu$$

$$\mathcal{L}_I^{\text{CC}} \xrightarrow{\text{CP}} - \frac{g}{\sqrt{2}} \overline{\ell_L} \gamma_\mu \nu_L W^{\mu\dagger} - \frac{g}{\sqrt{2}} \overline{\nu_L} \gamma_\mu \ell_L W^\mu$$

CP invariance OK!

CP parity of charged lepton is also imaginary!

Maximal Mixing

$$\tan 2\vartheta = \frac{2m_{\text{D}}}{m_R - m_L} \qquad m'_{2,1} = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

$$m_L = m_R \quad \Longrightarrow \quad \vartheta = \pi/4\,, \quad m'_{2,1} = m_L \pm |m_{\text{D}}|$$

$$|m_{\text{D}}| > m_L \geq 0 \Rightarrow \left\{ \begin{array}{lll} m_1 = |m_{\text{D}}| - m_L\,, & \rho_1^2 = -1\,, & \nu_{1L} = \frac{-i}{\sqrt{2}}\,(\nu_L - \nu_R^c) \\ m_2 = |m_{\text{D}}| + m_L\,, & \rho_2^2 = +1\,, & \nu_{2L} = \frac{1}{\sqrt{2}}\,(\nu_L + \nu_R^c) \end{array} \right.$$

Majorana Neutrino Fields:

$$\left\{ \begin{array}{l} \nu_1 = \nu_{1L} + \nu_{1L}^c = \frac{-i}{\sqrt{2}}\left[(\nu_L + \nu_R) - (\nu_L^c + \nu_R^c)\right] \\ \nu_2 = \nu_{2L} + \nu_{2L}^c = \frac{1}{\sqrt{2}}\left[(\nu_L + \nu_R) + (\nu_L^c + \nu_R^c)\right] \end{array} \right.$$

$m_L = m_R = 0 \quad \Longrightarrow \quad$ Dirac Neutrino Field

ν_1 and ν_2 have the same mass $m_1 = m_2 = |m_\mathrm{D}|$ and opposite CP parities.

The two Majorana fields ν_1 and ν_2 can be combined to give one Dirac field ν

$$\nu = \frac{1}{\sqrt{2}} \left(i \nu_1 + \nu_2 \right) = \nu_L + \nu_R$$

Viceversa, one Dirac field ν can always be splitted in two Majorana fields

$$\nu = \frac{1}{2} \left[(\nu - \nu^c) + (\nu + \nu^c) \right] = \frac{i}{\sqrt{2}} \left(-i \, \frac{\nu - \nu^c}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left(\frac{\nu + \nu^c}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} \left(i \nu_1 + \nu_2 \right)$$

Majorana Neutrino Fields ($\nu_1 = \nu_1^c$, $\nu_2 = \nu_2^c$):

$$\begin{cases} \nu_1 = \dfrac{-i}{\sqrt{2}} \left(\nu - \nu^c \right) \\ \nu_2 = \dfrac{1}{\sqrt{2}} \left(\nu + \nu^c \right) \end{cases}$$

In general: one Dirac field $\equiv$ two Majorana fields with same mass and opposite CP parities

CP parity of Dirac = 2 Majorana neutrino field

$$\nu_1(t,\vec{x}) \xrightarrow{\text{CP}} -i\,\gamma^0\,\nu_1(t,-\vec{x}) \qquad \nu_2(t,\vec{x}) \xrightarrow{\text{CP}} i\,\gamma^0\,\nu_2(t,-\vec{x})$$

$$\nu = \frac{1}{\sqrt{2}}\left(i\nu_1+\nu_2\right) \xrightarrow{\text{CP}} i\,\gamma^0\,\frac{1}{\sqrt{2}}\left(-i\nu_1+\nu_2\right)$$

$$\nu_1 = \nu_{1L}+\nu_{1L}^c = \tfrac{-i}{\sqrt{2}}\left[(\nu_L+\nu_R)-(\nu_L^c+\nu_R^c)\right]$$

$$\nu_2 = \nu_{2L}+\nu_{2L}^c = \tfrac{1}{\sqrt{2}}\left[(\nu_L+\nu_R)+(\nu_L^c+\nu_R^c)\right]$$

$$\nu \xrightarrow{\text{CP}} i\,\gamma^0\left(\nu_L^c+\nu_R^c\right) = i\,\gamma^0\nu^c$$

Dirac neutrino field has definite CP parity $= i$

Pseudo-Dirac Neutrinos

$$m_L\,,\,m_R \ll |m_{\mathrm{D}}| \quad \Longrightarrow \quad m'_{2,1} \simeq \frac{m_L+m_R}{2} \pm |m_{\mathrm{D}}| \quad \Longrightarrow \quad \rho_1^2=-1\,, \quad \rho_2^2=+1$$

$$m_1 \simeq |m_{\mathrm{D}}| - \frac{m_L+m_R}{2}\,, \quad m_2 \simeq |m_{\mathrm{D}}| + \frac{m_L+m_R}{2} \quad \Longrightarrow \quad \Delta m^2 \simeq |m_{\mathrm{D}}|\,(m_L+m_R)$$

$$\tan 2\vartheta = \frac{2m_{\mathrm{D}}}{m_R-m_L} \gg 1 \quad \Longrightarrow \quad \vartheta \simeq \pi/4$$

practically maximal mixing!

$$\begin{aligned} \nu_{1L} &\simeq \tfrac{-i}{\sqrt{2}}\left(\nu_L - \nu_R^c\right) \\ \nu_{2L} &\simeq \tfrac{1}{\sqrt{2}}\left(\nu_L + \nu_R^c\right) \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \nu_L &\simeq \tfrac{1}{\sqrt{2}}\left(i\nu_{1L} + \nu_{2L}\right) \\ \nu_R^c &\simeq \tfrac{1}{\sqrt{2}}\left(-i\nu_{1L} + \nu_{2L}\right) \end{aligned}$$

$$U \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$$

active (ν_L) – sterile (ν_R) oscillations!

See-Saw Mechanism

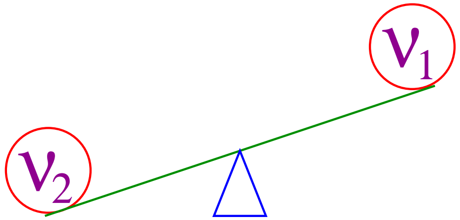
[Yanagida, 1979] [Gell-Mann, Ramond, Slansky, 1979] [Witten, Phys. Lett. B91 (1980) 81] [Mohapatra, Senjanovic, Phys. Rev. Lett. 44 (1980) 912]

$$\tan 2\vartheta = \frac{2m_{\mathrm{D}}}{m_R - m_L} \qquad m'_{2,1} = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4\,m_{\mathrm{D}}^2}\right]$$

$$m_L = 0\,, \quad |m_{\mathrm{D}}| \ll m_R \quad \Longrightarrow \quad \tan 2\vartheta = 2\,\frac{m_{\mathrm{D}}}{m_R}\,, \quad m'_1 \simeq -\frac{(m_{\mathrm{D}})^2}{m_R}\,, \quad m'_2 \simeq m_R$$

$$m_1 \simeq \frac{(m_{\mathrm{D}})^2}{m_R} \ll |m_{\mathrm{D}}| \qquad \rho_1^2 = -1$$

$$m_2 \simeq m_R \qquad \rho_2^2 = +1$$



$$\tan\vartheta \simeq \frac{m_{\mathrm{D}}}{m_R} \ll 1 \quad \Longrightarrow \quad \nu_{1L} \simeq -\nu_L\,, \quad \nu_{2L} \simeq \nu_R^c$$

Example: $|m_{\mathrm{D}}| \sim M_{\mathrm{EW}} \sim 10^2\,\mathrm{GeV}\,, \quad m_R \sim M_{\mathrm{GUT}} \sim 10^{15}\,\mathrm{GeV} \quad \Longrightarrow \quad m_1 \sim 10^{-2}\,\mathrm{eV}$

See-Saw Mass Matrix: $M = \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix}$ Why $m_L = 0$?

$$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \leftarrow I_3{=}1/2$$

doublet

$$\mathcal{L}^{\mathrm{M}} \sim \nu_L^T \, \nu_L$$

$I_3{=}1$

triplet

$$(L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^{-1} \, (\Phi^T \, \sigma_2 \, L_L) \xrightarrow[\text{Breaking}]{\text{Symmetry}} \nu_L^T \, \nu_L$$

non-renormalizable

Effective Lagrangian

[Weinberg, Phys. Rev. Lett. 43 (1979) 1566, Phys. Rev. D22 (1980) 1694] [Weldon, Zee, Nucl. Phys. B173 (1980) 269]

minimum dimension lepton-number violating operator invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$\frac{g}{\mathcal{M}} (L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^{-1} \, (\Phi^T \, \sigma_2 \, L_L) + \mathrm{H.c.}$$

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}^{\mathrm{M}} = \frac{1}{2} \, \frac{g v^2}{\mathcal{M}} \nu_L^T \, \mathcal{C}^{-1} \nu_L + \mathrm{H.c.} \sim - \frac{m_{\mathrm{D}}^2}{\mathcal{M}} \, \overline{(\nu_L)^c} \, \nu_L + \mathrm{H.c.}$$

$$\boxed{m_L \sim \frac{m_{\mathrm{D}}^2}{\mathcal{M}}}$$

See-Saw Type

Plausible Cut-Off: $\mathcal{M} \lesssim M_{\mathrm{P}} \sim 10^{19} \, \mathrm{GeV}$

General Considerations on Fermion Masses

In Standard Model fermion masses are generated through Yukawa couplings

$$\mathcal{L}_{H,\ell} = - \sum_{\alpha,\beta=e,\mu,\tau} y^{\ell}_{\alpha\beta} \, \overline{L_{\alpha L}} \, \Phi \, \ell_{\beta R} + \text{H.c.}$$

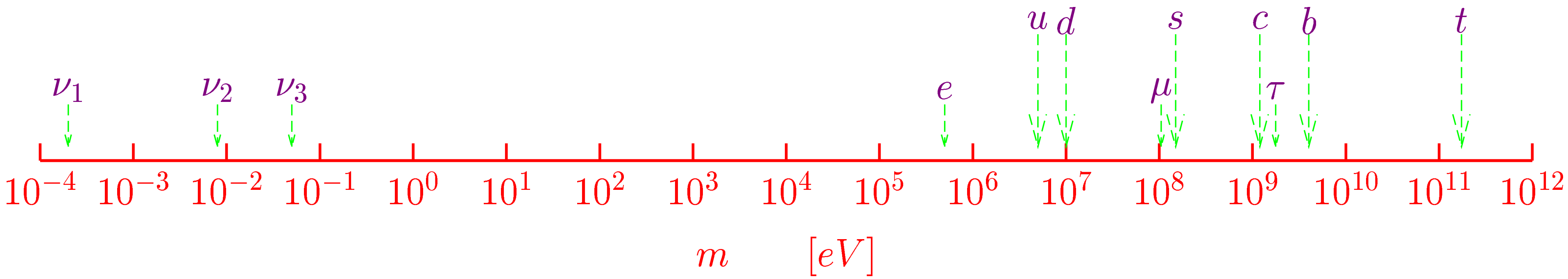
the coefficients $y_{\alpha,\beta}$ are parameters of the model

$\Downarrow$

explanation of parameters must come from new physics Beyond the SM

$\Downarrow$

all fermion masses give info on new physics BSM smallness of ν masses is additional mystery $\Longrightarrow$ more info?



known natural explanations of smallness of ν masses:

- ⋆ See-Saw Mechanism
- ⋆ Effective Lagrangian

both imply

- ⋆ Majorana ν masses!
- ⋆ see-saw type relation $m_{\text{light}} \sim \dfrac{m_{\text{D}}^2}{\mathcal{M}}$
- ⋆ New high energy scale $\mathcal{M}$

general features of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant models with additional scalars and fermions (unless special symmetries forbid all Majorana mass terms) neutrino masses provide a window on New Physics Beyond the Standard Model

most accessible window on NPBSM at low energy

the lepton-number violating dimension 5 operator $(L^T L)(\Phi^T \Phi) \to m_L \nu_L^T \nu_L$ is the operator beyond the Standard Model with minimum dimension (quarks are Dirac!)

$$Y(\Phi) = 1 \,, \quad Y(L_L) = -1 \,, \quad Y(\ell_R) = -2 \,, \quad Y(Q_L) = 1/3 \,, \quad Y(q_R^U) = 4/3 \,, \quad Y(q_R^D) = -2/3$$

next: lepton and barion number violating dimension 6 operators $\sim qqq\ell$ $(\Delta L = \Delta B)$

$$\left({q_R^D}^T q_R^U \right) \left(Q_L^T L_L \right) \,, \quad \left(Q_L^T Q_L \right) \left({q_R^U}^T \ell_R \right) \,, \quad \left(Q_L^T Q_L \right) \left(Q_L^T L_L \right) \,,$$

$$\left({q_R^D}^T q_R^U \right) \left({q_R^U}^T \ell_R \right) \,, \quad \left({q_R^U}^T q_R^U \right) \left({q_R^D}^T \ell_R \right) \quad \Longrightarrow \quad p \to e^+ \pi^0 \,, \quad \text{etc.}$$

Majorana mass term for ν_R respects $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2} \, m \left(\overline{\nu_R^c} \, \nu_R + \overline{\nu_R} \, \nu_R^c \right)$$

Majorana mass term for ν_R breaks Lepton number conservation!

Three possibilities:

- Lepton number can be explicitly broken
- Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left(\overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle \neq 0]{} -m_{\text{D}} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left(\eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle \neq 0]{} -\tfrac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left(\langle\eta\rangle + \rho + i\, \chi \right) \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2} \left(\overline{\nu_L^c}\ \overline{\nu_R} \right) \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_{\text{D}}} \Longrightarrow \text{See-Saw: } \boxed{m_1 \simeq \frac{m_{\text{D}}^2}{m_R}}$$

ρ = massive scalar $\qquad \chi$ = massless pseudoscalar Goldstone boson = Majoron

Majoron weakly coupled to light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\, \chi \left[\overline{\nu_2} \gamma^5 \nu_2 - \frac{m_{\text{D}}}{m_R} \left[\overline{\nu_2} \gamma^5 \nu_1 + \overline{\nu_1} \gamma^5 \nu_2 \right) + \left(\frac{m_{\text{D}}}{m_R} \right)^2 \overline{\nu_1} \gamma^5 \nu_1 \right]$$

Majoron weakly coupled to matter through $W-\nu$ loop and $Z-\chi$ mixing

$$\mathcal{L}^{\text{eff}}_{\chi-f} = \pm \frac{y_s G_{\text{F}}}{16\pi^2}\, m_f\, \frac{m_{\text{D}}^2}{m_R}\, \chi\, \overline{f} \gamma^5 f$$

weak long-range force with spin-dependent potential $\sim 10^{-65}\ \text{cm}^2/r^3$

Three-Neutrino Mixing

[Bilenky & Petcov, Rev. Mod. Phys. 59 (1987) 671]

SM with ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$ $\Longrightarrow$ Dirac neutrino mass term generated by standard Higgs mechanism

$$\mathcal{L}^{\mathrm{D}} = - \sum_{\alpha,\beta} \overline{\nu_{\alpha R}} \, M^{\mathrm{D}}_{\alpha\beta} \, \nu_{\beta L} + \mathrm{H.c.} \qquad (\alpha, \beta = e, \mu, \tau)$$

M^{D} = complex 3×3 matrix

M^{D} can be diagonalized by the biunitary transformation $\boxed{V^{\dagger} \, M^{\mathrm{D}} \, U = M}$

$$V^{\dagger} = V^{-1} \, , \qquad U^{\dagger} = U^{-1} \, , \qquad M_{kj} = m_k \, \delta_{kj} \, , \qquad \text{real } m_k \geq 0$$

POSSIBLE?

Proof that M^{D} can be diagonalized by a biunitary transformation

consider $M^{\mathrm{D}}(M^{\mathrm{D}})^\dagger$: Hermitian $\Longrightarrow$ can be diagonalized by the unitary transformation

$$V^\dagger\, M^{\mathrm{D}}\, (M^{\mathrm{D}})^\dagger V = M^2\,, \qquad V^\dagger = V^{-1}\,, \qquad M^2_{kj} = m_k^2\, \delta_{kj}\,, \qquad \text{real } m_k^2$$

choosing an appropriate matrix U, it is always possible to write

$$M^{\mathrm{D}} = V\, M\, U^\dagger \qquad \text{with} \qquad M_{kj} = \sqrt{m_k^2\, \delta_{kj}} = m_k\, \delta_{kj} \qquad \Longrightarrow \qquad \boxed{V^\dagger\, M^{\mathrm{D}}\, U = M}$$

only problem: is U unitary?

$$U^\dagger = M^{-1}\, V^\dagger\, M^{\mathrm{D}}\,, \qquad U = (M^{\mathrm{D}})^\dagger\, V\, M^{-1} \qquad (M^\dagger = M)$$

magically U is unitary!

$$U^\dagger U = M^{-1} V^\dagger M^{\mathrm{D}} (M^{\mathrm{D}})^\dagger V M^{-1} = 1$$

$$U U^\dagger = (M^{\mathrm{D}})^\dagger V M^{-2} V^\dagger M^{\mathrm{D}} = (M^{\mathrm{D}})^\dagger V V^\dagger ((M^{\mathrm{D}})^\dagger)^{-1} (M^{\mathrm{D}})^{-1} V V^\dagger M^{\mathrm{D}} = 1$$

diagonalized Dirac mass term: $$\mathcal{L}^{\mathrm{D}} = - \sum_{k=1}^{3} m_k \, \overline{\nu_k} \, \nu_k$$

mixing: $$\left. \begin{aligned} \nu_{\alpha L} &= \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \\ \nu_{\alpha R} &= \sum_{k=1}^{3} V_{\alpha k} \, \nu_{kR} \end{aligned} \right\} \qquad (\alpha = e, \mu, \tau)$$

no right-handed fields in weak interaction Lagrangian

$\Downarrow$

right-handed singlets are sterile and not mixed with active neutrinos

weak charged current: $$j_\rho^{\mathrm{CC}\,\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha L}} \, \gamma_\rho \, \nu_{\alpha L} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma_\rho \, U_{\alpha k} \, \nu_{kL}$$

U = unitary 3×3 mixing matrix

we assumed for simplicity that the mass matrix of charged leptons is diagonal

otherwise $U = U^{(\ell)\,\dagger} \, U^{(\nu)}$

Physical Parameters in $N\times N$ Mixing Matrix

$N\times N$ Unitary Mixing Matrix ⇒ N^2 parameters $\begin{cases} \frac{N(N-1)}{2} & \text{Mixing Angles} \\ \frac{N(N+1)}{2} & \text{Phases} \end{cases}$

Weak Charged Current: $j_\rho^{\mathrm{CC}\dagger} = 2 \sum_\alpha \overline{\ell_{\alpha L}} \gamma_\rho \nu_{\alpha L} = 2 \sum_{\alpha,k} \overline{\ell_{\alpha L}} \gamma_\rho U_{\alpha k} \nu_{kL}$

Lagrangian is invariant under global phase transformations of Dirac fields

$$\left.\begin{array}{l} \ell_\alpha \to e^{i\theta_\alpha} \ell_\alpha \\ \nu_k \to e^{i\phi_k} \nu_k \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} j_\rho^{\mathrm{CC}\dagger} \to 2 \sum_{\alpha,k} \overline{\ell_{\alpha L}} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} e^{i\phi_k} \nu_{kL} \\ \qquad = 2 \sum_{\alpha,k} \overline{\ell_{\alpha L}} \underset{\substack{\uparrow \\ 1}}{e^{-i(\theta_e-\phi_1)}} \underset{\substack{\uparrow \\ N-1}}{e^{-i(\theta_\alpha-\theta_e)}} \gamma_\rho U_{\alpha k} \underset{\substack{\uparrow \\ N-1}}{e^{i(\phi_k-\phi_1)}} \nu_{kL} \end{array}\right.$$

number of independent phases that can be eliminated: $2N-1$ (not $2N$!)

number of physical phases: $\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$

remains global phase freedom of lepton fields ⟹ conservation of L

$N \times N$ Unitary Mixing Matrix: $\dfrac{N(N-1)}{2}$ Mixing Angles and $\dfrac{(N-1)(N-2)}{2}$ Phases

$N = 3 \Rightarrow$ 3 Mixing Angles and 1 Physical Phase (as in the quark sector)

standard parameterization (convenient) $(c_{ij} \equiv \cos\vartheta_{ij}\,, \quad s_{ij} \equiv \sin\vartheta_{ij})$

$$U = R_{23}\,W_{13}\,R_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

phase δ_{13} associated with s_{13} $\Rightarrow$ CP violation is small if ϑ_{13} is small

in other parameterizations phase can be associated with s_{12} or s_{23}

$\Downarrow$

CP violation is small if any mixing angle is small

if any element of U is zero the phase can be rotated away $\Rightarrow$ no CP violation

Dirac mass term allows L_e, L_μ, L_τ violating processes like

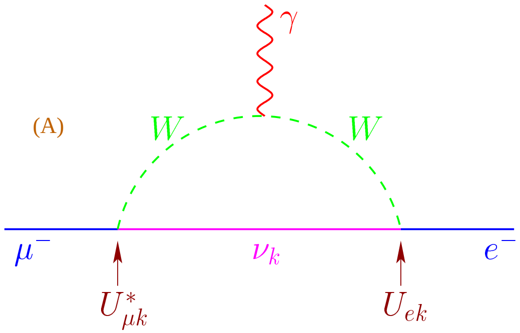
$$\mu^\pm \to e^\pm + \gamma$$

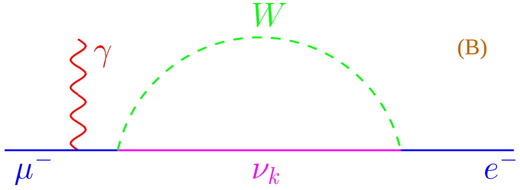
$$\mu^\pm \to e^\pm + e^+ + e^-$$

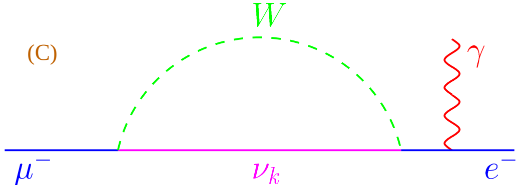
$$\mu^- \to e^- + \gamma$$

$$\sum_k U^*_{\mu k} U_{ek} = 0 \Rightarrow \text{GIM Mechanism}$$

$$\Gamma = \frac{G_{\text{F}} m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi}\left|\sum_k U^*_{\mu k} U_{ek} \frac{m_k}{m_W}\right|^2}_{\text{BR}}$$







Suppression factor: $\dfrac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1\,\text{eV}$

$(\text{BR})_{\text{exp}} \lesssim 10^{-11}$

$(\text{BR})_{\text{the}} \lesssim 10^{-25}$

14 orders of magnitude smaller!

NUMBER OF MASSIVE NEUTRINOS?

$Z \to \nu\bar{\nu} \quad \Rightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$ active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$

no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$

ACTIVE STERILE

STERILE NEUTRINOS

singlets of SM $\Longrightarrow$ no interactions!

active $\to$ sterile transitions are possible if $\nu_4, \ldots$ are light (no see-saw)

$\Downarrow$

disappearance of active neutrinos

Dirac-Majorana mass term

active $\nu_{\alpha L}$ $(\alpha = e, \mu, \tau)$ + sterile ν_{sR} $(s = s_1, s_2, \ldots, s_{\mathcal{N}})$

$$\mathcal{L}^{\mathrm{D+M}} = \mathcal{L}_L^{\mathrm{M}} + \mathcal{L}^{\mathrm{D}} + \mathcal{L}_R^{\mathrm{M}}$$

$$\mathcal{L}^{\mathrm{D}} = -\sum_{s,\alpha} \overline{\nu_{sR}}\, M^{\mathrm{D}}_{s\alpha}\, \nu_{\alpha L} + \mathrm{H.c.}$$

$$\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2} \sum_{\alpha,\beta} \overline{\nu^c_{\alpha L}}\, M^L_{\alpha\beta}\, \nu_{\beta L} + \mathrm{H.c.}$$

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2} \sum_{s,s'} \overline{\nu_{sR}}\, M^R_{ss'}\, \nu^c_{s'R} + \mathrm{H.c.}$$

M^{D}, M^L, M^R are complex matrices

M^L, M^R are symmetric

example:

$$\boxed{\nu^c_{\alpha L} = \mathcal{C}\,\overline{\nu_{\alpha L}}^T \,, \qquad \overline{\nu^c_{\alpha L}} = -\nu^T_{\alpha L}\,\mathcal{C}^\dagger}$$

$$\left.\begin{aligned}
\sum_{\alpha,\beta} \overline{\nu^c_{\alpha L}}\, M^L_{\alpha\beta}\, \nu_{\beta L} &= -\sum_{\alpha,\beta} \nu^T_{\alpha L}\, \mathcal{C}^\dagger\, M^L_{\alpha\beta}\, \nu_{\beta L} \\
&= \sum_{\alpha,\beta} \nu^T_{\beta L}\, (\mathcal{C}^\dagger)^T\, M^L_{\alpha\beta}\, \nu_{\alpha L} \\
\boxed{\mathcal{C}^T = -\mathcal{C}} \rightarrow \quad &= -\sum_{\alpha,\beta} \nu^T_{\beta L}\, \mathcal{C}^\dagger\, M^L_{\alpha\beta}\, \nu_{\alpha L} \\
&= \sum_{\alpha,\beta} \overline{\nu^c_{\beta L}}\, M^L_{\alpha\beta}\, \nu_{\alpha L} \\
\boxed{\alpha \leftrightarrows \beta} \rightarrow \quad &= \sum_{\alpha,\beta} \overline{\nu^c_{\alpha L}}\, M^L_{\beta\alpha}\, \nu_{\beta L}
\end{aligned}\right\} \Longrightarrow$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha}$$

$$\Updownarrow$$

M^L is symmetric!

$$\begin{aligned}\mathcal{L}^{\mathrm{D+M}} &= \mathcal{L}_L^{\mathrm{M}} + \mathcal{L}^{\mathrm{D}} + \mathcal{L}_R^{\mathrm{M}} \\ &= -\frac{1}{2} \sum_{\alpha,\beta} \overline{\nu_{\alpha L}^{c}} \, M_{\alpha\beta}^{L} \, \nu_{\beta L} - \sum_{s,\alpha} \overline{\nu_{sR}} \, M_{s\alpha}^{\mathrm{D}} \, \nu_{\alpha L} - \frac{1}{2} \sum_{s,s'} \overline{\nu_{sR}} \, M_{ss'}^{R} \, \nu_{s'R}^{c} + \mathrm{H.c.}\end{aligned}$$

write Lagrangian in compact form for mass diagonalization

column matrix of left-handed fields: $N_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ $\qquad \nu_L \equiv \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \qquad \nu_R^c \equiv \begin{pmatrix} \nu_{s_1 R}^c \\ \vdots \\ \nu_{s_{\mathcal{N}} R}^c \end{pmatrix}$

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \overline{N_L^c} \, M^{\mathrm{D+M}} \, N_L + \mathrm{H.c.} = \frac{1}{2} N_L^T \, \mathcal{C}^\dagger \, M^{\mathrm{D+M}} \, N_L + \mathrm{H.c.}$$

$(3+\mathcal{N}) \times (3+\mathcal{N})$ symmetric mass matrix: $\qquad M^{\mathrm{D+M}} \equiv \begin{pmatrix} M^L & (M^{\mathrm{D}})^T \\ M^{\mathrm{D}} & M^R \end{pmatrix}$

diagonalization: $\quad N_L = U \, n_L \,, \quad U^T \, M^{\mathrm{D+M}} \, U = M \,, \quad M_{kj} = m_k \, \delta_{kj} \,, \quad m_k \geq 0 \,,$

$U^\dagger = U^{-1}$ $\qquad$ POSSIBLE?

Proof that $M^{\mathrm{D+M}} = (M^{\mathrm{D+M}})^T$ can be diagonalized by $U^T\, M^{\mathrm{D+M}}\, U = M$

an arbitrary complex matrix can be diagonalized by the biunitary transformation

$$V^\dagger\, M^{\mathrm{D+M}}\, W = M\,, \quad M_{kj} = m_k\, \delta_{kj}\,, \quad m_k \geq 0\,, \quad V^\dagger = V^{-1}\,, \quad W^\dagger = W^{-1}$$

$$\left.\begin{array}{l} M^{\mathrm{D+M}} = V\, M\, W^\dagger \\ \quad \| \\ (M^{\mathrm{D+M}})^T = (W^\dagger)^T\, M\, V^T \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} M^{\mathrm{D+M}}(M^{\mathrm{D+M}})^\dagger = V\, M^2\, V^\dagger \\ M^{\mathrm{D+M}}(M^{\mathrm{D+M}})^\dagger = (W^\dagger)^T\, M^2\, W^T \end{array}\right.$$

$$V\, M^2\, V^\dagger = (W^\dagger)^T\, M^2\, W^T \quad \Rightarrow \quad W^T V\, M^2 = M^2\, W^T V$$

$$W^T V = D\,, \quad D_{kj} = e^{2i\lambda_k}\, \delta_{kj}$$

$$\begin{aligned} M^{\mathrm{D+M}} &= V\, M\, W^\dagger = (W^\dagger)^T\, W^T\, V\, M\, W^\dagger = (W^\dagger)^T\, D\, M\, W^\dagger \\ &= (W^\dagger)^T\, D^{1/2}\, M\, D^{1/2}\, W^\dagger = (D^{1/2} W^\dagger)^T\, M\, (D^{1/2} W^\dagger) = (U^\dagger)^T\, M\, U^\dagger \end{aligned}$$

$$\Downarrow$$

$$\boxed{U^T\, M^{\mathrm{D+M}}\, U = M}$$

left-handed

components

of fields with

definite mass

$$n_L \equiv \begin{pmatrix} \nu_{1L} \\ \vdots \\ \nu_{(3+\mathcal{N})L} \end{pmatrix} = U^\dagger N_L \qquad N_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \equiv \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu^c_{s_1 R} \\ \vdots \\ \nu^c_{s_\mathcal{N} R} \end{pmatrix} = U \, n_L$$

$$\begin{aligned} \mathcal{L}^{\text{D+M}} &= -\frac{1}{2} \, \overline{N_L^c} \, M^{\text{D+M}} \, N_L + \text{H.c.} \\ &= -\frac{1}{2} \, \overline{n_L^c} \, M \, n_L + \text{H.c.} = -\frac{1}{2} \sum_{k=1}^{3+\mathcal{N}} m_k \, \overline{\nu^c_{kL}} \, \nu_{kL} + \text{H.c.} \end{aligned}$$

fields with definite mass are Majorana: $n \equiv \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_{3+\mathcal{N}} \end{pmatrix} = n_L + n_L^c = U^\dagger \, N_L + U^T \, N_L^c$

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2} \, \overline{n} \, M \, n = -\frac{1}{2} \sum_{k=1}^{3+\mathcal{N}} m_k \, \overline{\nu_k} \, \nu_k$$

mixing relations:

$$\nu_{\alpha L} = \sum_{k=1}^{3+\mathcal{N}} U_{\alpha k} \, \nu_{kL} \quad (\alpha = e, \mu, \tau)$$

$$\nu^{c}_{sR} = \sum_{k=1}^{3+\mathcal{N}} U_{sk} \, \nu_{kL} \quad (s = s_1, \ldots, s_{\mathcal{N}})$$

Sterile neutrino fields ν_{sR} are connected to Active neutrino fields $\nu_{\alpha L}$ trough the Massive neutrino fields ν_{kL}

$$\Downarrow$$

Active $\leftrightarrows$ Sterile oscillations are possible!

$$\Downarrow$$

disappearance of active neutrinos

Physical Parameters in $N \times N$ Mixing Matrix for Majorana Neutrinos

$N \times N$ Unitary Mixing Matrix $\Rightarrow$ N^2 parameters $\frac{N(N-1)}{2}$ angles, $\frac{N(N+1)}{2}$ phases

not rephasable

$$\text{Weak Charged Current:} \quad j_\rho^{\text{CC}\dagger} = 2 \sum_{\alpha,k} \underset{\substack{\uparrow \\ \text{rephasable}}}{\overline{\ell_{\alpha L}}} \, \gamma_\rho \, U_{\alpha k} \, \overset{\downarrow}{\nu_{kL}}$$

Lagrangian is not invariant under global phase transformations $\nu_k \to e^{i\phi_k} \nu_k$

Majorana mass term: $\nu_{kL}^T \mathcal{C}^{-1} \nu_{kL} \to e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^{-1} \nu_{kL}$ Lepton number is not conserved!

only N phases in the mixing matrix can be eliminated rephasing the charged lepton fields

$$j_\rho^{\text{CC}\dagger} \to 2 \sum_{\alpha,k} \overline{\ell_{\alpha L}} \, \underset{\substack{\uparrow \\ N}}{e^{-i\theta_\alpha}} \, \gamma_\rho \, U_{\alpha k} \, \nu_{kL}$$

number of physical phases: $$\frac{N(N+1)}{2} - N = \frac{N(N-1)}{2}$$ $\left(\begin{array}{c}\text{same number as}\\ \text{mixing angles}\end{array}\right)$

$$\frac{N(N-1)}{2} = \underbrace{\frac{(N-1)(N-2)}{2}}_{\text{“Dirac phases”}} + \underbrace{N-1}_{\text{“Majorana phases”}}$$

$$U_{\alpha k} = U^{(\mathrm{D})}_{\alpha k} e^{i\lambda_{k1}} \,, \quad \underset{\text{overall phase}}{\lambda_{11} = 0} \implies U = U^{(\mathrm{D})} D(\lambda) \,, \quad D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix}$$

Three Light Majorana Neutrinos ($\Leftarrow$ See-Saw)

$N = 3$ $\Longrightarrow$ 3 Mixing Angles 1 Dirac Phase 2 Majorana Phases

standard parameterization (convenient) $(c_{ij} \equiv \cos\vartheta_{ij}\,, \quad s_{ij} \equiv \sin\vartheta_{ij})$

$$
\begin{aligned}
U &= R_{23}\, W_{13}\, R_{12}\, D(\lambda) \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}
\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}
\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}
\end{aligned}
$$

Majorana phases are relevant only in processes involving Lepton number violation

$$\beta\beta_{0\nu}, \qquad \nu_\alpha \leftrightarrows \bar{\nu}_\beta, \qquad \ldots$$

these processes are suppressed by smallness of neutrino masses because of helicity mismatch

in the limit of negligible neutrino massess Dirac = Majorana!

CP invariance

CP invariance of $\mathcal{L}_I^{\mathrm{CC}}$ $\quad\Rightarrow\quad N_L \xrightarrow{\mathrm{CP}} i\,\gamma^0\,N_L^c \quad\Rightarrow\quad N_L^c \xrightarrow{\mathrm{CP}} i\,\gamma^0\,N_L$

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\,\overline{N_L^c}\,M^{\mathrm{D+M}}\,N_L - \frac{1}{2}\,\overline{N_L}\,M^{\mathrm{D+M}*}\,N_L^c \qquad \left(M^{\mathrm{D+M}\,T} = M^{\mathrm{D+M}}\right)$$

$$\mathcal{L}^{\mathrm{D+M}} \xrightarrow{\mathrm{CP}} -\frac{1}{2}\,\overline{N_L}\,M^{\mathrm{D+M}}\,N_L^c - \frac{1}{2}\,\overline{N_L^c}\,M^{\mathrm{D+M}*}\,N_L$$

CP invariance $\quad\Longleftrightarrow\quad M^{\mathrm{D+M}} = M^{\mathrm{D+M}*}$ $\quad$ real!

$$N_L = U\,n_L \qquad n_L = U^\dagger\,N_L \qquad U = \mathcal{O}\,D \qquad D_{kj} = D_k\,\delta_{kj}$$

$$N_L^c = U^*\,n_L^c \qquad n_L^c = U^T\,N_L^c \qquad \mathcal{O}^T\,\mathcal{O} = I \qquad D_k^2 = \pm 1$$

$$n_L = U^\dagger\,N_L \xrightarrow{\mathrm{CP}} i\,U^\dagger\,\gamma^0\,N_L^c = \underbrace{i\,U^\dagger\,U^*}_{\eta}\,\gamma^0\,n_L^c$$

η_k = CP parity of ν_k

$$\eta = i\,U^\dagger\,U^* = i\left(U^T\,U\right)^* = i\left(D\,\mathcal{O}^T\,\mathcal{O}\,D\right)^* = i\,D^2$$

$$\boxed{\eta_k = iD_k^2 = \pm i}$$

important: relative CP parities $\eta_{kj} \equiv \eta_k/\eta_j = D_k^2/D_j^2 = \pm 1$

standard parameterization of CP-invariant Majorana mixing matrix

$$U = R_{23}\, R_{13}\, R_{12}\, D(\lambda)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\lambda_{kj} = 0\,,\, \frac{\pi}{2} \quad \Longleftrightarrow \quad \eta_{kj} = e^{2i\lambda_{kj}} = \pm 1$$

equal or opposite

CP parities

if $\lambda_{kj} = \frac{\pi}{2} \quad \Longrightarrow \quad e^{i\lambda_{kj}} = i \quad \Longrightarrow$ complex U!

Neutrinoless Double-β Decay ($\beta\beta_{0\nu}$): $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

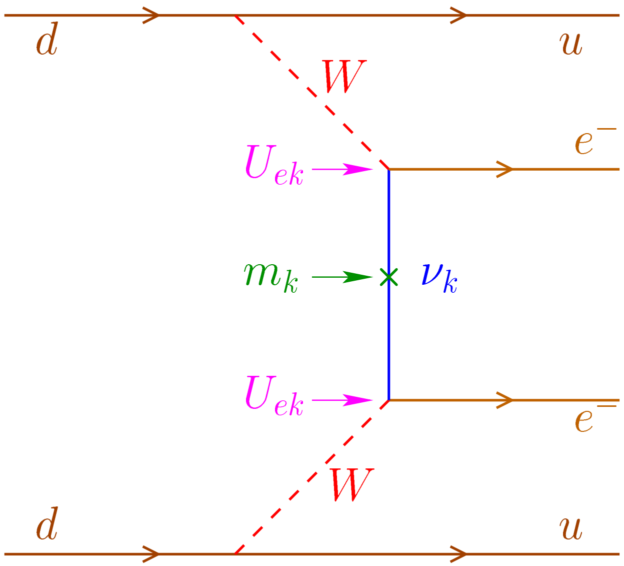
$$\Gamma_{\beta\beta_{0\nu}} \propto |\langle m \rangle|^2$$

effective
Majorana
mass

$$|\langle m \rangle| = \left| \sum_k U_{ek}^2 \, m_k \right|$$

examples:

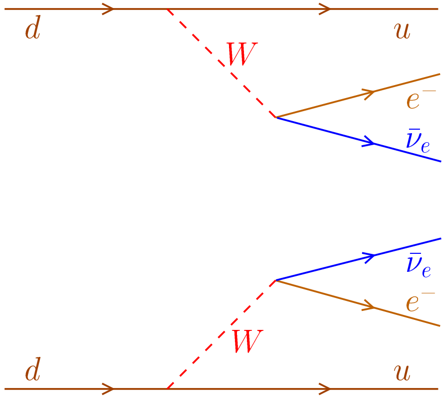
$$\begin{cases} {}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^- \\ {}^{100}\mathrm{Mo} \to {}^{100}\mathrm{Ru} + e^- + e^- \\ {}^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe} + e^- + e^- \\ {}^{136}\mathrm{Xe} \to {}^{136}\mathrm{Ba} + e^- + e^- \end{cases}$$



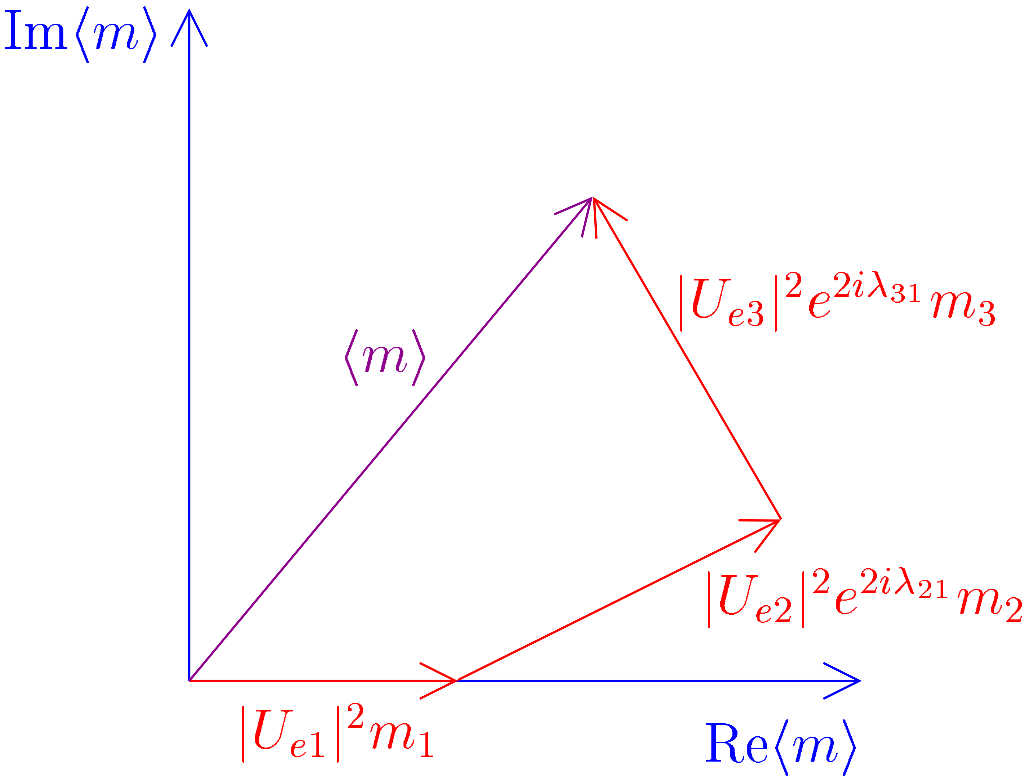
Two-Neutrino Double-β Decay ($\Delta L = 0$)

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

second order weak interaction process



$$|\langle m \rangle| = \left| \sum_k U_{ek}^2 \, m_k \right|$$



complex U_{ek} ⇒ possible cancellations among m_1, m_2, m_3 contributions!

$$|\langle m \rangle| = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{2i\lambda_{21}} m_2 + |U_{e3}|^2 e^{2i\lambda_{31}} m_3 \right|$$

conserved CP $\Longrightarrow$ $\lambda_{kj} = 0 \, , \, \frac{\pi}{2}$ $\Longrightarrow$ $e^{2i\lambda_{kj}} = \eta_{kj} = \pm 1$

opposite CP parities of ν_k and ν_j $\Longrightarrow$ $e^{2i\lambda_{kj}} = -1$ $\Longrightarrow$ maximal cancellation!

EXAMPLE: 2 MASSIVE NEUTRINOS

$$|\langle m \rangle| = \left| |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{2i\lambda_{21}} \, m_2 \right|$$

$$\lambda_{21} = \frac{\pi}{2} \quad \Longrightarrow \quad |\langle m \rangle| = \left| |U_{e1}^2| \, m_1 \underset{\uparrow}{-} |U_{e2}^2| \, m_2 \right|$$

conserved CP
opposite CP parities

cancellation

if $m_1 \simeq m_2$ and $|U_{e1}^2| \simeq |U_{e2}^2| \simeq 1/2 \Longrightarrow |\langle m \rangle|$ can be extremely small!

Dirac neutrino: perfect cancellation

$$1 \text{ Dirac neutrino} \equiv 2 \text{ Majorana neutrinos with} \begin{cases} \text{equal mass} \\ \text{maximal mixing} \\ \text{opposite CP parities} \end{cases}$$

$$\left.\begin{aligned} m_1 &= m_2 \\ |U_{e1}|^2 = |U_{e2}|^2 &= 1/2 \\ \lambda_{21} &= \pi/2 \end{aligned}\right\} \quad \Longrightarrow \quad |\langle m \rangle| = 0$$

See-Saw Mechanism

$$M^L = 0 \Longrightarrow M^{\text{D+M}} = \begin{pmatrix} 0 & (M^{\text{D}})^T \\ M^{\text{D}} & M^R \end{pmatrix}$$

eigenvalues of M^R $\gg$ eigenvalues of M^{D} $\Longrightarrow$ $M^{\text{D+M}}$ is block-diagonalized

$$W^T\, M^{\text{D+M}}\, W \simeq \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix} \qquad W^\dagger \simeq W^{-1}$$

corrections $\sim (M^R)^{-1} M^{\text{D}}$

$$W = 1 - \frac{1}{2} \begin{pmatrix} (M^{\text{D}})^\dagger (M^R (M^R)^\dagger)^{-1} M^{\text{D}} & 2 (M^{\text{D}})^\dagger (M^R)^{\dagger\,-1} \\ -2 (M^R)^{-1} M^{\text{D}} & (M^R)^{-1} M^{\text{D}} (M^{\text{D}})^\dagger (M^R)^{\dagger\,-1} \end{pmatrix}$$

$$\boxed{M_{\text{light}} \simeq -(M^{\text{D}})^T\, (M^R)^{-1}\, M^{\text{D}}} \qquad \boxed{M_{\text{heavy}} \simeq M^R}$$

$$\boxed{M_{\text{light}} \simeq -(M^{\text{D}})^T \, (M^R)^{-1} \, M^{\text{D}}}$$

$M^R = \mathcal{M} \, I \quad \Longrightarrow$ QUADRATIC SEE-SAW $\qquad \mathcal{M} =$ high energy scale

$$M_{\text{light}} \simeq - \frac{(M^{\text{D}})^T \, M^{\text{D}}}{\mathcal{M}} \qquad \Longrightarrow \qquad m_k \sim \frac{(m_k^f)^2}{\mathcal{M}}$$

$$m_1 : m_2 : m_3 \sim (m_1^f)^2 : (m_2^f)^2 : (m_3^f)^2$$

$M^R = \dfrac{\mathcal{M}}{\mathcal{M}_{\text{D}}} \, M_{\text{D}} \quad \Longrightarrow$ LINEAR SEE-SAW $\qquad \mathcal{M}_{\text{D}} =$ scale of M_{D}

$$M_{\text{light}} \simeq - \frac{\mathcal{M}_{\text{D}}}{\mathcal{M}} \, M^{\text{D}} \qquad \Longrightarrow \qquad m_k \sim \frac{\mathcal{M}_{\text{D}}}{\mathcal{M}} \, m_k^f$$

$$m_1 : m_2 : m_3 \sim m_1^f : m_2^f : m_3^f$$

Summary of Part 1: Neutrino Masses and Mixing

in the "Standard Model" neutrino are massless by construction

implementation of "two-component theory"

"Standard Model" can be naturally extended to include neutrino masses

add $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$

surprise: Majorana Masses

known natural explanations of smallness of ν masses

See-Saw Mechanism, Effective Lagrangian

$\Downarrow$

Majorana ν Masses, New High Energy Scale

$\Downarrow$

Neutrino Masses are powerful window on New Physics Beyond Standard Model

Part 2: Neutrino Oscillations in Vacuum and in Matter

Detectable Neutrinos are Extremely Relativistic

Only neutrinos with energy larger than some fraction of MeV are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered} \nu + A \to B + C \\ \Downarrow \\ s = 2 E m_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2 m_A} - \frac{m_A}{2} \end{gathered}}$$

☼ $\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$	$E_{\text{th}} = 0.81\,\mathrm{MeV}$
☼ $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$	$E_{\text{th}} = 0.233\,\mathrm{MeV}$
♁ $\bar{\nu}_e + p \to n + e^+$	$E_{\text{th}} = 1.8\,\mathrm{MeV}$
♁ $\nu_\mu + n \to p + \mu^-$	$E_{\text{th}} = 110\,\mathrm{MeV}$
♁ $\nu_\mu + e^- \to \nu_e + \mu^-$	$E_{\text{th}} \simeq \frac{m_\mu^2}{2 m_e} = 10.9\,\mathrm{GeV}$

Elastic Scattering Processes: Cross Section $\propto$ Energy

☼ $\nu + e^- \to \nu + e^-$ $\qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\mathrm{cm}^2$

Background $\Rightarrow$ $E_{\text{th}} \simeq 5\,\mathrm{MeV}$ (SK, SNO)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\mathrm{eV}}$

Easy Example of Neutrino Production: $\pi^+ \to \mu^+ + \nu_\mu \quad \pi^- \to \mu^- + \bar{\nu}_\mu$

two-body decay $\Longrightarrow$ fixed kinematics

$$\boxed{E_k^2 = p_k^2 + m_k^2}$$

π at rest:
$$\begin{cases} p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \\ E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \end{cases}$$

0^{th} order: $m_k = 0 \Rightarrow p_k = E_k = E = \dfrac{m_\pi}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1^{st} order: $\boxed{E_k \simeq E + \xi\,\dfrac{m_k^2}{2E}}$ $\boxed{p_k \simeq E - (1-\xi)\,\dfrac{m_k^2}{2E}}$ $\xi = \dfrac{1}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$

$\nwarrow \quad \nearrow$

general!

Neutrino Oscillations in Vacuum: Plane Wave Model

Neutrino Production: $j_\rho^{\mathrm{CC}} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\, \gamma_\rho\, \ell_{\alpha L}$ $\qquad \nu_{\alpha L} = \sum_k U_{\alpha k}\, \nu_{kL}$ Fields

$$\langle 0|\nu_{\alpha L}|\nu_\beta\rangle = \sum_{k,j} U_{\alpha k} U_{\beta j}^* \underbrace{\langle 0|\nu_{kL}|\nu_j\rangle}_{\propto \delta_{kj}} \propto \sum_k U_{\alpha k} U_{\beta k}^* = \delta_{\alpha\beta}$$

$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^*\, |\nu_k\rangle$ States

$$|\nu_k(x,t)\rangle = e^{-iE_k t + i p_k x}|\nu_k\rangle \quad \Longrightarrow \quad |\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k}^*\, e^{-iE_k t + i p_k x}\, |\nu_k\rangle$$

$\uparrow$

$$\boxed{|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k}\, |\nu_\beta\rangle}$$

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(x,t)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha \to \nu_\beta}(x,t) = |\langle \nu_\beta|\nu_\alpha(x,t)\rangle|^2 = \left|\mathcal{A}_{\nu_\alpha \to \nu_\beta}(x,t)\right|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right|^2$$

ultrarelativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_k U_{\alpha k}^* \, e^{-i m_k^2 L / 2E} \, U_{\beta k} \right|^2$$

$$= \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2$$ $\Leftarrow$ constant term

$$+ \, 2 \mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$ $\Leftarrow$ oscillating term

$\Updownarrow$

coherence

$$\boxed{\Delta m_{kj}^2 \equiv m_k^2 - m_j^2}$$

NEUTRINOS AND ANTINEUTRINOS

antineutrinos are described by CP-conjugated fields: $\nu^{\text{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Righ-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{CP} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L,E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\Longrightarrow$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

indeed, $$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

is invariant under CPT: $U \leftrightarrows U^*$ $\quad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

in particular $\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$\boxed{\text{CPT} \quad \Rightarrow \quad A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}$$

$$A_{\alpha\beta}^{\text{CP}}(L,E) = 2\text{Re} \sum_{k>j} U_{\alpha k}^{*} U_{\beta k} U_{\alpha j} U_{\beta j}^{*} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) - 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^{*} U_{\alpha j}^{*} U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\boxed{A_{\alpha\beta}^{\text{CP}}(L,E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \, \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}$$

Jarlskog rephasing $\left(U_{\alpha k} \to e^{i\lambda_\alpha} U_{\alpha k} e^{i\eta_k}\right)$ invariants: $\boxed{J_{\alpha\beta;kj} = \text{Im}\left[U_{\alpha k}^{*} U_{\beta k} U_{\alpha j} U_{\beta j}^{*}\right]}$

violation of CP symmetry depends only on Dirac phases

(three neutrinos: $J_{\alpha\beta;kj} = \pm c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$)

$\left\langle A_{\alpha\beta}^{\text{CP}} \right\rangle = 0 \quad \Longrightarrow$ observation of CP violation needs measurement of oscillations

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

CPT $\Longrightarrow$

$$\begin{aligned} 0 &= A^{\text{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} = A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}} \end{aligned}$$

$$\boxed{A^{\text{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \, \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

violation of T symmetry depends only on Dirac phases

$\langle A^{\text{T}}_{\alpha\beta} \rangle = 0 \quad \Longrightarrow$ observation of T violation needs measurement of oscillations

Two Generations $(k = 1, 2)$

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \Delta m^2 \equiv \Delta m^2_{21} \equiv m^2_2 - m^2_1$$

Transition Probability $(\alpha \neq \beta)$: $$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probability $(\alpha = \beta)$: $$P_{\nu_\alpha \to \nu_\alpha}(L, E) = 1 - P_{\nu_\alpha \to \nu_\beta}(L, E)$$

Averaged Transition Probability: $$\langle P_{\nu_\alpha \to \nu_\beta} \rangle = \frac{1}{2} \sin^2 2\vartheta$$

TYPES OF EXPERIMENTS

Two-Neutrino Mixing $\Rightarrow$ $\boxed{P_{\nu_\alpha \to \nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$ observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL (high statistics)

$L/E \lesssim 1\,\text{eV}^{-2} \implies \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor SBL: $L \sim 10\,\text{m},\ E \sim 1\,\text{MeV}$

Accelerator SBL: $L \sim 1\,\text{km},\ E \gtrsim 1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor LBL: $L \sim 1\,\text{km},\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator LBL: $L \sim 10^3\,\text{km},\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km},\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \implies \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,, \quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO

Matter Effect (MSW) $\implies$ $10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1 \qquad 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

MSW effect (resonant transitions in matter)

a flavor neutrino ν_α with momentum p is described by $|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$

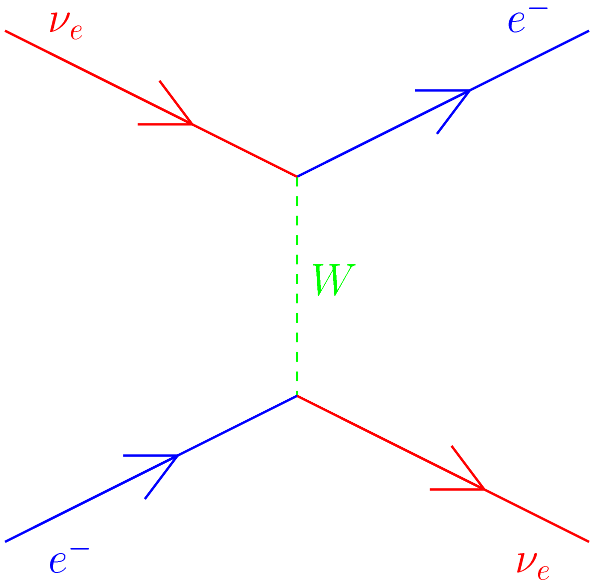
$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

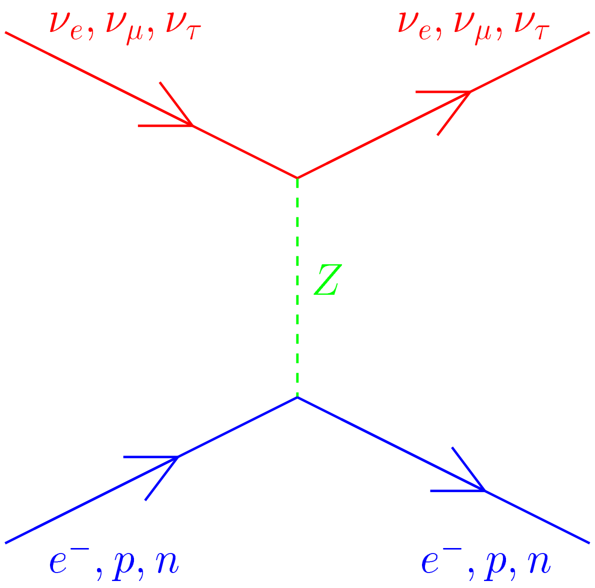
in matter $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$ $\qquad \mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$

$V_\alpha =$ effective potential due to coherent interactions with medium

forward elastic CC and NC scattering

EFFECTIVE POTENTIAL IN MATTER





$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$V_e = V_{\text{CC}} + V_{\text{NC}}$ $\qquad V_\mu = V_\tau = V_{\text{NC}}$ (common phase) $\Longrightarrow$ $V_e - V_\mu = V_{\text{CC}}$

antineutrinos: $\qquad \overline{V}_{\text{CC}} = -V_{\text{CC}} \qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Schrödinger picture: $$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\nu_\alpha(p,t)\rangle = \mathcal{H}|\nu_\alpha(p,t)\rangle\,, \qquad |\nu_\alpha(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes: $$\varphi_{\alpha\beta}(p,t) = \langle\nu_\beta(p)|\nu_\alpha(p,t)\rangle\,, \qquad \varphi_{\alpha\beta}(p,0) = \delta_{\alpha\beta}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_{\alpha\beta}(p,t) = \langle\nu_\beta(p)|\mathcal{H}|\nu_\alpha(p,t)\rangle = \langle\nu_\beta(p)|\mathcal{H}_0|\nu_\alpha(p,t)\rangle + \langle\nu_\beta(p)|\mathcal{H}_I|\nu_\alpha(p,t)\rangle$$

$$\begin{aligned}\langle\nu_\beta(p)|\mathcal{H}_0|\nu_\alpha(p,t)\rangle &= \sum_\rho \langle\nu_\beta(p)|\mathcal{H}_0|\nu_\rho(p)\rangle\,\underbrace{\langle\nu_\rho(p)|\nu_\alpha(p,t)\rangle}_{\varphi_{\alpha\rho}(p,t)} \\ &= \sum_\rho\sum_{k,j} U_{\beta k}\,\underbrace{\langle\nu_k(p)|\mathcal{H}_0|\nu_j(p)\rangle}_{\delta_{kj}E_k}\,U^*_{\rho j}\,\varphi_{\alpha\rho}(p,t)\end{aligned}$$

$$\langle\nu_\beta(p)|\mathcal{H}_I|\nu_\alpha(p,t)\rangle = \sum_\rho \underbrace{\langle\nu_\beta(p)|\mathcal{H}_I|\nu_\rho(p)\rangle}_{\delta_{\beta\rho}V_\beta}\,\varphi_{\alpha\rho}(p,t) = V_\beta\,\varphi_{\alpha\beta}(p,t)$$

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_{\alpha\beta} = \sum_\rho\left(\sum_k U_{\beta k}\,E_k\,U^*_{\rho k} + \delta_{\beta\rho}\,V_\beta\right)\varphi_{\alpha\rho}}$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $\quad E = p \quad t = x$

$$V_e = V_{\mathrm{CC}} + V_{\mathrm{NC}} \qquad V_\mu = V_\tau = V_{\mathrm{NC}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \varphi_{\alpha\beta}(p,x) = (p + V_{\mathrm{NC}})\, \varphi_{\alpha\beta}(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e}\, \delta_{\rho e}\, V_{\mathrm{CC}} \right) \varphi_{\alpha\rho}(p,x)$$

$$\psi_{\alpha\beta}(p,x) = \varphi_{\alpha\beta}(p,x)\, e^{ipx + i\int_0^x V_{\mathrm{NC}}(x')\,\mathrm{d}x'}$$

$$\Downarrow$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \psi_{\alpha\beta} = e^{ipx + i\int_0^x V_{\mathrm{NC}}(x')\,\mathrm{d}x'} \left(-p - V_{\mathrm{NC}} + i \frac{\mathrm{d}}{\mathrm{d}x} \right) \varphi_{\alpha\beta}$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \psi_{\alpha\beta} = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e}\, \delta_{\rho e}\, V_{\mathrm{CC}} \right) \psi_{\alpha\rho}}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_{\alpha\beta}|^2 = |\psi_{\alpha\beta}|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\,\Psi_\alpha = \frac{1}{2E}\left(U\,\mathbb{M}^2\,U^\dagger + \mathbb{A}\right)\Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha\mu} \\ \psi_{\alpha\tau} \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} A_{\mathrm{CC}} &= 2EV_{\mathrm{CC}} \\ &= 2\sqrt{2}EG_{\mathrm{F}}N_e \end{aligned}$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U\,\mathbb{M}^2\,U^\dagger \xrightarrow{\text{matter}} U\,\mathbb{M}^2\,U^\dagger + 2\,E\,\underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

potential due to coherent forward elastic scattering

effective mass-squared matrix in matter

simplest case: $\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$ (two-neutrino mixing)

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta m_1^2 + \sin^2\vartheta m_2^2 & \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) \\ \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) & \sin^2\vartheta m_1^2 + \cos^2\vartheta m_2^2 \end{pmatrix} = \underset{\uparrow}{\frac{1}{2}\,\Sigma m^2} + \frac{1}{2}\begin{pmatrix} -\Delta m^2\cos2\vartheta & \Delta m^2\sin2\vartheta \\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta \end{pmatrix}$$

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_{ee}\\\psi_{e\mu}\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin 2\vartheta\\\Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_{ee}\\\psi_{e\mu}\end{pmatrix}}$$

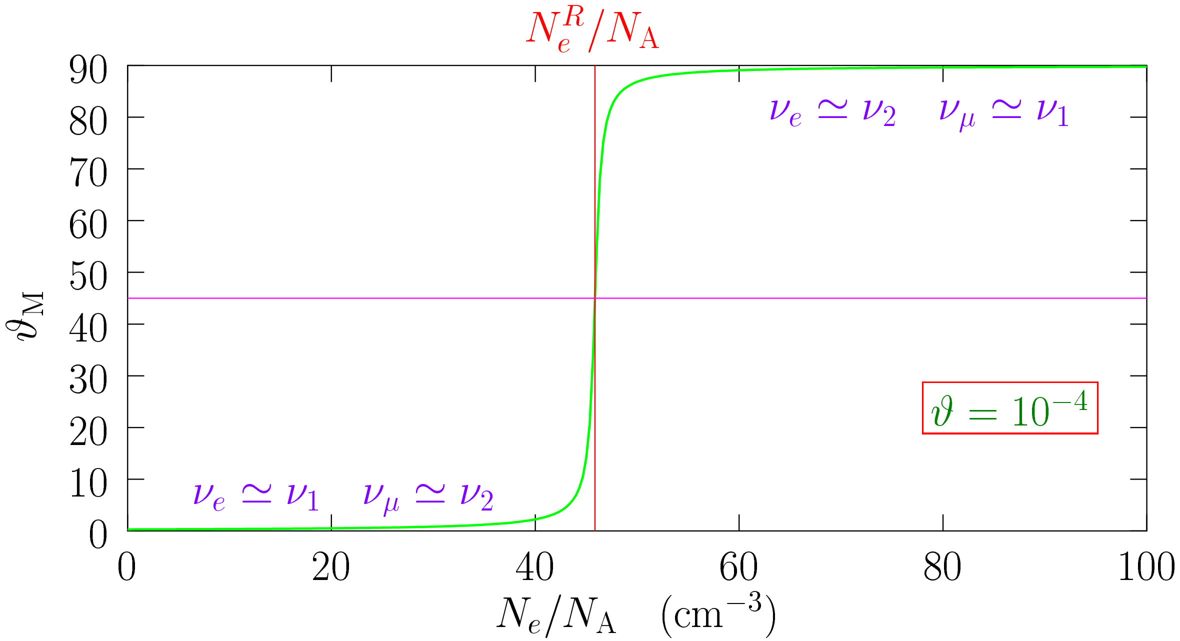
initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_{ee}(0)\\\psi_{e\mu}(0)\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$

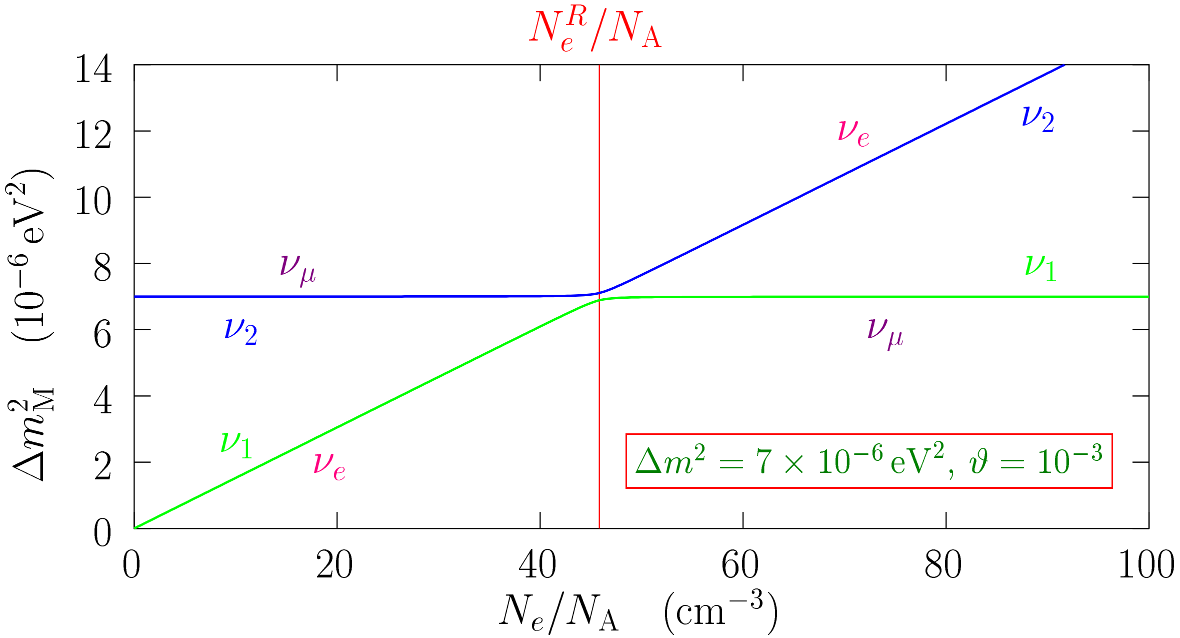
$$\boxed{\begin{aligned}&P_{\nu_e\to\nu_\mu}(x)=|\psi_{e\mu}(x)|^2\\&P_{\nu_e\to\nu_e}(x)=|\psi_{ee}(x)|^2=1-P_{\nu_e\to\nu_\mu}(x)\end{aligned}}$$

Diagonalization $\Longrightarrow$ Effective Mixing Angle in Matter: $\tan 2\vartheta_{\mathrm{M}}=\dfrac{\tan 2\vartheta}{1-\dfrac{A_{\mathrm{CC}}}{\Delta m^2\cos 2\vartheta}}$

Resonance ($\vartheta_{\mathrm{M}}=\pi/4$): $A_{\mathrm{CC}}^{\mathrm{R}}=\Delta m^2\cos 2\vartheta$ $\Longrightarrow$ $N_e^{\mathrm{R}}=\dfrac{\Delta m^2\cos 2\vartheta}{2\sqrt{2}EG_{\mathrm{F}}}$

Effective Squared-Mass Difference: $\Delta m^2_{\mathrm{M}}=\sqrt{\left(\Delta m^2\cos 2\vartheta-A_{\mathrm{CC}}\right)^2+\left(\Delta m^2\sin 2\vartheta\right)^2}$





$$\begin{aligned} \nu_e &= \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2 \\ \nu_\mu &= -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2 \end{aligned}$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

$$\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E} \begin{pmatrix} -\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}} \end{pmatrix} + \underset{\substack{\uparrow \\ \text{maximum near resonance}}}{\begin{pmatrix} 0 & -i \dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} \\ i \dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0 \end{pmatrix}} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} & -\sin\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} & \cos\vartheta^0_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} \end{pmatrix}$$

$$\begin{aligned} \psi_1(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{11} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{21} \right] \\ & \times \exp\left(i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \\ \psi_2(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{12} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{22} \right] \\ & \times \exp\left(-i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \end{aligned}$$

$$\psi_{ee}(x) = \cos\vartheta_{\text{M}}^{x}\,\psi_1(x) + \sin\vartheta_{\text{M}}^{x}\,\psi_2(x)$$

neglect phases (averaged over energy spectrum)

$$\begin{aligned}\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_{ee}(x)\rangle| = {} & \cos^2\vartheta_{\text{M}}^{x}\,\cos^2\vartheta_{\text{M}}^{0}\,|\mathcal{A}_{11}^{\text{R}}|^2 + \cos^2\vartheta_{\text{M}}^{x}\,\sin^2\vartheta_{\text{M}}^{0}\,|\mathcal{A}_{21}^{\text{R}}|^2 \\ & + \sin^2\vartheta_{\text{M}}^{x}\,\cos^2\vartheta_{\text{M}}^{0}\,|\mathcal{A}_{12}^{\text{R}}|^2 + \sin^2\vartheta_{\text{M}}^{x}\,\sin^2\vartheta_{\text{M}}^{0}\,|\mathcal{A}_{22}^{\text{R}}|^2\end{aligned}$$

$|\mathcal{A}_{11}^{\text{R}}|^2 = |\mathcal{A}_{22}^{\text{R}}|^2 = 1 - P_{\text{c}}$ $\quad$ $|\mathcal{A}_{12}^{\text{R}}|^2 = |\mathcal{A}_{21}^{\text{R}}|^2 = P_{\text{c}}$ $\quad$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos2\vartheta_{\text{M}}^{0}\,\cos2\vartheta_{\text{M}}^{x}}$$

[Parke, PRL 57 (1986) 1275]

CROSSING PROBABILITY

$$P_{\mathrm{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $\gamma = \left.\frac{\Delta m^2_{\mathrm{M}}/2E}{2|\mathrm{d}\vartheta_{\mathrm{M}}/\mathrm{d}x|}\right|_{\mathrm{R}} = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{\mathrm{d}\ln A_{\mathrm{CC}}}{\mathrm{d}x}\right|_{\mathrm{R}}}$

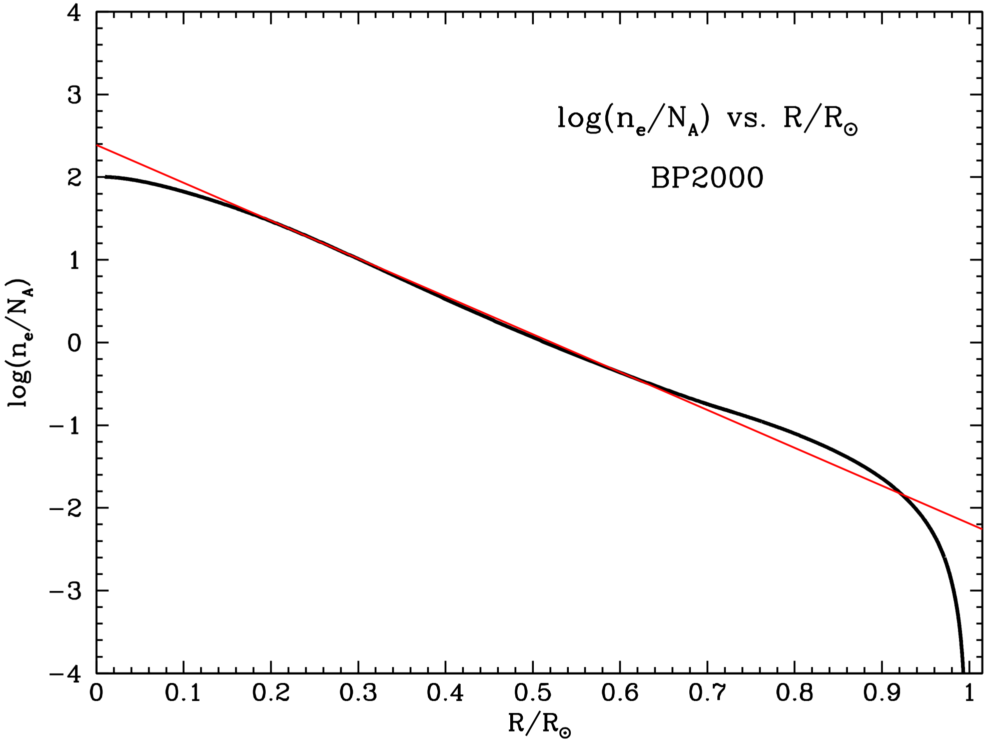
$A \propto x$ $\quad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $\quad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $\quad F = 1 - \tan^2\vartheta$ [Pizzochero, PRD 36 (1987) 2293, Toshev, PLB 196 (1987) 170, Petcov, PLB 200 (1988) 373]

[Kuo, Pantaleone, RMP 61 (1989) 937]

SUN: $$N_e(x) \simeq N_e^{\mathrm{c}} \exp\left(-\frac{x}{x_0}\right) \qquad N_e^{\mathrm{c}} = 245\, N_{\mathrm{A}}/\mathrm{cm}^3 \qquad x_0 = \frac{R_\odot}{10.54}$$



$$\overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right) \cos 2\vartheta^0_{\mathrm{M}}\, \cos 2\vartheta$$

$$P_{\mathrm{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{\mathrm{d} \ln A_{\mathrm{CC}}}{\mathrm{d}x}\right|_{\mathrm{R}}}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

practical prescription:

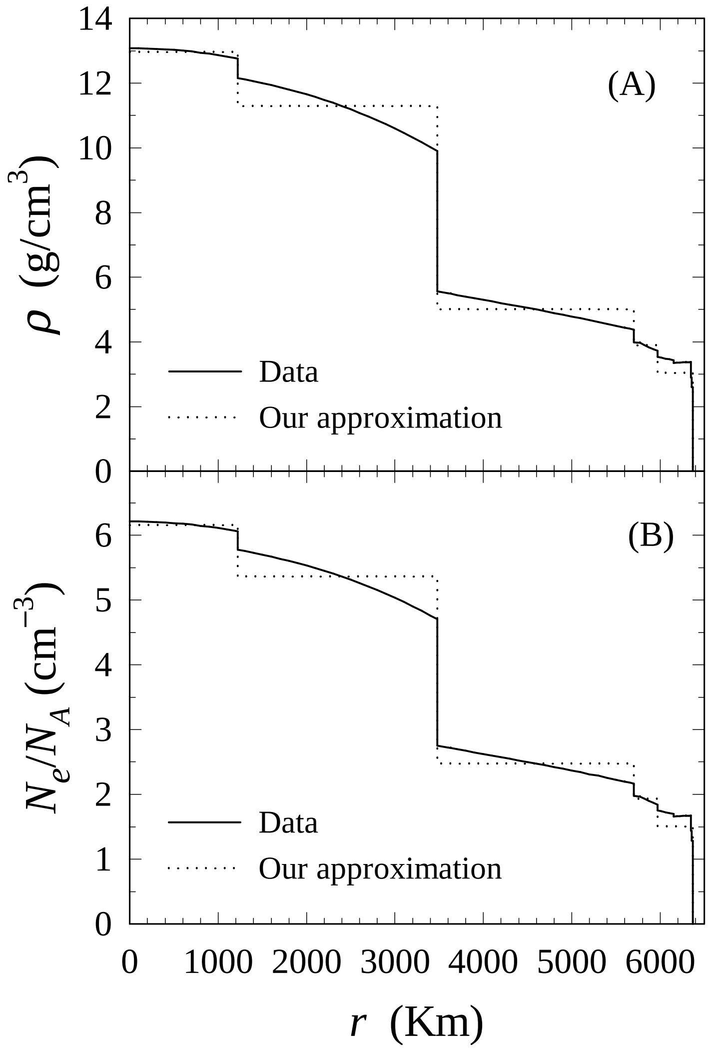
[Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } \left|\mathrm{d}\ln A_{\mathrm{CC}}/\mathrm{d}x\right|_{\mathrm{R}} & \text{for } x \leq 0.904 R_\odot \\ \left|\mathrm{d}\ln A_{\mathrm{CC}}/\mathrm{d}x\right|_{\mathrm{R}} \to \dfrac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Earth Matter Effect:

$$P^{\text{sun+earth}}_{\nu_e\to\nu_e} = \overline{P}^{\text{sun}}_{\nu_e\to\nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e\to\nu_e}\right)\left(P^{\text{earth}}_{\nu_2\to\nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



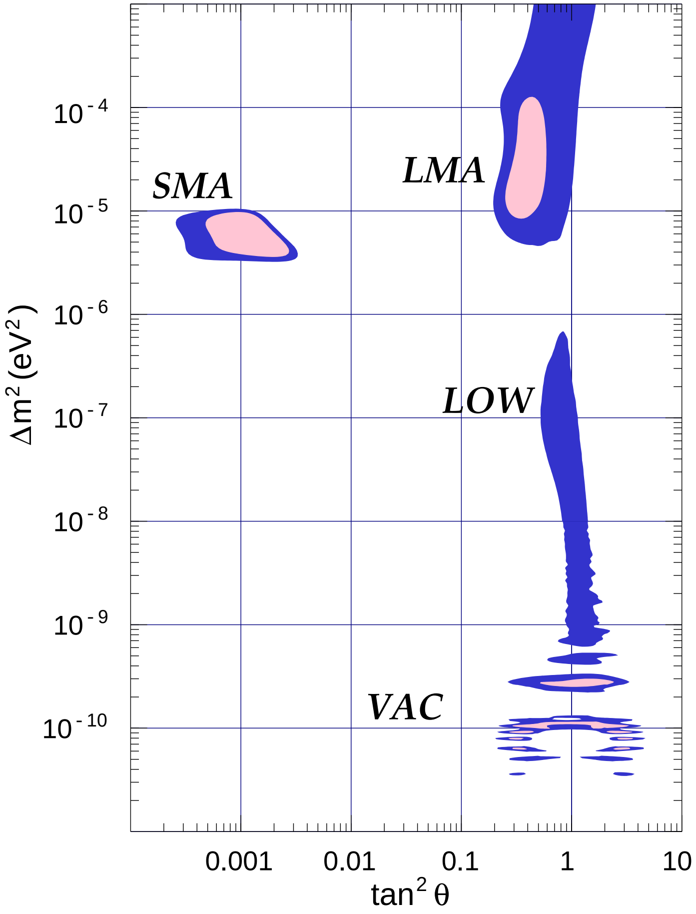
[Giunti, Kim, Monteno, NP B 521 (1998) 3]

$P^{\text{earth}}_{\nu_2\to\nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

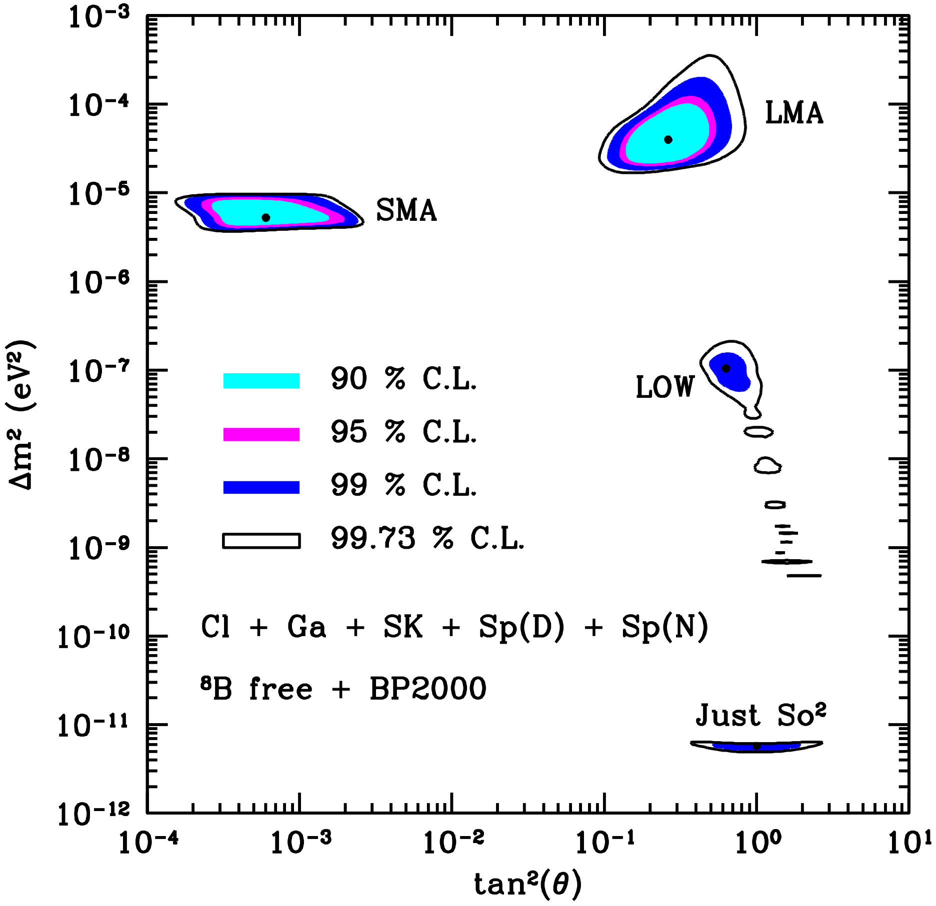
Effective massive neutrinos propagate as plane waves in regions of constant density.

Wave functions of flavor neutrinos are joined at the boundaries of steps.

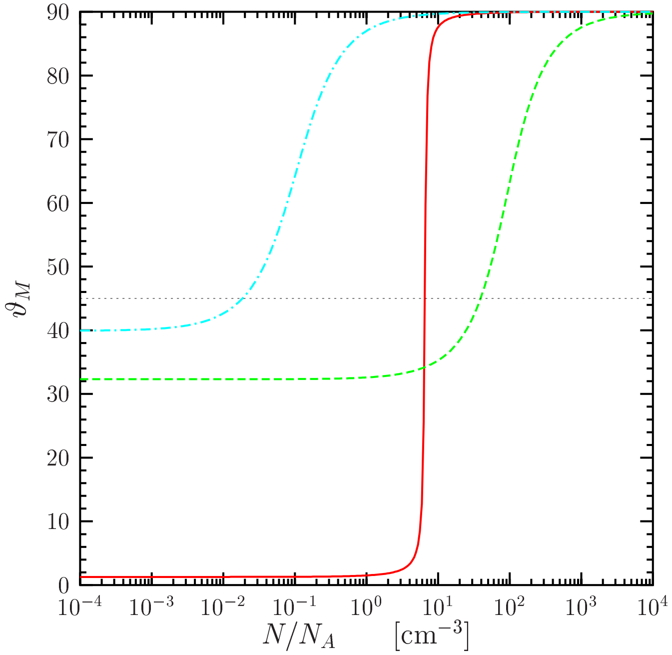
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\text{eV}^2\,,$	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\text{eV}^2\,,$	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\text{eV}^2\,,$	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\text{eV}^2\,,$	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\text{eV}^2\,,$	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



solid line: $\Delta m^2 = 5 \times 10^{-6}\,\mathrm{eV}^2$

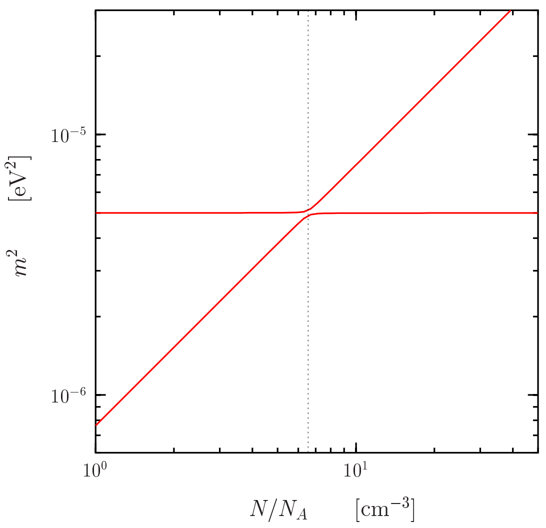
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\mathrm{eV}^2$

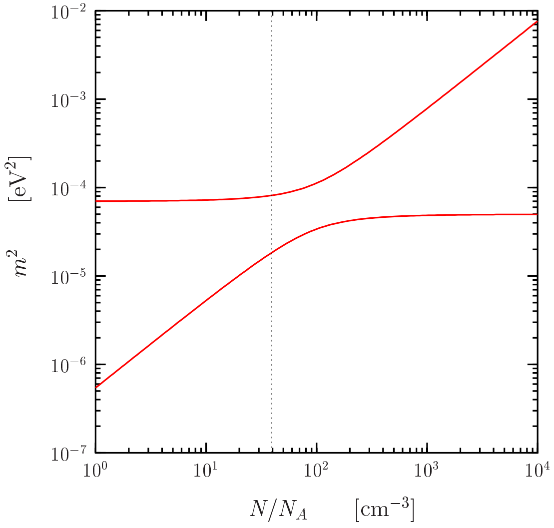
(typical LMA) $\tan^2\vartheta = 0.4$

dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\mathrm{eV}^2$

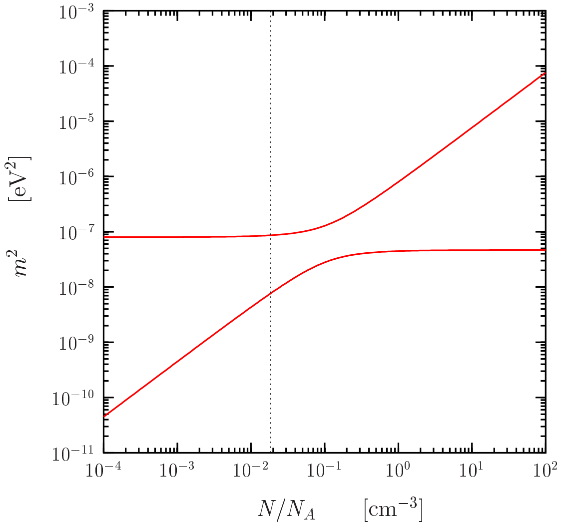
(typical LOW) $\tan^2\vartheta = 0.7$



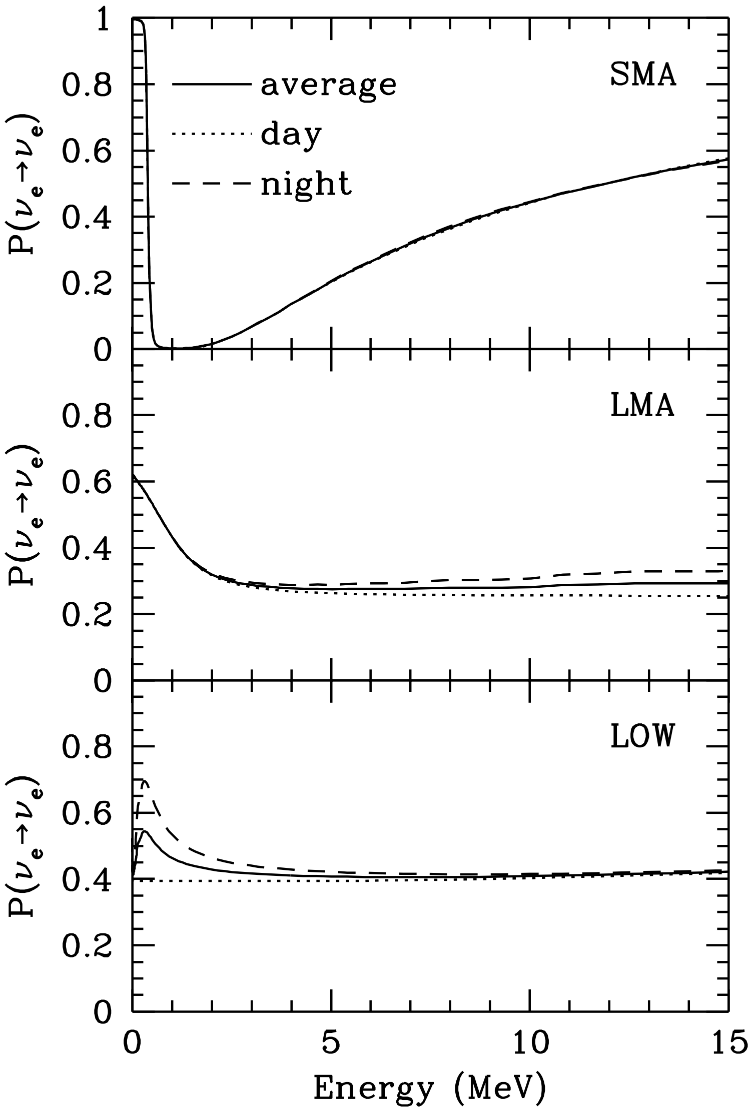
typical SMA



typical LMA



typical LOW

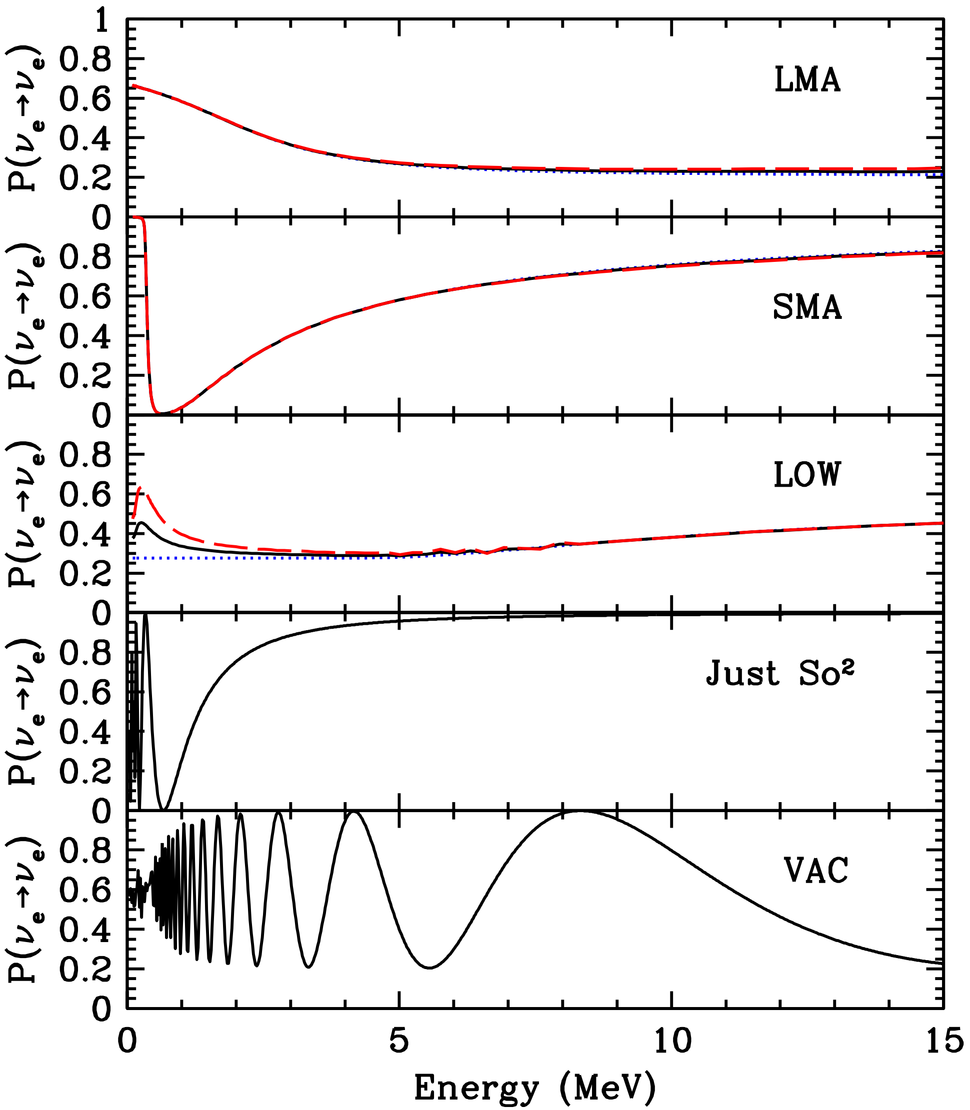


[Bahcall, Krastev, Smirnov, PRD 58 (1998) 096016]

SMA: $\Delta m^2 = 5.0 \times 10^{-6}\,\text{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$

LMA: $\Delta m^2 = 1.6 \times 10^{-5}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.57$

LOW: $\Delta m^2 = 7.9 \times 10^{-8}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.95$



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]

LMA: $\Delta m^2 = 4.2 \times 10^{-5}\,\text{eV}^2$ $\tan^2\vartheta = 0.26$

SMA: $\Delta m^2 = 5.2 \times 10^{-6}\,\text{eV}^2$ $\tan^2\vartheta = 5.5 \times 10^{-4}$

LOW: $\Delta m^2 = 7.6 \times 10^{-8}\,\text{eV}^2$ $\tan^2\vartheta = 0.72$

Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\,\text{eV}^2$ $\tan^2\vartheta = 1.0$

VAC: $\Delta m^2 = 1.4 \times 10^{-10}\,\text{eV}^2$ $\tan^2\vartheta = 0.38$

IN NEUTRINO OSCILLATIONS DIRAC ~ MAJORANA

Evolution of Amplitudes: $\frac{\mathrm{d}\nu_\alpha}{\mathrm{d}t} = \frac{1}{2E}\left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\nu_\beta$

difference: $\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})}D(\lambda) \end{cases}$

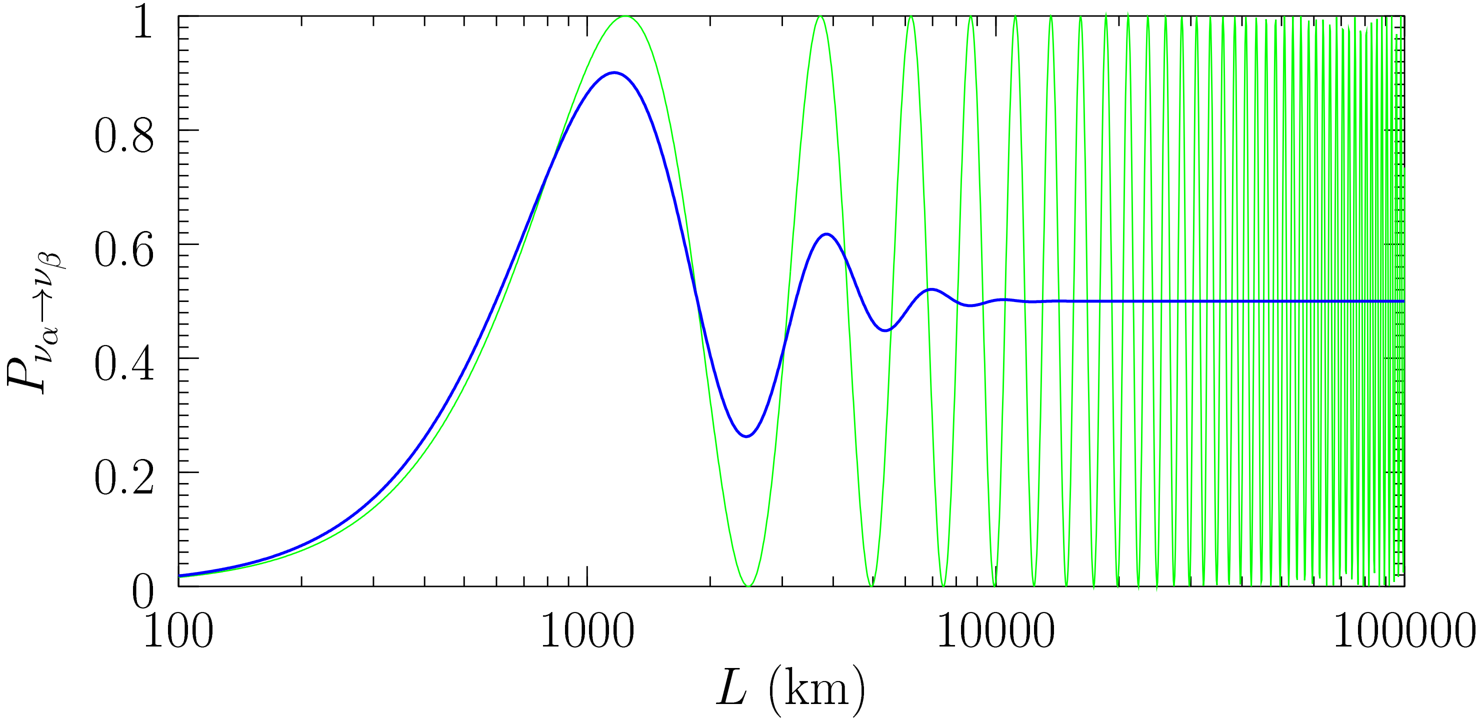
$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2D \quad \Longrightarrow \quad DM^2D^\dagger = M^2$$

$$U^{(\mathrm{M})}M^2(U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})}DM^2D^\dagger(U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})}M^2(U^{(\mathrm{D})})^\dagger$$

AVERAGE OVER ENERGY SPECTRUM

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\sin^2 2\vartheta\left[1-\cos\left(\frac{\Delta m^2 L}{2E}\right)\right] \qquad (\alpha\neq\beta)$$

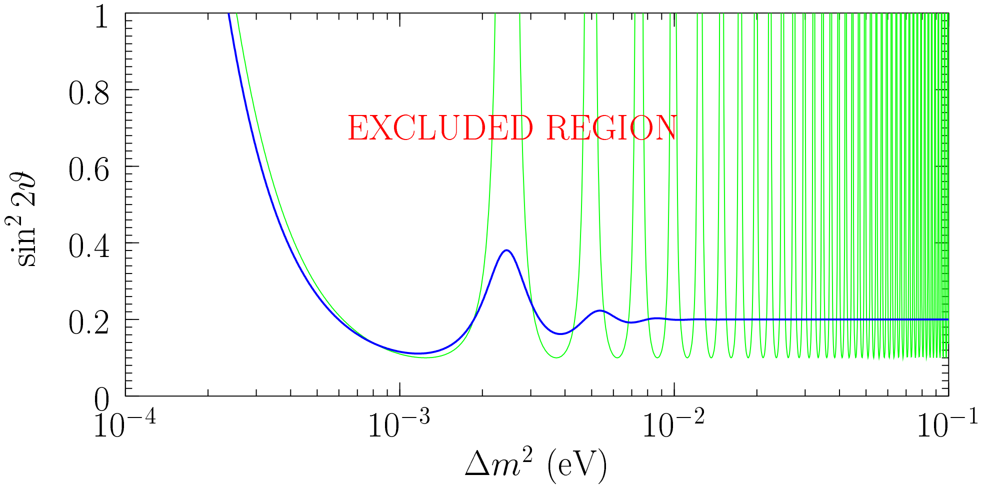


$\Delta m^2 = 10^{-3}\,\text{eV}$ $\qquad \sin^2 2\vartheta = 1$ $\qquad \langle E\rangle = 1\,\text{GeV}$ $\qquad \Delta E = 0.2\,\text{GeV}$

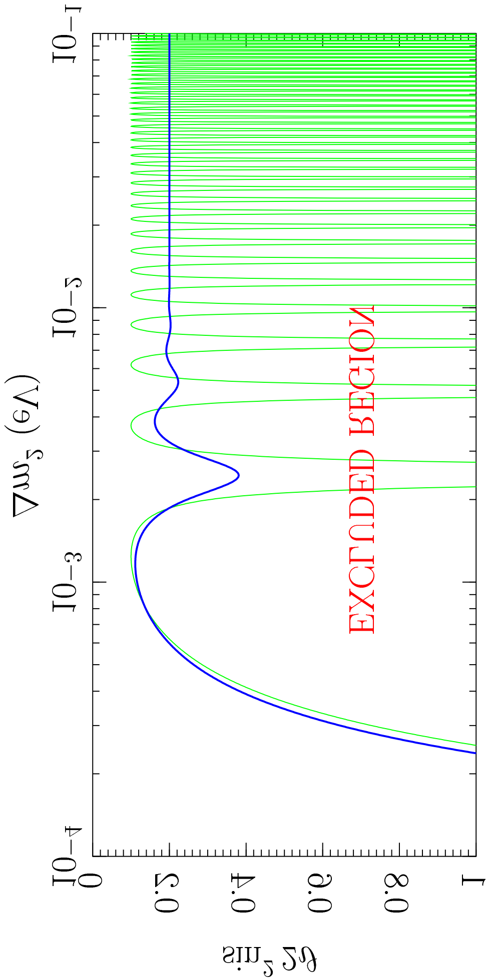
$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha\neq\beta)$$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\,\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right]\qquad(\alpha\neq\beta)$$

experiment: $\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle \leq P^{\max}_{\nu_\alpha\to\nu_\beta}$ $\Longrightarrow$ $\sin^2 2\vartheta \leq \dfrac{2\,P^{\max}_{\nu_\alpha\to\nu_\beta}}{1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E}$



rotate
and
mirror



Summary of Part 2: Neutrino Oscillations in Vacuum and in Matter

detectable neutrinos are extremely relativistic

$\Downarrow$

standard expression for the neutrino oscillation probabilities $(\Delta m^2_{kj}, U_{\alpha k})$

Neutrino Oscillations can test CPT, CP, T symmetries

Matter Effects are important for Solar neutrinos and VLBL experiments

in Neutrino Oscillations Dirac $\sim$ Majorana

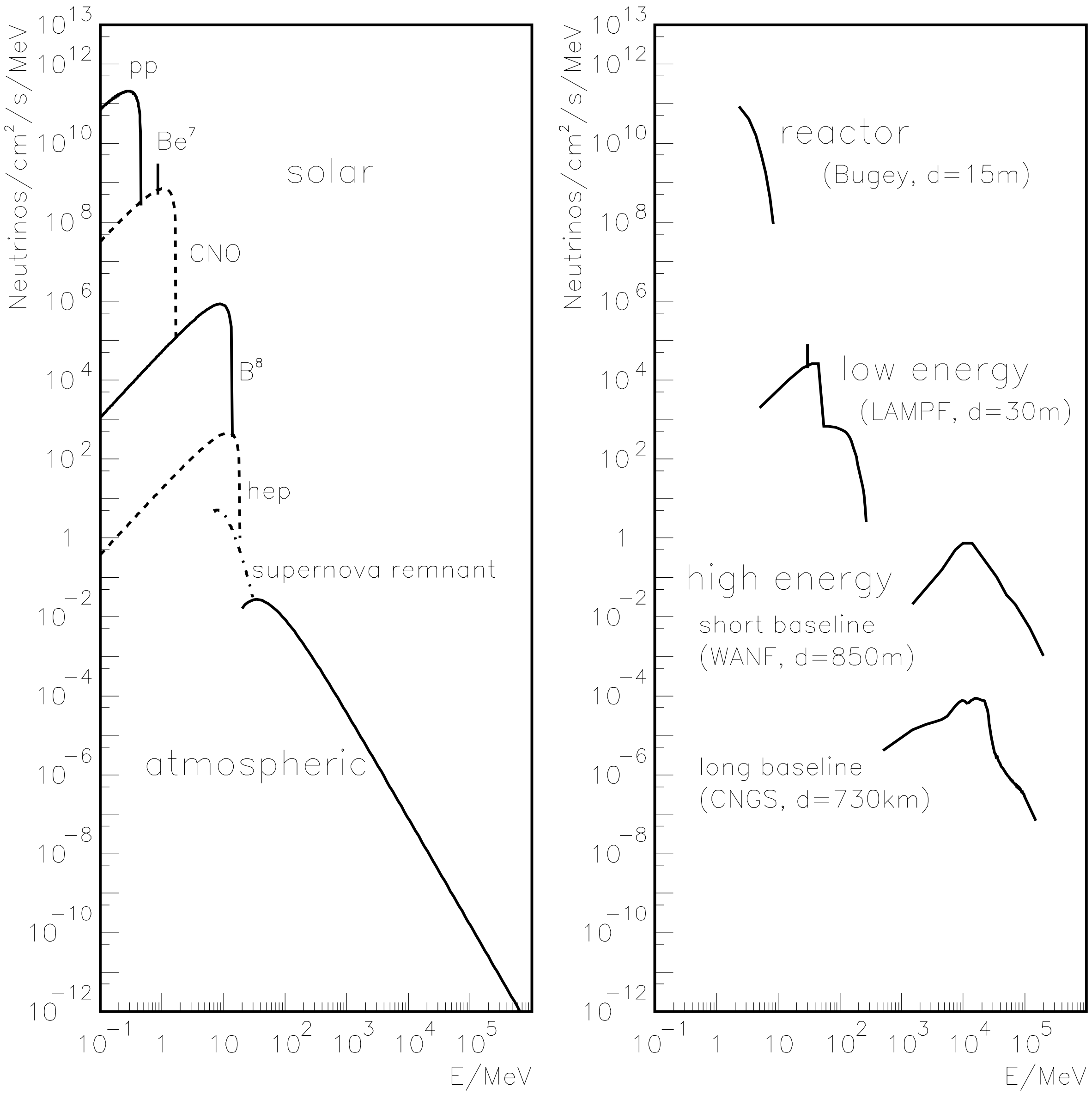
average over energy spectrum

$\Downarrow$

constant flavor changing probability

Part 3: Experimental Results and Theoretical Implications

Neutrino Fluxes



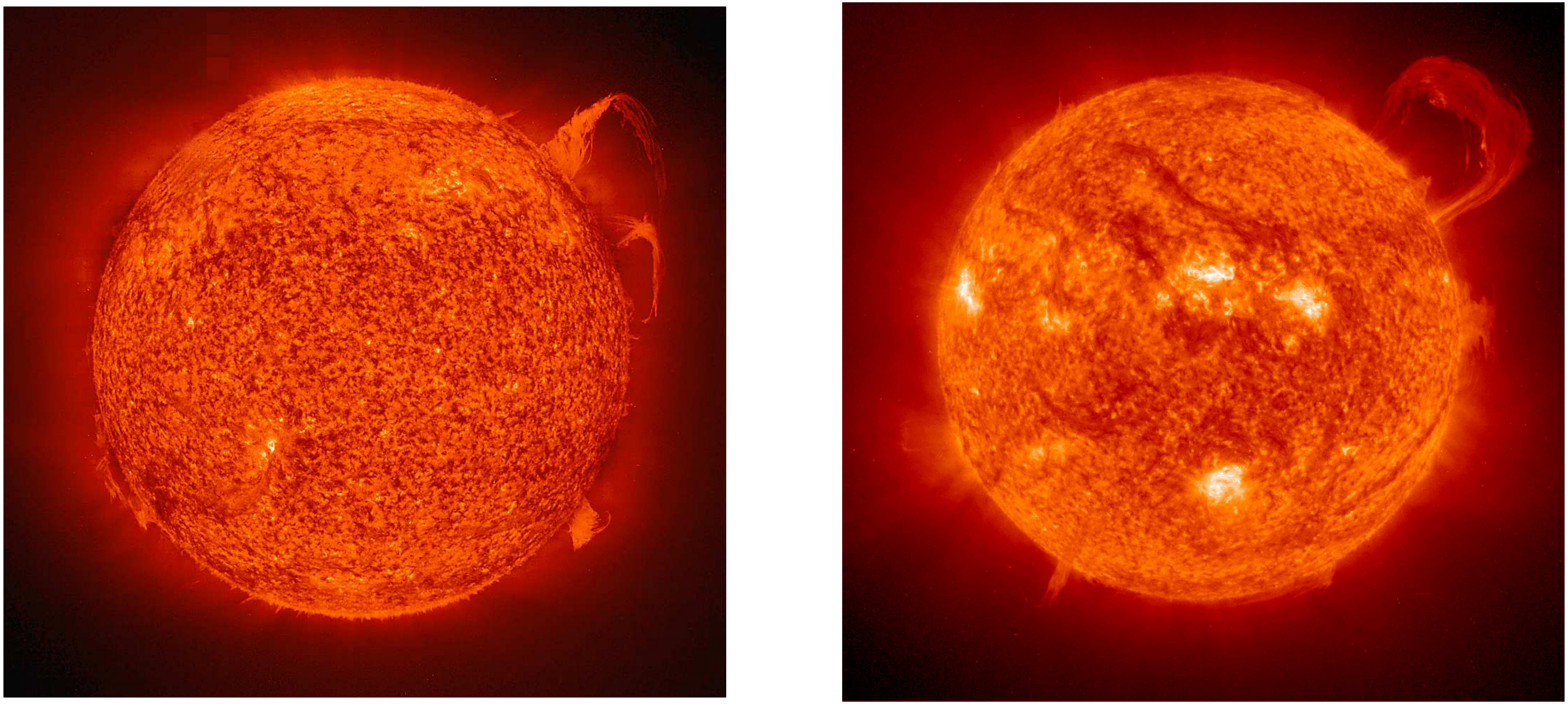
LAMPF = Los Alamos

WANF = CERN

CNGS = CERN⟶Gran Sasso

[A. Geiser, Rept. Prog. Phys. 63 (2000) 1779]

SOLAR NEUTRINOS

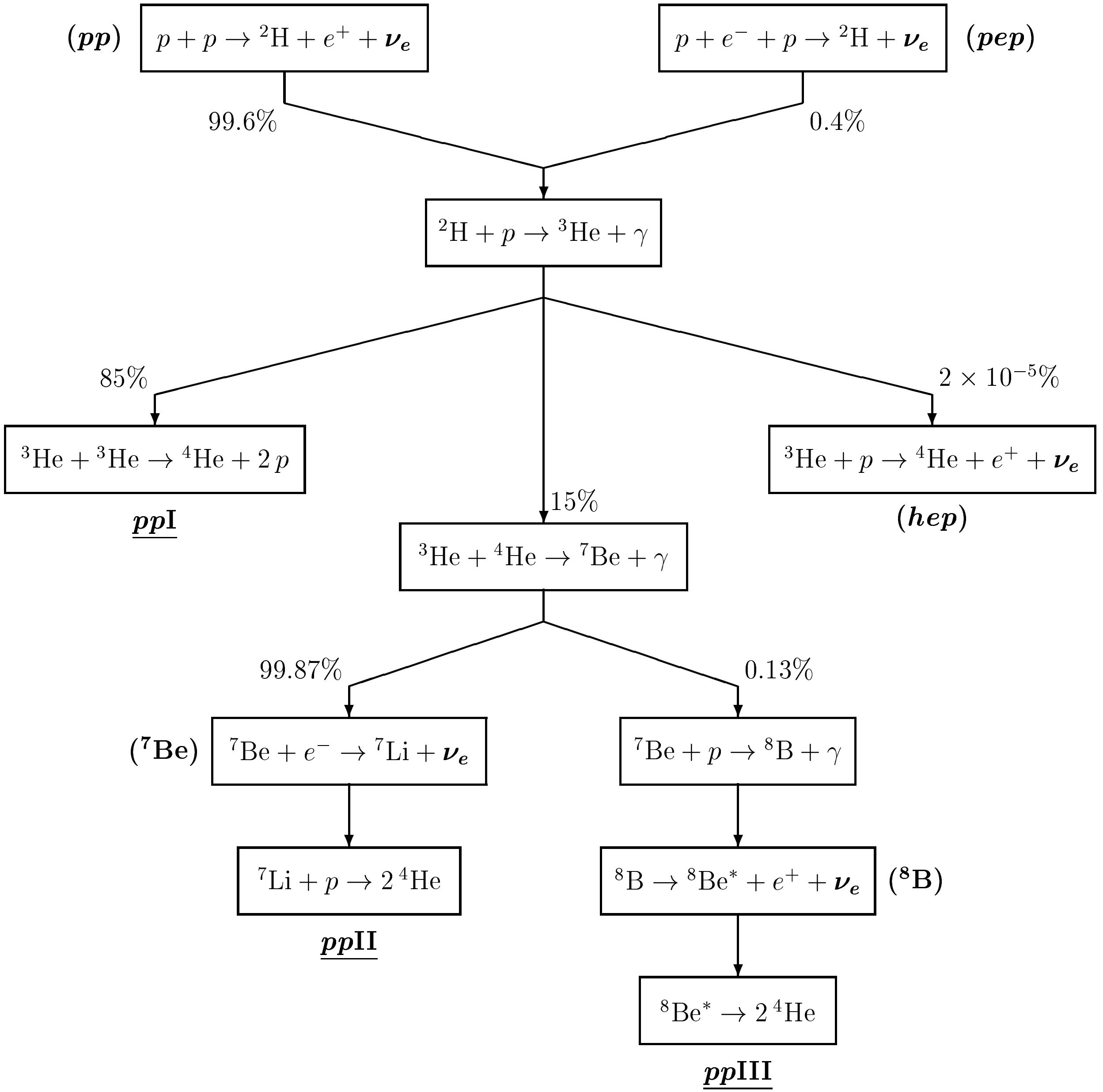


Extreme ultraviolet Imaging Telescope (EIT) $304\,\mathring{A}$ images of the Sun

emission in this spectral line (He II) shows the upper chromosphere at a temperature of about 60,000 K

[The Solar and Heliospheric Observatory (SOHO), http://sohowww.nascom.nasa.gov/]

Standard Solar Model (SSM)



(*pp*) $p + p \to {}^{2}\mathrm{H} + e^{+} + \nu_e$

$p + e^{-} + p \to {}^{2}\mathrm{H} + \nu_e$ (*pep*)

99.6%

0.4%

${}^{2}\mathrm{H} + p \to {}^{3}\mathrm{He} + \gamma$

85%

15%

$2 \times 10^{-5}\%$

${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \to {}^{4}\mathrm{He} + 2\,p$

***pp*I**

${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \to {}^{7}\mathrm{Be} + \gamma$

${}^{3}\mathrm{He} + p \to {}^{4}\mathrm{He} + e^{+} + \nu_e$

(*hep*)

99.87%

0.13%

(^{7}**Be**) ${}^{7}\mathrm{Be} + e^{-} \to {}^{7}\mathrm{Li} + \nu_e$

${}^{7}\mathrm{Be} + p \to {}^{8}\mathrm{B} + \gamma$

${}^{7}\mathrm{Li} + p \to 2\,{}^{4}\mathrm{He}$

***pp*II**

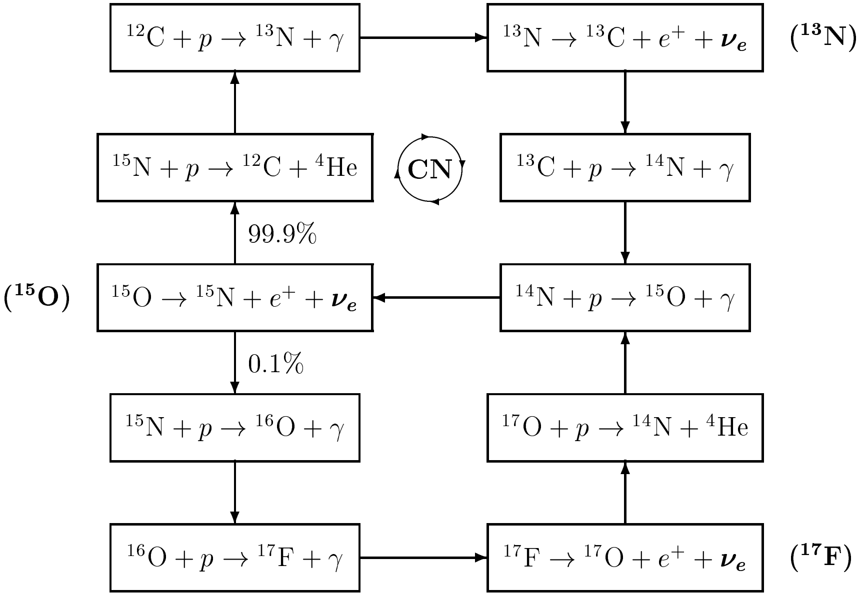
${}^{8}\mathrm{B} \to {}^{8}\mathrm{Be}^{*} + e^{+} + \nu_e$ (^{8}**B**)

${}^{8}\mathrm{Be}^{*} \to 2\,{}^{4}\mathrm{He}$

***pp*III**

pp and CNO cycles

$$4\,p + 2\,e^{-} \longrightarrow {}^{4}\mathrm{He} + 2\,\nu_e + 26.731\,\mathrm{MeV}$$



${}^{12}\mathrm{C} + p \to {}^{13}\mathrm{N} + \gamma$

${}^{13}\mathrm{N} \to {}^{13}\mathrm{C} + e^{+} + \nu_e$ (^{13}**N**)

${}^{15}\mathrm{N} + p \to {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$

CN

${}^{13}\mathrm{C} + p \to {}^{14}\mathrm{N} + \gamma$

99.9%

(^{15}**O**) ${}^{15}\mathrm{O} \to {}^{15}\mathrm{N} + e^{+} + \nu_e$

${}^{14}\mathrm{N} + p \to {}^{15}\mathrm{O} + \gamma$

0.1%

${}^{15}\mathrm{N} + p \to {}^{16}\mathrm{O} + \gamma$

${}^{17}\mathrm{O} + p \to {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$

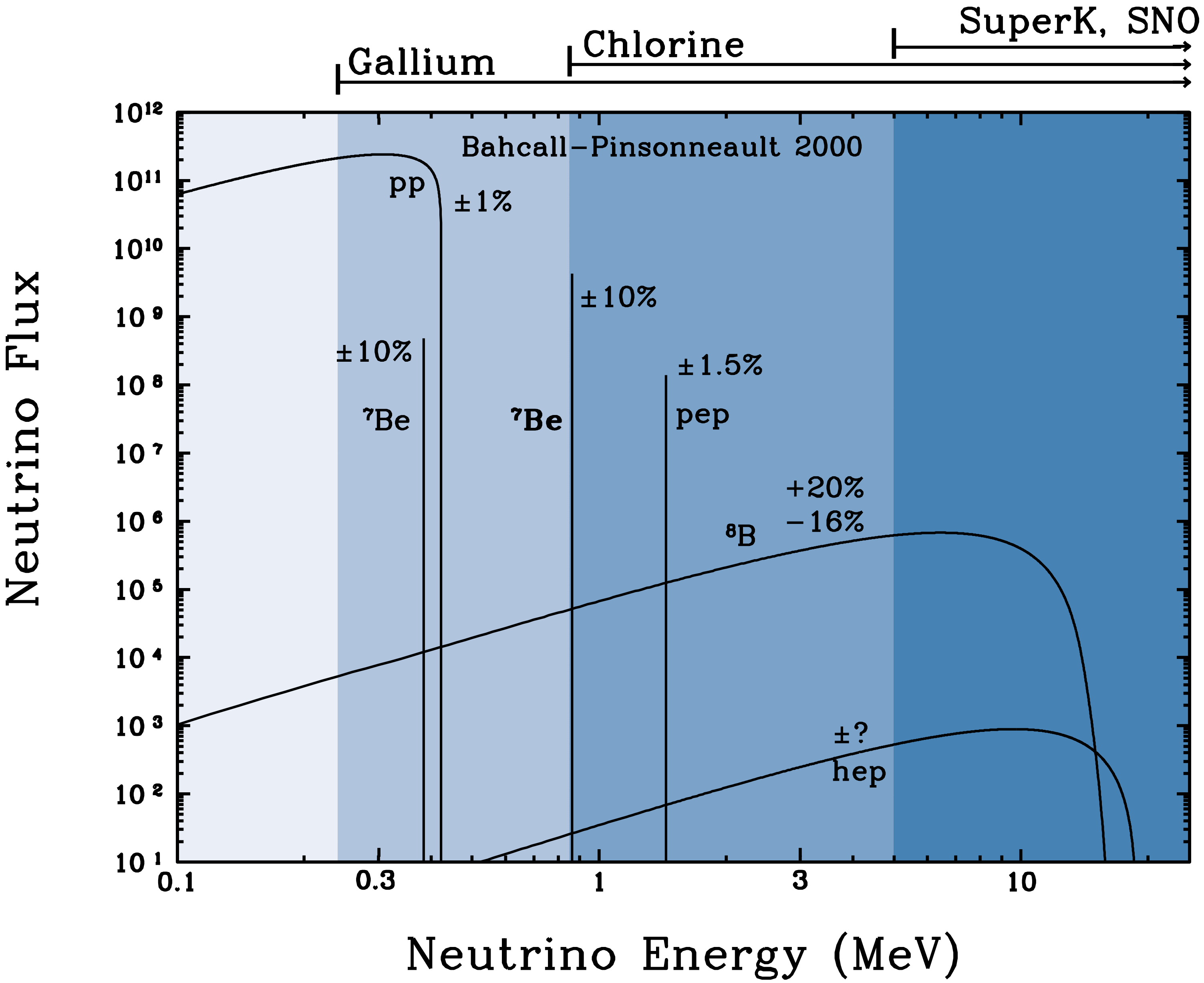
${}^{16}\mathrm{O} + p \to {}^{17}\mathrm{F} + \gamma$

${}^{17}\mathrm{F} \to {}^{17}\mathrm{O} + e^{+} + \nu_e$ (^{17}**F**)

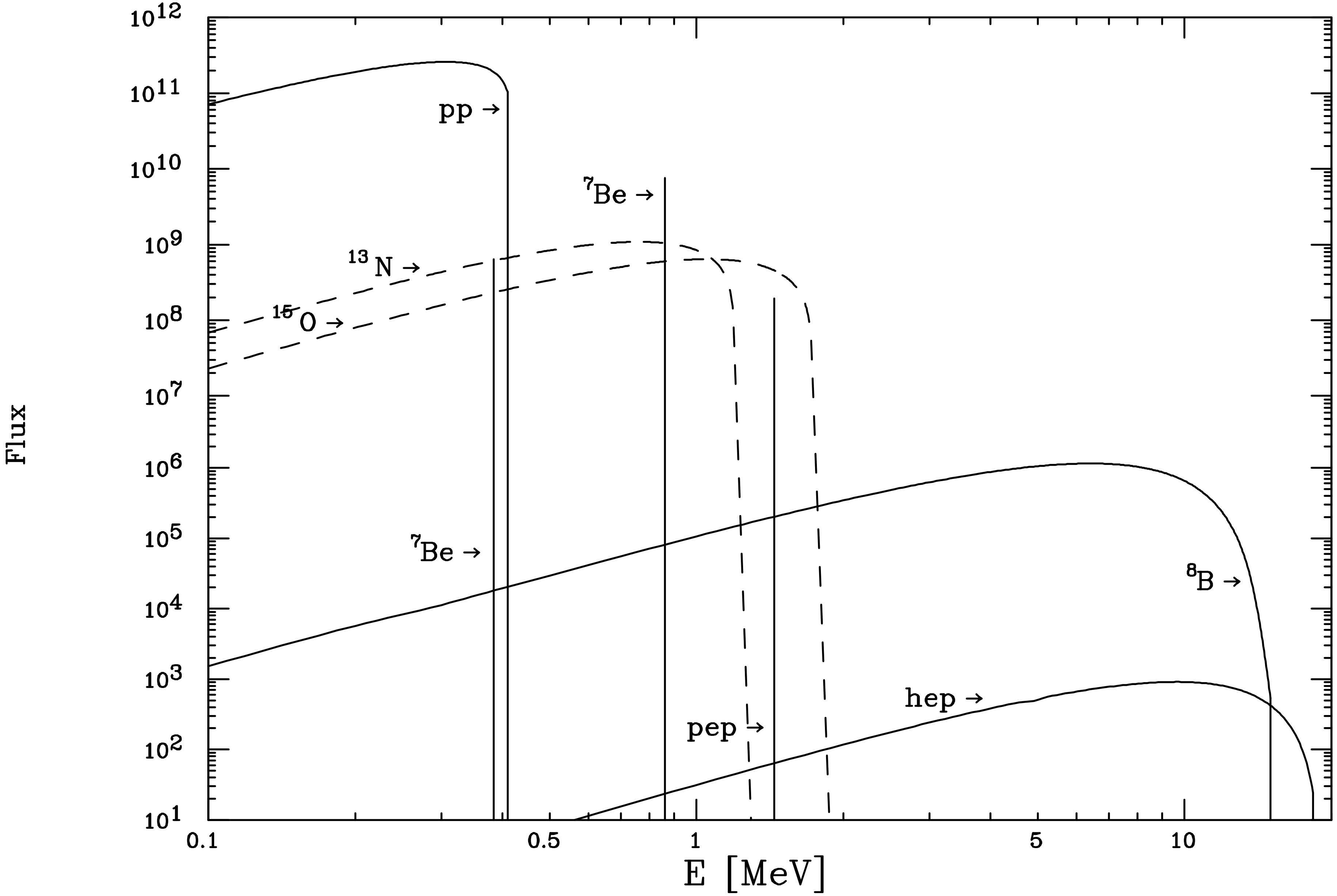
Current SSM: BP2000

[Bahcall, Pinsonneault, Basu, AJ 555 (2001) 990]

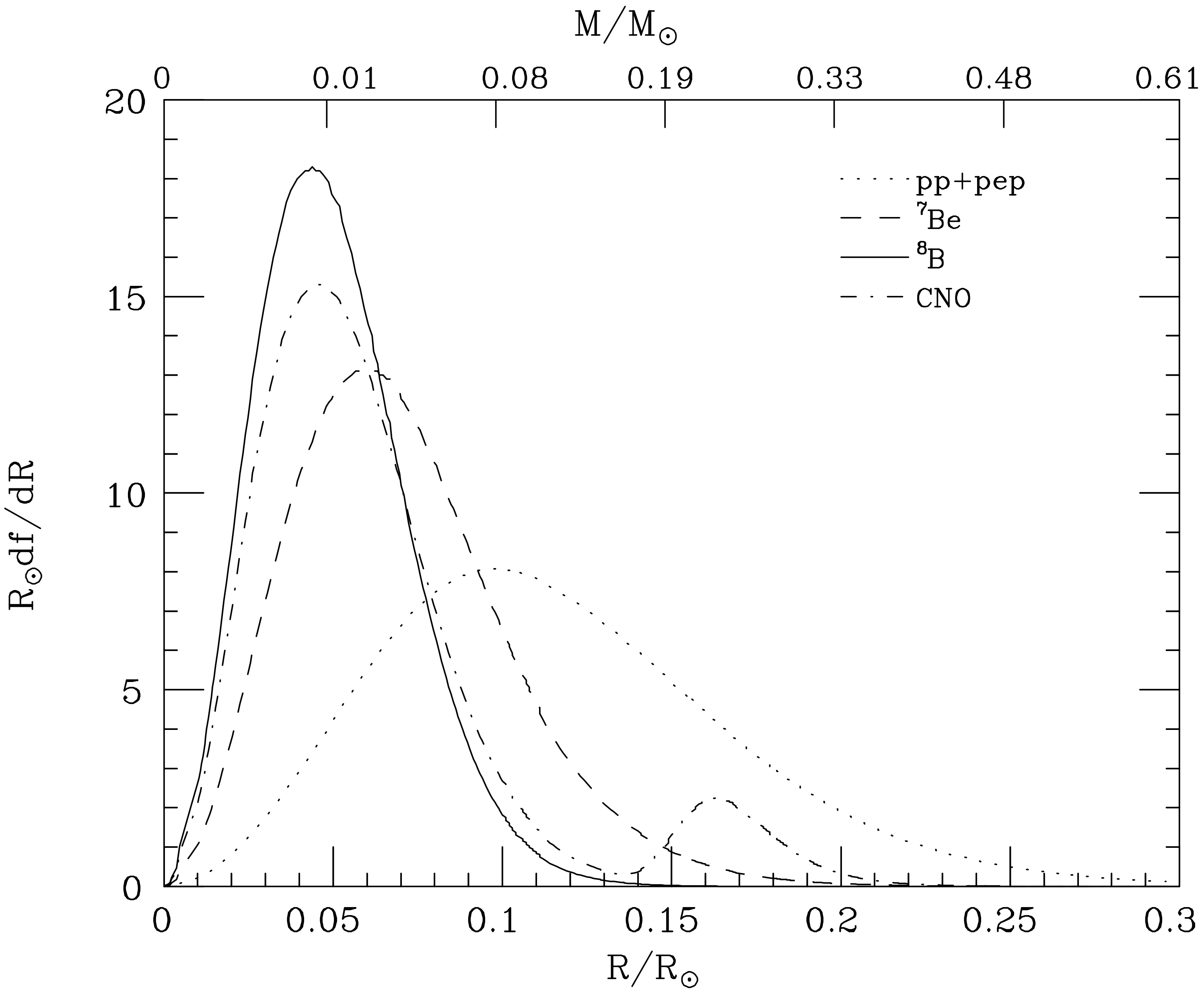
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



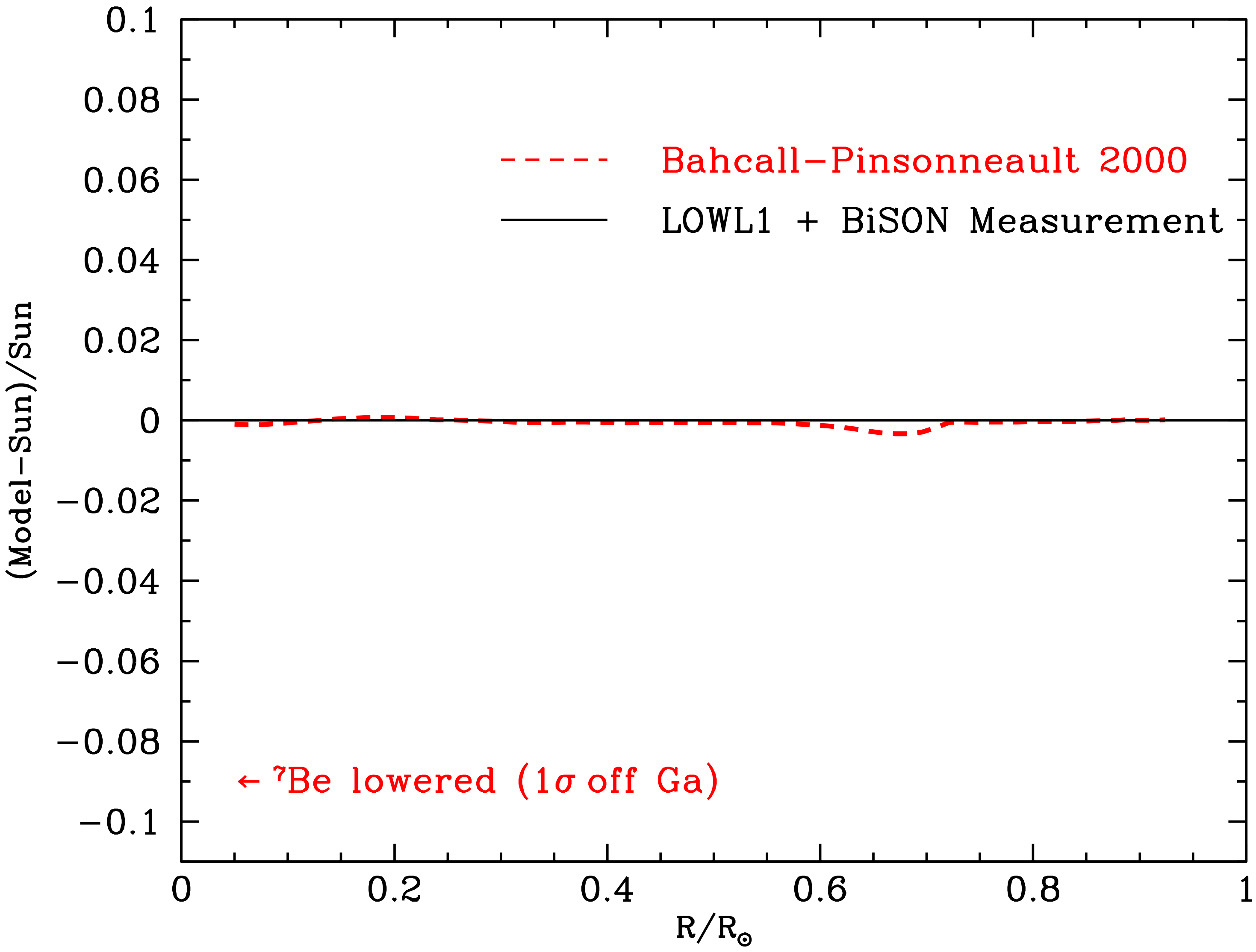
[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between $0.05\,R_{\odot}$ and $0.95\,R_{\odot}$ and is 0.08% for the deep interior region, $r < 0.25\,R_{\odot}$, in which neutrinos are produced

HOMESTAKE

$\nu_e + ^{37}\mathrm{Cl} \to ^{37}\mathrm{Ar} + e^-$ [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota), 1478 m deep, 4200 m.w.e. $\Longrightarrow \Phi_\mu \simeq 4\,\mathrm{m}^{-2}\,\mathrm{day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

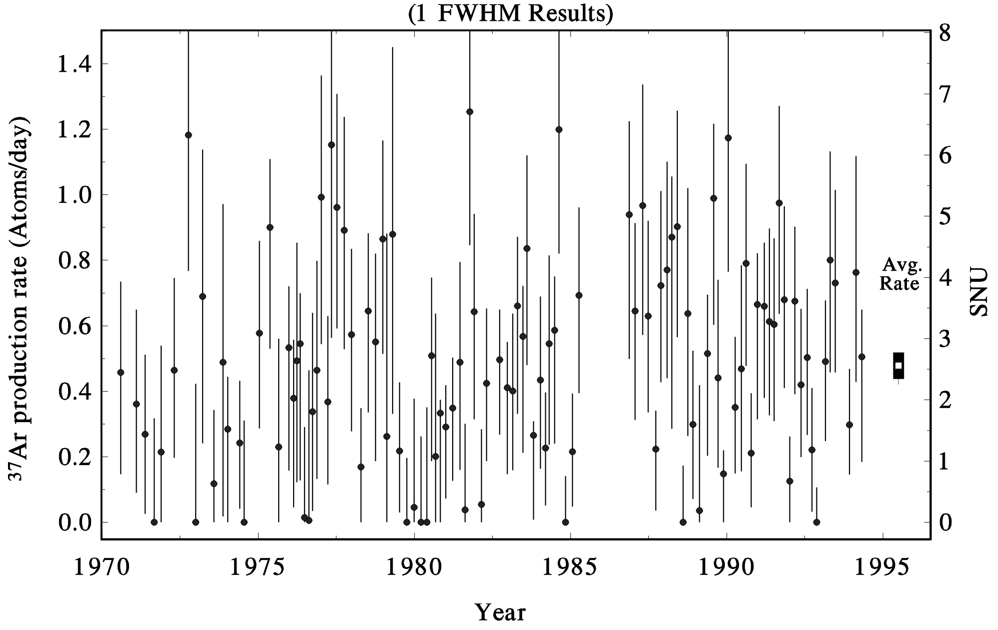
615 tons of tetrachloroethylene ($\mathrm{C_2Cl_4}$), 2.16×10^{30} atoms of $^{37}\mathrm{Cl}$ (133 tons)

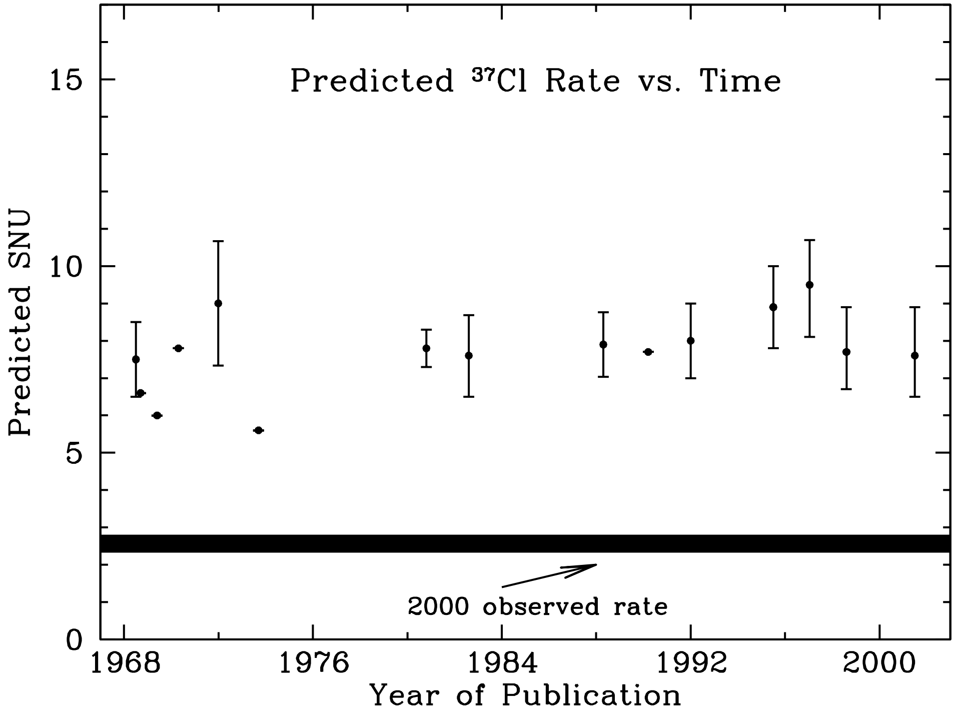
energy threshold: $E_{\mathrm{th}}^{\mathrm{Cl}} = 0.814\,\mathrm{MeV} \Longrightarrow {}^8\mathrm{B},\ {}^7\mathrm{Be},\ pep,\ hep,\ {}^{13}\mathrm{N},\ {}^{15}\mathrm{O},\ {}^{17}\mathrm{F}$

1970–1994, 108 extractions $\Longrightarrow$ $R_{\mathrm{Cl}}^{\mathrm{exp}} / R_{\mathrm{Cl}}^{\mathrm{SSM}} = 0.34 \pm 0.03$ [APJ 496 (1998) 505]

$$R_{\mathrm{Cl}}^{\mathrm{exp}} = 2.56 \pm 0.23\,\mathrm{SNU} \qquad R_{\mathrm{Cl}}^{\mathrm{SSM}} = 7.6^{+1.3}_{-1.1}\,\mathrm{SNU}$$

1 SNU $= 10^{-36}$ events atom^{-1} s^{-1}





GALLIUM EXPERIMENTS

SAGE, GALLEX, GNO

$\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$ [Kuzmin (1965)] radiochemical experiments

threshold: $E_{\mathrm{th}}^{\mathrm{Ga}} = 0.233\,\mathrm{MeV} \Longrightarrow pp,\ {}^{7}\mathrm{Be},\ {}^{8}\mathrm{B},\ pep,\ hep,\ {}^{13}\mathrm{N},\ {}^{15}\mathrm{O},\ {}^{17}\mathrm{F}$

SAGE+GALLEX+GNO $\Longrightarrow$ $R_{\mathrm{Ga}}^{\mathrm{exp}} / R_{\mathrm{Ga}}^{\mathrm{SSM}} = 0.56 \pm 0.03$

$$R_{\mathrm{Ga}}^{\mathrm{exp}} = 72.4 \pm 4.7\,\mathrm{SNU} \qquad R_{\mathrm{Ga}}^{\mathrm{SSM}} = 128^{+9}_{-7}\,\mathrm{SNU}$$

SAGE: Soviet-American Gallium Experiment

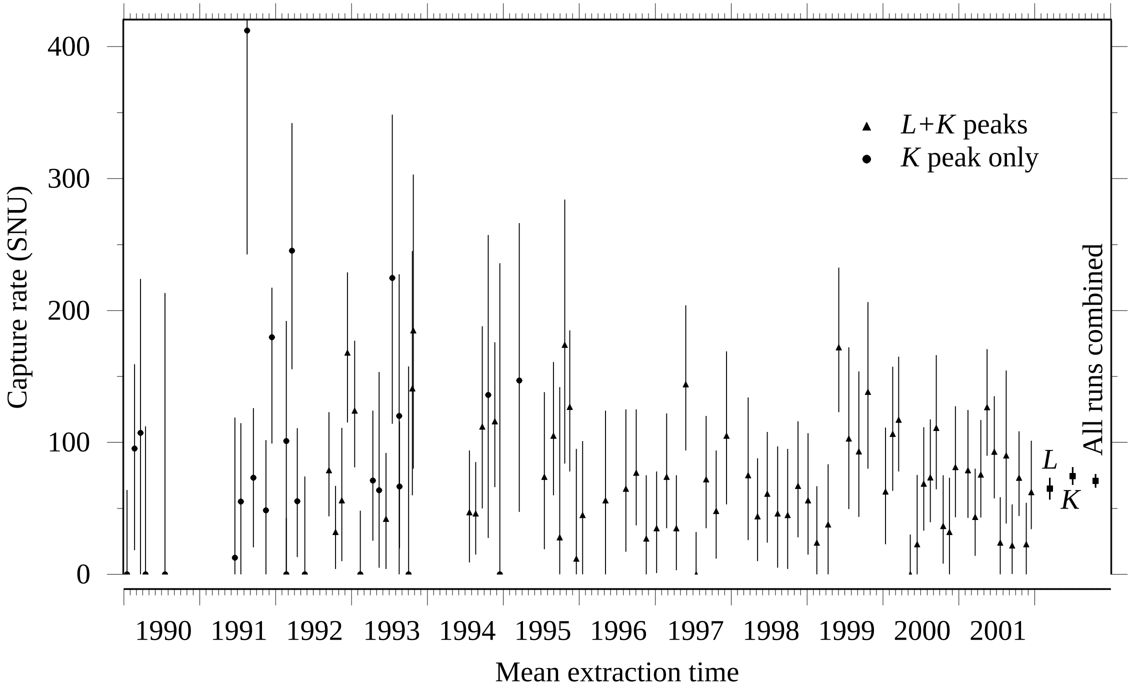
Baksan Neutrino Observatory, northern Caucasus, 3.5 km from entrance of horizontal adit

50 tons of metallic $^{71}\mathrm{Ga}$, 2000 m deep, 4700 m.w.e. $\Longrightarrow$ $\Phi_\mu \simeq 2.6\,\mathrm{m}^{-2}\,\mathrm{day}^{-1}$

detector test: $^{51}\mathrm{Cr}$ Source: $R = 0.95^{+0.11+0.06}_{-0.10-0.05}$ [PRC 59 (1999) 2246]

1990 – 2001 $\Longrightarrow$ $R^{\mathrm{SAGE}}_{\mathrm{Ga}} / R^{\mathrm{SSM}}_{\mathrm{Ga}} = 0.54 \pm 0.05$ [astro-ph/0204245]

$R^{\mathrm{SAGE}}_{\mathrm{Ga}} = 70.8^{+6.5}_{-6.1}\,\mathrm{SNU}$ $R^{\mathrm{SSM}}_{\mathrm{Ga}} = 128^{+9}_{-7}\,\mathrm{SNU}$



GALLium EXperiment (GALLEX)

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

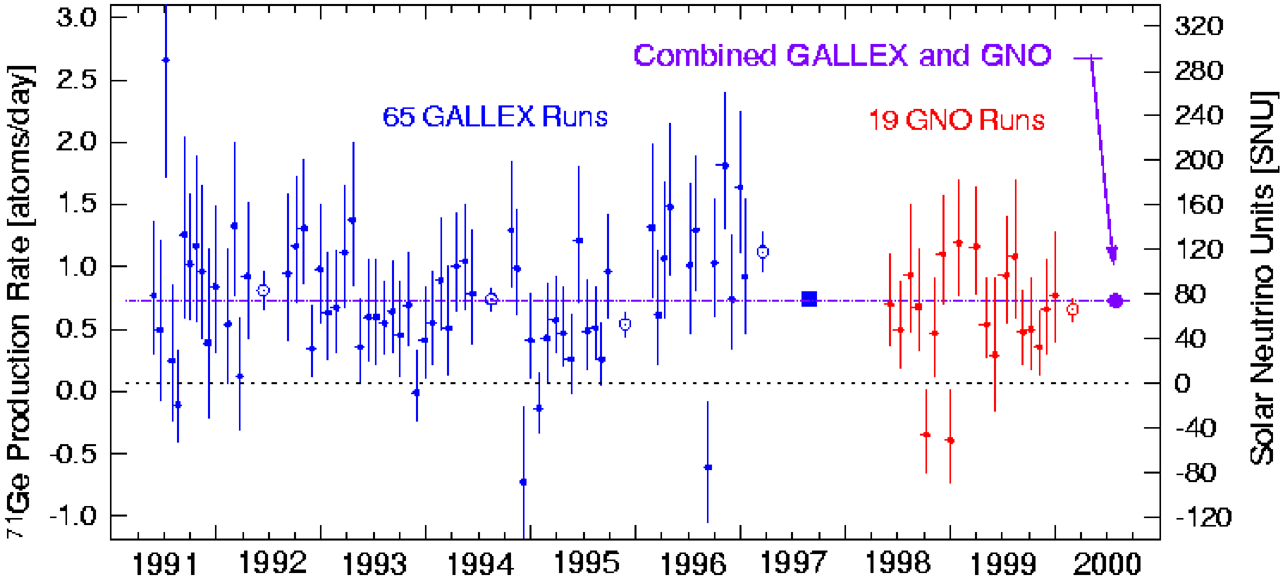
30.3 tons of gallium in 101 tons of gallium chloride ($GaCl_3$-HCl) solution

May 1991 – Jan 1997 $\Longrightarrow$ $R_{\text{Ga}}^{\text{GALLEX}}/R_{\text{Ga}}^{\text{SSM}} = 0.61 \pm 0.06$ [PLB 477 (1999) 127]

Gallium Neutrino Observatory (GNO)

continuation of GALLEX, GNO30: 30.3 tons of gallium

May 1998 – Jan 2000 $\Longrightarrow$ $R_{\text{Ga}}^{\text{GNO}}/R_{\text{Ga}}^{\text{SSM}} = 0.51 \pm 0.08$ [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{G+G}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

Kamiokande

water Cherenkov detector $\nu + e^- \to \nu + e^-$

Sensitive to ν_e, ν_μ, ν_τ, but $\sigma(\nu_e) \simeq 6\,\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}$, *hep*

Jan 1987 – Feb 1995 (2079 days) $\Longrightarrow$ $\dfrac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{SSM}}} = 0.55 \pm 0.08$ [PRL 77 (1996) 1683]

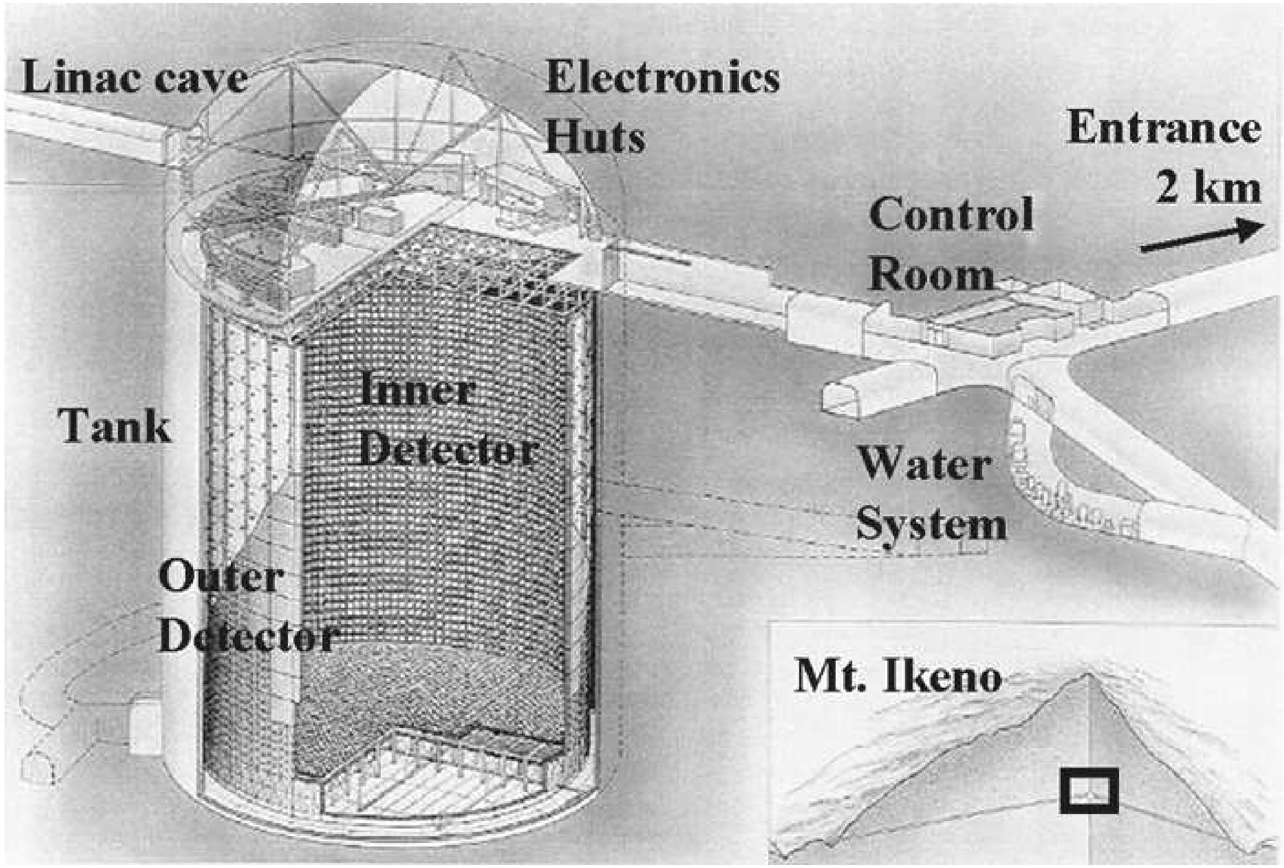
Super-Kamiokande

continuation of Kamiokande, 50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}$, *hep*

1996 – 2001 (1496 days) $\Longrightarrow$ $\dfrac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{SSM}}} = 0.465 \pm 0.015$ [SK, PLB 539 (2002) 179]

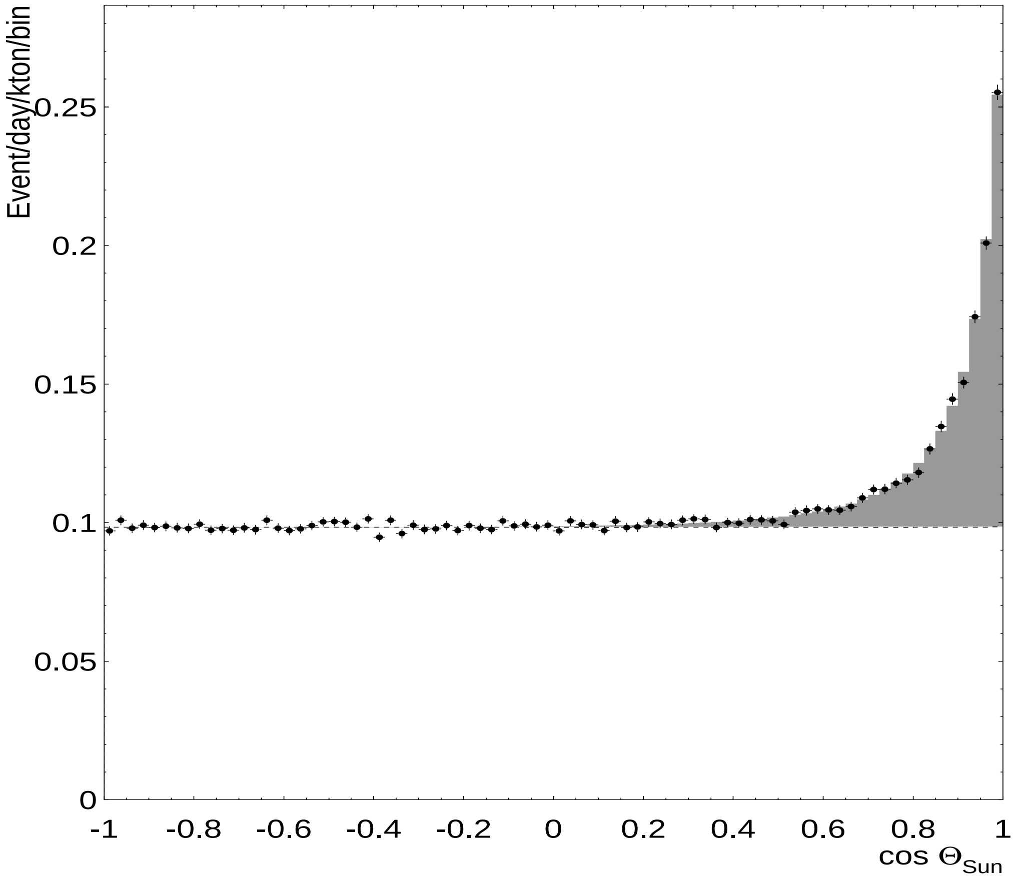
Super-Kamiokande



the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

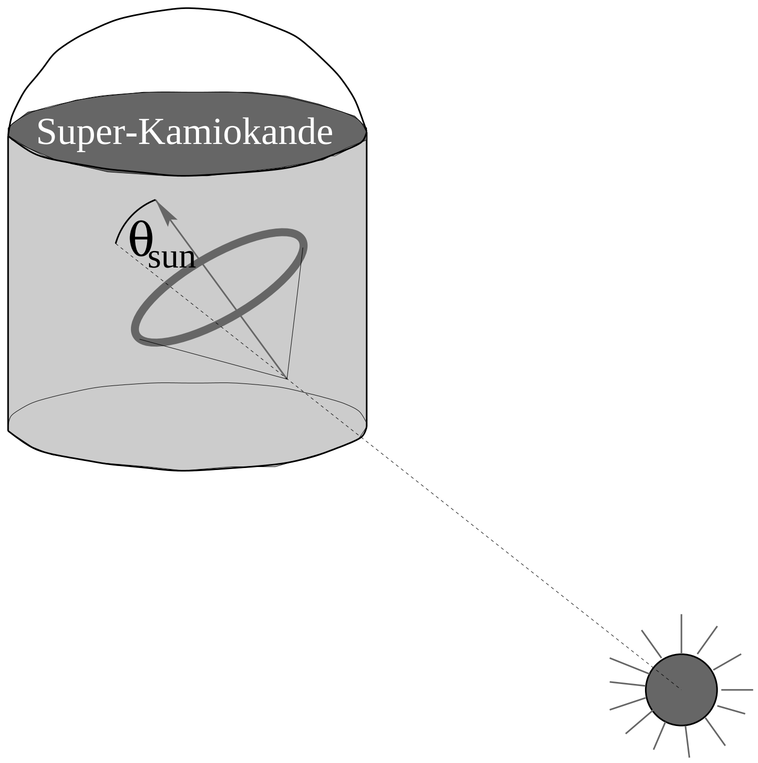
Super-Kamiokande $\cos\theta_{\text{sun}}$ distribution



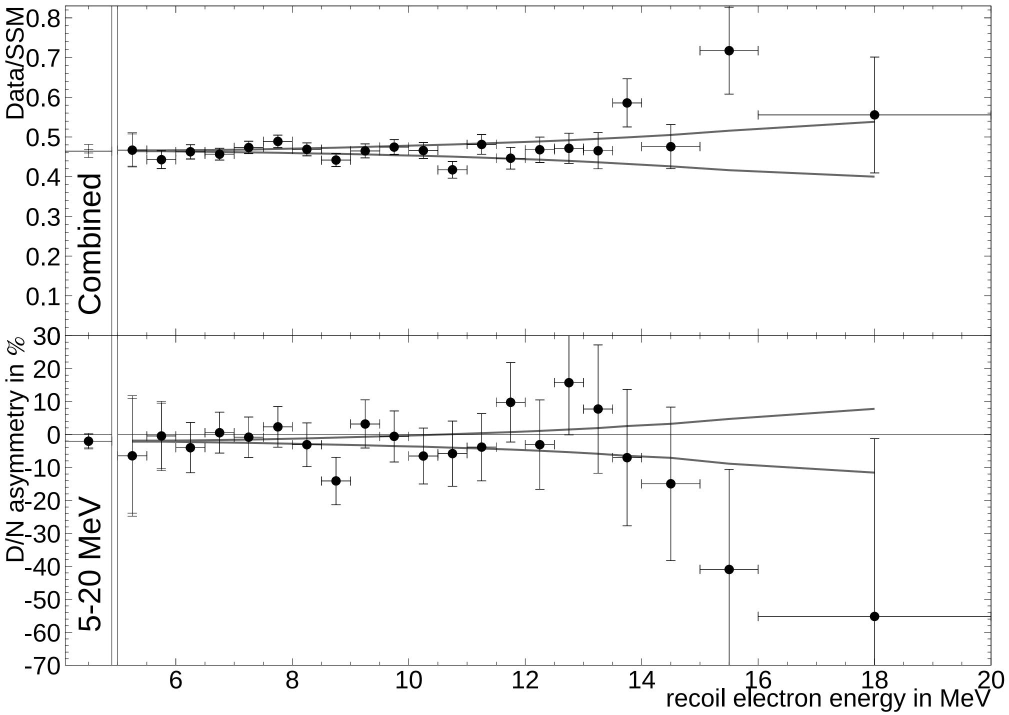
[Smy, hep-ex/0208004]

the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

the peak at $\cos\theta_{\text{sun}} = 1$ is due to solar neutrinos

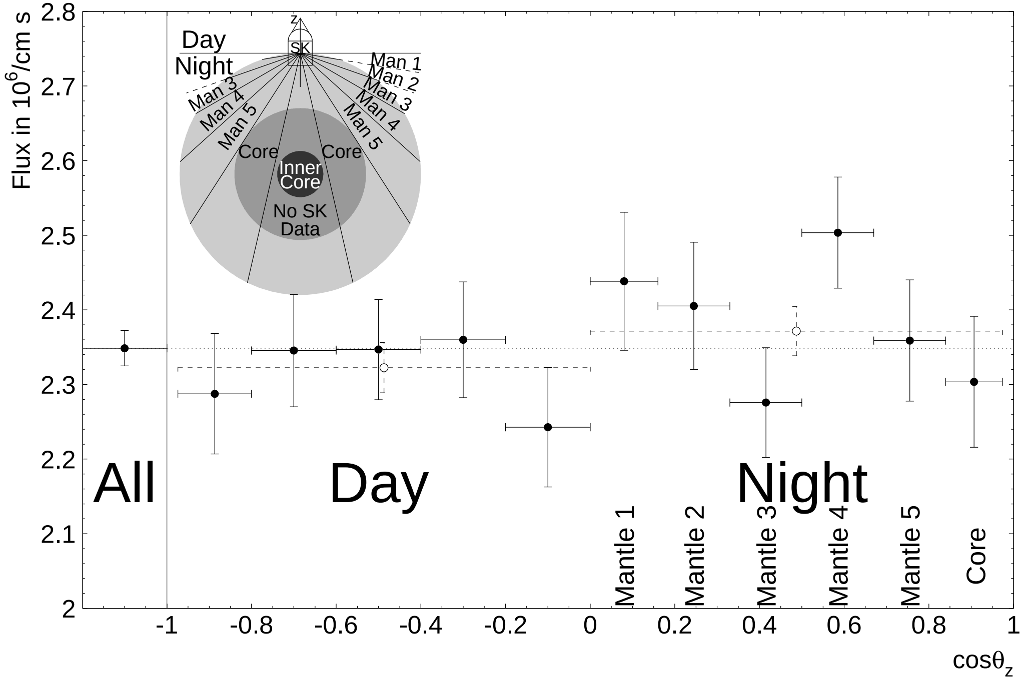


Super-Kamiokande energy spectrum normalized to BP2000 SSM



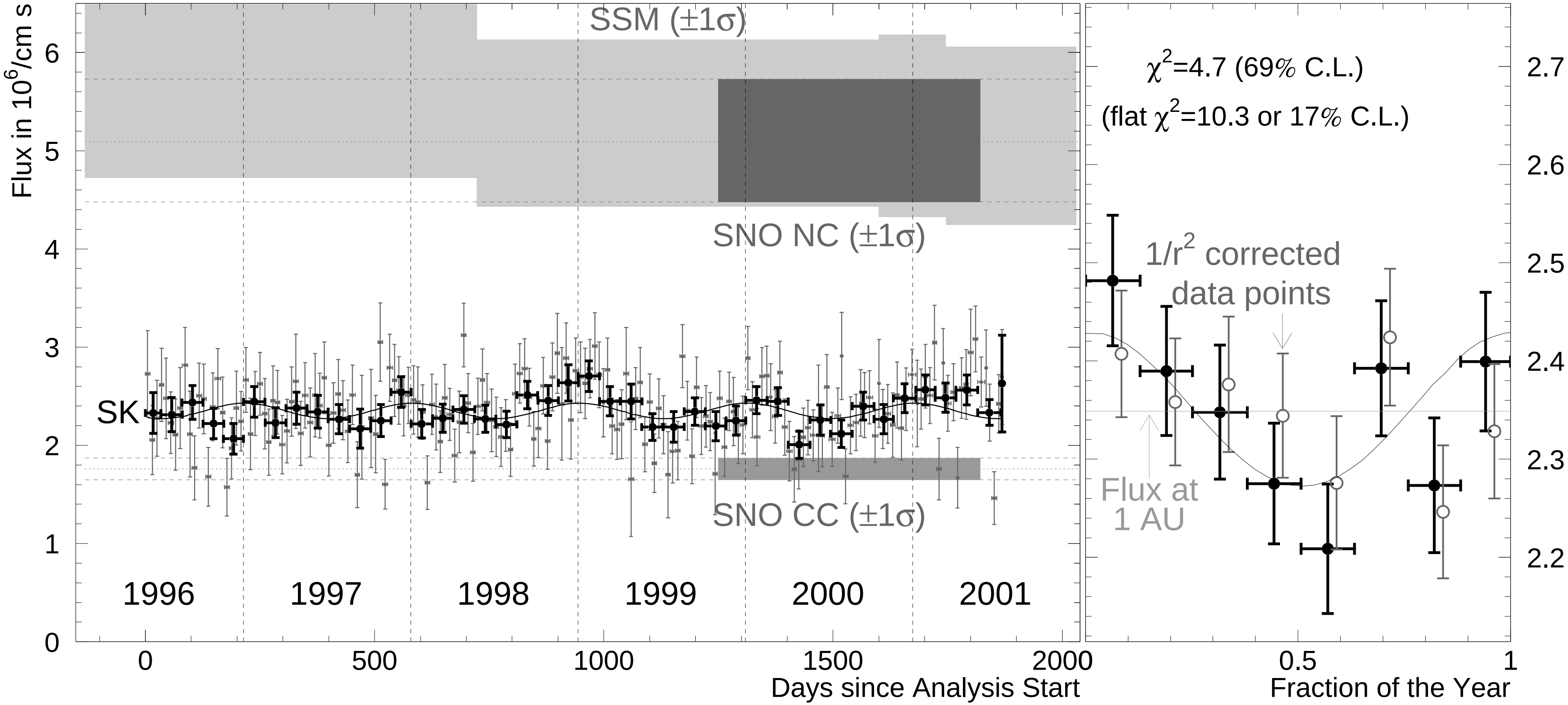
Day-Night asymmetry as a function of energy

solar zenith angle (θ_z) dependence of Super-Kamiokande data



[Smy, hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days, the black data points every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points

by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]

Sudbury Neutrino Observatory (SNO)

water Cherenkov detector, Creighton mine (INCO Ltd.), Sudbury, Ontario, Canada

1 kton of $\mathrm{D_2O}$, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

CC: $\nu_e + d \to p + p + e^-$
NC: $\nu + d \to p + n + \nu$
ES: $\nu + e^- \to \nu + e^-$

$$\left.\begin{array}{l}\text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\mathrm{CC}) \simeq 8.2\,\mathrm{MeV} \\ \text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\mathrm{NC}) \simeq 2.2\,\mathrm{MeV} \\ \text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\mathrm{ES}) \simeq 7.0\,\mathrm{MeV}\end{array}\right\} \Longrightarrow {}^{8}\mathrm{B},\ hep$$

$\mathrm{D_2O}$ phase: 1999 – 2001 (306.4 days)

$$\frac{R_{\mathrm{CC}}^{\mathrm{SNO}}}{R_{\mathrm{CC}}^{\mathrm{SSM}}} = 0.35 \pm 0.02$$
$$\frac{R_{\mathrm{NC}}^{\mathrm{SNO}}}{R_{\mathrm{NC}}^{\mathrm{SSM}}} = 1.01 \pm 0.13$$
$$\frac{R_{\mathrm{ES}}^{\mathrm{SNO}}}{R_{\mathrm{ES}}^{\mathrm{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002 (254.2 days)

$$\frac{R_{\mathrm{CC}}^{\mathrm{SNO}}}{R_{\mathrm{CC}}^{\mathrm{SSM}}} = 0.31 \pm 0.02$$
$$\frac{R_{\mathrm{NC}}^{\mathrm{SNO}}}{R_{\mathrm{NC}}^{\mathrm{SSM}}} = 1.03 \pm 0.09$$
$$\frac{R_{\mathrm{ES}}^{\mathrm{SNO}}}{R_{\mathrm{ES}}^{\mathrm{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

MAIN CHARACTERISTICS OF SOLAR ν DATA

Experiment	Reaction	E_{th} (MeV)	ν Flux Sensitivity	Operating Time	$\frac{R^{\text{exp}}}{R^{\text{BP2000}}}$
SAGE	$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ (CC)	0.233	pp, ${}^7\text{Be}$, ${}^8\text{B}$, pep, hep, ${}^{13}\text{N}$, ${}^{15}\text{O}$, ${}^{17}\text{F}$	$1990 - 2001$	0.54 ± 0.05
GALLEX				$1991 - 1997$	0.61 ± 0.06
GNO				$1998 - 2000$	0.51 ± 0.08
Homestake	$\nu_e +^{37}\text{Cl} \to^{37}\text{Ar} + e^-$ (CC)	0.814	${}^7\text{Be}$, ${}^8\text{B}$, pep, hep, ${}^{13}\text{N}$, ${}^{15}\text{O}$, ${}^{17}\text{F}$	$1970 - 1994$	0.34 ± 0.03
Kamiokande	$\nu + e^- \to \nu + e^-$ (ES)	6.75	${}^8\text{B}$	$1987 - 1995$ 2079 days	0.55 ± 0.08
Super-Kam.		4.75		$1996 - 2001$ 1496 days	0.465 ± 0.015
SNO D_2O phase	$\nu_e + d \to p + p + e^-$ (CC)	6.9		$1999 - 2001$ 306.4 days	0.35 ± 0.02
	$\nu + d \to p + n + \nu$ (NC)	2.2			1.01 ± 0.13
	$\nu + e^- \to \nu + e^-$ (ES)	5.2			0.47 ± 0.05
SNO NaCl phase	$\nu_e + d \to p + p + e^-$ (CC)	6.9		$2001 - 2002$ 254.2 days	0.31 ± 0.02
	$\nu + d \to p + n + \nu$ (NC)	2.2			1.03 ± 0.09
	$\nu + e^- \to \nu + e^-$ (ES)	5.2			0.44 ± 0.06

SNO SOLVED SOLAR NEUTRINO PROBLEM

⇓

NEUTRINO PHYSICS

OKKAM'S RAZOR

⇓

CONSIDER SIMPLEST HYPOTHESIS

⇓

$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

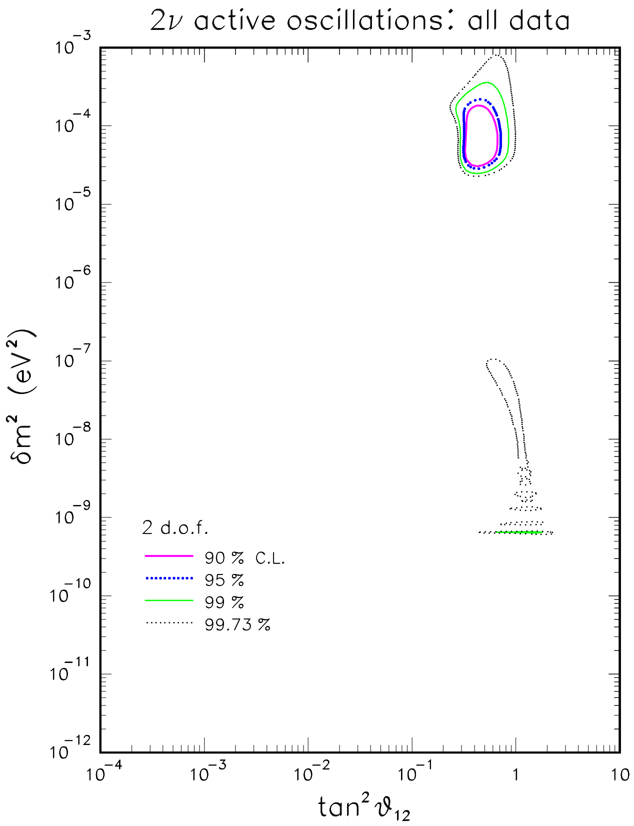
⇓

Large Mixing Angle solution

LMA

$$\Delta m^2 \simeq 5 \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\vartheta \simeq 0.4$$



90%, 95%, 99%, 99.73% (3σ) C.L.

[Fogli, Lisi, Marrone, Montanino, Palazzo, PRD 66 (2002) 053010]

see also

[SNO, PRL 89 (2002) 011302]

[Barger, Marfatia, Whisnant, Wood, PLB 537 (2002) 179]

[Bahcall, Gonzalez-Garcia, Peña-Garay, JHEP 07 (2002) 054]

[SK, PLB 539 (2002) 179]

[de Holanda, Smirnov, PRD66 (2002) 113005]

[Aliani et al., PRD 67 (2003) 013006]

[Bandyopadhyay et al., PLB 540 (2002) 14]

[Creminelli, Signorelli, Strumia, hep-ph/0102234]

[Maltoni, Schwetz, Tortóla, Valle, PRD 67 (2003) 013011]

KamLAND ⇒ spectacular confirmation of LMA

Kamioka Liquid scintillator Anti-Neutrino Detector, long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

average distance from reactors: 180 km

6.7% of flux from one reactor at 88 km

79% of flux from 26 reactors at 138–214 km

14.3% of flux from other reactors at $>$295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

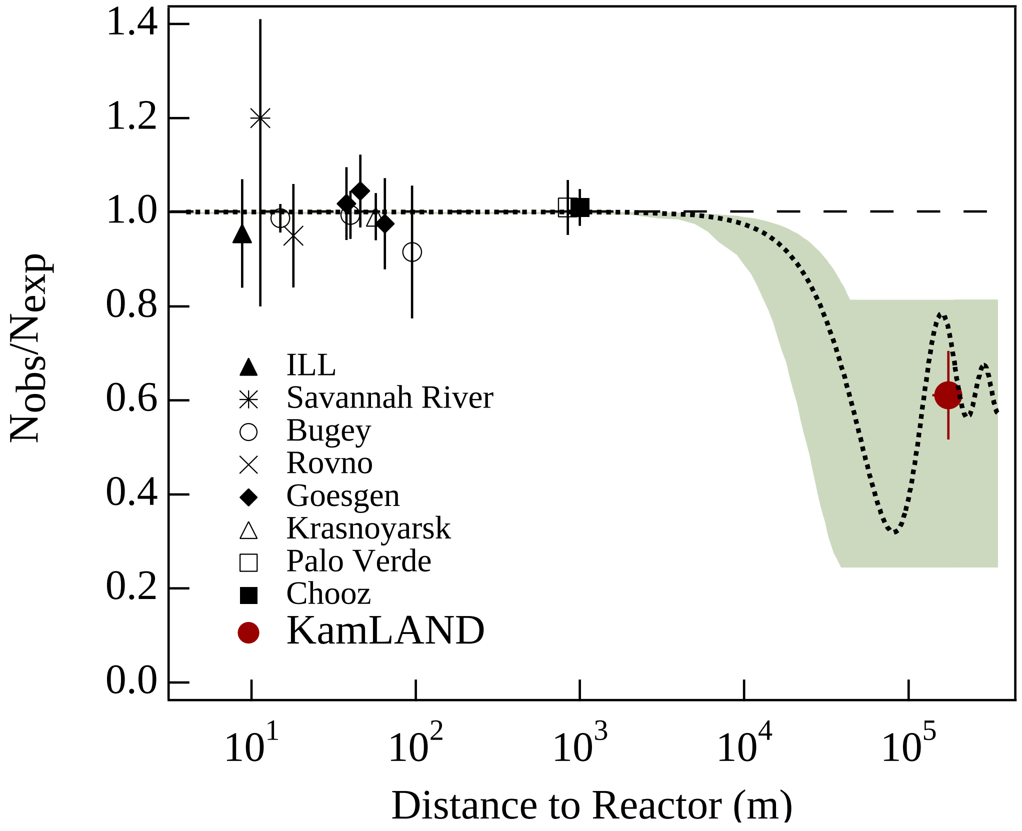
expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$

expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$

observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

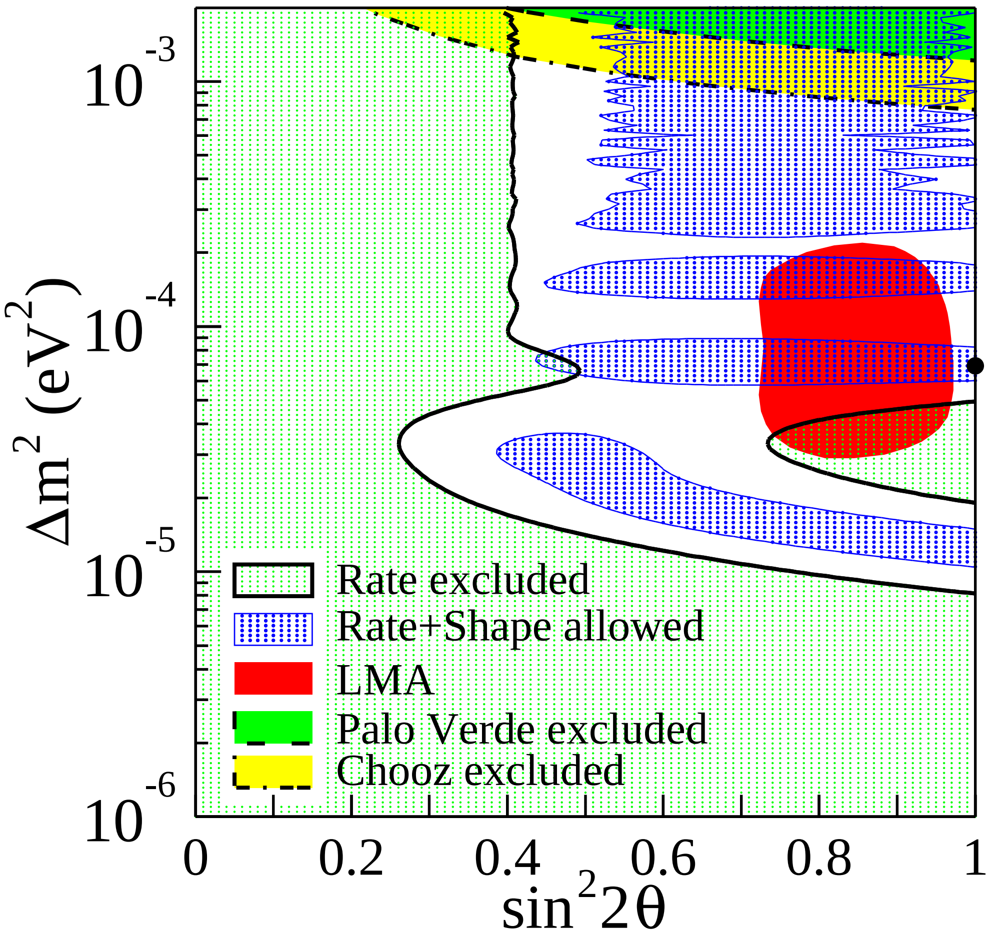
$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

99.95% C.L. evidence
of $\bar{\nu}_e$ disappearance



Shade: 95% C.L. LMA

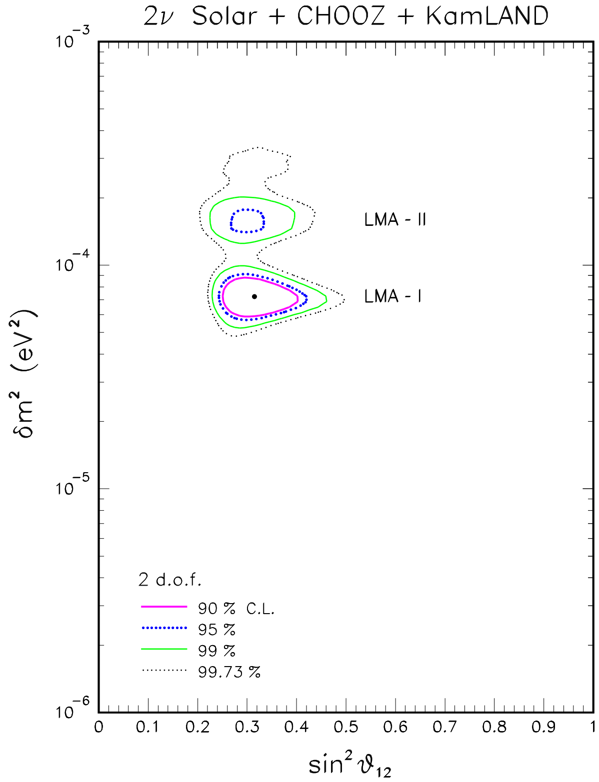
Curve: $\begin{cases} \Delta m^2_{\text{sol}} = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta_{\text{sol}} = 0.83 \end{cases}$



95% C.L.

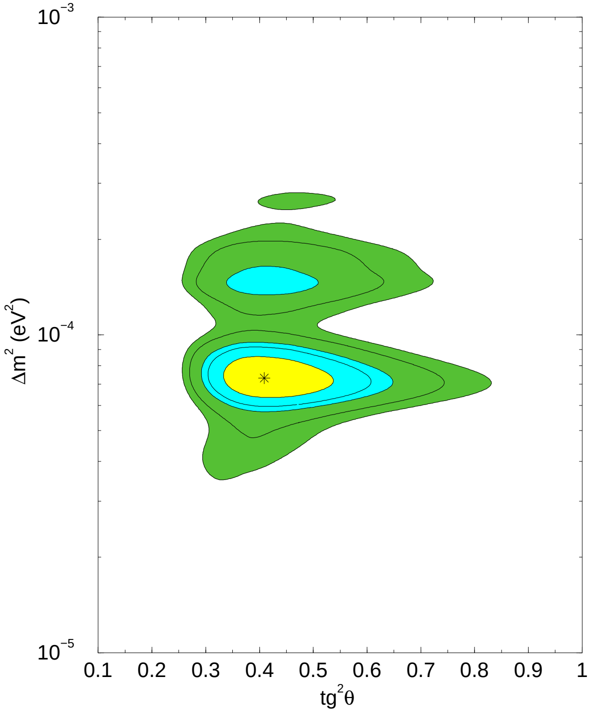
[KamLAND, PRL 90 (2003) 021802]

Fits of reactor + solar neutrino data



[Fogli et al., hep-ph/0212127]

see also
[Barger, Marfatia, hep-ph/0212126]
[Maltoni, Schwetz, Valle, hep-ph/0212129]
[Bandyopadhyay et al., hep-ph/0212146]
[Bahcall, Gonzalez-Garcia, Pena-Garay, hep-ph/0212147]
[Nunokawa, Teves, Zukanovich Funchal, hep-ph/0212202]
[Aliani, Antonelli, Picariello, Torrente-Lujan, hep-ph/0212212]
[Balantekin, Yuksel, hep-ph/0301072]



68.3% (1σ) 90%, 95%, 99%, 99.73% (3σ) C.L.

[de Holanda, Smirnov, hep-ph/0212270]

Best Fit: LMA-I

$$\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2$$

$$\tan^2\vartheta \simeq 0.4$$

$\tan^2\vartheta < 1$ at 3.5σ [Bahcall, Peña-Garay, hep-ph/0305159]

Sudbury Neutrino Observatory (SNO)

D_2O phase

[PRL 89 (2002) 011301, nucl-ex/0204008]

$$n + d \to {}^3\text{H} + \gamma\,(6.25\,\text{MeV})$$

2 Nov 1999 – 28 May 2001: 306.4 live days

$$\begin{aligned} N_{\text{NC}}^{\text{SNO}} &= 576.5_{-48.9}^{+49.5} \\ N_{\text{CC}}^{\text{SNO}} &= 1967.7_{-60.9}^{+61.9} \\ N_{\text{ES}}^{\text{SNO}} &= 263.6_{-25.6}^{+26.4} \end{aligned}$$

$$\begin{aligned} \Phi_{\text{NC}}^{\text{SNO}} &= 5.09_{-0.43}^{+0.44}{}_{-0.43}^{+0.46} \\ \Phi_{\text{CC}}^{\text{SNO}} &= 1.76_{-0.05}^{+0.06} \pm 0.09 \\ \Phi_{\text{ES}}^{\text{SNO}} &= 2.39_{-0.23}^{+0.24} \pm 0.12 \end{aligned}$$

$$\frac{\Phi_{\text{CC}}^{\text{SNO}}}{\Phi_{\text{NC}}^{\text{SNO}}} = 0.346 \pm 0.032 \pm 0.036$$

NaCl phase

[nucl-ex/0309004, 6 September 2003]

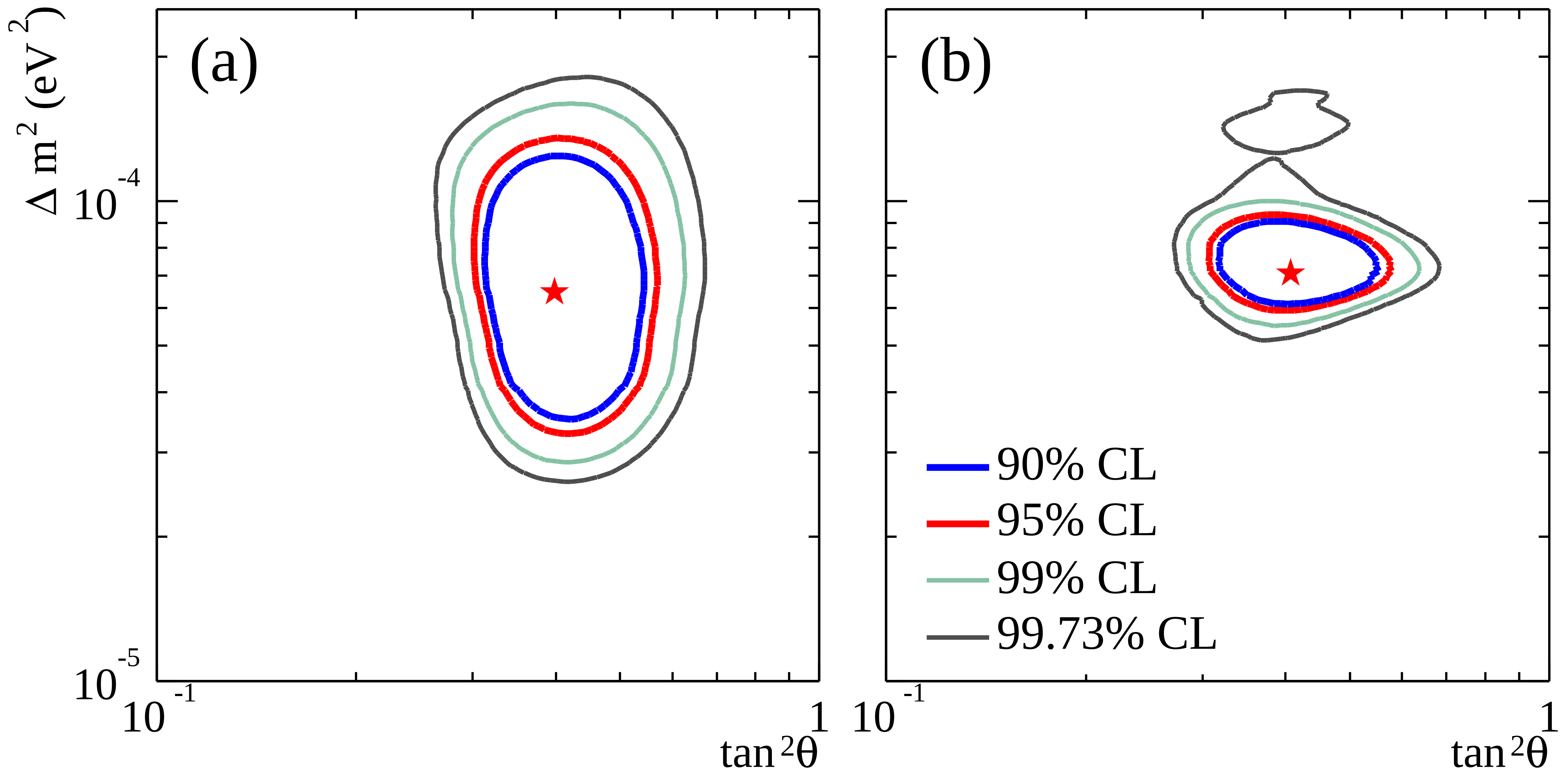
$$n + {}^{35}\text{Cl} \to {}^{36}\text{Cl} + \text{several } \gamma\text{'s}$$

26 Jul 2001 – 10 Oct 2002: 254.2 live days

$$\begin{aligned} N_{\text{NC}}^{\text{SNO}} &= 1344.2_{-69.0}^{+69.8} \\ N_{\text{CC}}^{\text{SNO}} &= 1339.6_{-61.5}^{+63.8} \\ N_{\text{ES}}^{\text{SNO}} &= 170.3_{-20.1}^{+23.9} \end{aligned}$$

$$\begin{aligned} \Phi_{\text{NC}}^{\text{SNO}} &= 5.21 \pm 0.27 \pm 0.38 \\ \Phi_{\text{CC}}^{\text{SNO}} &= 1.59_{-0.07}^{+0.08}{}_{-0.08}^{+0.06} \\ \Phi_{\text{ES}}^{\text{SNO}} &= 2.21_{-0.26}^{+0.31} \pm 0.10 \end{aligned}$$

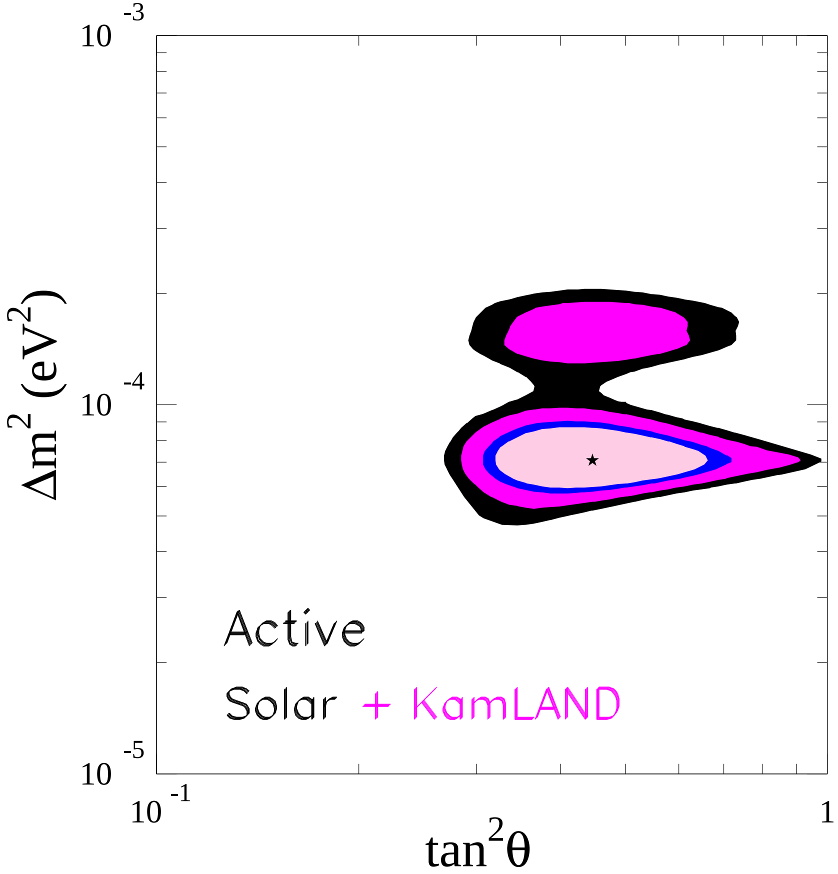
$$\frac{\Phi_{\text{CC}}^{\text{SNO}}}{\Phi_{\text{NC}}^{\text{SNO}}} = 0.306 \pm 0.026 \pm 0.024$$



$\Delta m^2 = 7.1^{+1.0}_{-0.3} \times 10^{-5}\,\text{eV}^2$ $\vartheta = 32.5^{+1.7}_{-1.6}$ $\vartheta < 90$ at $5.4\,\sigma$

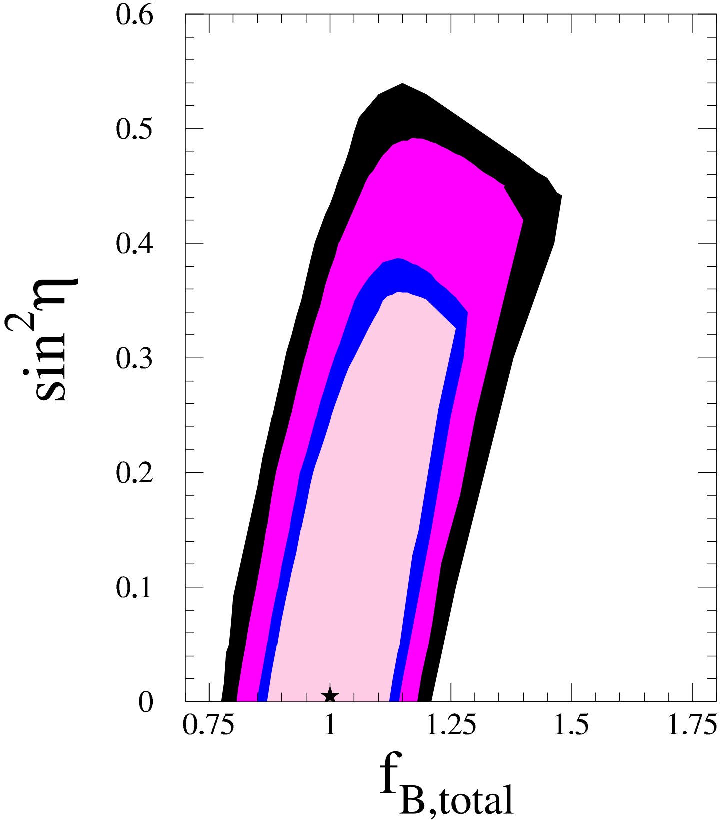
[SNO, nucl-ex/0309004]

Sterile Neutrinos in Solar Neutrino Flux?



90%, 95%, 99%, 99.73% (3σ) C.L.

[Bahcall, Gonzalez-Garcia, Pena-Garay, JHEP 0302 (2003) 009]



$$\nu_e \to \cos\eta\,\nu_a + \sin\eta\,\nu_s$$

$$\sin^2\eta < 0.52\,(3\sigma)$$

$$f_{\mathrm{B,total}} = \frac{\Phi_{^8\mathrm{B}}}{\Phi_{^8\mathrm{B}}^{\mathrm{SSM}}} = 1.00 \pm 0.06$$

Determination of Solar Neutrino Fluxes

[Bahcall, Peña-Garay, hep-ph/0305159]

fit of solar and KamLAND neutrino data with fluxes as free parameters

+ luminosity constraint

$$\sum_r \alpha_r \, \Phi_r = K_\odot \qquad (r = pp, pep, hep, {}^7\mathrm{Be}, {}^8\mathrm{B}, {}^{13}\mathrm{N}, {}^{15}\mathrm{O}, {}^{17}\mathrm{F})$$

$$K_\odot \equiv \mathcal{L}_\odot / 4\pi(1\mathrm{a.u.})^2 = 8.534 \times 10^{11} \, \mathrm{MeV} \, \mathrm{cm}^{-2} \, \mathrm{s}^{-1}$$

solar constant

$$\Delta m^2 = 7.3^{+0.4}_{-0.6} \, \mathrm{eV}^2 \qquad \tan^2\vartheta = 0.42^{+0.08}_{-0.06} \, ({}^{+0.39}_{-0.19})$$

$$\frac{\Phi_{{}^8\mathrm{B}}}{\Phi^{\mathrm{SSM}}_{{}^8\mathrm{B}}} = 1.01^{+0.06}_{-0.06} \, ({}^{+0.22}_{-0.17})$$

moderate uncertainty

will improve with new SNO

NC data (salt phase)

$$\frac{\Phi_{{}^7\mathrm{Be}}}{\Phi^{\mathrm{SSM}}_{{}^7\mathrm{Be}}} = 0.97^{+0.28}_{-0.54} \, ({}^{+0.85}_{-0.97})$$

large uncertainty

needs ${}^7\mathrm{Be}$ experiment

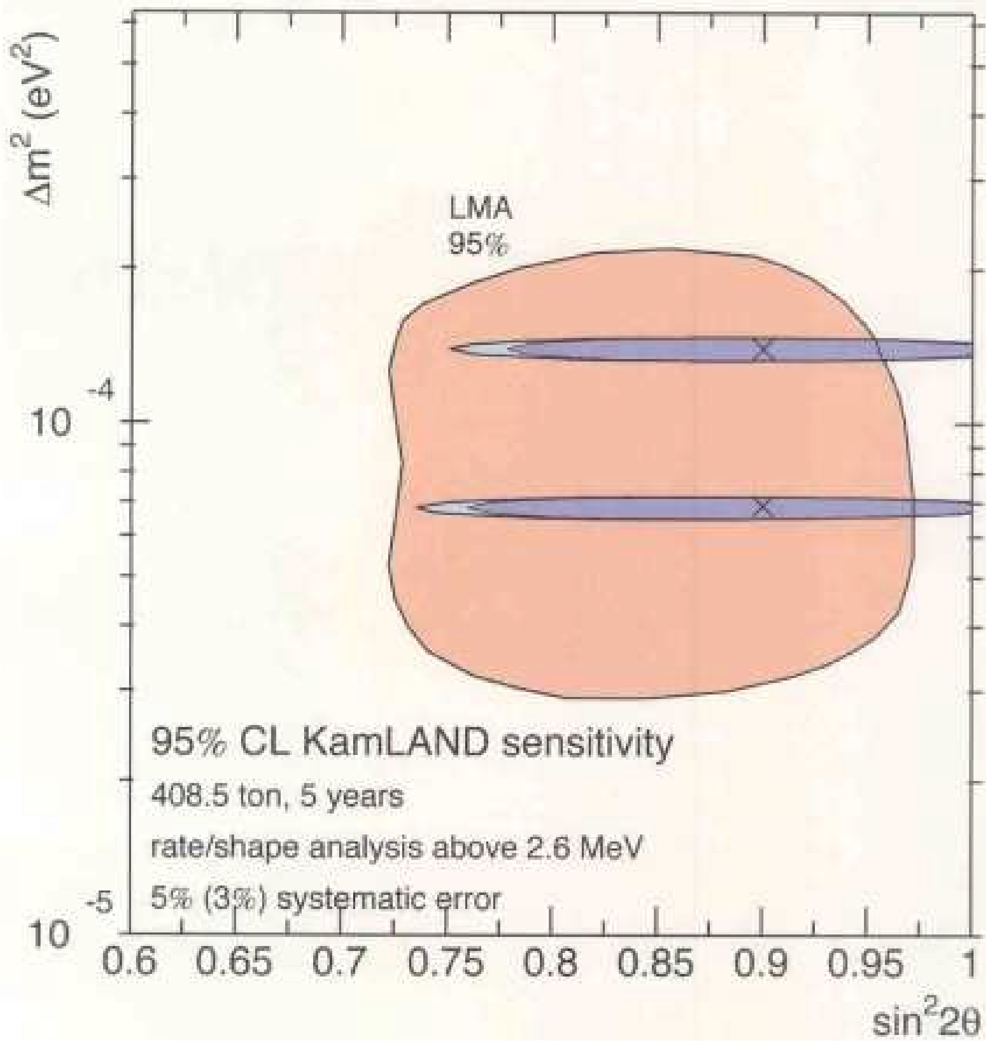
(KamLAND, Borexino?)

$$\frac{\Phi_{pp}}{\Phi^{\mathrm{SSM}}_{pp}} = 1.02^{+0.02}_{-0.02} \, ({}^{+0.07}_{-0.07})$$

small uncertainty

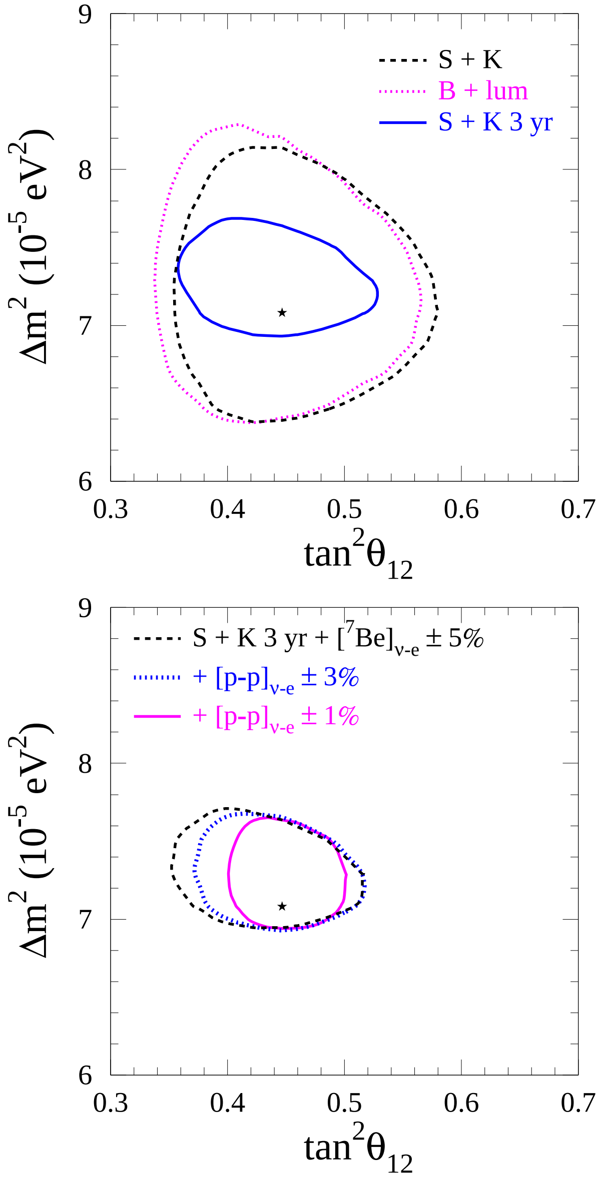
CNO luminosity: $\mathcal{L}_{\mathrm{CNO}} / \mathcal{L}_\odot = 0.0^{+2.8}_{-0.0} \, ({}^{+7.3}_{-0.0})$ [Bahcall, Gonzalez-Garcia, Peña-Garay, PRL 90 (2003) 131301]

Future Determination of Solar Mixing Parameters?



precise Δm^2 will be determined by KamLAND

[Inoue (KamLAND), Moriond 2003]



[Bahcall, Peña-Garay, hep-ph/0305159]

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\mathrm{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right) \cos 2\vartheta^0_{\text{M}} \, \cos 2\vartheta}$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{\mathrm{d}\ln A}{\mathrm{d}x}\right|_{\text{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2} E G_{\text{F}} N_e^{\text{c}} \exp\left(-\frac{x}{x_0}\right) \implies \left|\frac{\mathrm{d}\ln A}{\mathrm{d}x}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15}\,\mathrm{eV}$$

$$\tan^2\vartheta \simeq 0.4 \implies \sin^2 2\vartheta \simeq 0.82\,, \; \cos 2\vartheta \simeq 0.43 \qquad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \implies P_{\text{c}} \ll 1 \implies \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2} \cos 2\vartheta^0_{\text{M}} \, \cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta^0_{\rm M} = \frac{\Delta m^2 \cos 2\vartheta - A^0_{\rm CC}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A^0_{\rm CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter:

see [Bahcall, Peña-Garay, hep-ph/0305159]

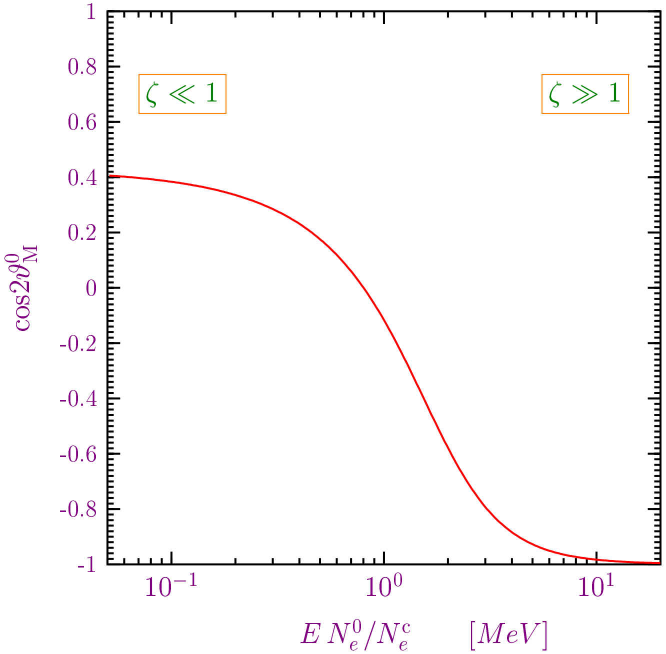
$$\zeta = \frac{A^0_{\rm CC}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\rm F} N^0_e}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\rm MeV}\right) \left(\frac{N^0_e}{N^c_e}\right)$$

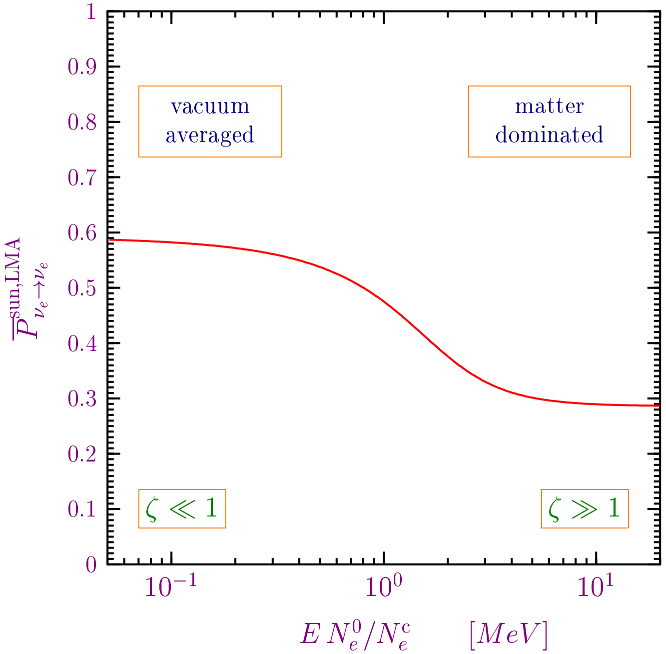
$$\zeta \ll 1 \quad \Longrightarrow \quad \vartheta^0_{\rm M} \simeq \vartheta \quad \Longrightarrow \quad \overline{P}^{\rm sun}_{\nu_e \to \nu_e} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \quad \Longrightarrow \quad \vartheta^0_{\rm M} \simeq \pi/2 \quad \Longrightarrow \quad \overline{P}^{\rm sun}_{\nu_e \to \nu_e} \simeq \sin^2 \vartheta$$

matter dominated survival probability

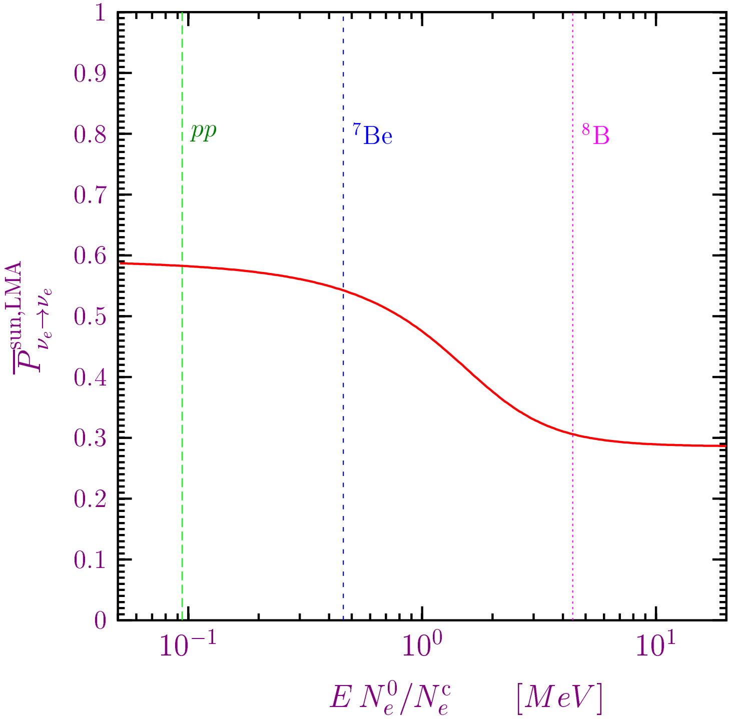




$$\langle E \rangle_{pp} \simeq 0.27\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot \qquad \Longrightarrow \qquad \langle E\,N_e^0/N_e^{\mathrm{c}} \rangle_{pp} \simeq 0.094\,\mathrm{MeV}$$

$$E_{^7\mathrm{Be}} \simeq 0.86\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{^7\mathrm{Be}} \simeq 0.06\,R_\odot \qquad \Longrightarrow \qquad \langle E\,N_e^0/N_e^{\mathrm{c}} \rangle_{^7\mathrm{Be}} \simeq 0.46\,\mathrm{MeV}$$

$$\langle E \rangle_{^8\mathrm{B}} \simeq 6.7\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{^8\mathrm{B}} \simeq 0.04\,R_\odot \qquad \Longrightarrow \qquad \langle E\,N_e^0/N_e^{\mathrm{c}} \rangle_{^8\mathrm{B}} \simeq 4.4\,\mathrm{MeV}$$



each neutrino experiment is mainly sensitive to one flux

each neutrino experiment is mainly sensitive to ϑ

accurate pp experiment can improve determination of ϑ [Bahcall, Peña-Garay, hep-ph/0305159]

Goals of Future Solar Neutrino Experiments

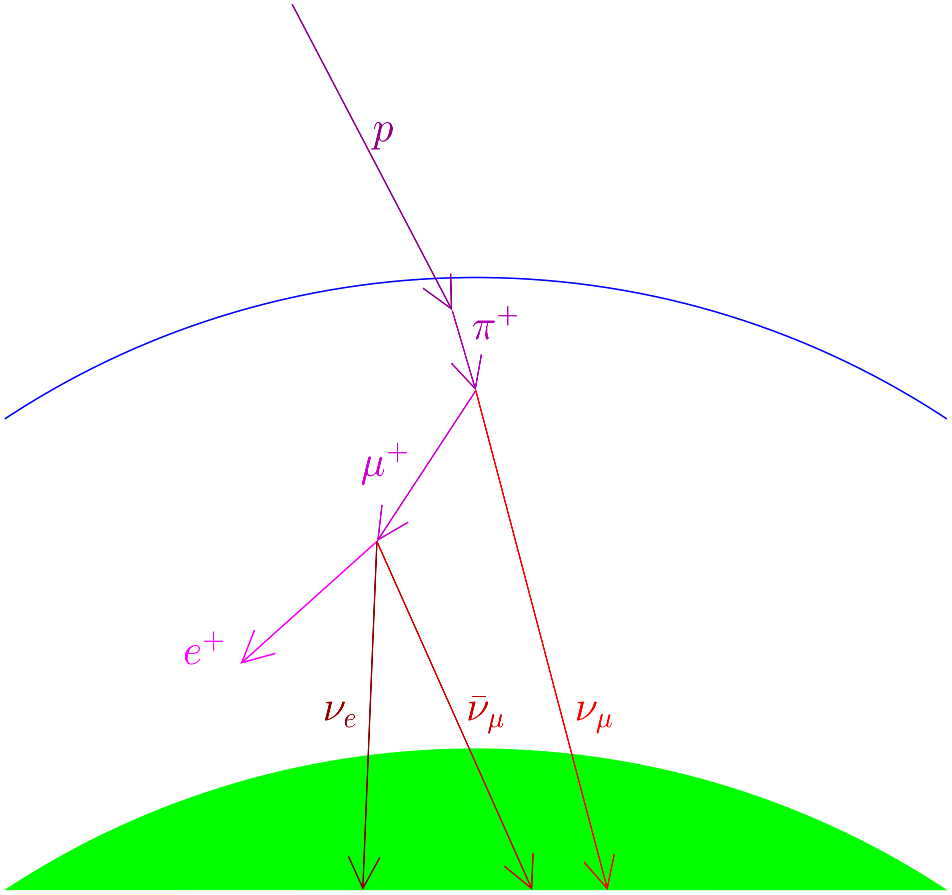
[Bahcall, Peña-Garay, hep-ph/0305159]

- ⋆ Improve the determination of ϑ
- ⋆ Accurate measure of solar neutrino fluxes
- ⋆ Discover or constraint subdominant neutrino conversion mechanisms

Precise Determination of Δm^2 and $\tan^2\vartheta$ with New Reactor Experiment

- ⋆ LMA-I: $L \simeq 70 - 80\,\text{km}$ [Bandyopadhyay, Choubey, Goswami, PRD 67 (2003) 113011] [Bouchiat, hep-ph/0304253]
- ⋆ LMA-II: $L \simeq 20 - 30\,\text{km}$ [Schoenert, Lasserre, Oberauer, Astropart. Phys. 18 (2003)], [Choubey, Petcov, Piai, hep-ph/0306017]

ATMOSPHERIC NEUTRINOS



$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

theoretical error on ratios: $\sim$ 5%

theoretical error on absolute fluxes: $\sim$ 30%

ratio of ratios

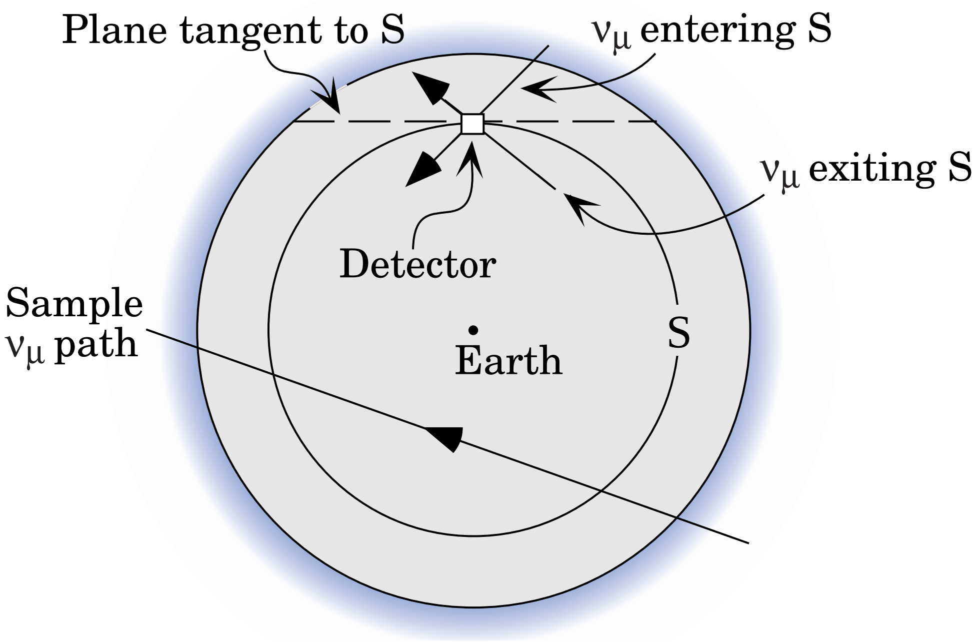
$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

$R = 0.638^{+0.017}_{-0.017} \pm 0.050$ at $E < 1\,\text{GeV}$

$R = 0.675^{+0.034}_{-0.032} \pm 0.080$ at $E > 1\,\text{GeV}$

[Super-Kamiokande, hep-ex/0105023]

Super-Kamiokande Up-Down Asymmetry

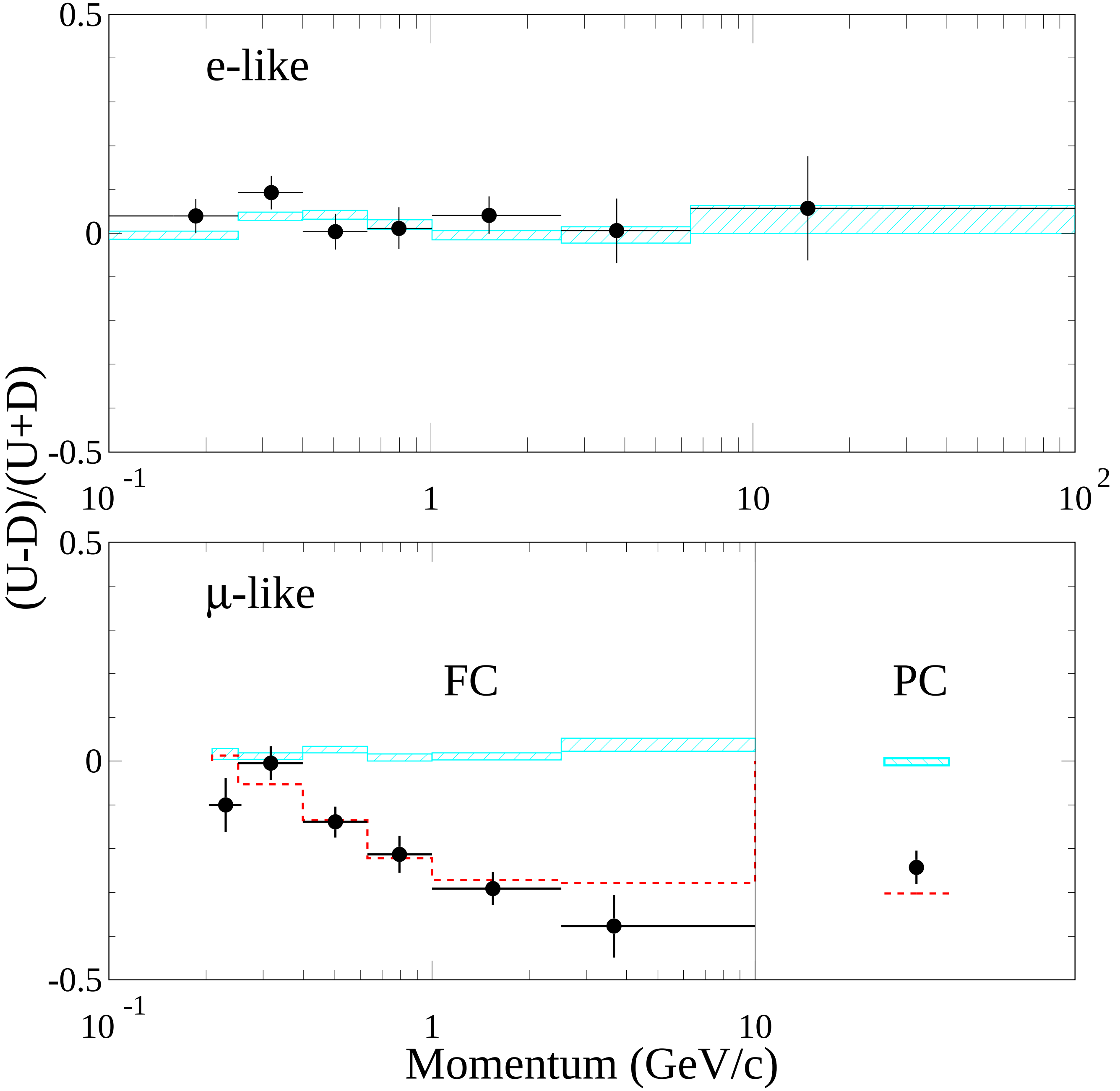


- any ν entering the sphere S later exits it
- steady state $\Rightarrow$ $\Phi^{\text{in}}(S) = \Phi^{\text{out}}(S)$
- $E_\nu \gtrsim 1\,\text{GeV}$ $\Rightarrow$ isotropic flux
- isotropy $\Rightarrow$ $\Phi^{\text{in}}(s) = \Phi^{\text{out}}(s)$, $\forall s \in S$
- $\text{D} \in S \Rightarrow \Phi^{\text{up}}(\text{D}) = \Phi^{\text{down}}(\text{D})$,

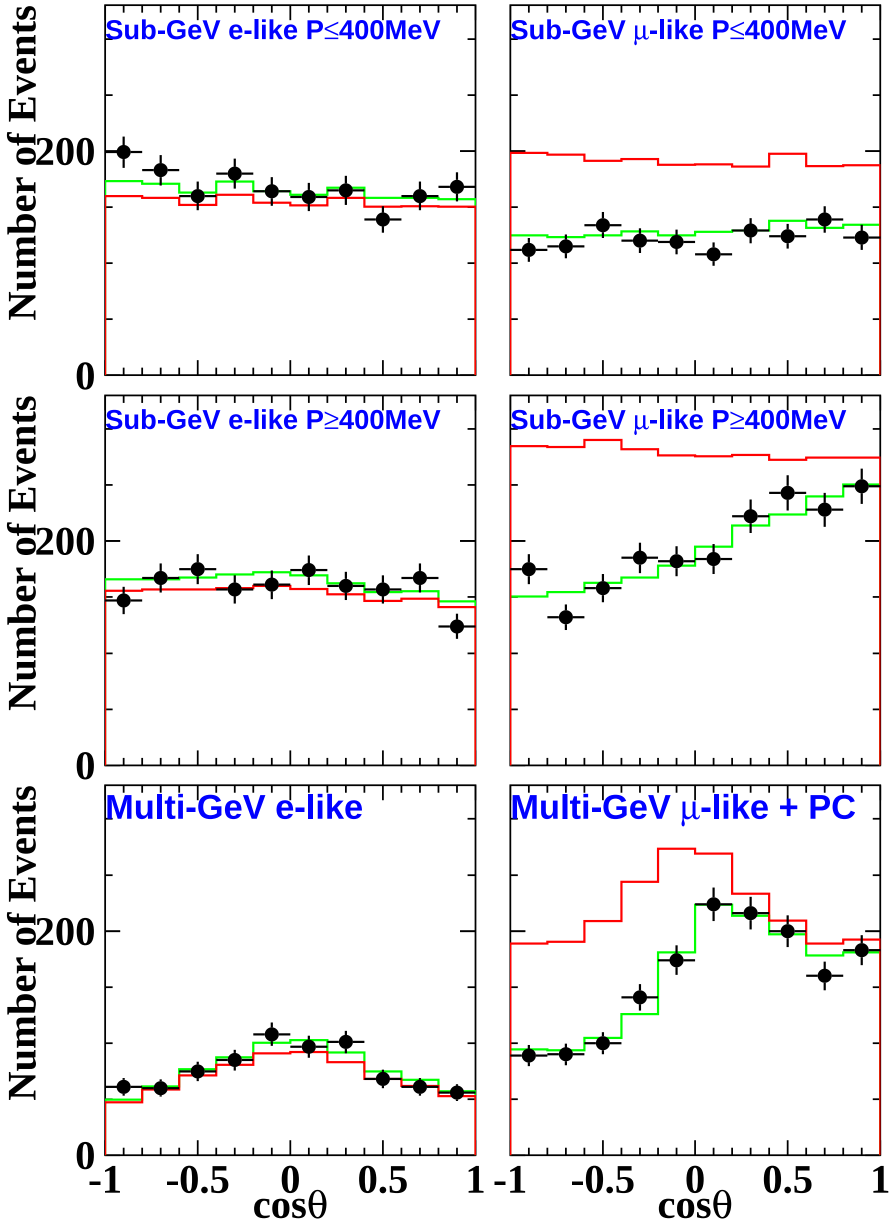
[B. Kayser, Review of Particle Properties, PRD 66 (2002) 010001]

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}} \right) = -0.311 \pm 0.043 \pm 0.01} \qquad \underline{7\sigma!}$$

MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

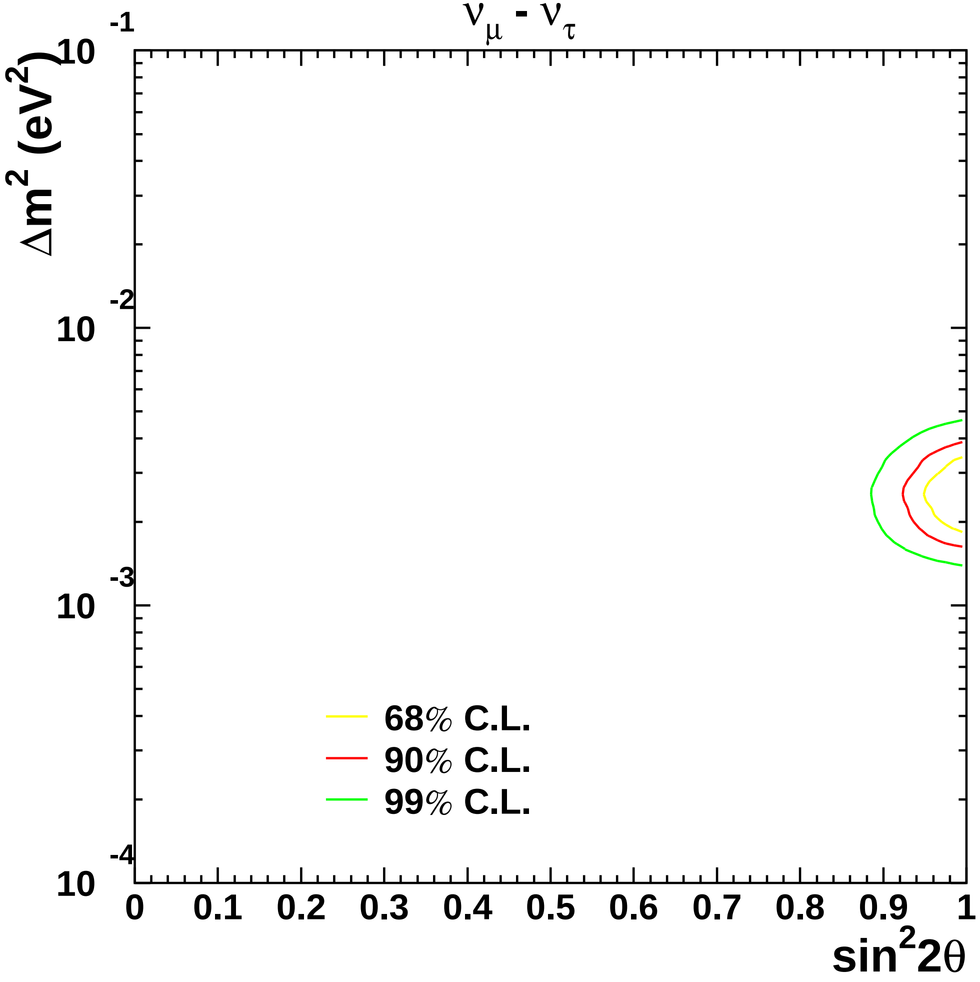


[R. J. Wilkes, SK, hep-ex/0212035]



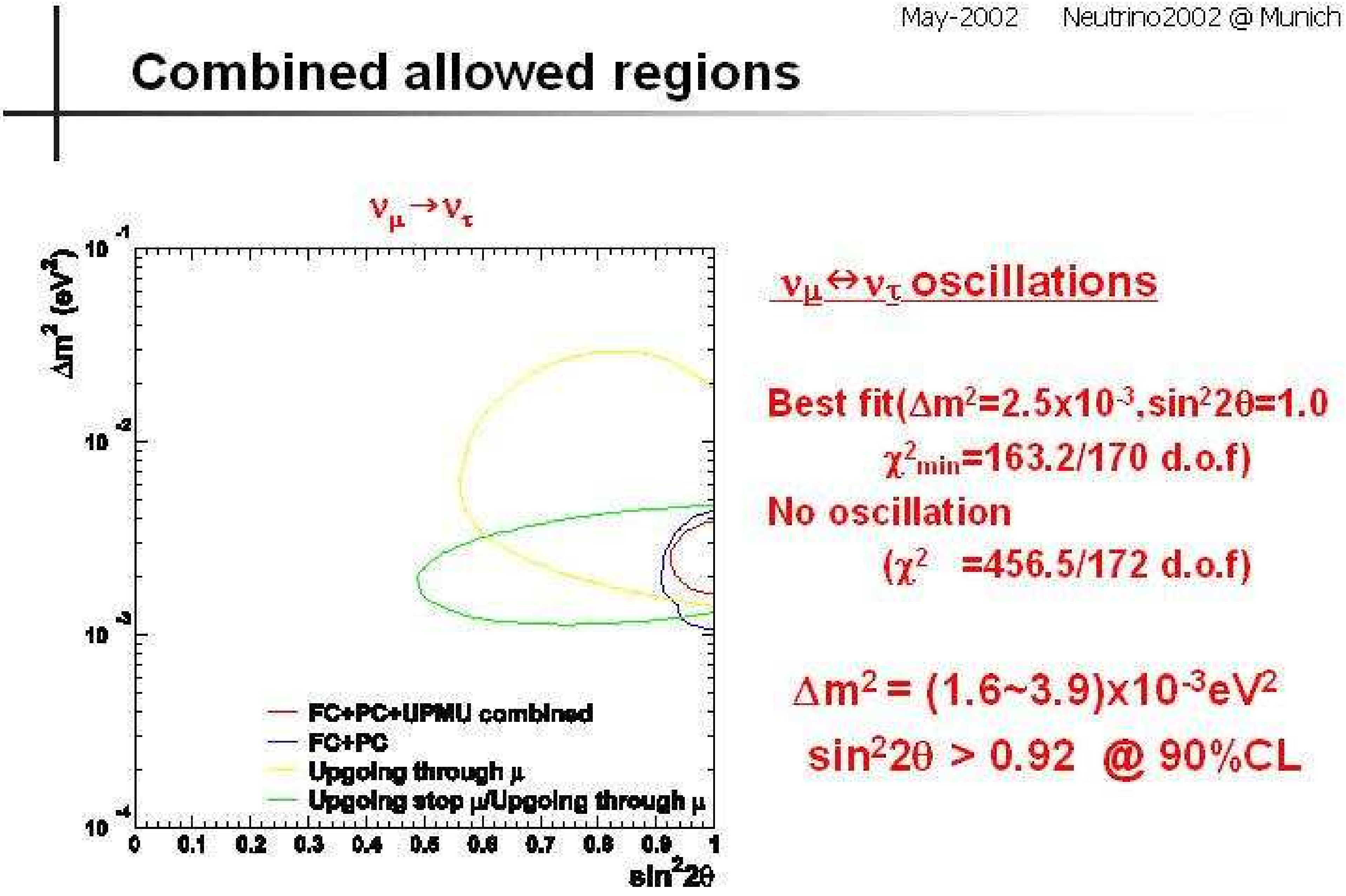
[R. J. Wilkes, SK, hep-ex/0212035]

Two-Neutrino Oscillation Fit of Super-Kamiokande Atmospheric Data



[R. J. Wilkes, SK, hep-ex/0212035]

Best Fit: $\Delta m^2 = 2.5 \times 10^{-3}\,\text{eV}^2 \quad \sin^2 2\theta = 1.0 \quad \chi^2_{\text{min}} = 163.2 \quad \text{d.o.f.} = 172$



May-2002 Neutrino2002 @ Munich

Combined allowed regions

$\nu_\mu \leftrightarrow \nu_\tau$ oscillations

Best fit($\Delta m^2=2.5\text{x}10^{-3}$, $\sin^2 2\theta=1.0$

$\chi^2_{\text{min}}=163.2/170$ d.o.f)

No oscillation

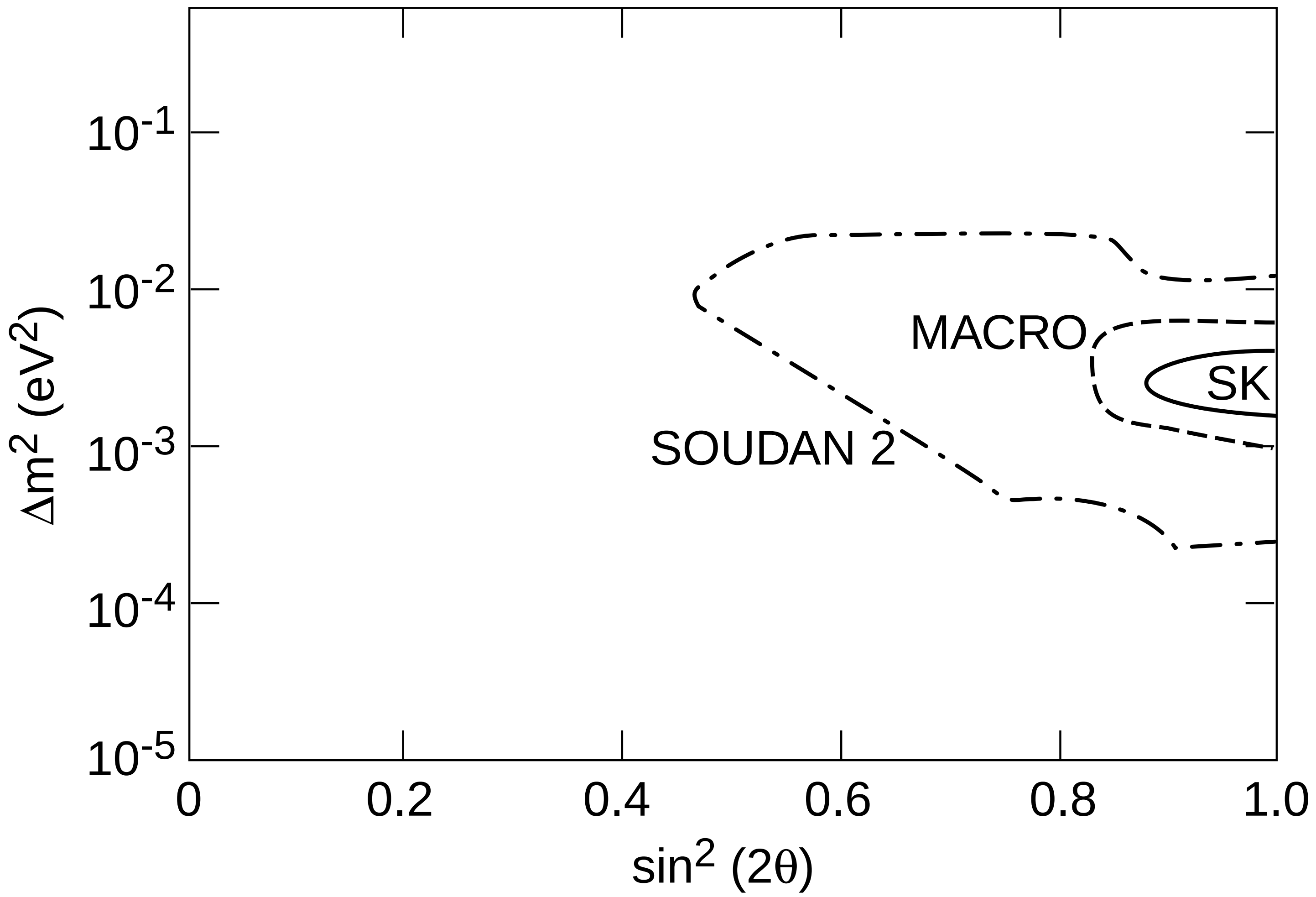
(χ^2 $=456.5/172$ d.o.f)

$\Delta m^2 = (1.6\sim3.9)\text{x}10^{-3}\text{eV}^2$

$\sin^2 2\theta > 0.92$ @ 90%CL

[Shiozawa (SK), Neutrino 2002]

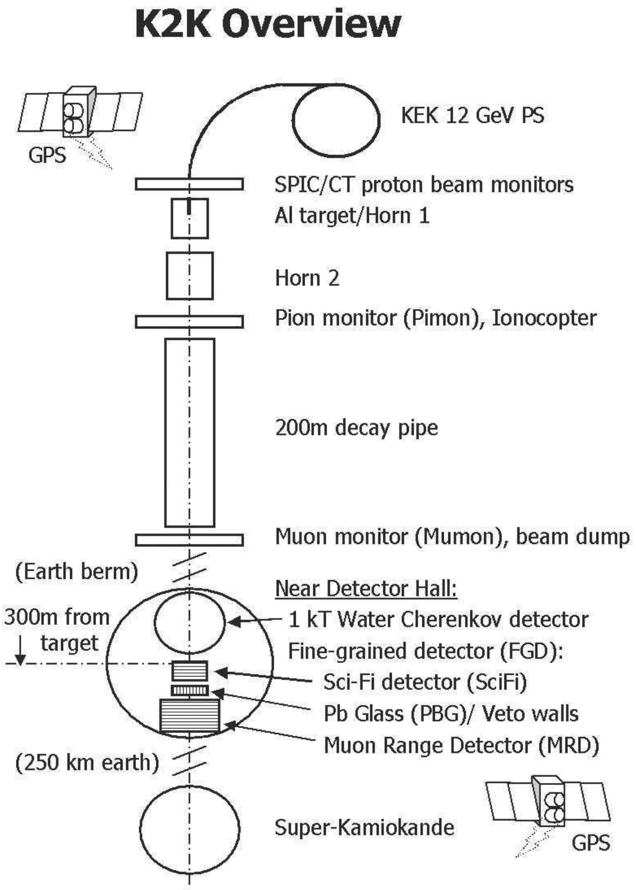
Soudan-2 & MACRO

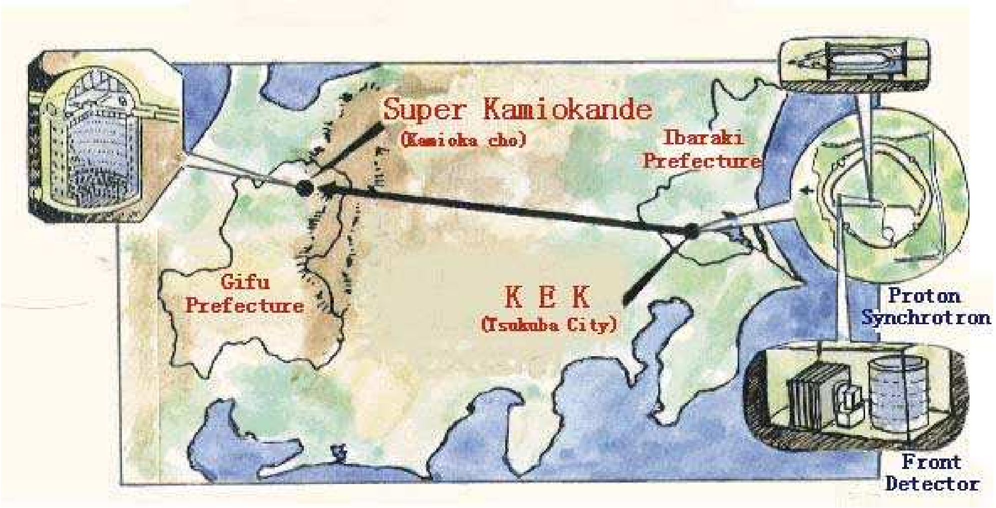


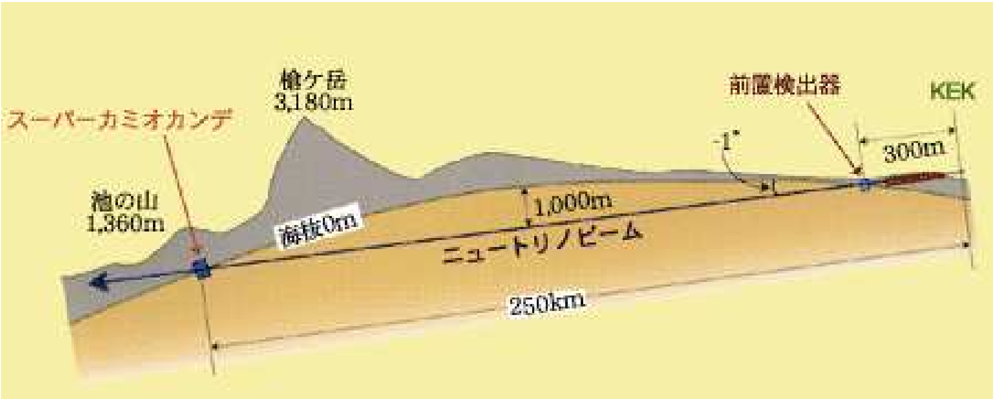
[Giacomelli, Giorgini, Spurio, hep-ex/0201032]

K2K

KEK to Super-Kamiokande long-baseline accelerator ν_μ disappearance experiment ($L = 250\,\mathrm{km}$)



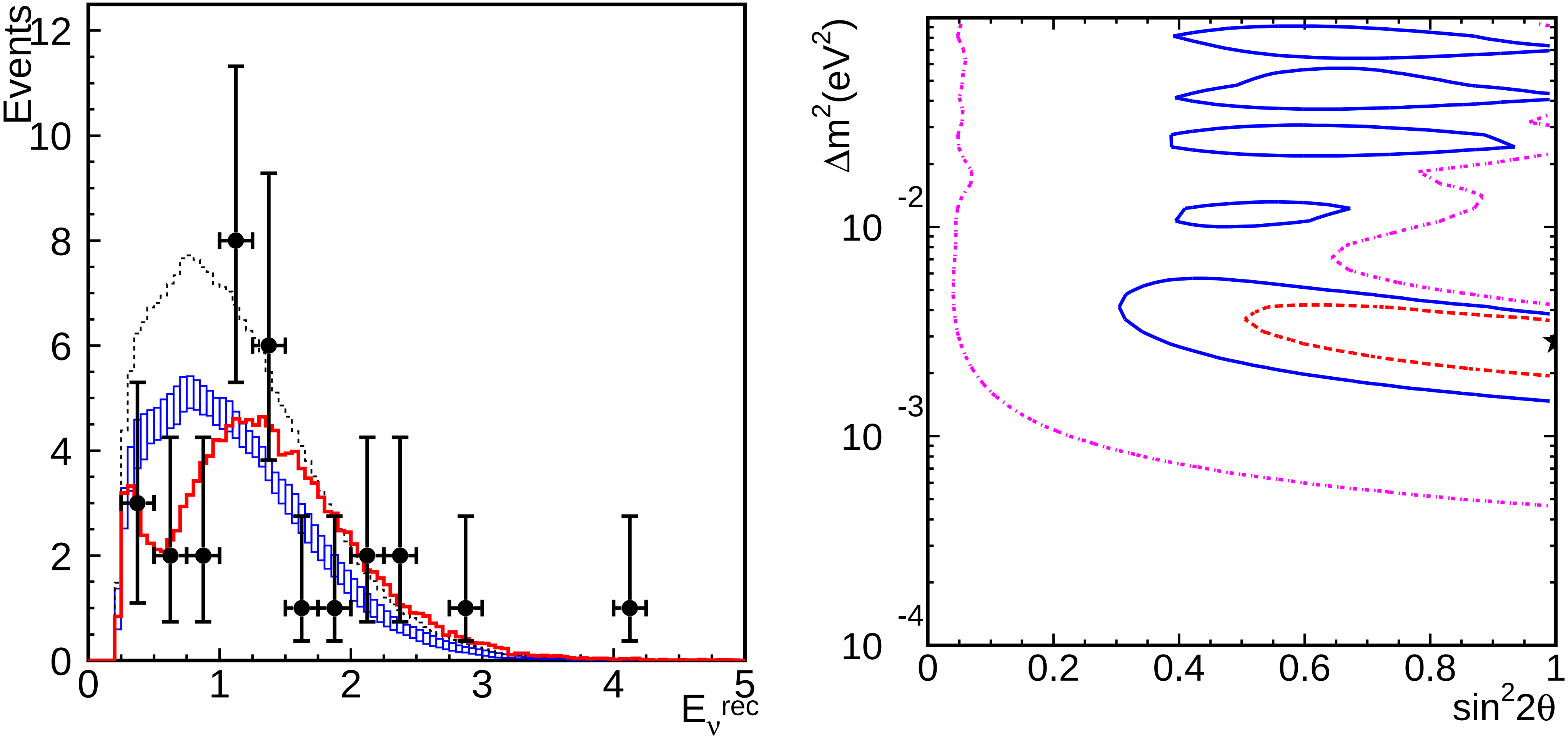




[R. J. Wilkes, SK, hep-ex/0212035]

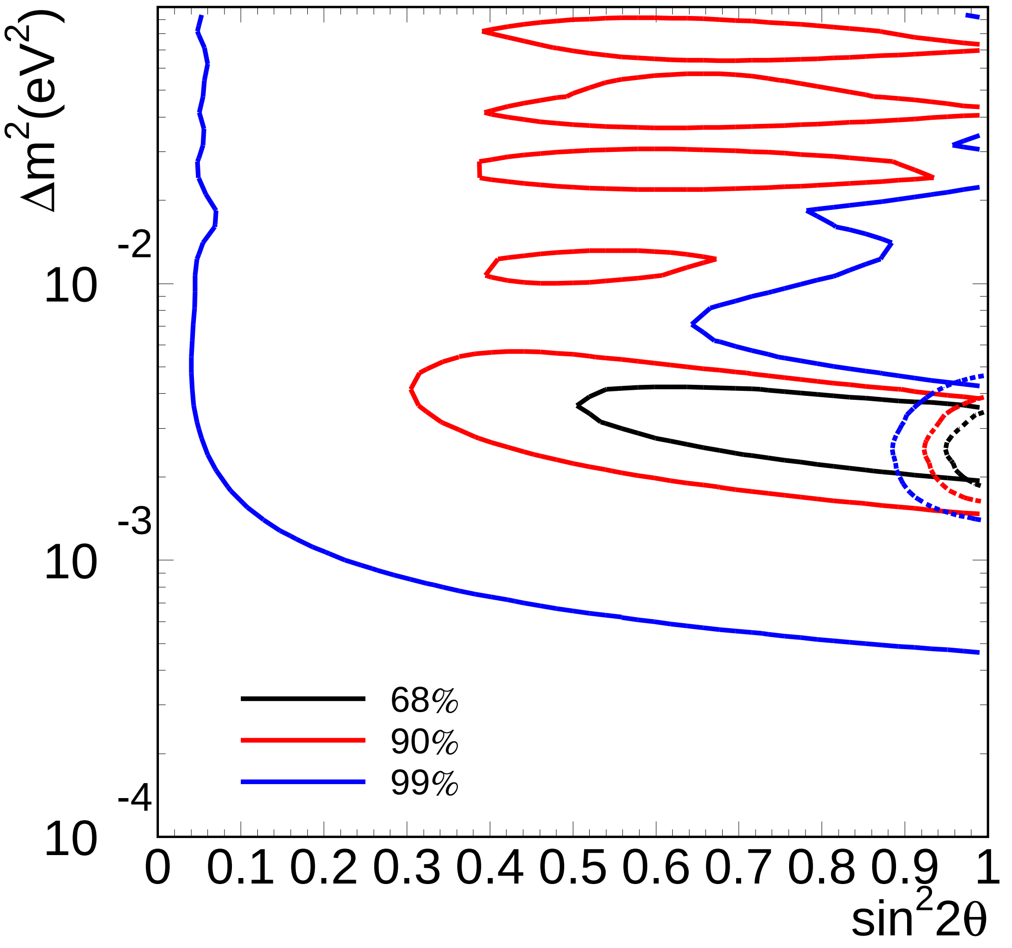
[http://neutrino.kek.jp]

Expected: $80.1^{+6.2}_{-5.4}$ events Observed: 56 events Probability $< 1\%$

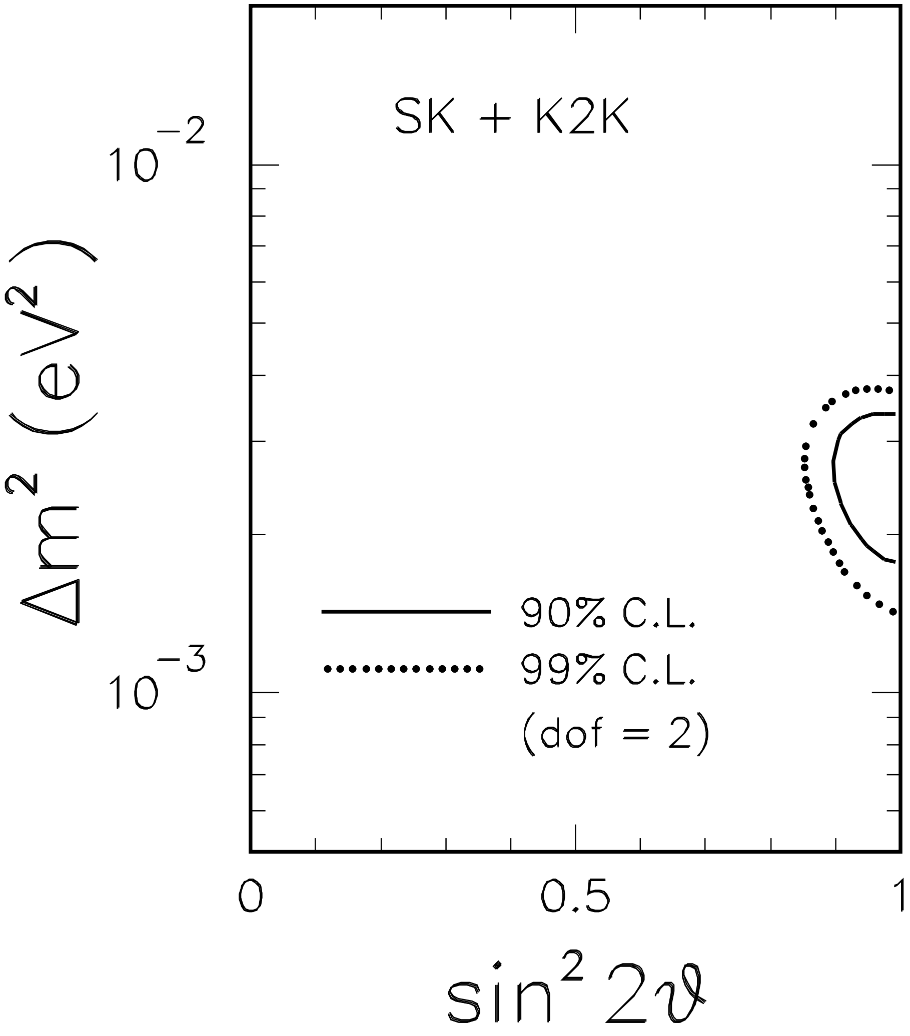


[K2K, PRL 90 (2003) 041801]

K2K ⇒ confirmation of atmospheric allowed region

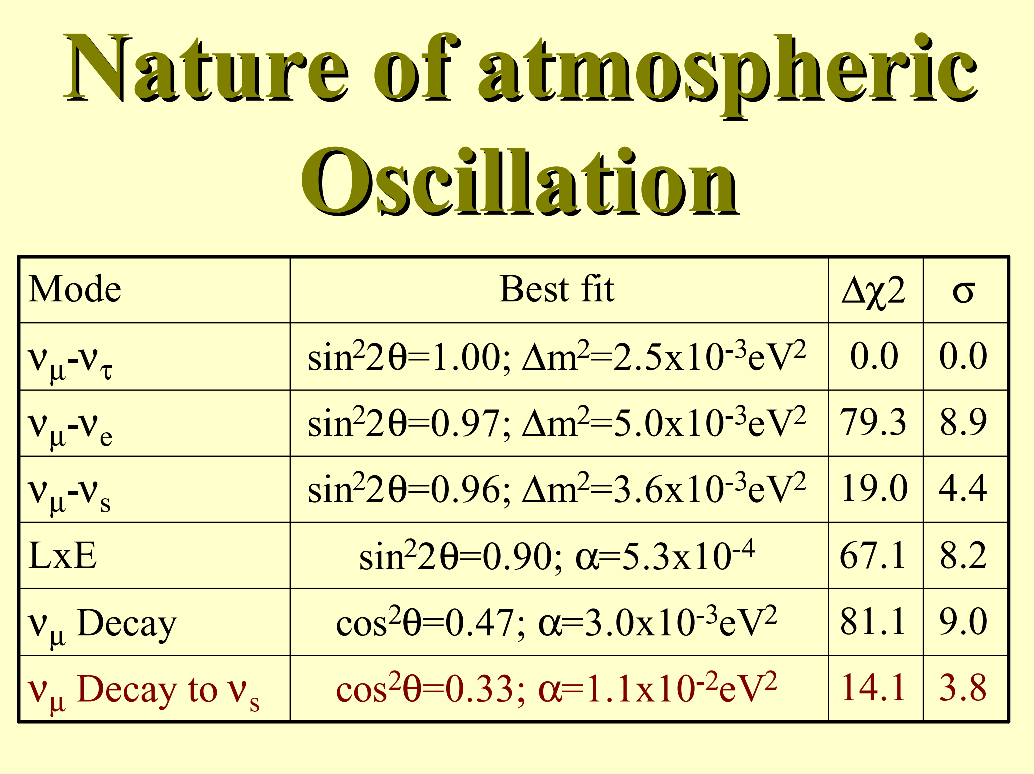


[Oyama, hep-ex/0210030]



[Fogli,Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

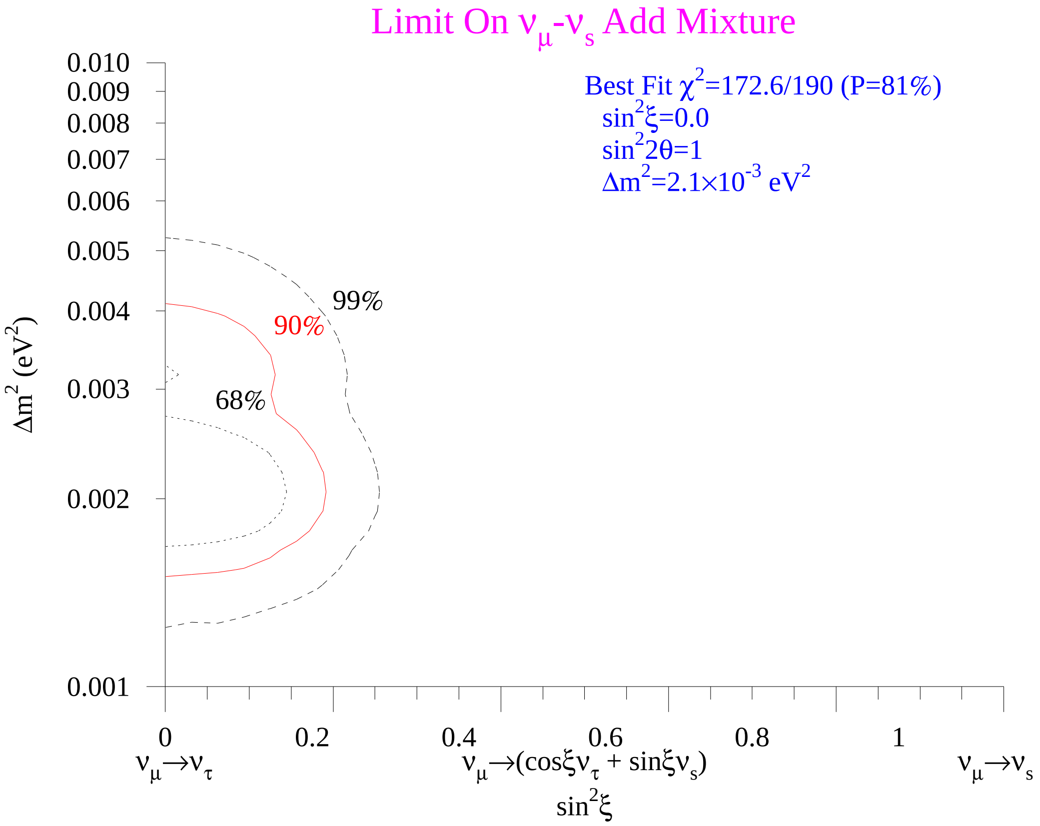
Sterile Neutrinos in Atmospheric Neutrino Flux?



Nature of atmospheric Oscillation

Mode	Best fit	$\Delta\chi2$	σ
ν_μ-ν_τ	$\sin^2 2\theta=1.00$; $\Delta m^2=2.5\text{x}10^{-3}\text{eV}^2$	0.0	0.0
ν_μ-ν_e	$\sin^2 2\theta=0.97$; $\Delta m^2=5.0\text{x}10^{-3}\text{eV}^2$	79.3	8.9
ν_μ-ν_s	$\sin^2 2\theta=0.96$; $\Delta m^2=3.6\text{x}10^{-3}\text{eV}^2$	19.0	4.4
LxE	$\sin^2 2\theta=0.90$; $\alpha=5.3\text{x}10^{-4}$	67.1	8.2
ν_μ Decay	$\cos^2\theta=0.47$; $\alpha=3.0\text{x}10^{-3}\text{eV}^2$	81.1	9.0
ν_μ Decay to ν_s	$\cos^2\theta=0.33$; $\alpha=1.1\text{x}10^{-2}\text{eV}^2$	14.1	3.8

[Smy (SK), Moriond 2002]



[Nakaya (SK), hep-ex/0209036]

FUTURE

MINOS: $\nu_\mu \to \nu_\mu$, $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_{e,\mu,\tau}$ (NC)

CNGS: ICARUS: $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$ OPERA: $\nu_\mu \to \nu_\tau$

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$ (Homestake, Kamiokande, GALLEX, SAGE, GNO, Super-Kamiokande, SNO)

Reactor $\bar{\nu}_e$ disappearance (KamLAND)

$\Longrightarrow$

$$\Delta m^{2\,\text{best-fit}}_{\text{SUN}} = 6.9 \times 10^{-5}$$

$$5.4 \times 10^{-5} < \Delta m^2_{\text{SUN}} < 9.4 \times 10^{-5}$$

$[\text{eV}^2]$ (99.73% C.L.)

[Maltoni, Schwetz, Tortola, Valle, hep-ph/0309130]

Atmospheric $\nu_\mu \to \nu_\tau$ (Kamiokande, IMB, Super-Kamiokande, MACRO, SOUDAN 2)

Accelerator ν_μ disappearance (K2K)

$\Longrightarrow$

$$\Delta m^{2\,\text{best-fit}}_{\text{ATM}} = 2.6 \times 10^{-3}$$

$$1.4 \times 10^{-3} < \Delta m^2_{\text{ATM}} < 5.1 \times 10^{-3}$$

$[\text{eV}^2]$ (99.73% C.L.)

[Fogli, Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

THREE-NEUTRINO MIXING

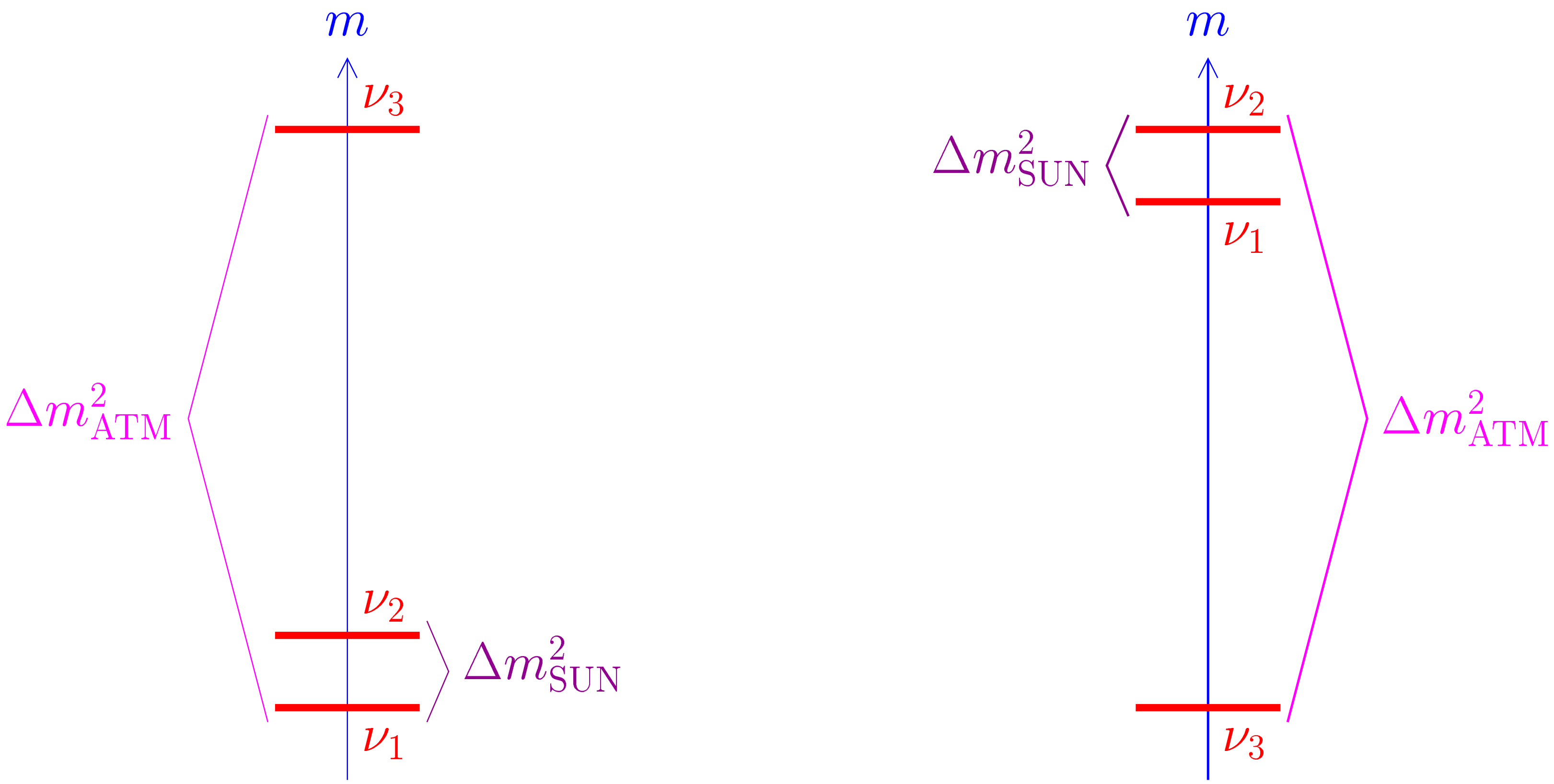
flavor fields ν_α, $\alpha = e, \mu, \tau$

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL}$$

massive fields $\nu_k \to m_k$

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \qquad \Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$$

ALLOWED THREE-NEUTRINO SCHEMES



"normal" "inverted"

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

SUN →

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

↑
ATM

CHOOZ:

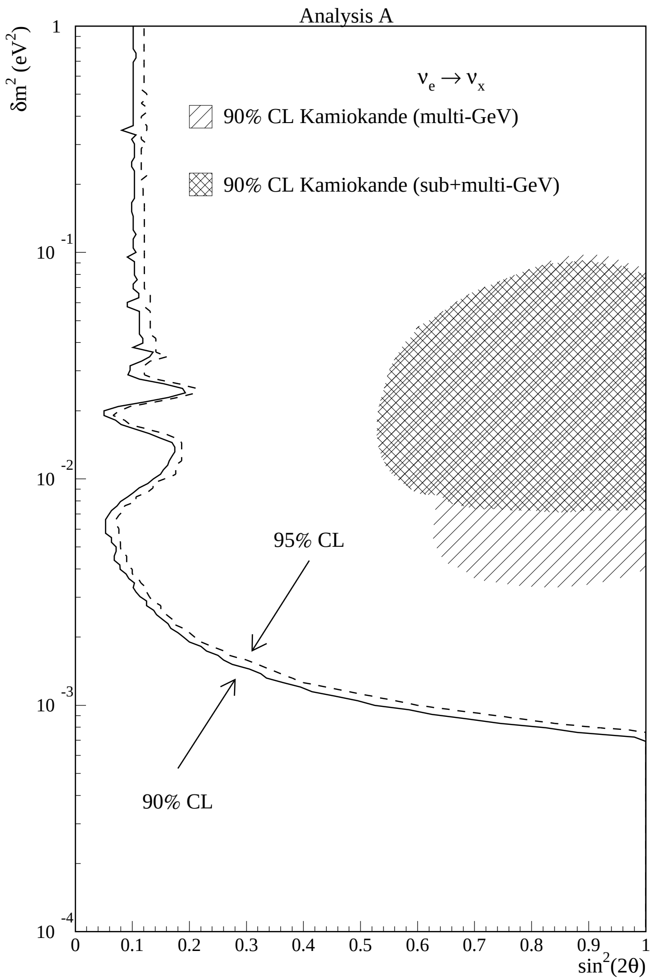
$$\begin{cases} \Delta m^2_{\mathrm{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\mathrm{ATM}} \\ \sin^2 2\vartheta_{\mathrm{CHOOZ}} = 4|U_{e3}|^2(1-|U_{e3}|^2) \end{cases}$$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 < 5 \times 10^{-2} \ (99.73\%\ \mathrm{C.L.})}$$

[Fogli et al., PRD 66 (2002) 093008]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!

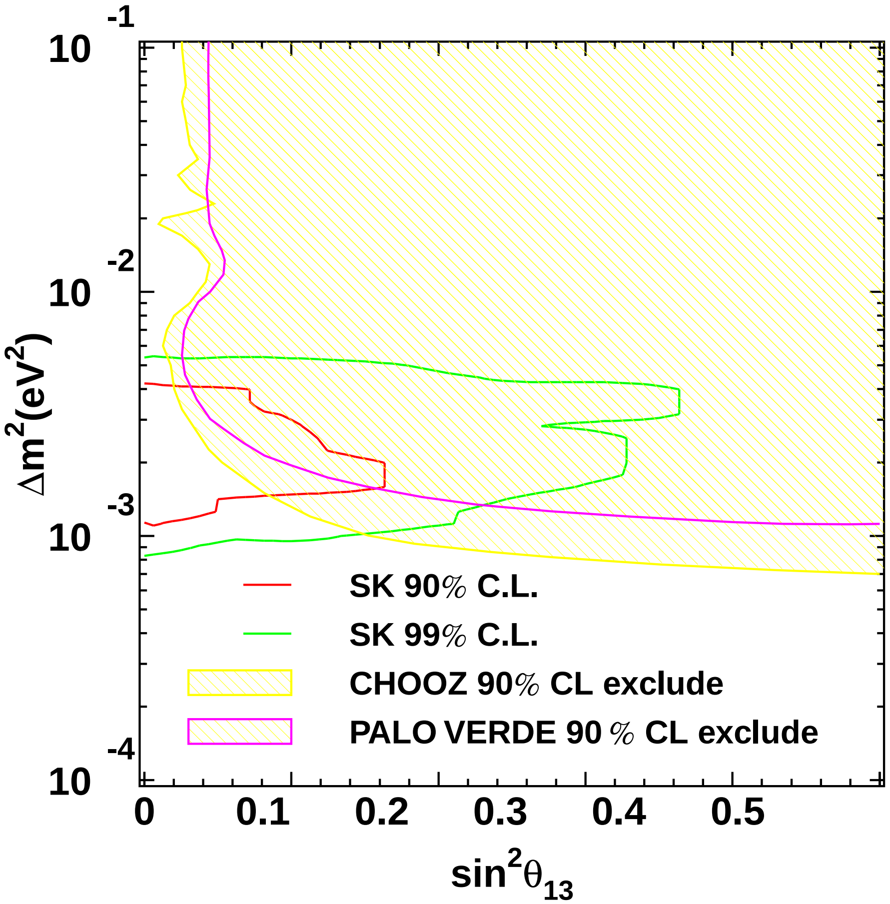


[CHOOZ, PLB 466 (1999) 415]
see also [Palo Verde, PRD 64 (2001) 112001]

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2\vartheta_{\mathrm{SUN}} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2\vartheta_{\mathrm{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, Giunti, PLB 444 (1998) 379]
[Guo, Xing, PRD 67 (2003) 053002]



[Nakaya (SK), hep-ex/0209036]

FUTURE

MINOS: sensitivity $|U_{e3}|^2 \sim 10^{-2}$

JHF-Kamioka: sensitivity $|U_{e3}|^2 \sim 2 \times 10^{-3}$ ($|U_{e3}|^2 \sim 10^{-4}$ with Hyper-Kamiokande) [hep-ex/0106019]

Reactor Experiments: sensitivity $|U_{e3}|^2 \sim 3 \times 10^{-3}$ [NuFact 03, `http://www.cap.bnl.gov/nufact03`]

Neutrino Factory: sensitivity $|U_{e3}|^2 \sim 10^{-5}$

$|U_{e3}| > 0 \Rightarrow$ normal or inverted scheme (Earth matter effects) and (maybe) CP violation

Standard Parameterization of Mixing Matrix

$$U = R_{23}\, W_{13}\, R_{12}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} = \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} = \vartheta_{\text{SUN}}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$\sin^2\vartheta_{\text{CHOOZ}} = |U_{e3}|^2 = \sin^2\vartheta_{13}$$

$$\sin^2\vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{s_{12}^2 c_{13}^2}{1 - s_{13}^2} = \sin^2\vartheta_{12}$$

$$\sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2 = s_{23}^2 c_{13}^2 \simeq \sin^2\vartheta_{23}$$

BILARGE MIXING

$$|U_{e3}|^2 \ll 1 \;\Rightarrow\; U \simeq \begin{pmatrix} c_{\vartheta_\mathrm{S}} & s_{\vartheta_\mathrm{S}} & 0 \\ -s_{\vartheta_\mathrm{S}} c_{\vartheta_\mathrm{A}} & c_{\vartheta_\mathrm{S}} c_{\vartheta_\mathrm{A}} & s_{\vartheta_\mathrm{A}} \\ s_{\vartheta_\mathrm{S}} s_{\vartheta_\mathrm{A}} & -c_{\vartheta_\mathrm{S}} s_{\vartheta_\mathrm{A}} & c_{\vartheta_\mathrm{A}} \end{pmatrix} \;\Rightarrow\; \left\{ \begin{aligned} \nu_e &= c_{\vartheta_\mathrm{S}} \nu_1 + s_{\vartheta_\mathrm{S}} \nu_2 \\ \nu_a^{(\mathrm{S})} &= -s_{\vartheta_\mathrm{S}} \nu_1 + c_{\vartheta_\mathrm{S}} \nu_2 \\ &= c_{\vartheta_\mathrm{A}} \nu_\mu - s_{\vartheta_\mathrm{A}} \nu_\tau \end{aligned} \right.$$

$$\sin^2 2\vartheta_\mathrm{A} \simeq 1 \Rightarrow \vartheta_\mathrm{A} \simeq \frac{\pi}{4} \Rightarrow U \simeq \begin{pmatrix} c_{\vartheta_\mathrm{S}} & s_{\vartheta_\mathrm{S}} & 0 \\ -s_{\vartheta_\mathrm{S}}/\sqrt{2} & c_{\vartheta_\mathrm{S}}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_\mathrm{S}}/\sqrt{2} & -c_{\vartheta_\mathrm{S}}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Solar $\nu_e \to \nu_a^{(\mathrm{S})} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$

$\frac{\Phi_{\mathrm{CC}}^{\mathrm{SNO}}}{\Phi_{\nu_\mathrm{e}}^{\mathrm{SSM}}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_\mathrm{e}} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau}$ for $E \gtrsim 6\,\mathrm{MeV}$

$$\text{LMA} \Rightarrow \tan^2 \vartheta_\mathrm{S} \simeq 0.4 \Rightarrow \vartheta_\mathrm{S} \simeq \frac{\pi}{6} \Rightarrow U \simeq \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

INFERENCE OF MIXING MATRIX

$$\sin^2\vartheta_{\rm SUN} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} \qquad \sin^2\vartheta_{\rm ATM} = |U_{\mu3}|^2 \qquad \sin^2\vartheta_{\rm CHOOZ} = |U_{e3}|^2$$

$$\tan^2\vartheta_{\rm SUN}^{\text{best-fit}} = 0.43 \qquad 0.30 < \tan^2\vartheta_{\rm SUN} < 0.64 \quad (99.73\%\ \text{C.L.})$$

[Maltoni, Schwetz, Tortola, Valle, hep-ph/0309130]

$$\sin^2 2\vartheta_{\rm ATM}^{\text{best-fit}} = 1 \qquad \sin^2 2\vartheta_{\rm ATM} > 0.86 \quad (99.73\%\ \text{C.L.})$$

[Fogli, Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

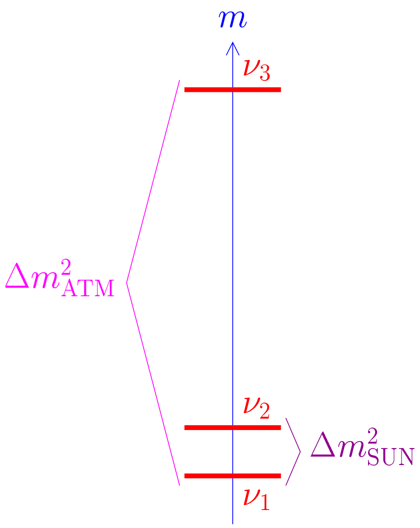
$$\sin^2 2\vartheta_{\rm CHOOZ}^{\text{best-fit}} = 0 \qquad \sin^2 2\vartheta_{\rm CHOOZ} < 5\times10^{-2} \quad (99.73\%\ \text{C.L.})$$

[Fogli et al., PRD 66 (2002) 093008]

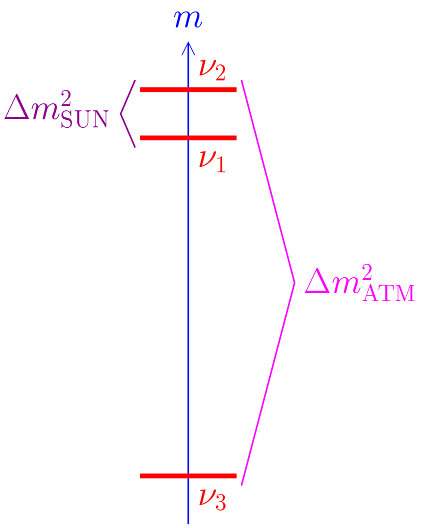
$$U_{\rm bf} \simeq \begin{pmatrix} 0.84 & 0.55 & 0.00 \\ -0.39 & 0.59 & 0.71 \\ 0.39 & -0.59 & 0.71 \end{pmatrix}$$

$$|U| \simeq \begin{pmatrix} 0.76-0.88 & 0.47-0.62 & 0.00-0.22 \\ 0.09-0.62 & 0.29-0.79 & 0.55-0.85 \\ 0.11-0.62 & 0.32-0.80 & 0.51-0.83 \end{pmatrix}$$

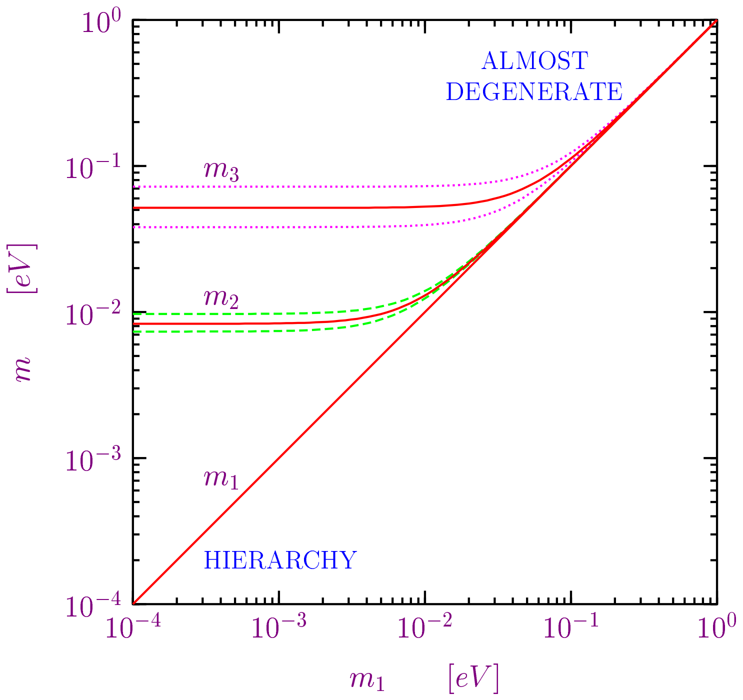
ABSOLUTE SCALE OF NEUTRINO MASSES

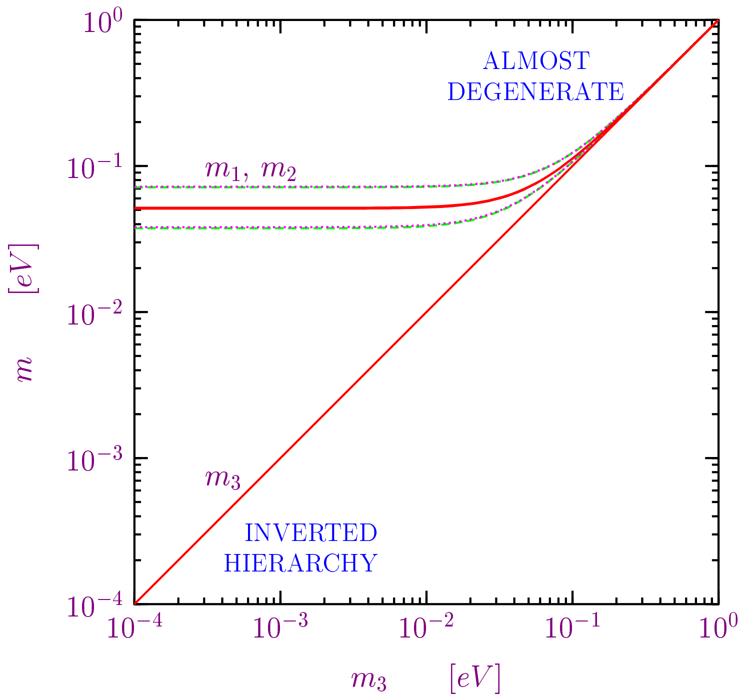


normal



inverted



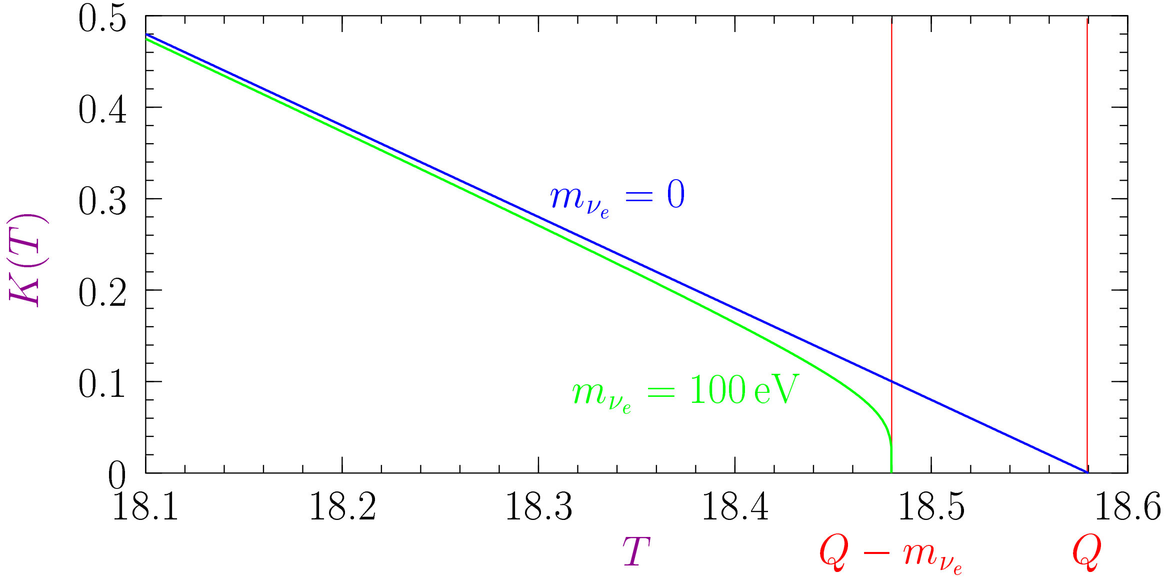


Tritium β Decay: ${}^{3}\mathrm{H} \to {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_{e}$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_\mathrm{F})^2}{2\pi^3} \, |\mathcal{M}|^2 \, F(E) \, pE \, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^3\mathrm{H}} - M_{{}^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot: $K(T) = \sqrt{\dfrac{\mathrm{d}\Gamma/\mathrm{d}T}{\dfrac{(\cos\vartheta_C G_\mathrm{F})^2}{2\pi^3} \, |\mathcal{M}|^2 \, F(E) \, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$



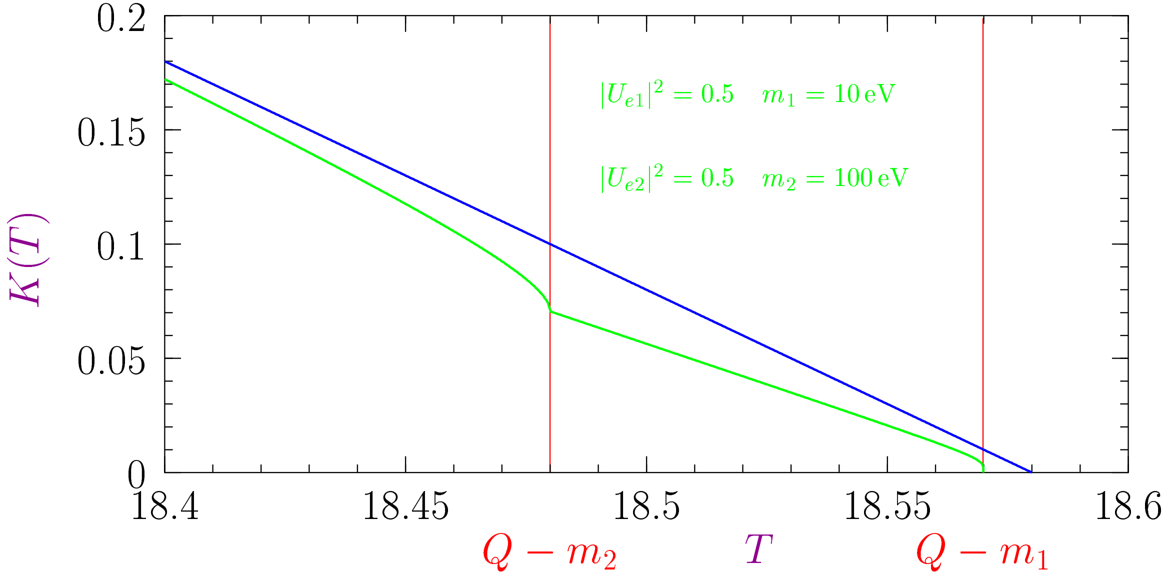
$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad \text{(95\% C.L.)}}$

[Mainz, Troitsk, hep-ex/0210050]

Future: KATRIN [hep-ex/0109033]

sensitivity: $m_{\nu_e} \gtrsim 0.3\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



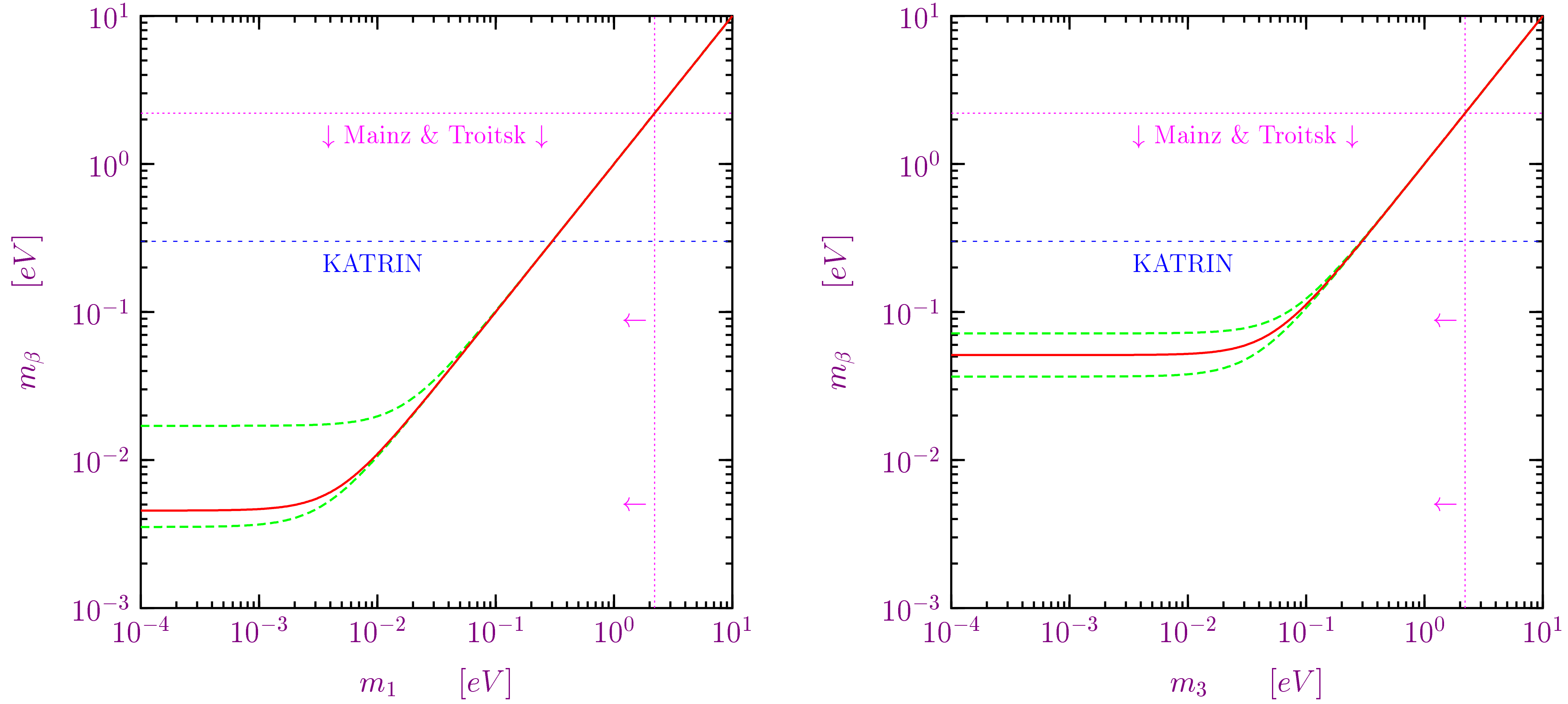
analysis of data is different from the no-mixing case: $2N-1$ parameters $\left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses $(m_k \ll Q - T)$ $\Longrightarrow$ effective mass

$$\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$$

$$\begin{aligned} K^2 &= (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right] \\ &= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2} \end{aligned}$$

$m_{\nu_e} < 2.2\,\text{eV}$ (95% C.L.) $\Longrightarrow$ $m_\beta < 2.2\,\text{eV}$ (95% C.L.)



normal scheme

inverted scheme

almost degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu$ $\Longrightarrow$ $m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

VERY FAR FUTURE: IF $m_\beta \lesssim 3 \times 10^{-2}\,\text{eV}$ $\Longrightarrow$ NORMAL HIERARCHY

COSMOLOGICAL LIMIT ON NEUTRINO MASSES

neutrinos are in equilibrium in the primeval plasma through the weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \qquad \nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \qquad \Longrightarrow T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \simeq 1.676 \times 10^{-4}\,\text{eV}$ $\quad (T_\gamma{=}2.725{\pm}0.001\,\text{K})$

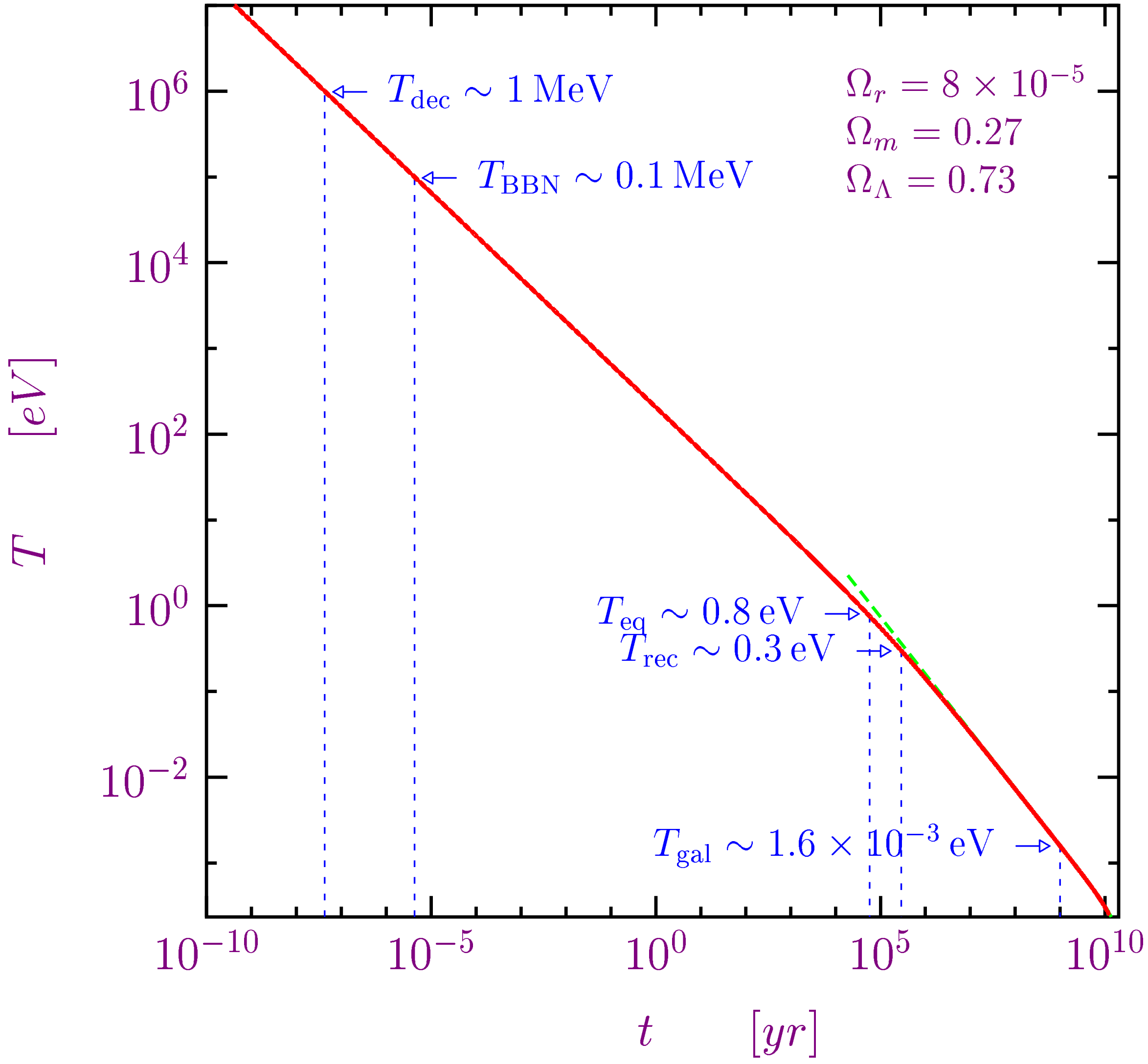
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\, h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$ $\quad \left(\rho_c{=}\frac{3\,H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

very weak assumptions: $h \lesssim 1$, $\Omega_\nu \lesssim 1 \quad \Longrightarrow \quad \sum_k m_k \lesssim 94\,\text{eV}$

reasonable assumptions: $h \lesssim 0.8$, $\Omega_\nu \lesssim 0.1 \quad \Longrightarrow \quad \sum_k m_k \lesssim 6\,\text{eV}$



massive neutrinos $=$ hot dark matter

$\Updownarrow$

relativistic at matter-radiation equality

$(z_{\text{eq}} \sim 3000)$

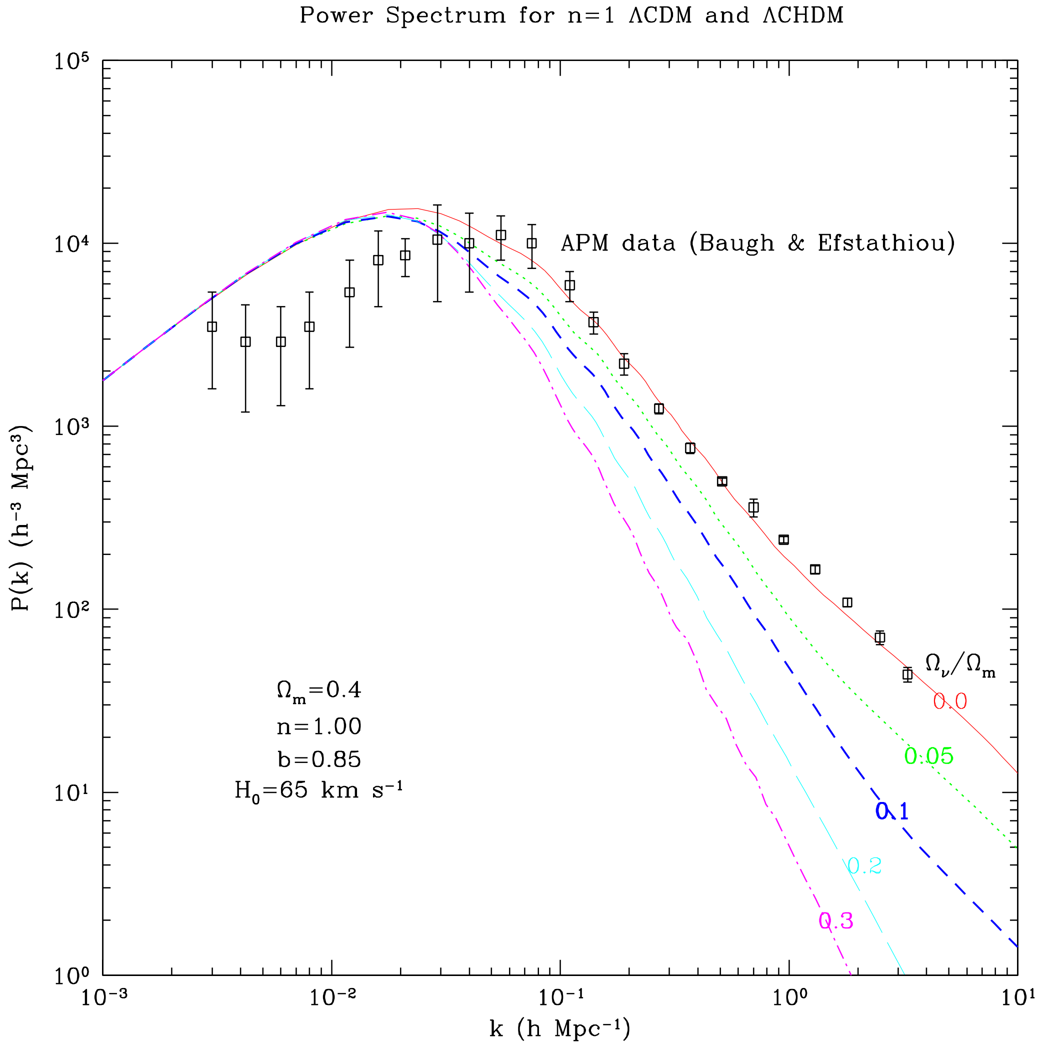
when structures start to form

last CMB Scattering (recombination)

$z_{\text{rec}} \sim 1300$, $T_{\text{rec}} \sim 3700\,\text{K} \sim 0.3\,\text{eV}$

galaxy formation at $z_{\text{gal}} \sim 6.8$

Power Spectrum of Density Fluctuations



[Primack, Gross, astro-ph/0007165]

massive neutrinos = hot dark matter

$\Updownarrow$

relativistic at matter-radiation equality
when structures start to form

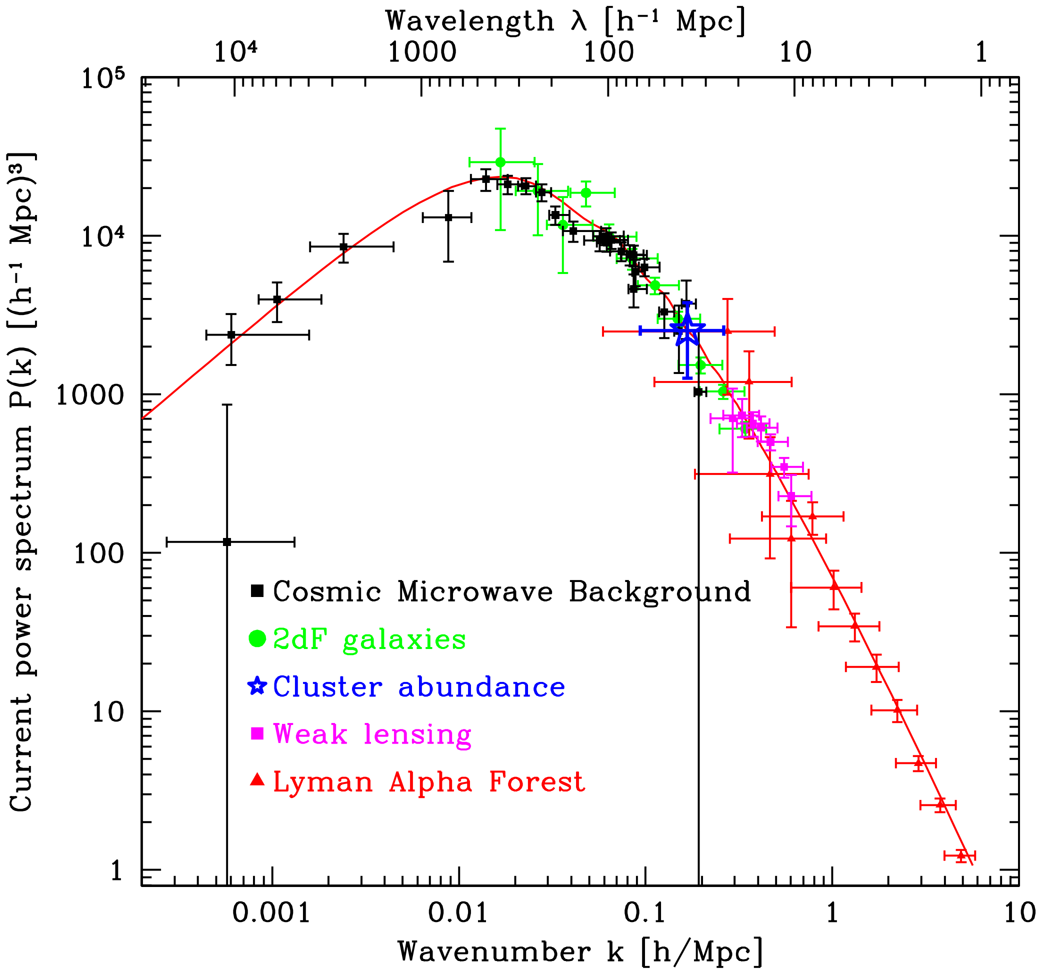
hot dark matter prevents early galaxy formation

small scale suppression

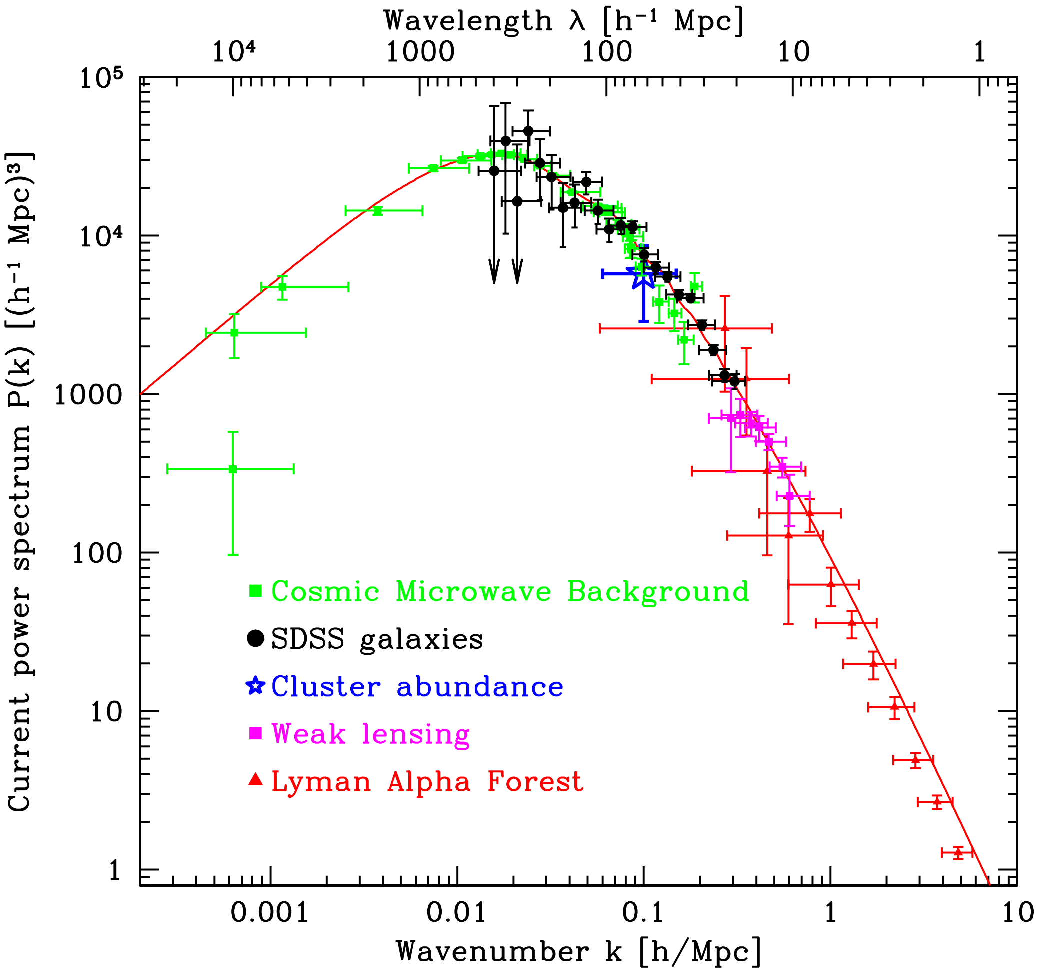
$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m\,h^2}\right)$$

for $k \gtrsim k_{\text{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

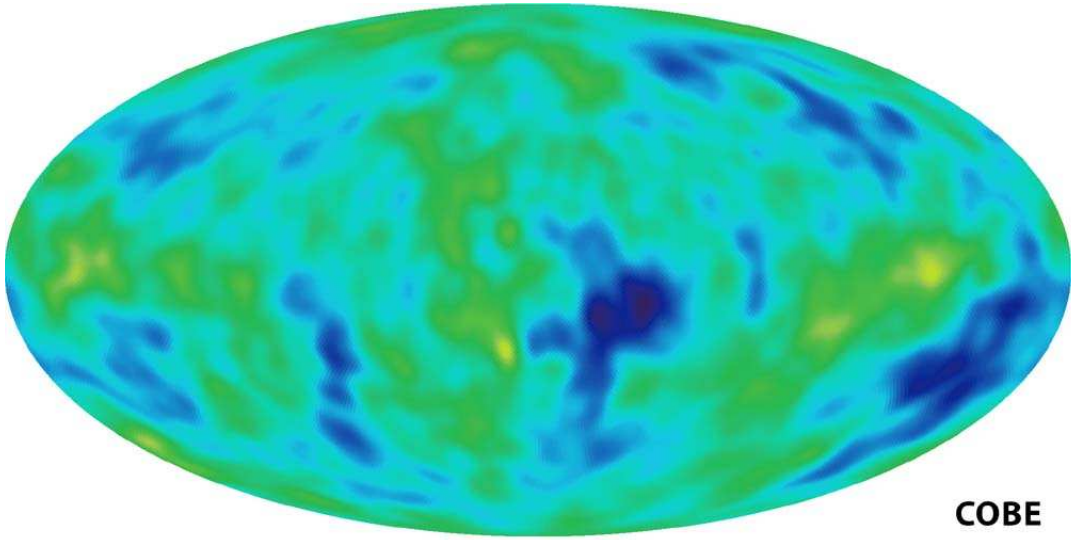


[Tegmark, Zaldarriaga, Phys. Rev. D66 (2002) 103508]

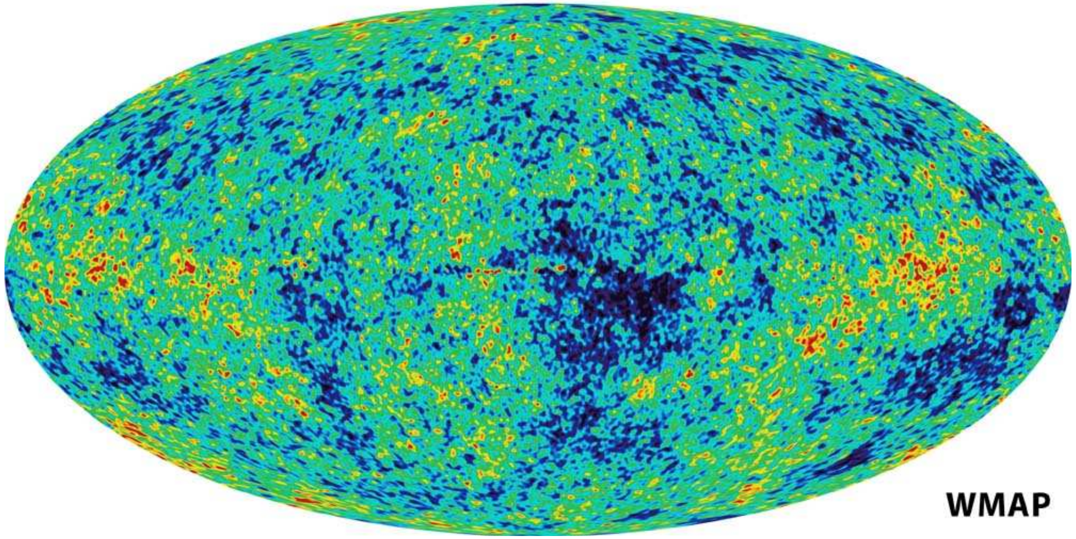


[SDSS, astro-ph/0310725]

Wilkinson Microwave Anisotropy Probe (WMAP)



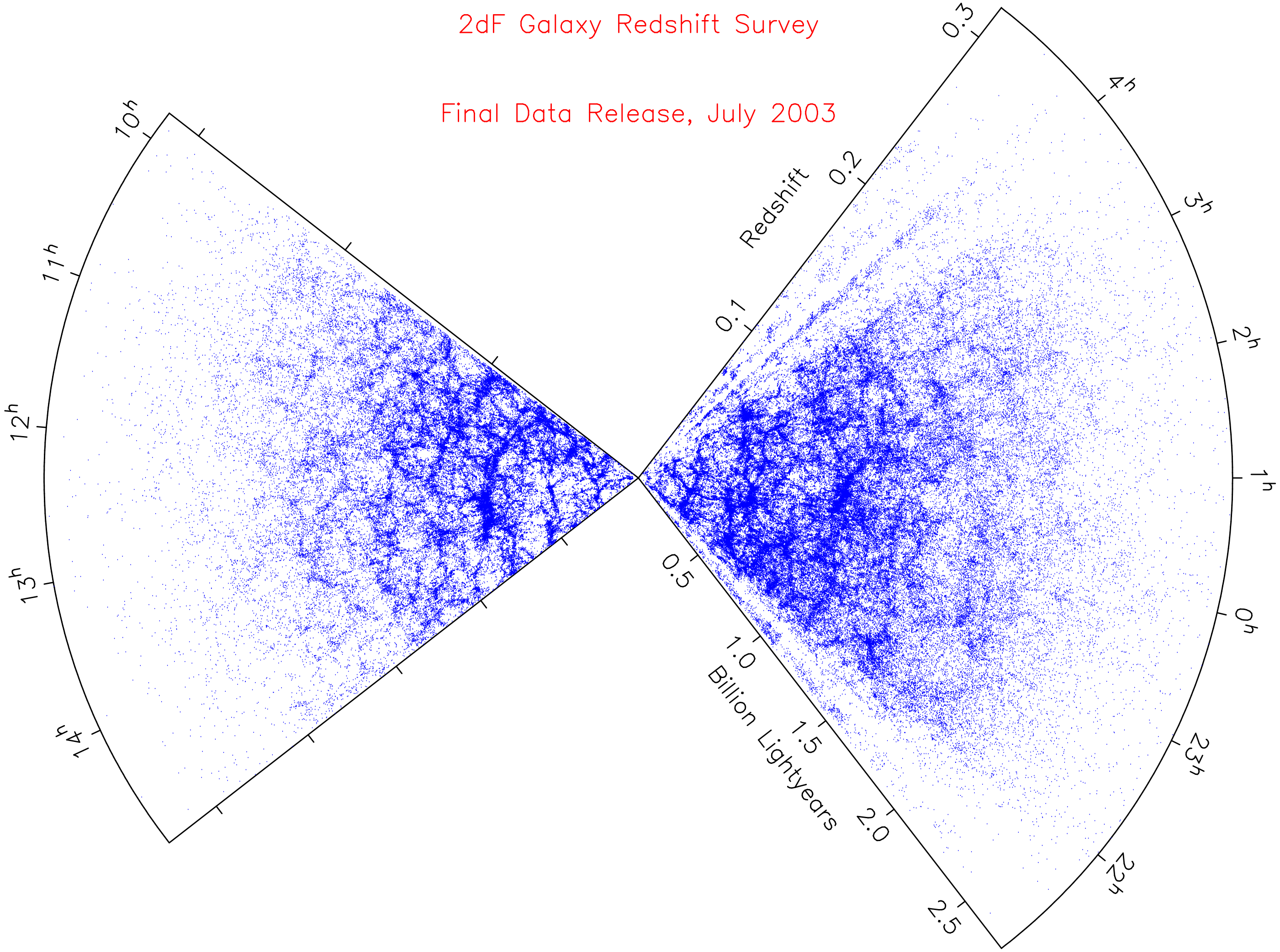
COBE



WMAP

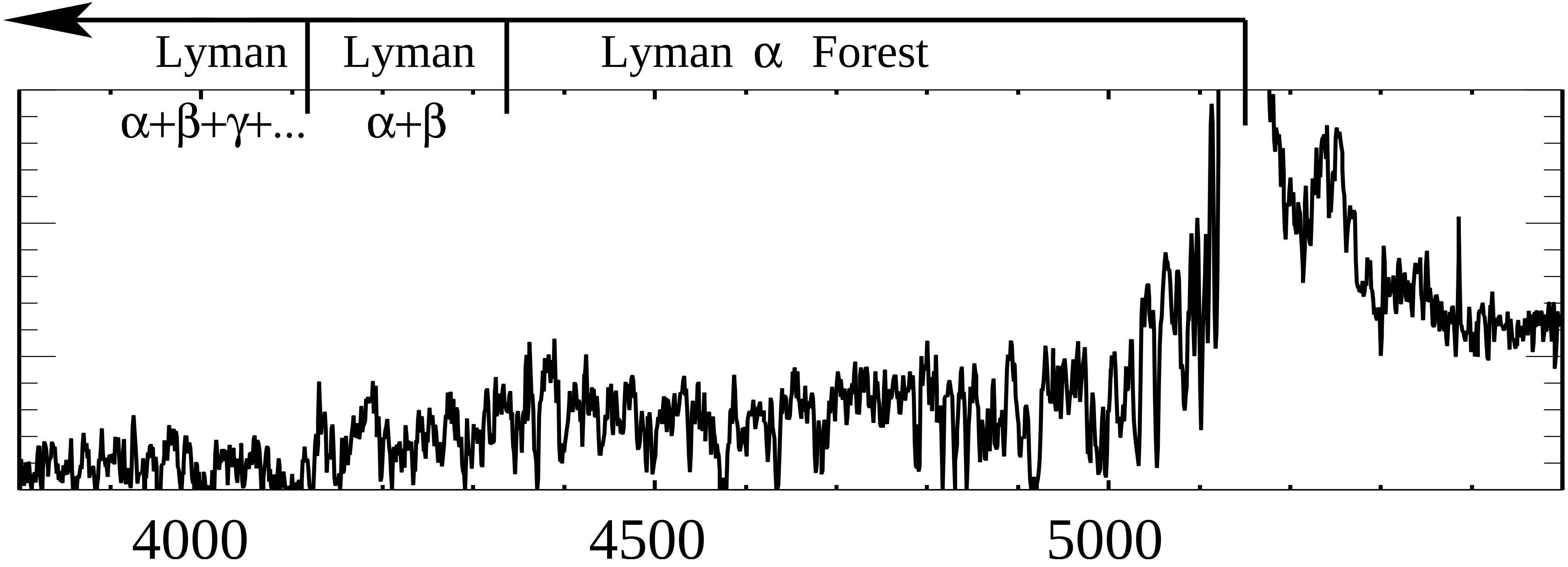
[WMAP, `http://map.gsfc.nasa.gov`]

2dF Galaxy Redshift Survey



[2dFGRS, `http://www.mso.anu.edu.au/2dFGRS`]

Lyman-α Forest



Spectrum of quasar Q2139-4434, at $z_q = 3.23$.

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda^0_\beta, (1+z_q)\lambda^0_\alpha]$.

Lyman-α+β region: $[(1+z_q)\lambda^0_\gamma, (1+z_q)\lambda^0_\beta]$.

Rest-frame Lyα, β, γ wavelengths: $\lambda^0_\alpha = 1215.67\,\text{Å}$, $\lambda^0_\beta = 1025.72\,\text{Å}$, $\lambda^0_\gamma = 972.54\,\text{Å}$.

The Lyman-α emission line (not fully shown) is at $\lambda = 5144\text{Å}$.

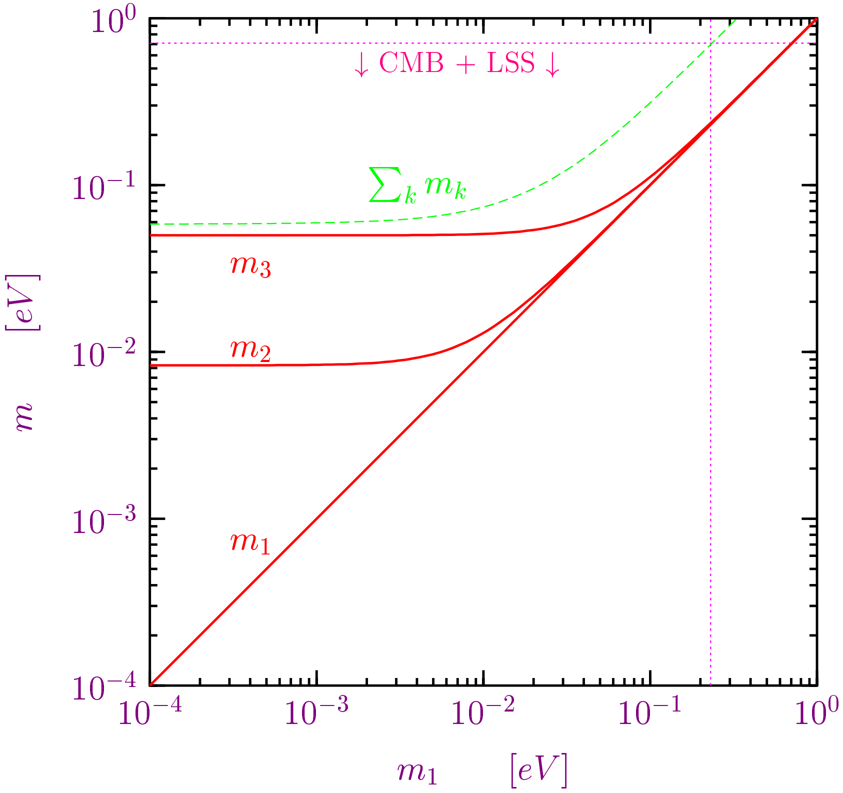
[Dijkstra, Lidz, Hui, astro-ph/0305498]

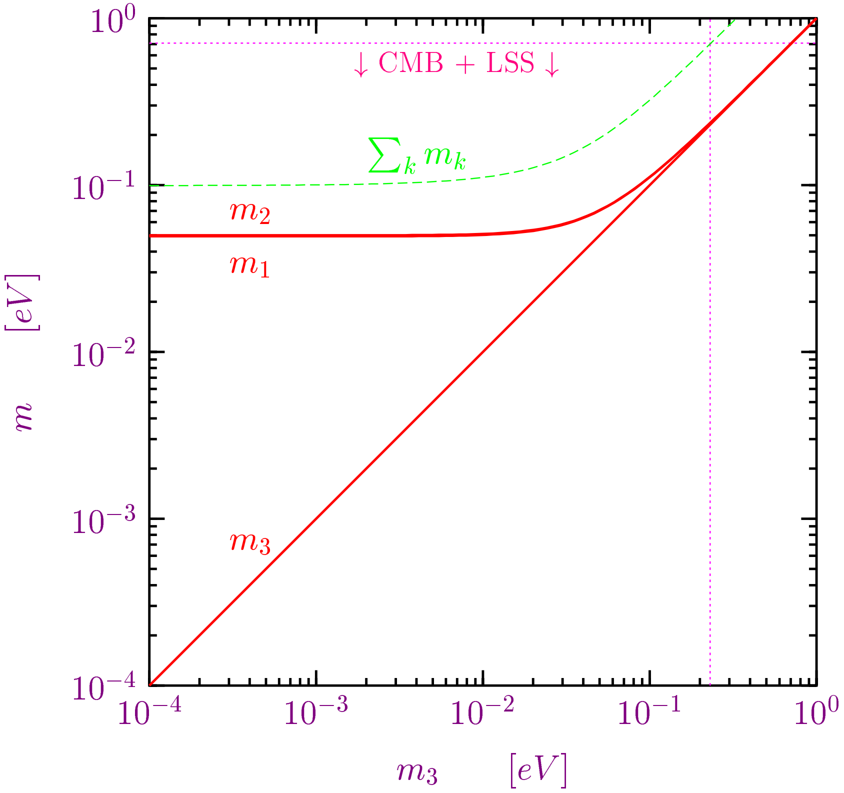
CMB (WMAP, CBI, ACBAR) + LSS (2dFGRS, Lyman-α) + HST + SN-Ia

[WMAP, astro-ph/0302207, astro-ph/0302209]

$$\Lambda\text{CDM:} \quad \left\{ \begin{array}{l} T_0 = 13.7 \pm 0.1\,\text{Gyr},\ h = 0.71^{+0.04}_{-0.03}, \\ \Omega_{\text{tot}} = 1.02 \pm 0.02,\ \Omega_b h^2 = 0.0224 \pm 0.0009,\ \Omega_m h^2 = 0.135^{+0.008}_{-0.009} \end{array} \right.$$

$$\Omega_\nu h^2 < 0.0076 \ \text{(95\% confidence)} \Longrightarrow \sum_k m_k < 0.71\,\text{eV} \Longrightarrow m_k < 0.23\,\text{eV}$$





Hannestad [astro-ph/0303076]

$\sum_k m_k < 1.01\,\text{eV}$ (95%) [WMAP+CBI+2dFGRS+HST+SN-Ia]

$\sum_k m_k < 1.20\,\text{eV}$ (95%) [WMAP+CBI+2dFGRS]

$\sum_k m_k < 2.12\,\text{eV}$ (95%) [WMAP+2dFGRS]

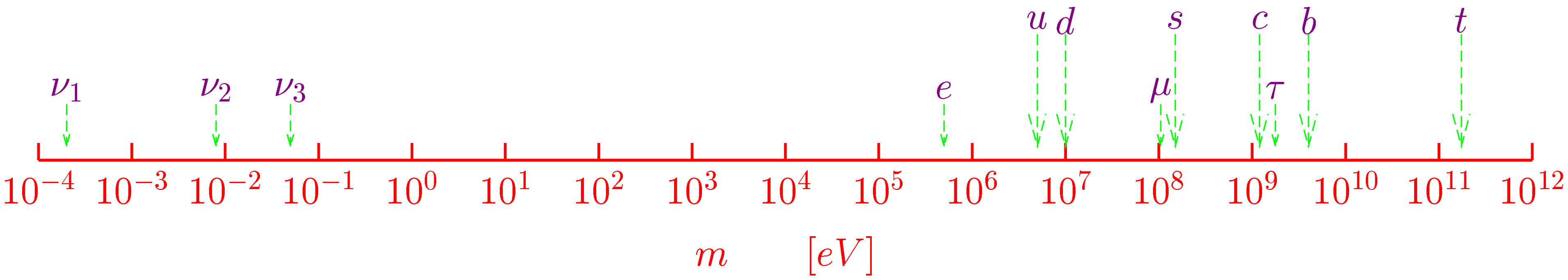
Elgaroy and Lahav [astro-ph/0303089]

$$\sum_k m_k < 1.1\,\text{eV} \quad (95\%)$$ [WMAP+2dFGRS+HST]

WMAP + SDSS [astro-ph/0310723]

$$h \approx 0.70^{+0.04}_{-0.03} \qquad \Omega_m \approx 0.30 \pm 0.04 \quad (1\sigma) \qquad \sum_k m_{\nu_k} < 1.7\,\text{eV} \quad (95\%)$$

MAJORANA NEUTRINOS?

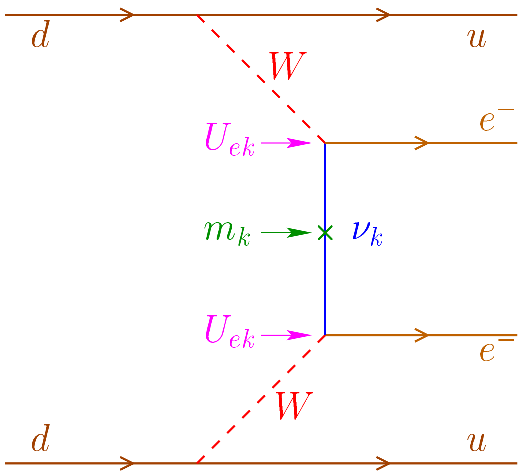


known natural explanations of smallness of ν masses:
- ⋆ See-Saw Mechanism
- ⋆ Penta-Dim. Non-Renorm. Effective Operator

both imply
- ⋆ Majorana ν masses
- ⋆ see-saw type relation $m_{\text{light}} \sim \dfrac{M_{\text{EW}}^2}{\mathcal{M}}$
- ⋆ new high energy scale $\mathcal{M}$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

MAJORANA NEUTRINOS $\Longleftrightarrow$ $\beta\beta_{0\nu}$ decay



$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

effective
Majorana
mass

$$|\langle m \rangle| = \left| \sum_k U_{ek}^2 m_k \right|$$

complex U_{ek} $\Rightarrow$ possible cancellations among m_1, m_2, m_3 contributions

$$|\langle m \rangle| = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 \right|$$

conserved CP

$\alpha_{21} = 0, \pi$ $\qquad$ $\alpha_{31} = 0, \pi$

$\eta_{kj} = e^{i\alpha_{kj}}$ relative CP parity

Heidelberg-Moscow ($^{76}\mathrm{Ge}$) $\quad |\langle m \rangle|_{\mathrm{exp}} < 0.35\,\mathrm{eV}$ (90% C.L.) [EPJA 12 (2001) 147]

IGEX ($^{76}\mathrm{Ge}$) $\quad |\langle m \rangle|_{\mathrm{exp}} < 0.33 - 1.35\,\mathrm{eV}$ (90% C.L.) [PRD 65 (2002) 092007]

serious problem: about factor 3 theoretical uncertainty on nuclear matrix element!

Neutrino Oscillations Implications for $\beta\beta_{0\nu}$ decay

$$|\langle m\rangle| = \left||U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3\right|$$

mass hierarchy without fine-tuned cancellations

among m_1, m_2, m_3 contributions

[Giunti, PRD 61 (2000) 036002]

$$|\langle m\rangle| \simeq \max_k |\langle m\rangle|_k \qquad |\langle m\rangle|_k \equiv |U_{ek}|^2 m_k$$

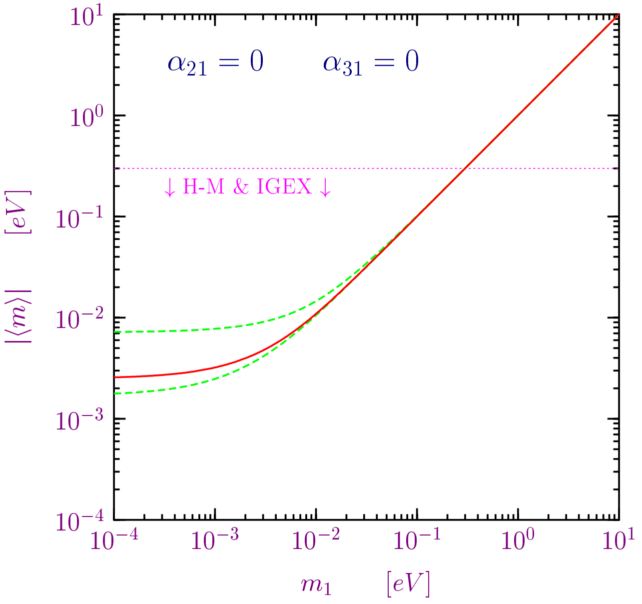
$$|U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SUN}},\ m_2 \simeq \sqrt{\Delta m^2_{\text{SUN}}} \qquad |U_{e3}|^2 \simeq \sin^2\vartheta_{\text{CHOOZ}},\ m_3 \simeq \sqrt{\Delta m^2_{\text{ATM}}}$$

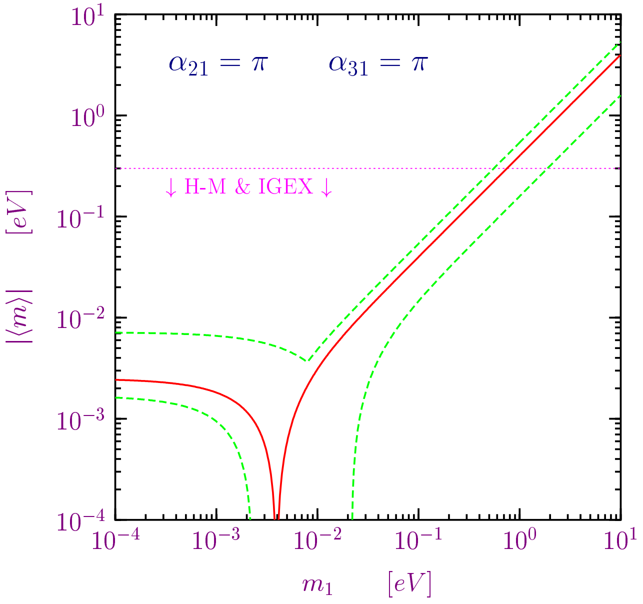
$$\left.\begin{array}{c} \Delta m^{2\,\text{best-fit}}_{\text{SUN}} = 6.9\times10^{-5}\,, \quad |U_{e2}|_{\text{best-fit}} = 0.56 \\ 5.1\times10^{-5} \lesssim \Delta m^2_{\text{SUN}} \lesssim 1.9\times10^{-4} \\ 0.46 \lesssim |U_{e2}| \lesssim 0.68 \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} |\langle m\rangle|_2^{\text{best-fit}} = 2.6\times10^{-3} \\ 1.5\times10^{-3} \lesssim |\langle m\rangle|_2 \lesssim 6.4\times10^{-3} \end{array}\right.$$

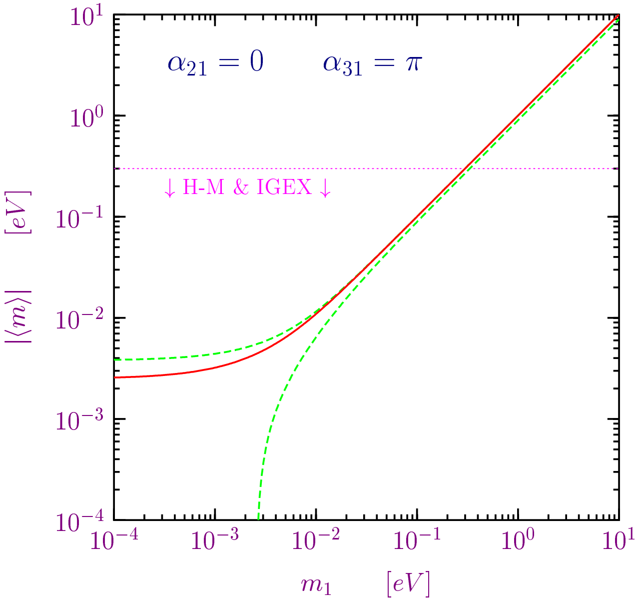
$$\left.\begin{array}{c} \Delta m^{2\,\text{best-fit}}_{\text{ATM}} = 2.6\times10^{-3}\,, \quad |U_{e3}|_{\text{best-fit}} = 0 \\ 1.4\times10^{-3} \lesssim \Delta m^2_{\text{ATM}} \lesssim 5.1\times10^{-3} \\ |U_{e2}| \lesssim 0.22 \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} |\langle m\rangle|_3^{\text{best-fit}} = 0 \\ |\langle m\rangle|_3 \lesssim 3.5\times10^{-3} \end{array}\right.$$

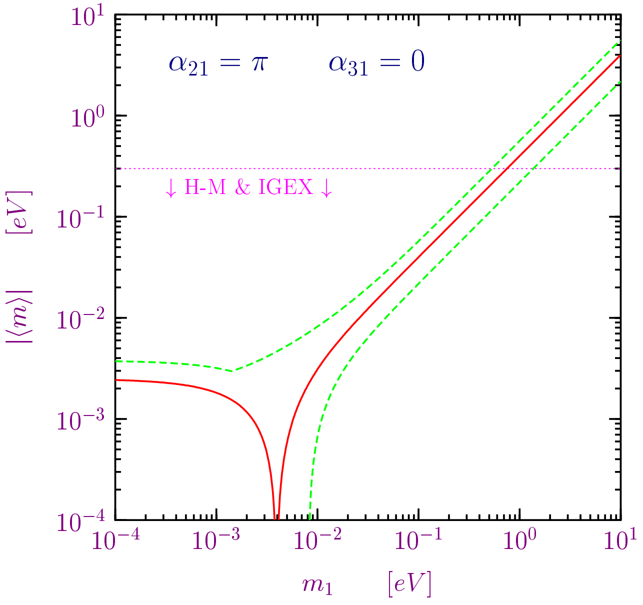
m_2 contribution $|\langle m\rangle|_2$ may be dominant! (lower limit for $|\langle m\rangle|$)

CP Conservation: Normal Scheme

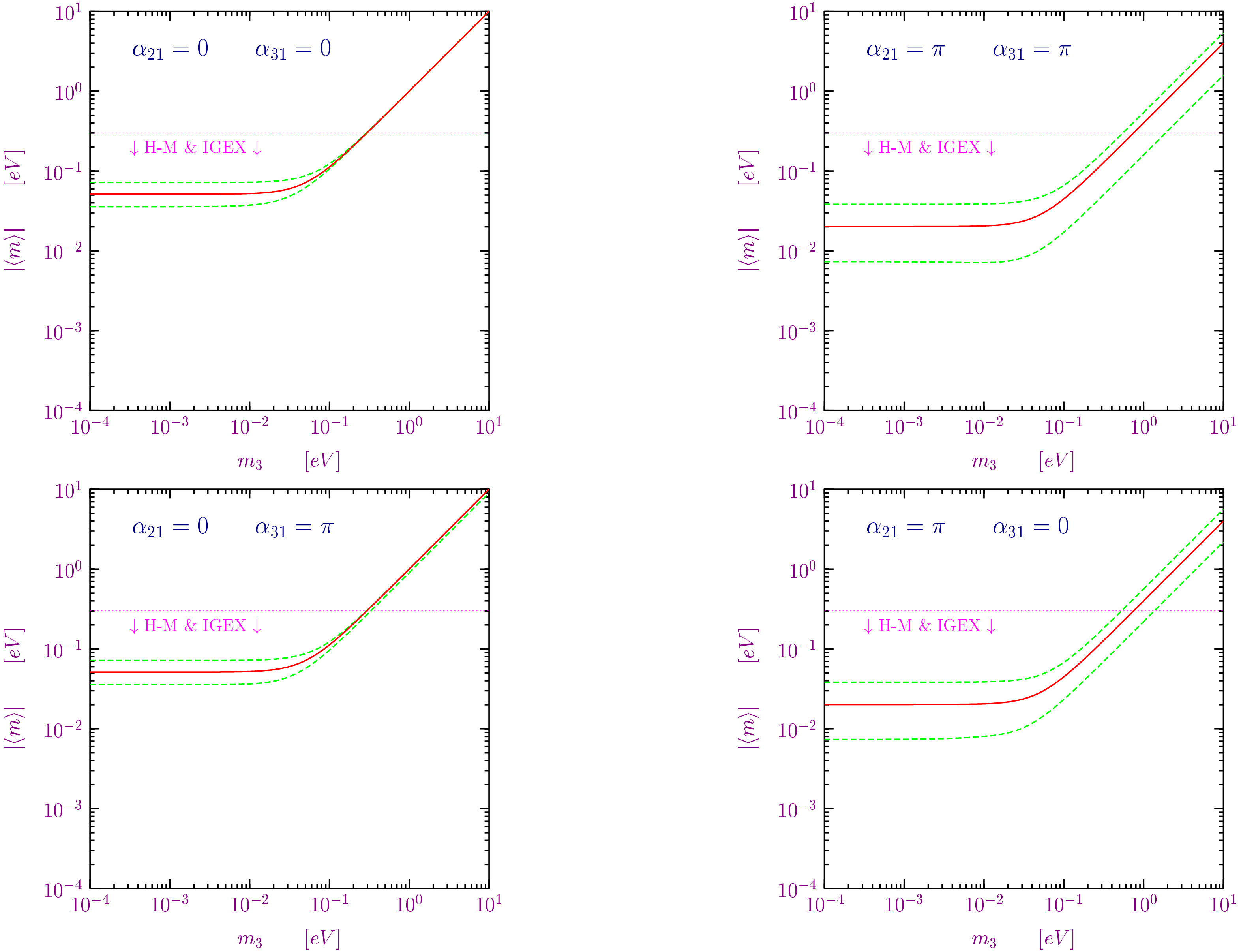




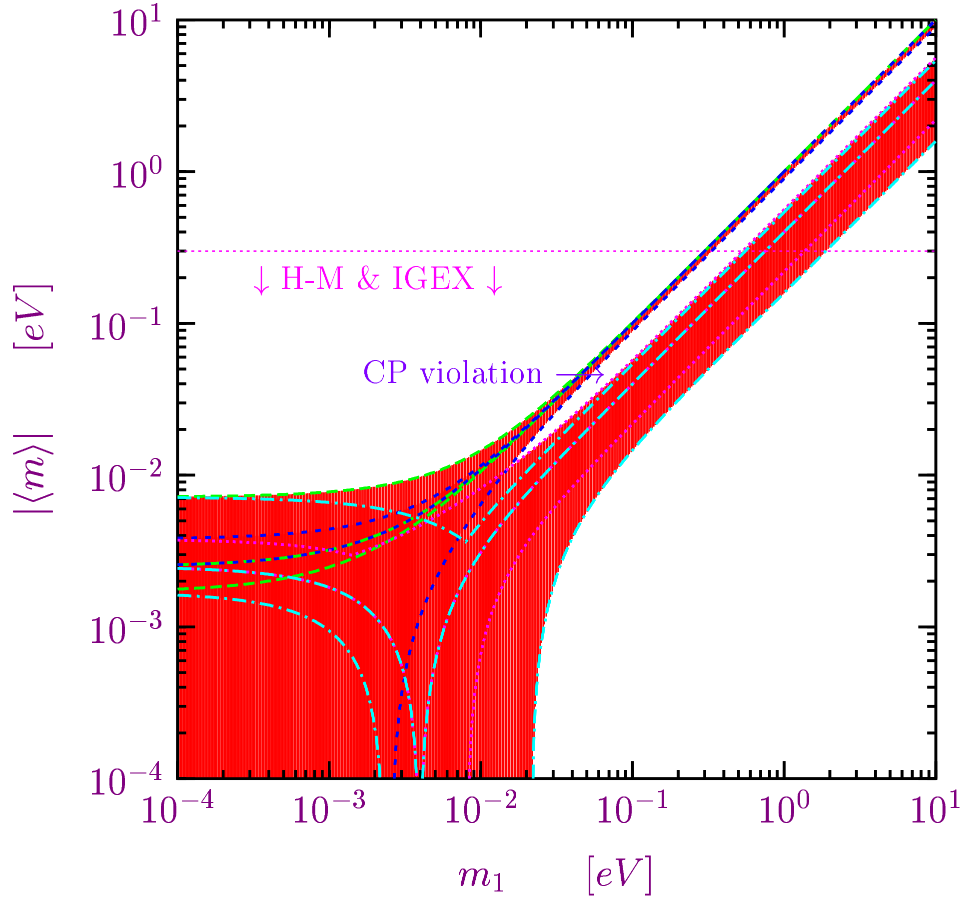




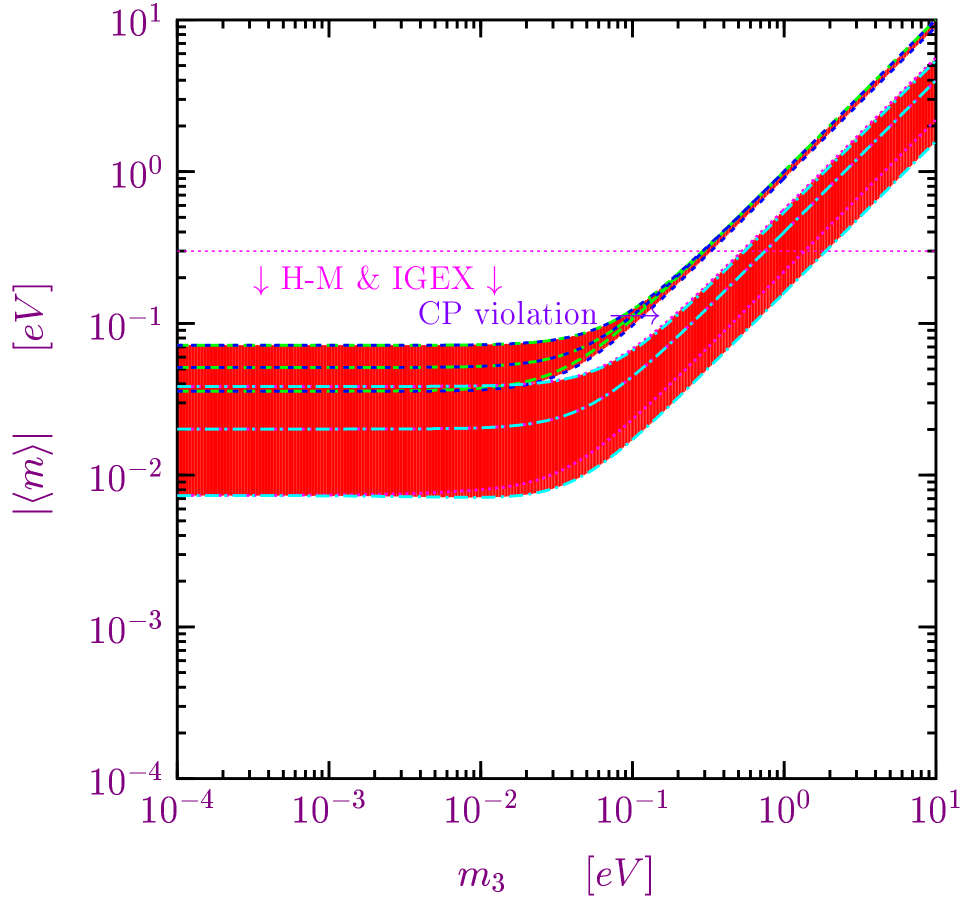
CP Conservation: Inverted Scheme



General Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ decay



"normal" scheme



"inverted" scheme

FUTURE: NEMO3, CAMEO, Majorana, CUORICINO, XMASS ($|\langle m \rangle| \sim 10^{-1}\,\mathrm{eV}$)
GENIUS, CUORE, EXO, MOON, GEM ($|\langle m \rangle| \sim 10^{-2}\,\mathrm{eV}$)

VERY FAR FUTURE: IF $|\langle m \rangle| \lesssim 7 \times 10^{-3}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

Summary of Part 3: Experimental Results and Theoretical Implications

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\mathrm{ATM}} \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\mathrm{SUN}} \simeq 7 \times 10^{-5}\,\mathrm{eV}^2$

Tritium and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\mathrm{eV}$

3ν mixing $\Longrightarrow$ bilarge mixing with $|U_{e3}|^2 \ll 1$

theory: why $|U_{e3}|^2$ is so small?

future exp.: measure $|U_{e3}| > 0$ $\Rightarrow$ normal or inverted scheme and CP violation

data disfavor Active → Sterile transitions

CONCLUSIONS

Neutrino Physics is a very active and interesting field of research

next years will hopefully bring new interesting results

OPEN FUNDAMENTAL QUESTIONS

Absolute Scale of Neutrino Masses?

Nature of Neutrinos (Dirac or Majorana)?

Are There Sterile Neutrinos?

Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ (LSND)? $\Longleftarrow$ MiniBooNE

Electromagnetic Properties of Neutrinos?

Neutrino Unbound

`http://www.nu.to.infn.it`

Carlo Giunti & Marco Laveder