Status of Neutrino Masses and Mixing

Carlo Giunti

INFN, Sezione di Torino, and
Dipartimento di Fisica Teorica, Università di Torino
giunti@to.infn.it

POSTECH, Pohang, Korea, 11 October 2004

Introduction to Neutrino Mass, Mixing and Oscillations

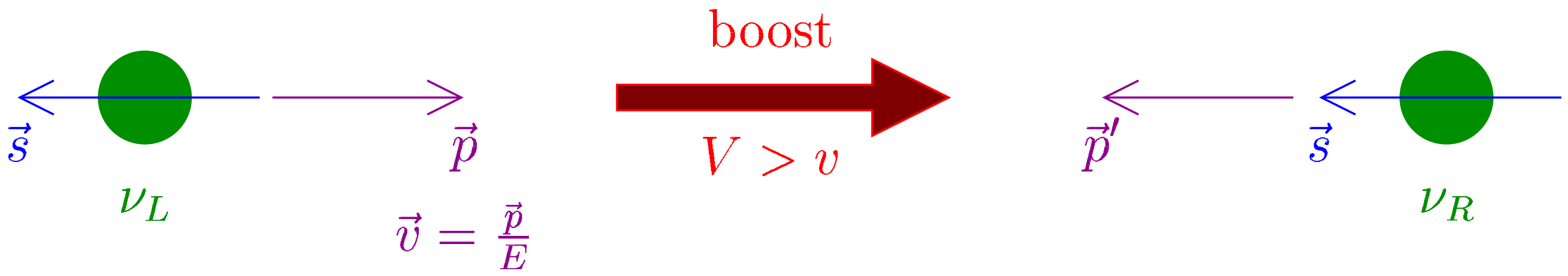
Solar $\nu_e \to \nu_\mu, \nu_\tau$ + Atmospheric $\nu_\mu \to \nu_\tau$ $\Longrightarrow$ Three-Neutrino Mixing

Absolute Scale of Neutrino Masses

Cosmological Bound on Neutrino Masses

Neutrinoless Double-β Decay $\Longleftrightarrow$ Majorana Mass

Neutrino Mass



Standard Model: ν_L , $\bar{\nu}_R$ $\Longrightarrow$ no Dirac mass term $\sim \nu_L \nu_R$

(no ν_R, $\bar{\nu}_L$)

Majorana neutrino: $\nu = \bar{\nu}$ $\Longrightarrow$ $\bar{\nu}_R = \nu_R$ $\Longrightarrow$ Majorana mass term $\sim \nu_L \bar{\nu}_R$

Standard Model: Majorana mass term is not allowed by $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(no Higgs triplet)

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	+1	0	0
$(\nu_\mu\,,\,\mu^-)$	0	+1	0
$(\nu_\tau\,,\,\tau^-)$	0	0	+1

	L_e	L_μ	L_τ
$(\bar{\nu}_e\,,\,e^+)$	−1	0	0
$(\bar{\nu}_\mu\,,\,\mu^+)$	0	−1	0
$(\bar{\nu}_\tau\,,\,\tau^+)$	0	0	−1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Dirac mass $\Longrightarrow$ L_e, L_μ, L_τ are not conserved,

$$M = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix}$$

but L is conserved $(\nu \neq \bar{\nu})$

Majorana mass $\Longrightarrow$ L, L_e, L_μ, L_τ are not conserved $(\nu = \bar{\nu} \Rightarrow L(\nu) = L(\bar{\nu}))$

no reason not to extend the Standard Model with ν_R $(e_L, e_R; u_L, u_R; d_L, d_R; \ldots)$

$\nu_L + \nu_R \Longrightarrow$ Dirac neutrino mass term $\sim \nu_L \nu_R$ (as all other particles)

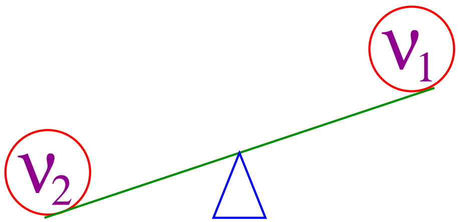
surprise: also Majorana neutrino mass for ν_R is allowed!

Lagrangian neutrino mass term $\sim \begin{pmatrix} \nu_L & \bar{\nu}_L \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \bar{\nu}_R \\ \nu_R \end{pmatrix}$

$m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$, but m_R^{M} can be arbitrarily large (not protected by any symmetry)

natural that $m_R^{\mathrm{M}} \sim$ scale of new physics beyond the Standard Model $\Longrightarrow m_R^{\mathrm{M}} \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \Longrightarrow m_1 \simeq \dfrac{(m^{\mathrm{D}})^2}{m_R^{\mathrm{M}}}, \qquad m_2 \simeq m_R^{\mathrm{M}}$



see-saw mechanism

natural explanation of
smallness of neutrino masses

massive neutrinos are Majorana!

[Minkowski, Phys. Lett. B 67 (1977) 42] [Yanagida, 1979] [Gell-Mann, Ramond, Slansky, 1979] [Mohapatra, Senjanovic, Phys. Rev. Lett. 44 (1980) 912]

Neutrino Oscillations

[Pontecorvo, Sov. Phys. JETP 6 (1957) 429] [Pontecorvo, Sov. Phys. JETP 7 (1958) 172] [Gribov, Pontecorvo, Phys. Lett. B 28 (1969) 49] [Eliezer,Swift, Nucl. Phys. B 105 (1976) 45] [Fritzsch, Minkowski,Phys. Lett. B 62 (1976) 72] [Bilenky, Pontecorvo, Sov. J. Nucl. Phys. 24 (1976) 316] [Bilenky, Pontecorvo, Nuovo Cim. Lett. 17 (1976) 56] [Bilenky, Pontecorvo, Phys. Rept. 41 (1978) 225]

$$\alpha = e, \mu, \tau \qquad \boxed{|\nu_\alpha\rangle = \sum_k U_{\alpha k}\, |\nu_k\rangle} \qquad \nu_k \to m_k$$

Neutrino Mixing

$$|\nu_k(x,t)\rangle = e^{-iE_k t + ip_k x}|\nu_k\rangle \implies |\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k}\, e^{-iE_k t + ip_k x}\, |\nu_k\rangle$$

$$\boxed{|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U^*_{\beta k}\, |\nu_\beta\rangle}$$

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k} e^{-iE_k t + ip_k x} U^*_{\beta k}\right)}_{\mathcal{A}_{\nu_\alpha\to\nu_\beta}(x,t)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha\to\nu_\beta}(x,t) = \left|\langle\nu_\beta|\nu_\alpha(x,t)\rangle\right|^2 = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(x,t)\right|^2 = \left|\sum_k U_{\alpha k} e^{-iE_k t + ip_k x} U^*_{\beta k}\right|^2$$

ultrarelativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_k U_{\alpha k}\, e^{-i m_k^2 L / 2E}\, U_{\beta k}^* \right|^2$$

$$= \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2$$ $\Leftarrow$ constant term

$$+ 2\mathrm{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$ $\Leftarrow$ oscillating term

$\Updownarrow$

coherence

$$\boxed{\Delta m_{kj}^2 \equiv m_k^2 - m_j^2}$$

[Nussinov, Phys. Lett. B 63 (1976) 201] [Kayser, Phys. Rev. D 24 (1981) 110], [Giunti, Kim, Lee, Phys. Rev. D44 (1991) 3635]
[Kiers,Nussinov, Weiss, Phys. Rev. D53 (1996) 53] [Giunti, Kim, Phys. Rev. D 58 (1998) 017301] [Giunti, Kim, Found. Phys. Lett. 14 (2001) 213],
[Beuthe, Phys. Rept. 375 (2003) 105], [Giunti, Found. Phys. Lett. 17 (2004) 103]

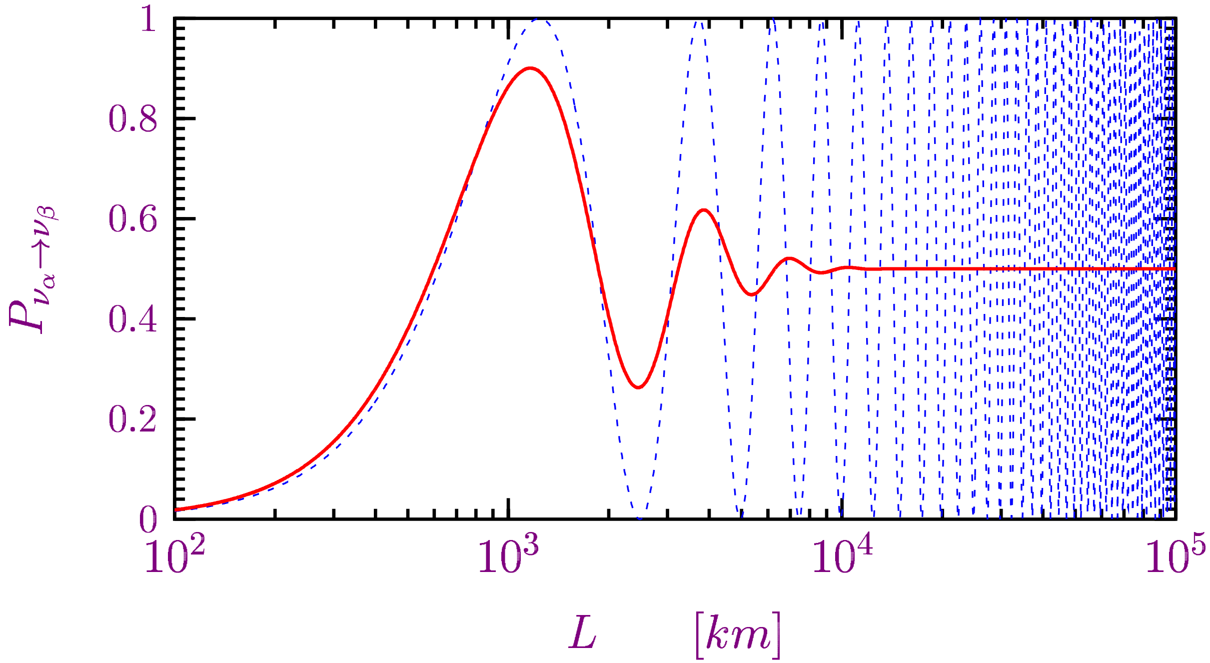
Two-Neutrino Mixing

$$k = 1,2 \implies U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \Delta m^2 \equiv \Delta m^2_{21} \equiv m^2_2 - m^2_1$$

Transition Probability ($\alpha \neq \beta$): $$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probability ($\alpha = \beta$): $$P_{\nu_\alpha \to \nu_\alpha}(L,E) = 1 - P_{\nu_\alpha \to \nu_\beta}(L,E)$$

Averaged Transition Probability: $$\langle P_{\nu_\alpha \to \nu_\beta} \rangle = \frac{1}{2} \sin^2 2\vartheta$$



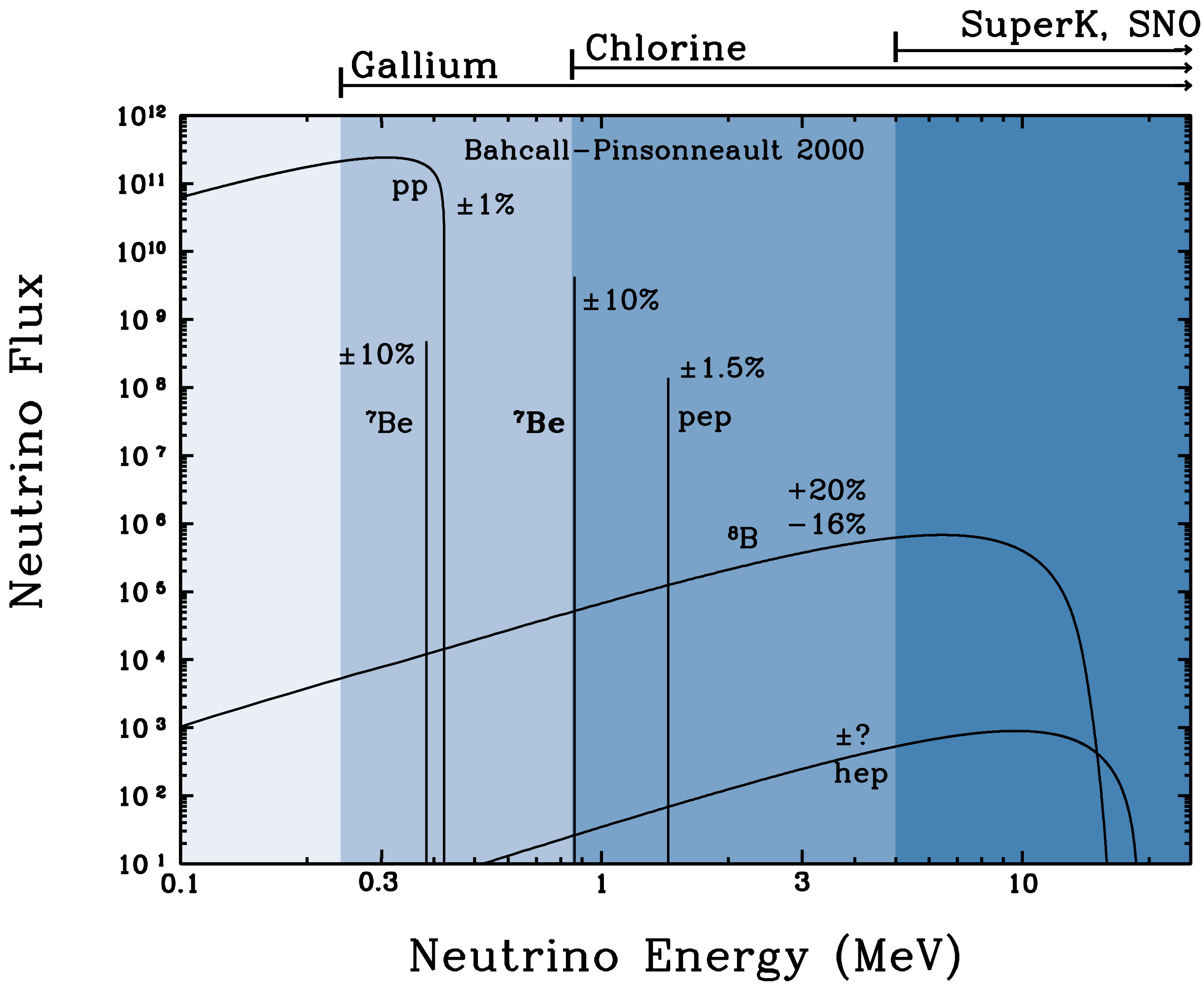
$\Delta m^2 = 10^{-3}\,\mathrm{eV}$

$\sin^2 2\vartheta = 1$

$\langle E \rangle = 1\,\mathrm{GeV}$

$\Delta E = 0.2\,\mathrm{GeV}$

Solar Neutrinos



[J.N. Bahcall, http://www.sns.ias.edu/~jnb]

Main Characteristics of Solar ν Data

Experiment	Reaction	E_{th} (MeV)	ν Flux Sensitivity	Operating Time	$\frac{R^{\mathrm{exp}}}{R^{\mathrm{BP04}}}$
SAGE	$\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$ (CC)	0.233	pp, ${}^7\mathrm{Be}$, ${}^8\mathrm{B}$, pep, hep, ${}^{13}\mathrm{N}$, ${}^{15}\mathrm{O}$, ${}^{17}\mathrm{F}$	$1990 - 2001$	0.52 ± 0.04
GALLEX				$1991 - 1997$	0.59 ± 0.06
GNO				$1998 - 2000$	0.50 ± 0.08
Homestake	$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$ (CC)	0.814	${}^7\mathrm{Be}$, ${}^8\mathrm{B}$, pep, hep, ${}^{13}\mathrm{N}$, ${}^{15}\mathrm{O}$, ${}^{17}\mathrm{F}$	$1970 - 1994$	0.30 ± 0.03
Kamiokande	$\nu + e^- \to \nu + e^-$ (ES)	6.75	${}^8\mathrm{B}$	$1987 - 1995$ 2079 days	0.49 ± 0.06
Super-Kam.		4.75		$1996 - 2001$ 1496 days	0.406 ± 0.013
SNO D_2O phase	$\nu_e + d \to p + p + e^-$ (CC)	6.9		$1999 - 2001$ 306.4 days	0.30 ± 0.02
	$\nu + d \to p + n + \nu$ (NC)	2.2			0.88 ± 0.11
	$\nu + e^- \to \nu + e^-$ (ES)	5.2			0.41 ± 0.05
SNO NaCl phase	$\nu_e + d \to p + p + e^-$ (CC)	6.9		$2001 - 2002$ 254.2 days	0.27 ± 0.02
	$\nu + d \to p + n + \nu$ (NC)	2.2			0.90 ± 0.08
	$\nu + e^- \to \nu + e^-$ (ES)	5.2			0.38 ± 0.05

SNO

$$\Phi^{\rm SNO}_{\rm CC} = 1.76 \pm 0.11 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$
$$\Phi^{\rm SNO}_{\rm NC} = 5.09 \pm 0.64 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$
$$\Downarrow$$
$$\Phi^{\rm SNO}_{\nu_e} = 1.76 \pm 0.11 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$
$$\Phi^{\rm SNO}_{\nu_\mu,\nu_\tau} = 5.41 \pm 0.66 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

SNO SOLVED SOLAR NEUTRINO PROBLEM

$$\Downarrow$$

NEUTRINO PHYSICS

(April 2002)

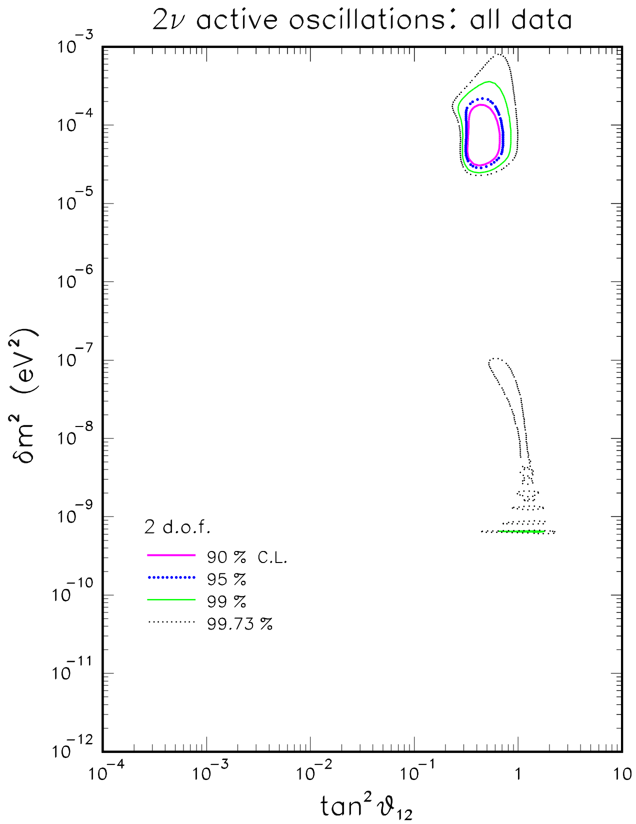
[SNO, Phys. Rev. Lett. 89 (2002) 011301, nucl-ex/0204008]

$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$$\Downarrow$$

Large Mixing Angle solution

$$\Delta m^2 \simeq 5 \times 10^{-5}\,{\rm eV}^2 \qquad \tan^2\vartheta \simeq 0.4$$



90%, 95%, 99%, 99.73% (3σ) C.L.

[Fogli, Lisi, Marrone, Montanino, Palazzo, PRD 66 (2002) 053010]

see also

[SNO, PRL 89 (2002) 011302]

[Barger, Marfatia, Whisnant, Wood, PLB 537 (2002) 179]

[Bahcall, Gonzalez-Garcia, Peña-Garay, JHEP 07 (2002) 054]

[SK, PLB 539 (2002) 179]

[de Holanda, Smirnov, PRD66 (2002) 113005]

[Aliani et al., PRD 67 (2003) 013006]

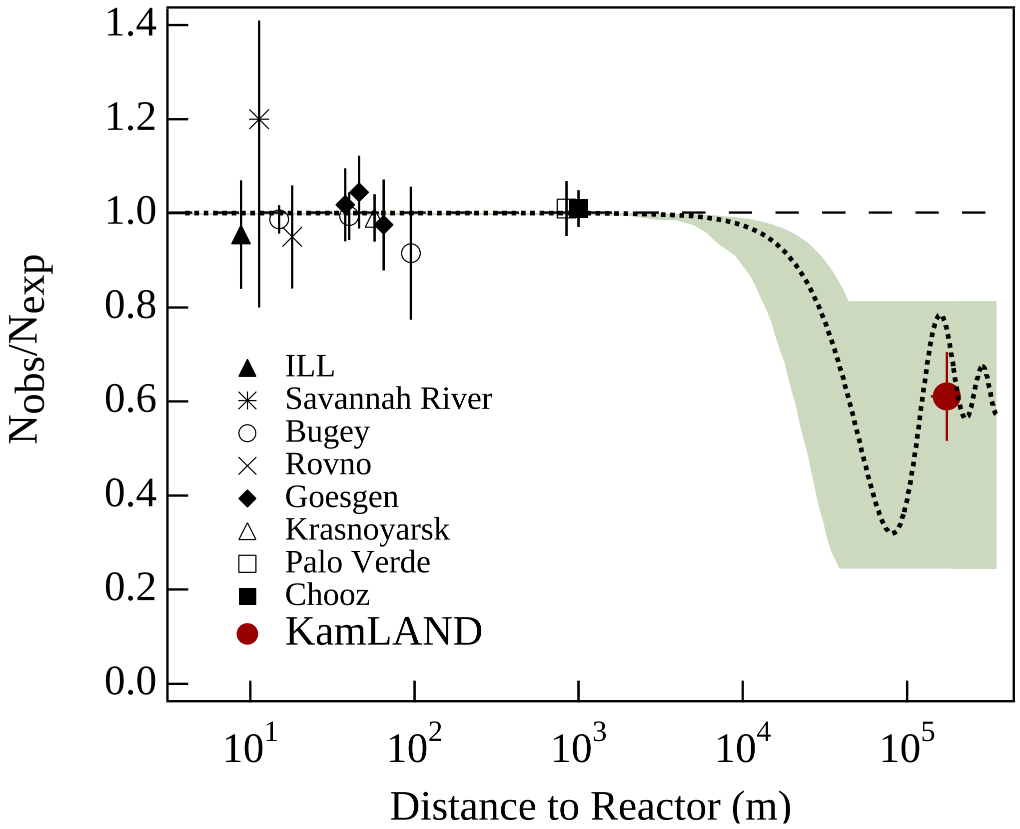
[Bandyopadhyay et al., PLB 540 (2002) 14]

[Creminelli, Signorelli, Strumia, hep-ph/0102234]

[Maltoni, Schwetz, Tortóla, Valle, PRD 67 (2003) 013011]

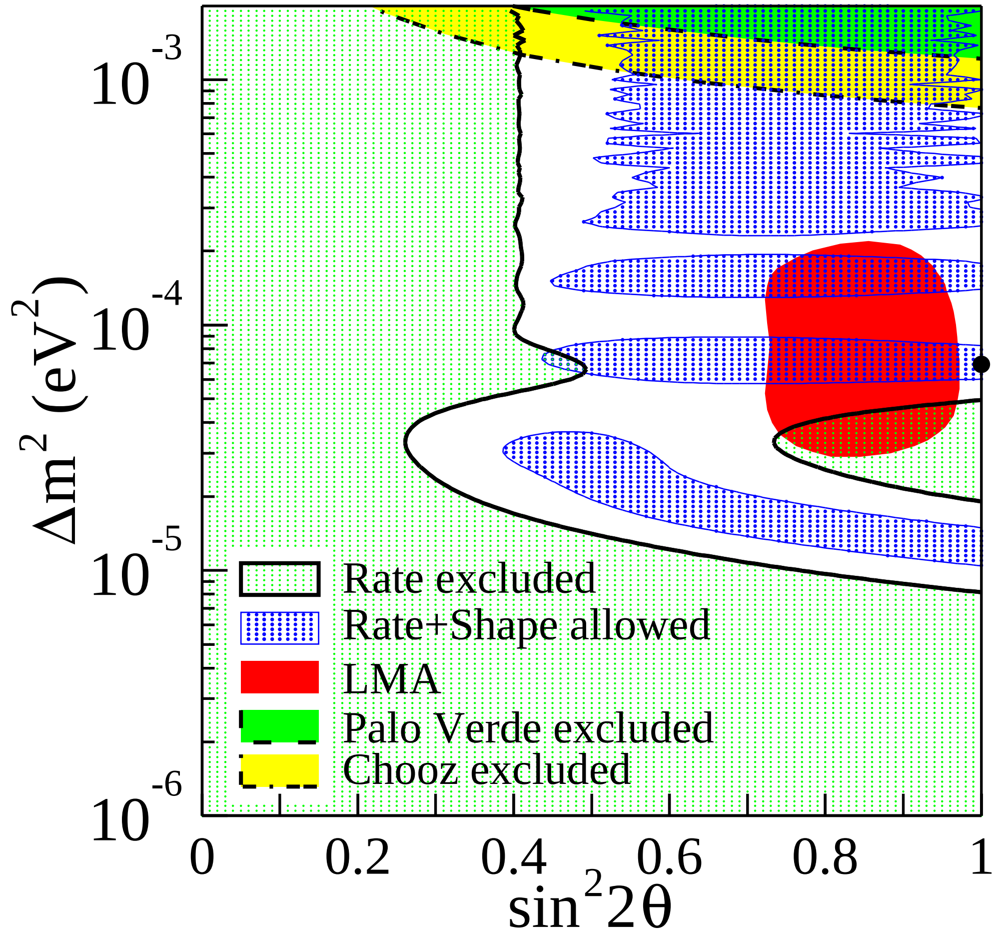
KamLAND

spectacular confirmation of LMA (December 2002)



Shade: 95% C.L. LMA

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\mathrm{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$

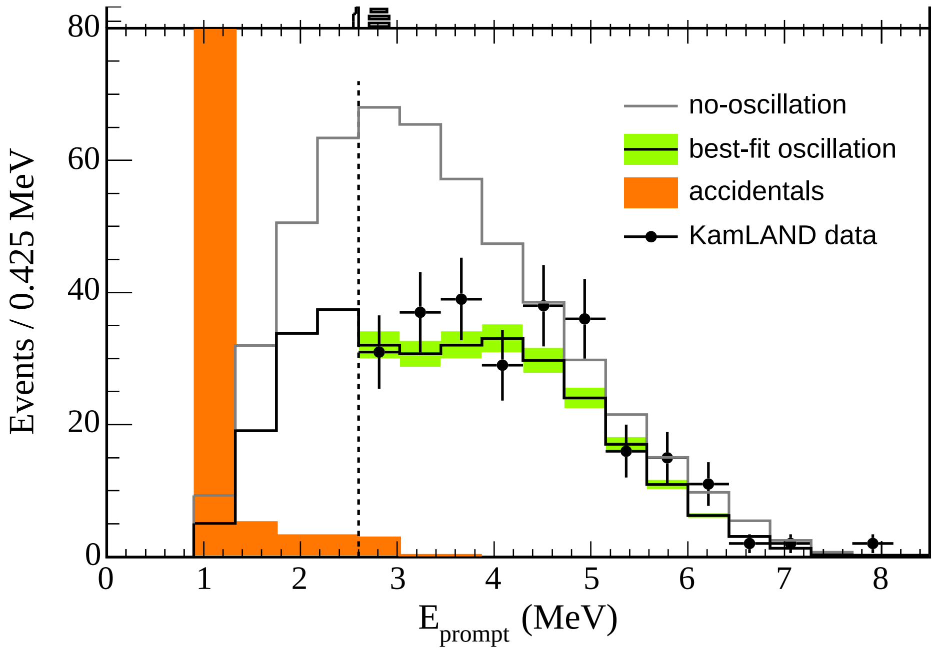


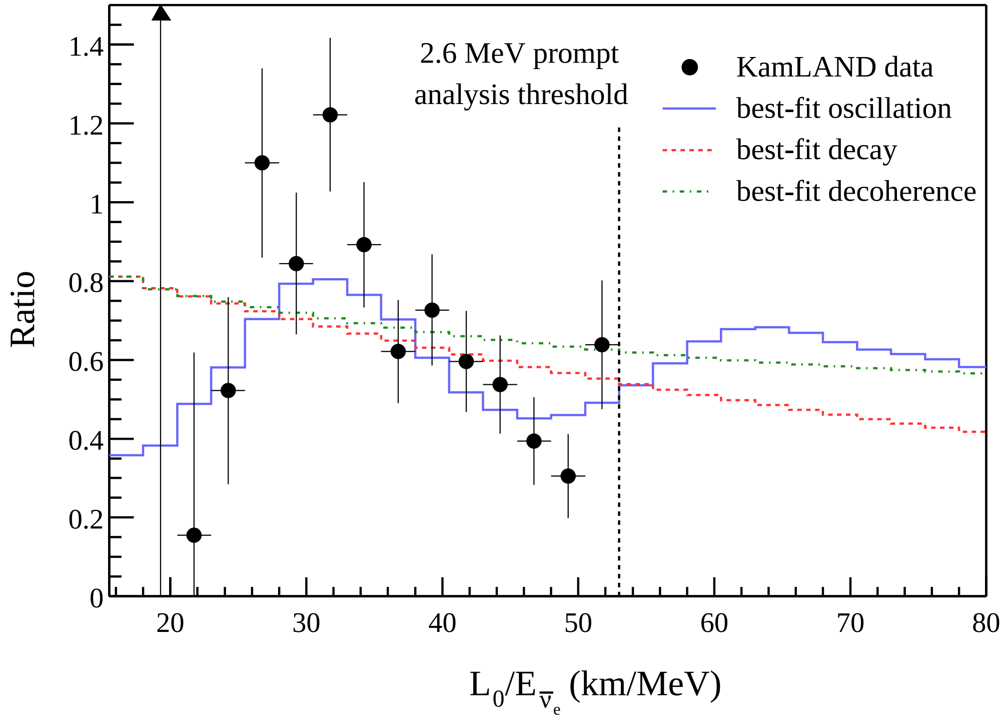
95% C.L.

[KamLAND, Phys. Rev. Lett. 90 (2003) 021802, hep-ex/0212021]

KamLAND: Evidence of Spectral Distortion

hep-ex/0406035, June 2004



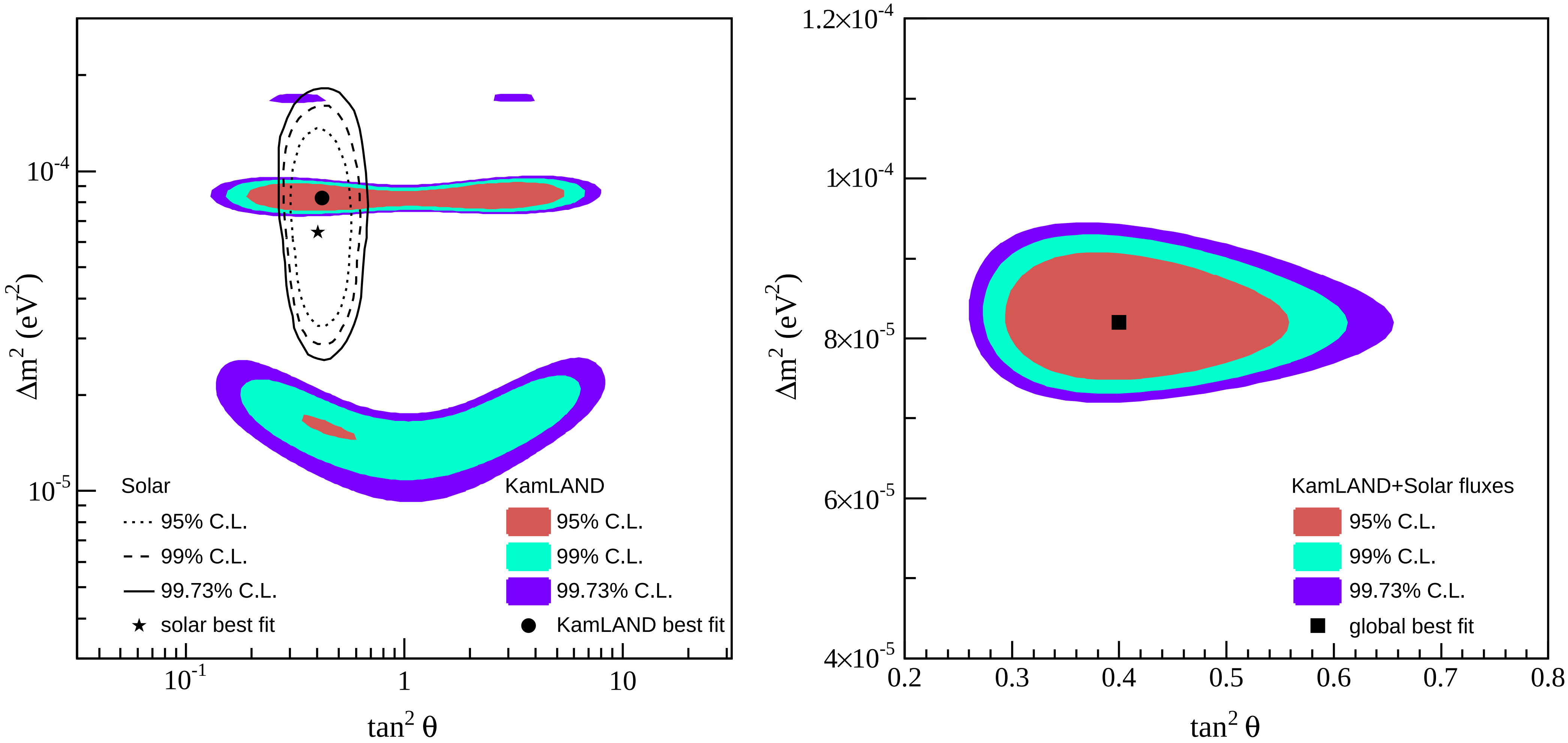


$L_0 = 180\,\text{km}$

The confidence level for reactor $\bar{\nu}_e$ disappearance is now 99.995%. The observed energy spectrum disagrees with the expected spectral shape in the absence of neutrino oscillation at the 99.9% confidence level but agrees with the distortion expected from $\bar{\nu}_e$ oscillation effects.

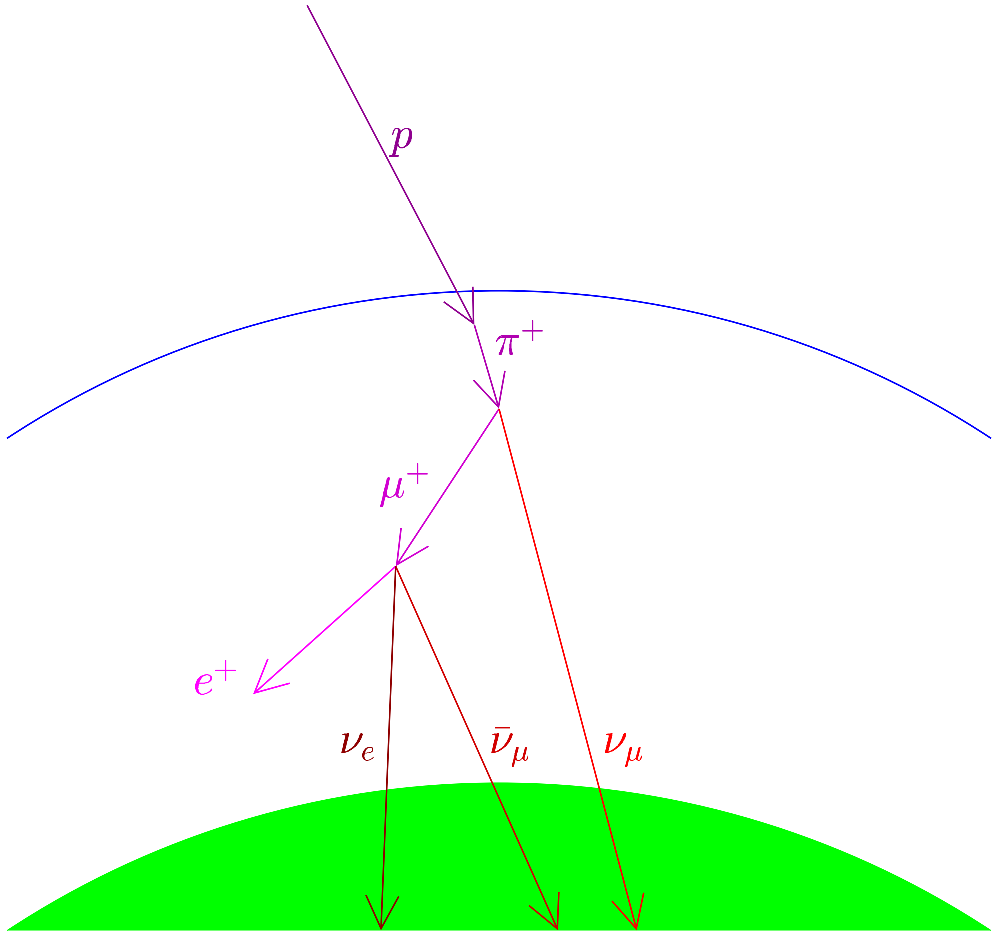
Combined Fit of Solar + Reactor Neutrino Data

[KamLAND, hep-ex/0406035]



Best Fit: $\Delta m^2 = 0.82^{+0.6}_{-0.5} \times 10^{-5}\,\mathrm{eV}^2$ $\tan^2\vartheta = 0.40^{+0.09}_{-0.07}$

Atmospheric Neutrinos



$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \qquad \text{at } E \lesssim 1\,\mathrm{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on absolute fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{\left[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)\right]_{\text{data}}}{\left[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)\right]_{\text{MC}}}$$

$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, Phys. Lett. B 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, Phys. Lett. B 335 (1994) 237]

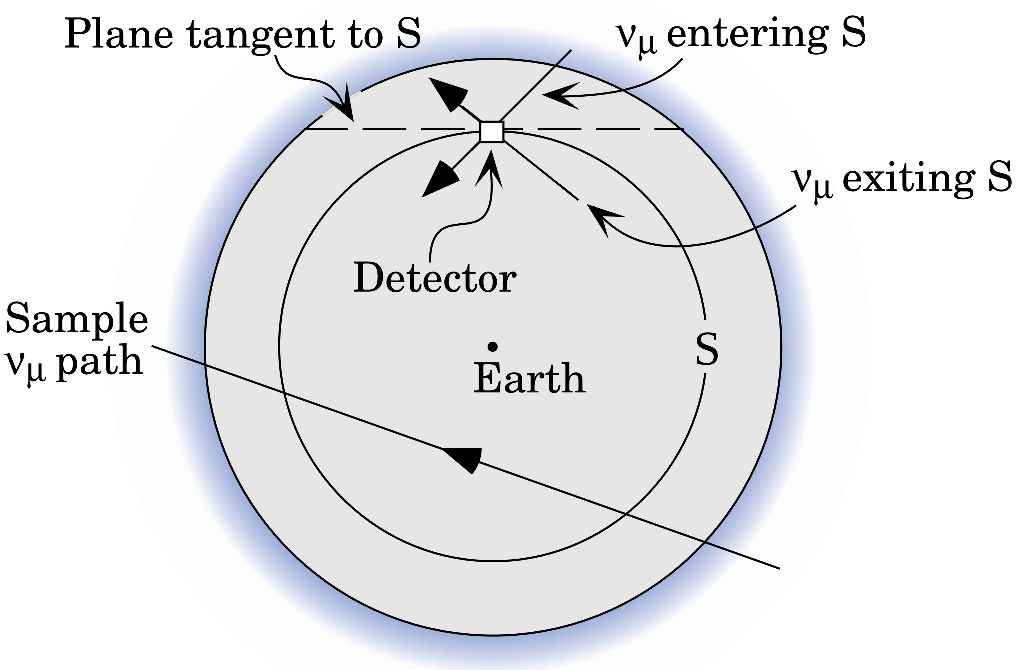
$$R^{\text{SK}}_{\text{sub-GeV}} = 0.61 \pm 0.03 \pm 0.05$$

[Super-Kamiokande, Phys. Lett. B 433 (1998) 9, hep-ex/9803006]

$$R^{\text{SK}}_{\text{multi-GeV}} = 0.66 \pm 0.06 \pm 0.08$$

[Super-Kamiokande, Phys. Lett. B 436 (1998) 33, hep-ex/9805006]

Super-Kamiokande Up-Down Asymmetry



- any ν entering the sphere S later exits it
- steady state $\Rightarrow$ $\Phi^{\text{in}}(S) = \Phi^{\text{out}}(S)$
- $E_\nu \gtrsim 1\,\text{GeV}$ $\Rightarrow$ isotropic flux of cosmic rays
- homogeneity $\Rightarrow$ $\Phi^{\text{in}}(s) = \Phi^{\text{out}}(s)$, $\forall s \in S$
- $\text{D} \in S$ $\Rightarrow$ $\Phi^{\text{up}}(\text{D}) = \Phi^{\text{down}}(\text{D})$,

[B. Kayser, Review of Particle Properties, PRD 66 (2002) 010001]

$$\boxed{A_{\nu_\mu}^{\text{up-down}}(\text{SK}) = \left(\frac{N_{\nu_\mu}^{\text{up}} - N_{\nu_\mu}^{\text{down}}}{N_{\nu_\mu}^{\text{up}} + N_{\nu_\mu}^{\text{down}}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

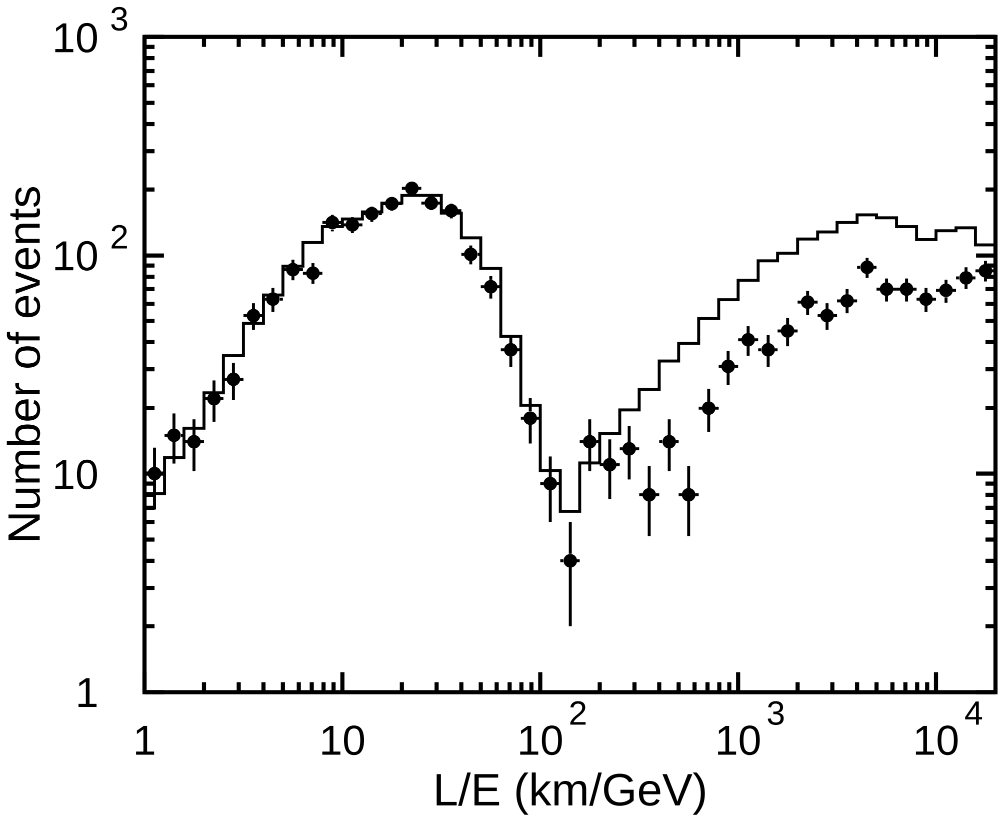
(December 1998)

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

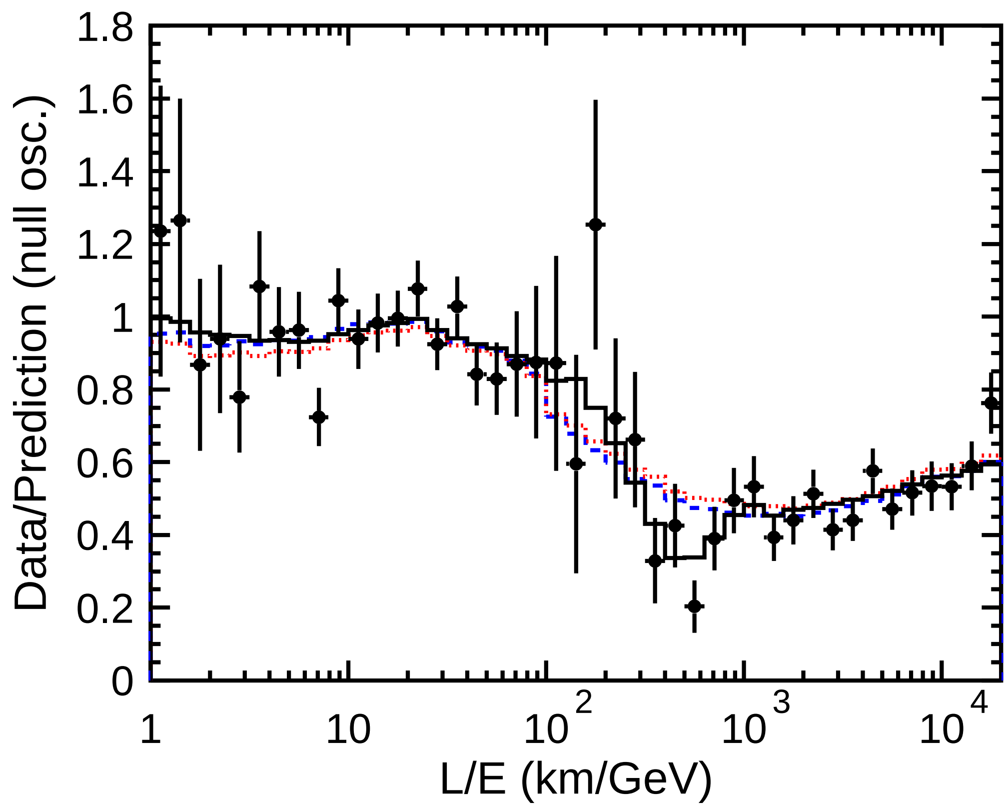
6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Super-Kamiokande Evidence of Oscillations

hep-ex/0404034, April 2004

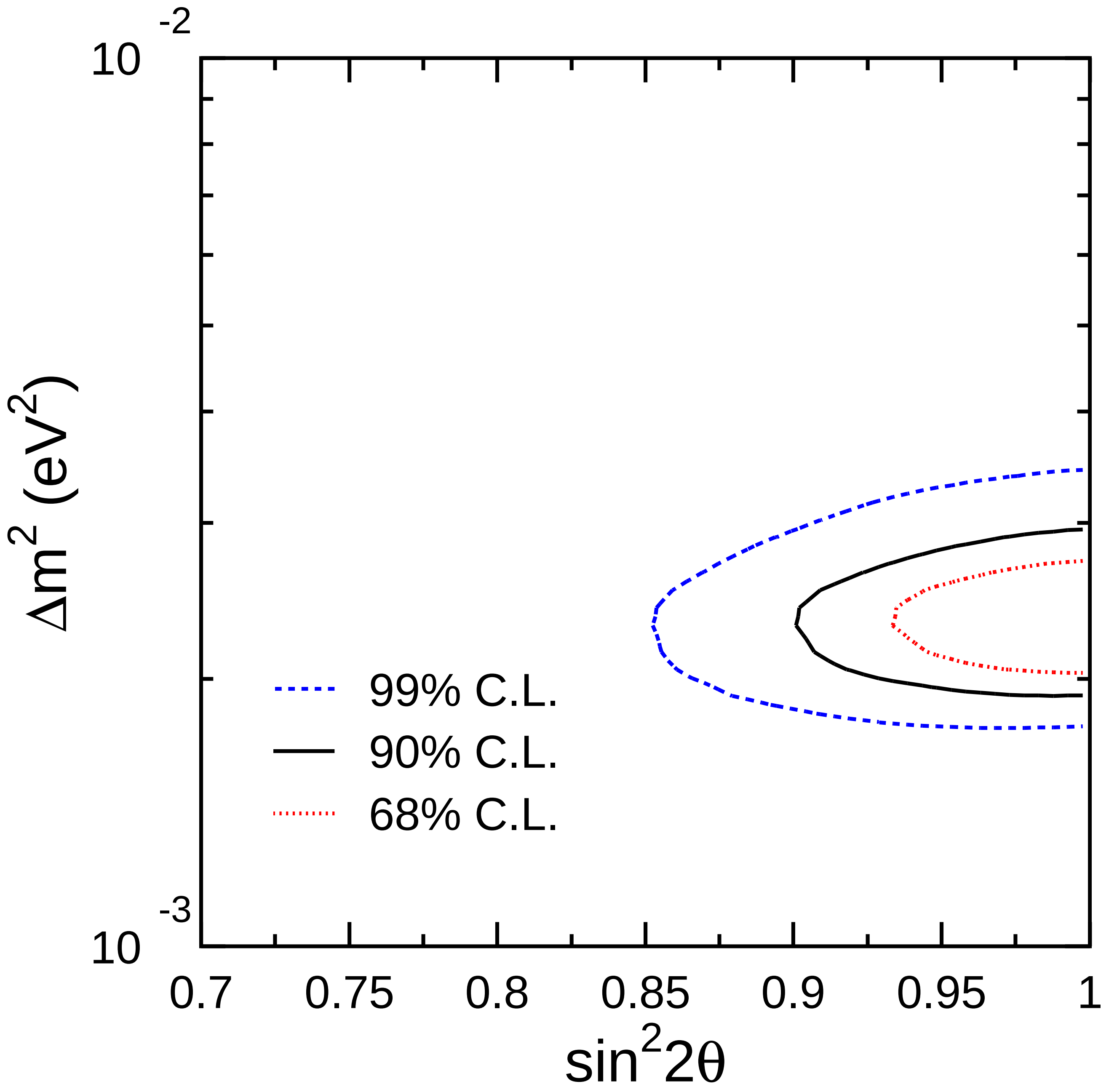


Number of events as a function of L/E for the data (points) and the atmospheric neutrino MC events without oscillations (histogram).



Ratio of the data to the MC events without neutrino oscillation (points) as a function of L/E together with the best-fi t expectations for $\nu_\mu \to \nu_\tau$ oscillations (solid line), neutrino decay (dashed line) and neutrino decoherence (dotted line).

$\nu_\mu \to \nu_\tau$ Fit of Super-Kamiokande Atmospheric Data



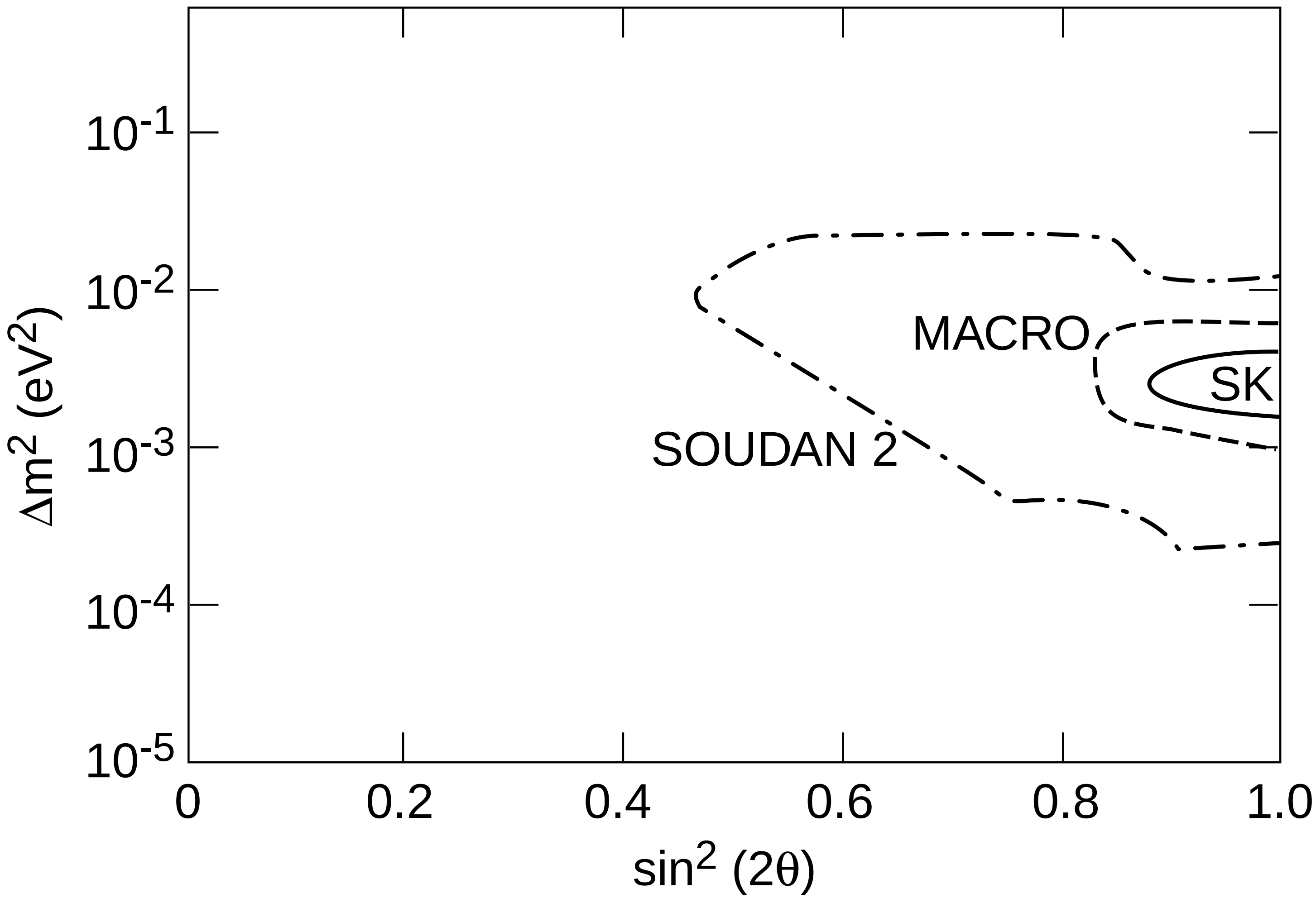
Best Fit:

$\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$\sin^2 2\theta = 1.0$

[Super-Kamiokande, hep-ex/0404034]

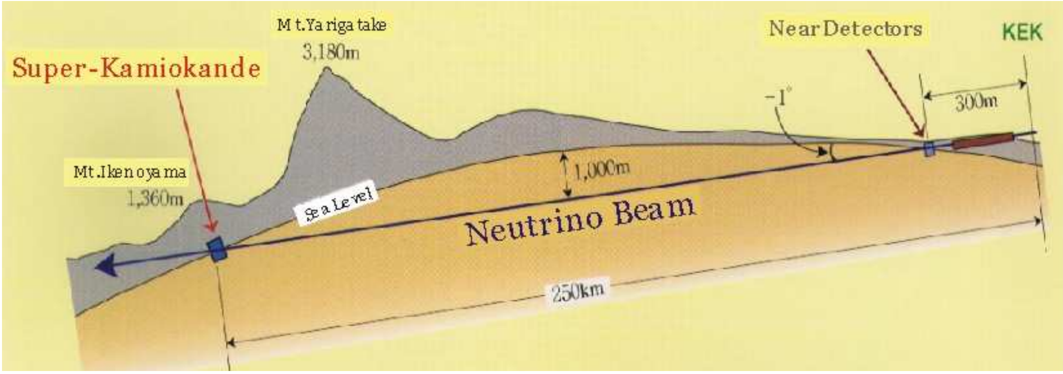
Soudan-2 & MACRO

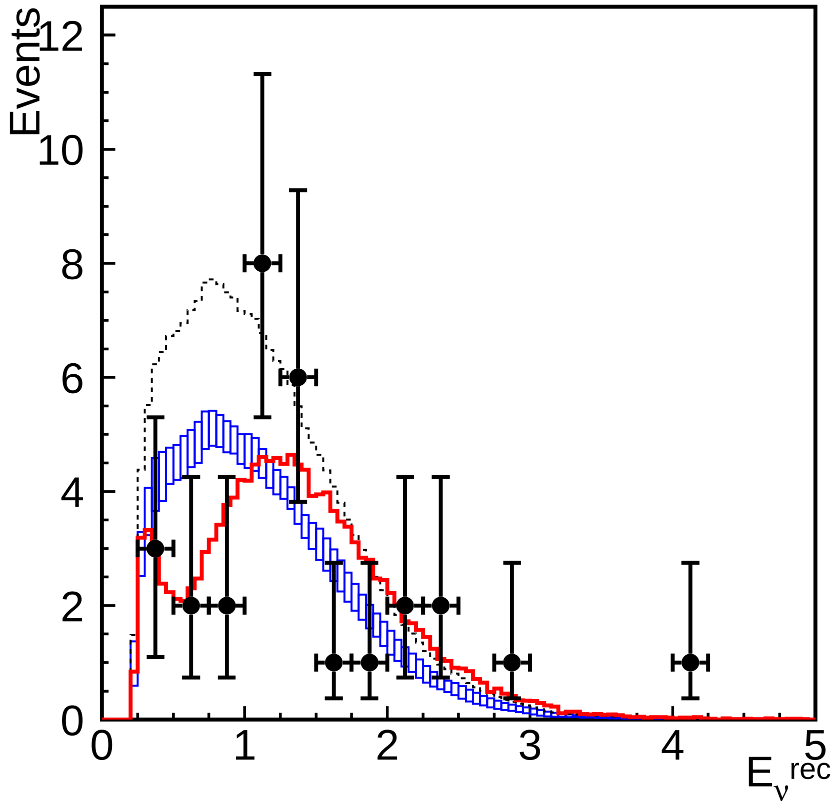


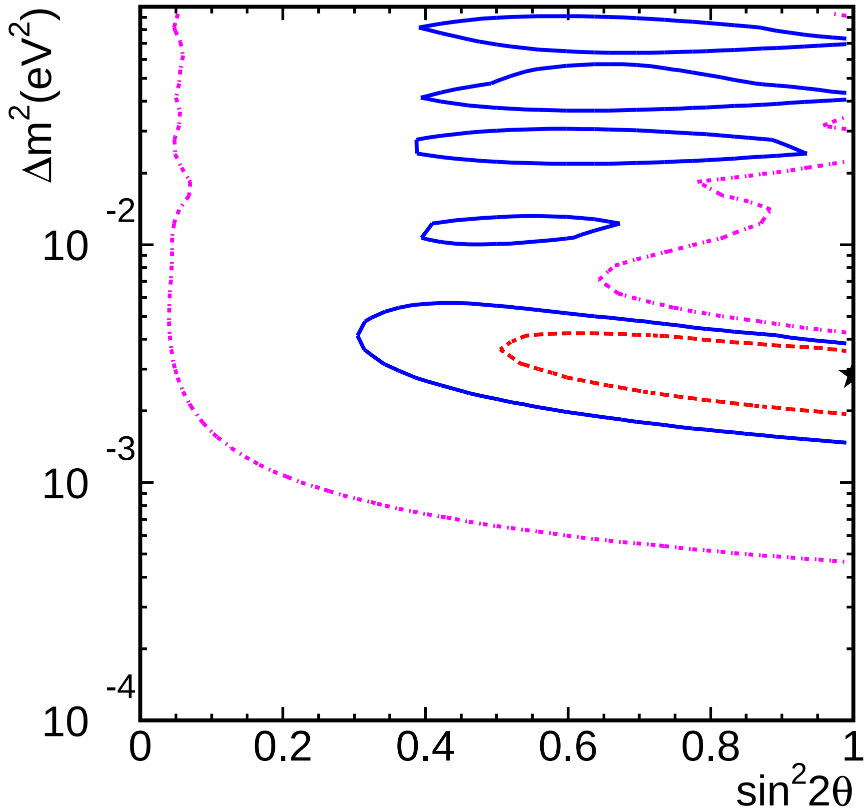
[Giacomelli, Giorgini, Spurio, hep-ex/0201032]

K2K

confirmation of atmospheric allowed region (June 2002)



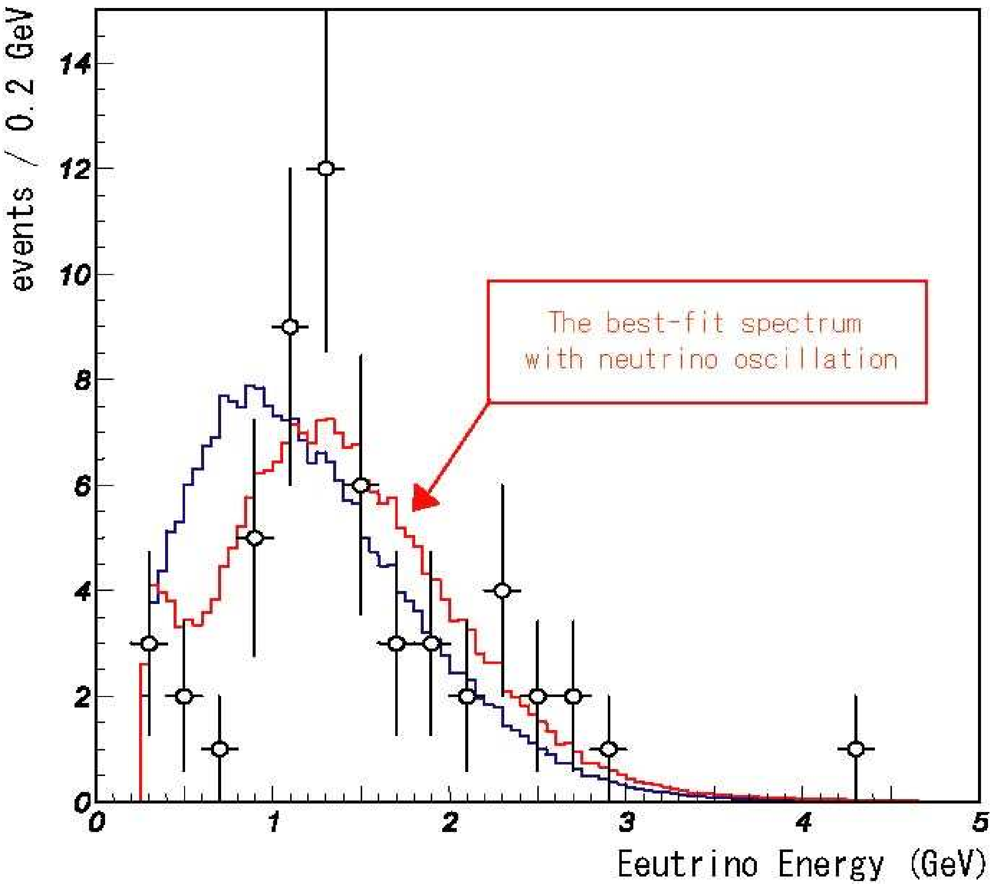




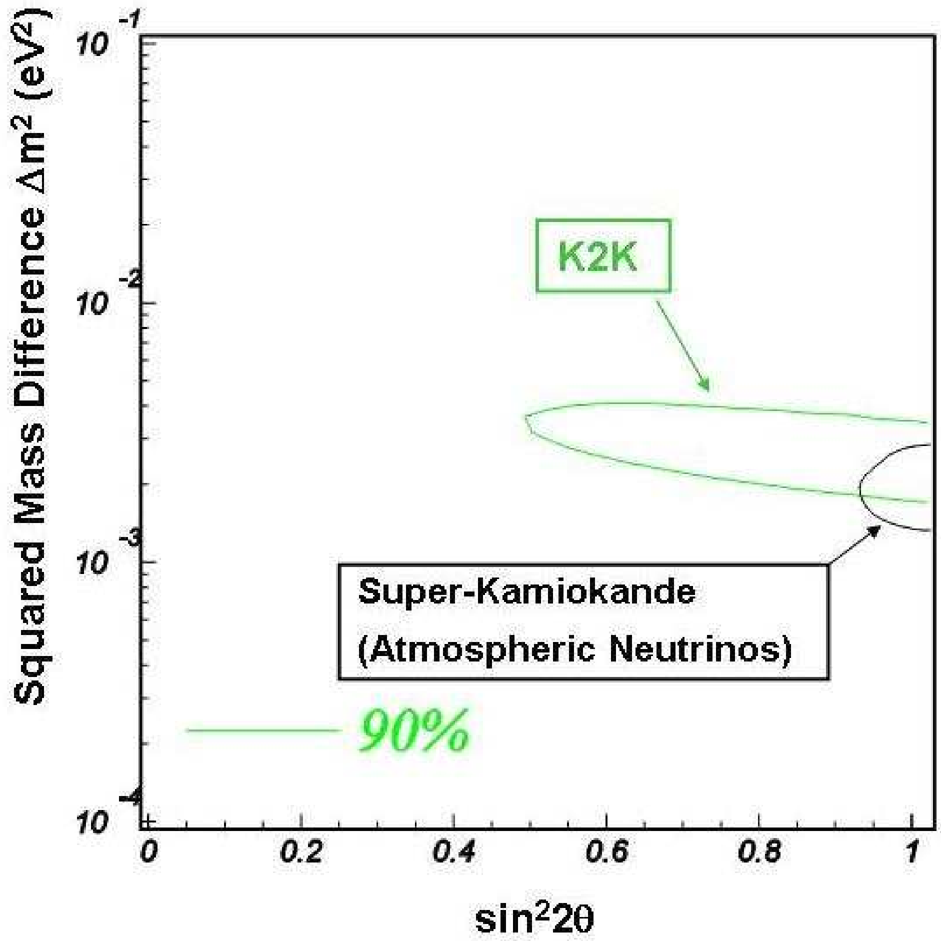
[K2K, Phys. Rev. Lett. 90 (2003) 041801]

K2K: Evidence of Spectral Distortion

http://neutrino.kek.jp/news/2004.06.10/index-e.html, June 2004



K2K energy spectrum (data points), compared to the scaled KEK beam spectrum (black histogram) and the best-fit expected spectrum taking into account neutrino oscillations (red histogram).



90% confidence regions for $\nu_\mu \to \nu_\tau$ oscillation from K2K (green) and Super-Kamiokande atmospheric neutrinos (black).

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$ $\left(\begin{array}{l} \text{Homestake, Kamiokande,} \\ \text{GALLEX, SAGE, GNO,} \\ \text{Super-Kamiokande, SNO} \end{array} \right)$

Reactor $\bar{\nu}_e$ disappearance (KamLAND)

$$\Longrightarrow \left\{ \begin{array}{c} \Delta m^{2\,\text{best-fit}}_{\text{SUN}} = 8.2 \times 10^{-5} \\ 7.4 \times 10^{-5} < \Delta m^2_{\text{SUN}} < 9.2 \times 10^{-5} \\ \left[\text{eV}^2\right] \qquad (3\sigma) \end{array} \right.$$

[Bahcall, Gonzalez-Garcia, Pena-Garay, hep-ph/0406294]

Atmospheric $\nu_\mu \to \nu_\tau$ $\left(\begin{array}{l} \text{Kamiokande, IMB,} \\ \text{Super-Kamiokande,} \\ \text{MACRO, SOUDAN 2} \end{array} \right)$

Accelerator ν_μ disappearance (K2K)

$$\Longrightarrow \left\{ \begin{array}{c} \Delta m^{2\,\text{best-fit}}_{\text{ATM}} = 2.6 \times 10^{-3} \\ 1.4 \times 10^{-3} < \Delta m^2_{\text{ATM}} < 5.1 \times 10^{-3} \\ \left[\text{eV}^2\right] \qquad (3\sigma) \end{array} \right.$$

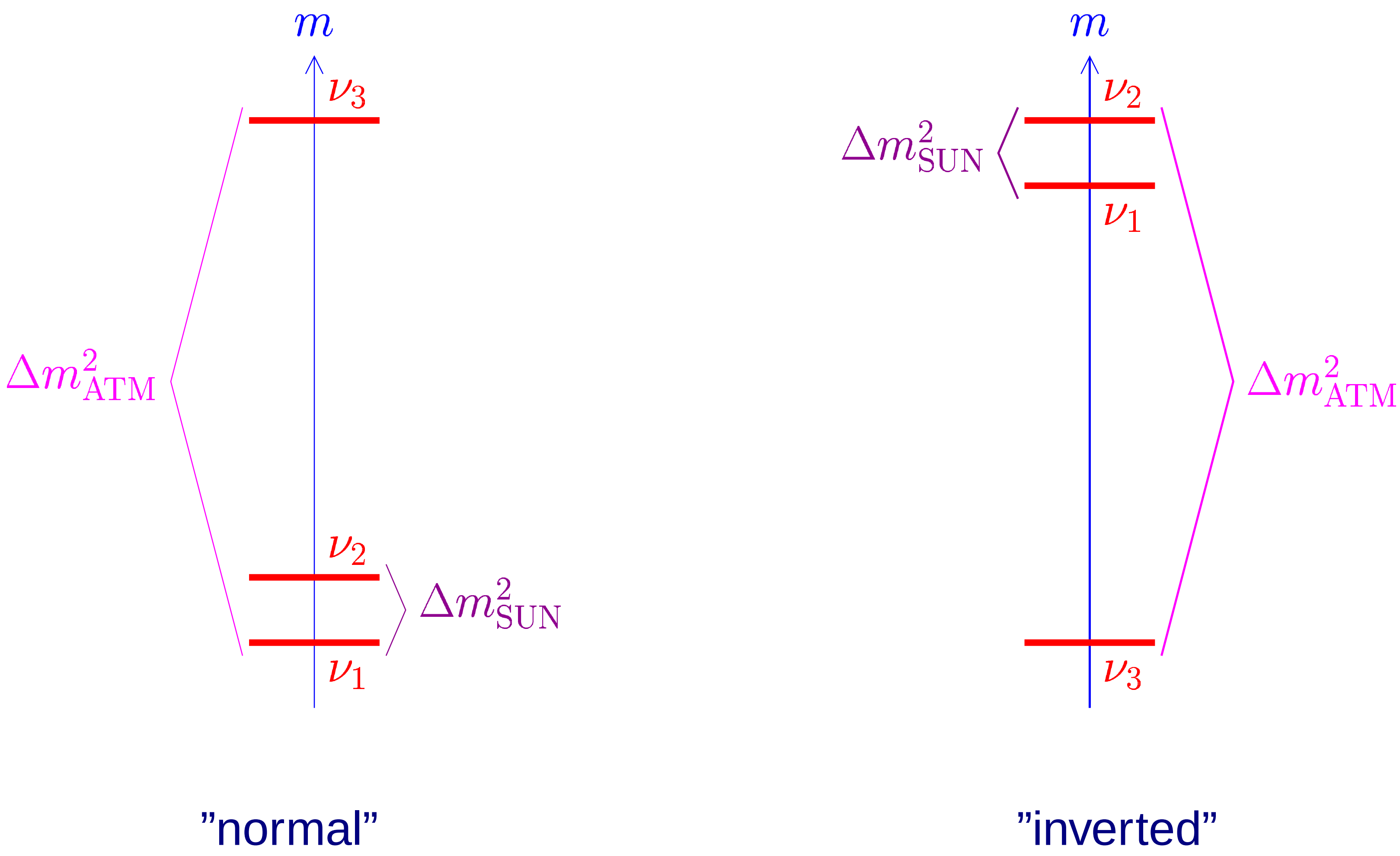
[Fogli, Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

Three-Neutrino Mixing

flavor fields $\nu_\alpha, \ \alpha = e, \mu, \tau$ $\qquad$ $\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL}$ $\qquad$ massive fields $\nu_k \to m_k$

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \qquad \Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$$

Allowed Three-Neutrino Schemes



absolute scale is not determined by neutrino oscillation data

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

$$\begin{array}{cc} \text{SUN} \rightarrow & U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ & \qquad\qquad\qquad\quad \uparrow \\ & \qquad\qquad\qquad\quad \text{ATM} \end{array}$$

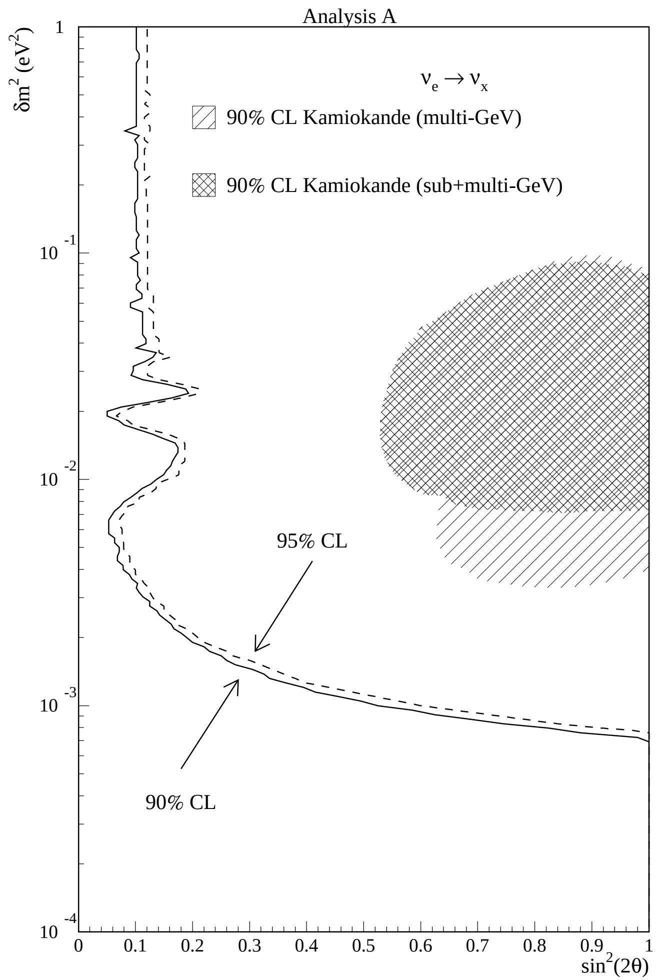
CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 < 5 \times 10^{-2} \text{ (99.73\% C.L.)}}$$

[Fogli et al., PRD 66 (2002) 093008]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!



[CHOOZ, PLB 466 (1999) 415]
see also [Palo Verde, PRD 64 (2001) 112001]

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2 \vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2 \vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, Giunti, PLB 444 (1998) 379]
[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

$$U = R_{23}\, W_{13}\, R_{12}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} = \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} = \vartheta_{\text{SUN}}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$\sin^2\vartheta_{\text{CHOOZ}} = |U_{e3}|^2 = \sin^2\vartheta_{13}$$

$$\sin^2\vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{s_{12}^2 c_{13}^2}{1 - s_{13}^2} = \sin^2\vartheta_{12}$$

$$\sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2 = s_{23}^2 c_{13}^2 \simeq \sin^2\vartheta_{23}$$

Bilarge Mixing

$$|U_{e3}|^2 \ll 1 \;\Rightarrow\; U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Rightarrow \left\{ \begin{aligned} \nu_e &= c_{\vartheta_S}\nu_1 + s_{\vartheta_S}\nu_2 \\ \nu_a^{(S)} &= -s_{\vartheta_S}\nu_1 + c_{\vartheta_S}\nu_2 \\ &= c_{\vartheta_A}\nu_\mu - s_{\vartheta_A}\nu_\tau \end{aligned} \right.$$

$$\sin^2 2\vartheta_A \simeq 1 \Rightarrow \vartheta_A \simeq \frac{\pi}{4} \Rightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Solar $\nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right)$

$\frac{\Phi_{CC}^{SNO}}{\Phi_{\nu_e}^{SSM}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau}$ for $E \gtrsim 6\,\mathrm{MeV}$

LMA $\Rightarrow$ $\tan^2\vartheta_S \simeq 0.4 \Rightarrow \vartheta_S \simeq \frac{\pi}{6} \Rightarrow U \simeq \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

Global Fit of Oscillation Data $\Rightarrow$ Bilarge Mixing

[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

$$\Delta m^{2\,\mathrm{bf}}_{21} \simeq 8.3 \times 10^{-5}\,\mathrm{eV}^2 \qquad 7.4 \times 10^{-5} \lesssim \Delta m^2_{21} \lesssim 9.3 \times 10^{-5} \quad (3\sigma)$$

$$\Delta m^{2\,\mathrm{bf}}_{31} \simeq 2.4 \times 10^{-3}\,\mathrm{eV}^2 \qquad 1.8 \times 10^{-3} \lesssim \Delta m^2_{31} \lesssim 3.2 \times 10^{-3} \quad (3\sigma)$$

$$\sin^2 \vartheta^{\mathrm{bf}}_{12} \simeq 0.28 \qquad 0.22 \lesssim \sin^2 \vartheta_{12} \lesssim 0.37 \quad (3\sigma)$$

$$\sin^2 \vartheta^{\mathrm{bf}}_{13} \simeq 0.01 \qquad \sin^2 \vartheta_{13} \lesssim 0.05 \quad (3\sigma)$$

$$\sin^2 2\vartheta^{\mathrm{bf}}_{\mathrm{ATM}} \simeq 1 \qquad \sin^2 2\vartheta_{\mathrm{ATM}} \gtrsim 0.86 \quad (3\sigma)$$

[Fogli, Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

$$\sin^2 \vartheta_{\mathrm{ATM}} = |U_{\mu 3}|^2 = \sin^2 \vartheta_{23} \, \cos^2 \vartheta_{13}$$

$$|U|_{\mathrm{bf}} \simeq \begin{pmatrix} 0.84 & 0.53 & 0.10 \\ 0.31 - 0.43 & 0.56 - 0.63 & 0.71 \\ 0.32 - 0.44 & 0.57 - 0.64 & 0.70 \end{pmatrix}$$

$$|U| \simeq \begin{pmatrix} 0.77 - 0.88 & 0.46 - 0.61 & 0.00 - 0.22 \\ 0.08 - 0.60 & 0.30 - 0.79 & 0.55 - 0.85 \\ 0.10 - 0.61 & 0.33 - 0.81 & 0.51 - 0.83 \end{pmatrix}$$

Open Questions on Neutrino Masses

Value of $\vert U_{e3}\vert$, J_{CP}?	$\Leftarrow$	ν Oscillations
Sterile Neutrinos?	$\Leftarrow$	ν Oscillations
		LSND indication $\Longrightarrow$ wait MiniBooNE
Absolute Scale of Neutrino Masses?	$\Leftarrow$	β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay
Pattern of Neutrino Masses?	$\Leftarrow$	ν Osc., β Dec., Cosmology, $\beta\beta_{0\nu}$ Dec.
Are Neutrinos Majorana?	$\Leftarrow$	$\beta\beta_{0\nu}$ Decay

Oscillations are Insensitive to Absolute Scale of Neutrino Masses

oscillations are due to the interference of different massive components of a flavor ν

$\Downarrow$

oscillations can be sensitive only to mass differences

In Neutrino Oscillations Dirac = Majorana

$$\frac{\mathrm{d}\nu_\alpha}{\mathrm{d}t} = \frac{1}{2E}\left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\nu_\beta \qquad M^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

Dirac: $U \to U_\mathrm{D}$

Majorana: $U \to U_\mathrm{M} = U_\mathrm{D}\, D$

$$U_\mathrm{D} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$D = \mathrm{diag}(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}) \quad \Rightarrow \quad U_\mathrm{M}M^2U_\mathrm{M}^\dagger = U_\mathrm{D}DM^2D^\dagger U_\mathrm{D}^\dagger = U_\mathrm{D}M^2U_\mathrm{D}^\dagger$$

Majorana phases

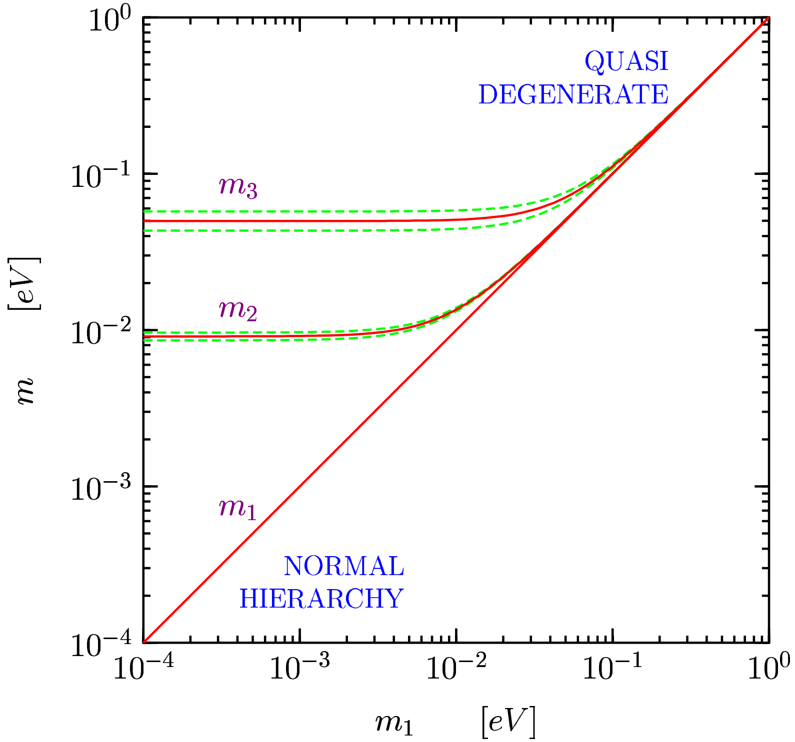
[Bilenky, Hosek, Petcov, PLB 94 (1980) 495]

[Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

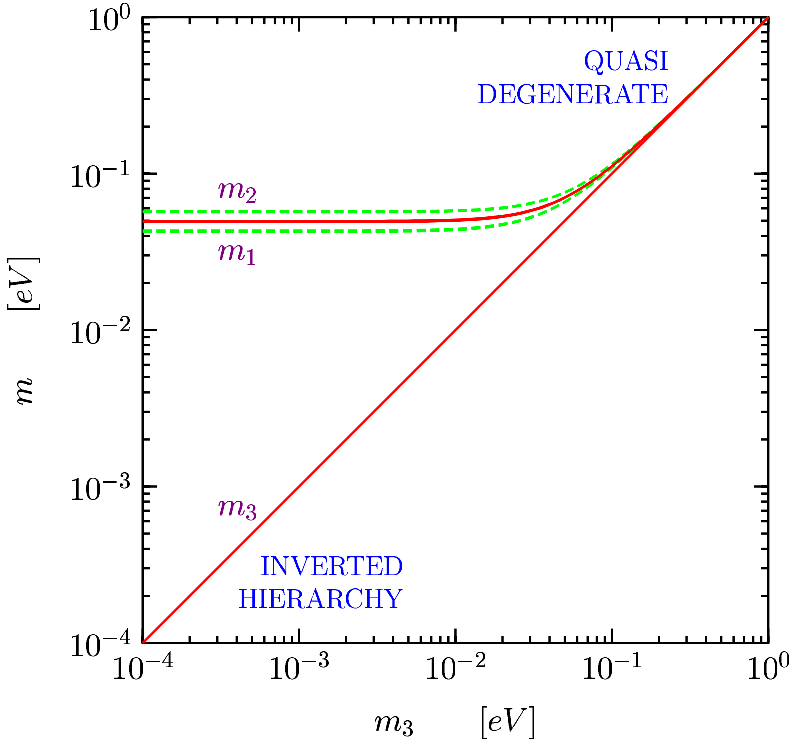
[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

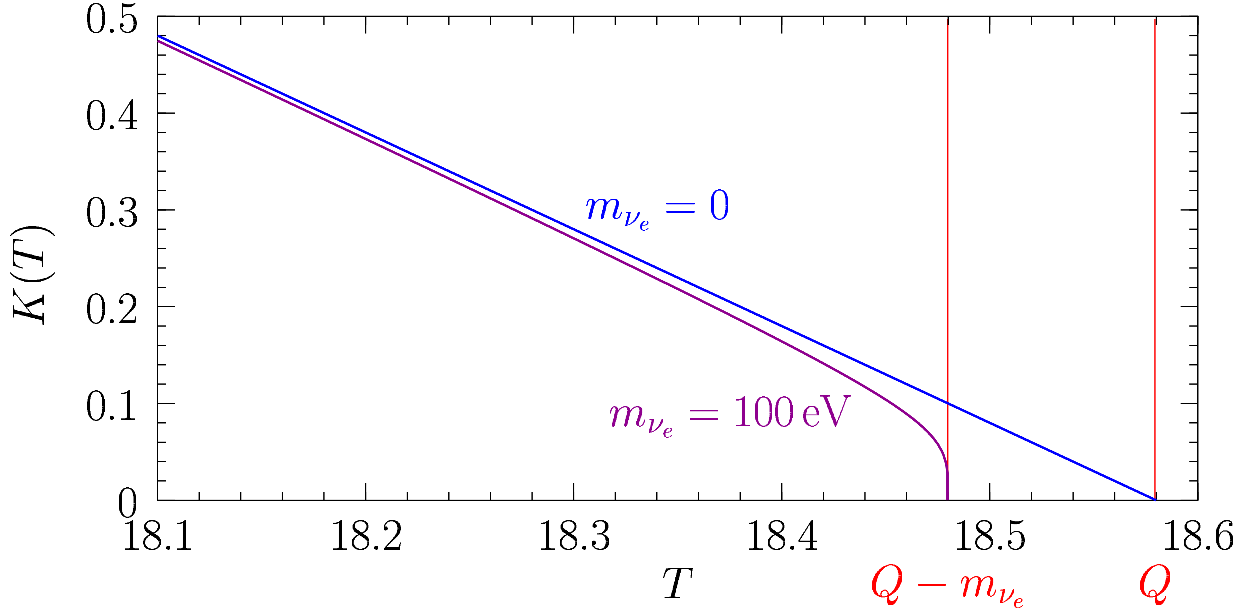
Tritium β Decay

$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot: $$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$

Mainz & Troitsk [Weinheimer, hep-ex/0210050]

$\boxed{m_{\nu_e} < 1.8\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$

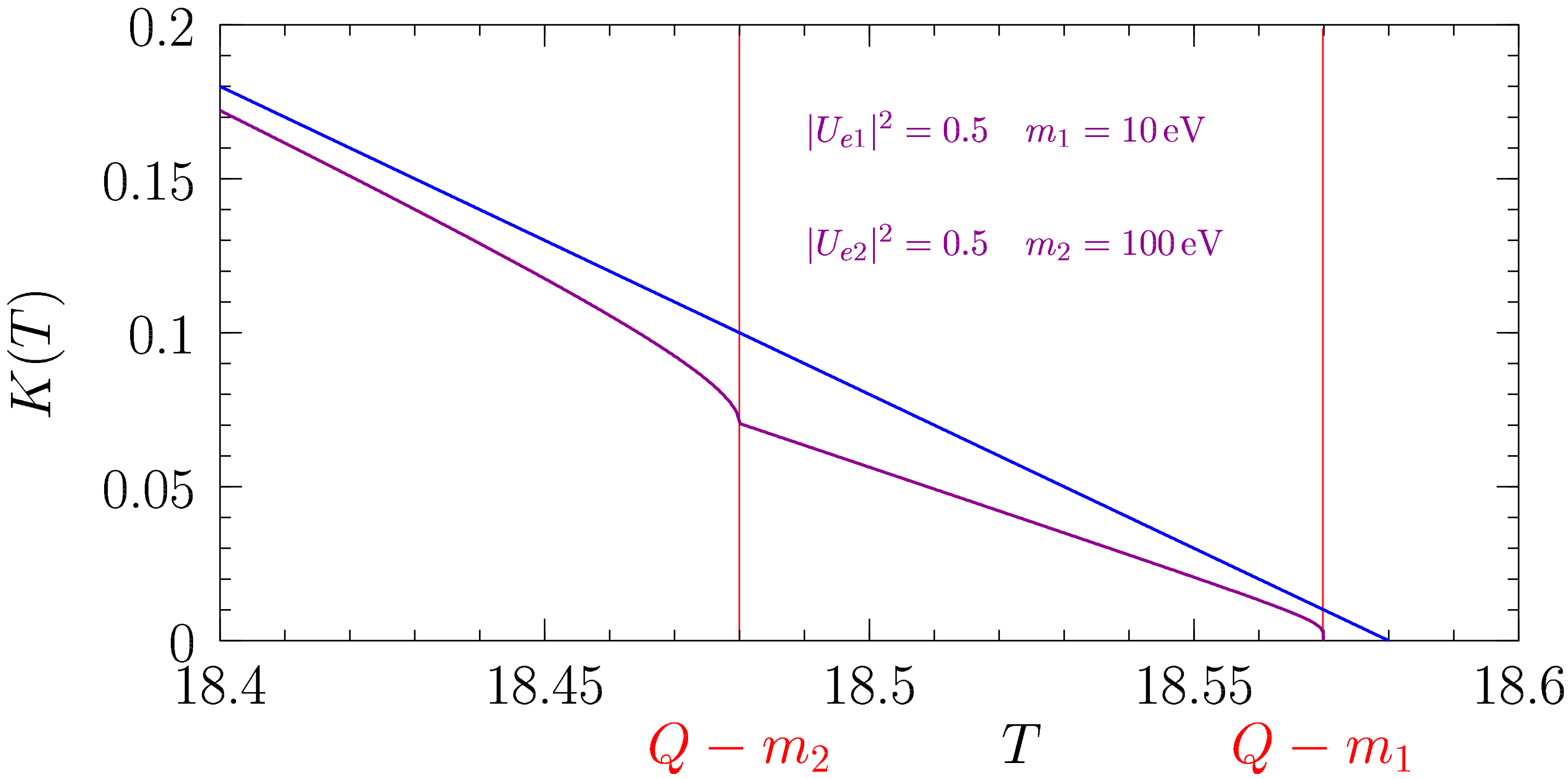
Mainz + Troitsk [Fogli et al., hep-ph/0408045]

future: KATRIN

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2 - 0.3\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case

$2N-1$ parameters $= N$ masses $+ \, N-1$ mixing matrix elements $\quad \left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses ($m_k \ll Q - T$) $\Longrightarrow$ effective mass

$$\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$$

[McKellar, PLB 97 (1980) 93] [Shrock, PLB 96 (1980) 159]

[Kobzarev, Martemyanov, Okun, Shchepkin, SJNP 32 (1980) 823]

[Holzschuh, RPP 55 (1992) 1035] [Weinheimer et al., PLB 460 (1999) 219]

[Vissani, NPB PS 100 (2001) 273 (NOW 2000)] [Farzan, Smirnov, PLB 557 (2003) 224]

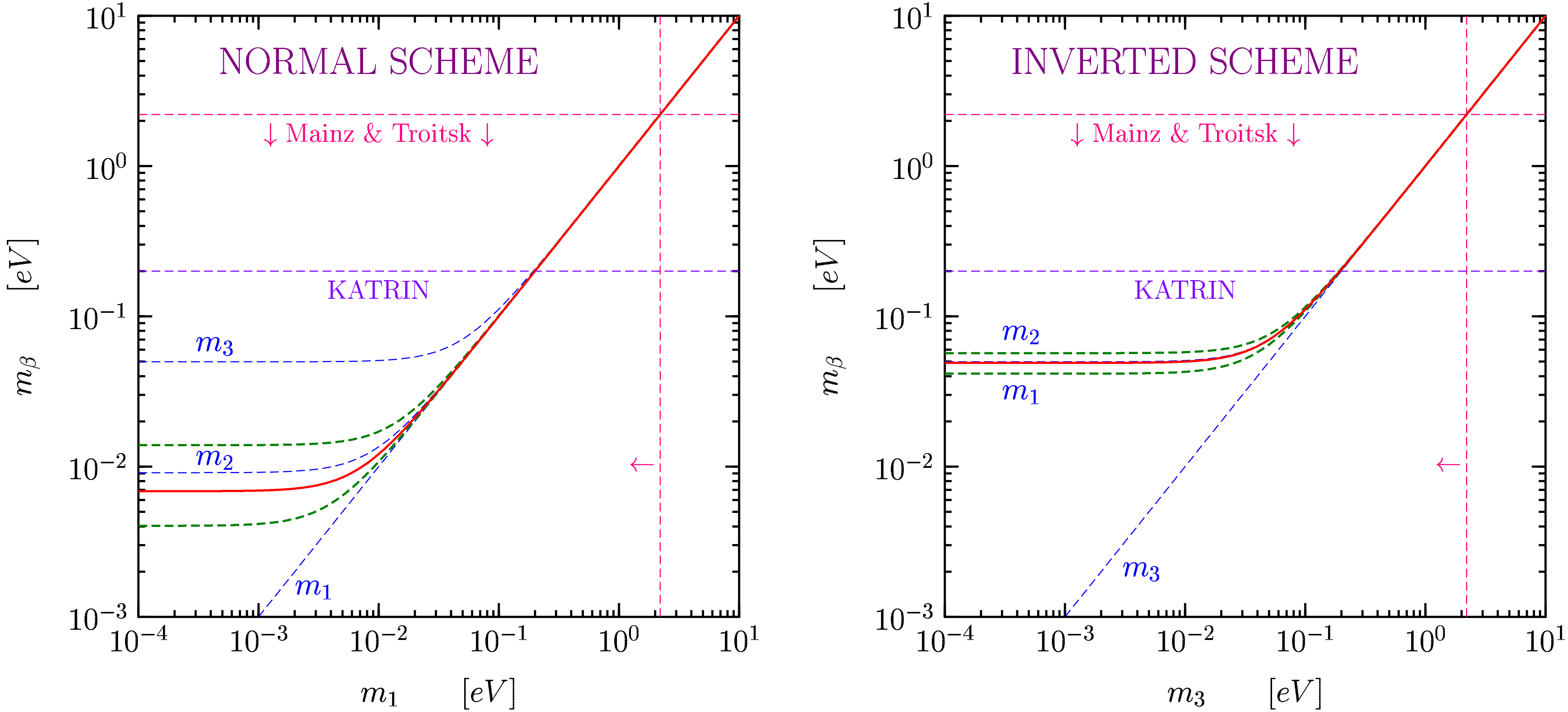
$$K^2 = (Q-T)\sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}}$$

$$\simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2}\frac{m_k^2}{(Q-T)^2}\right] = (Q-T)^2 \left[1 - \frac{1}{2}\frac{m_\beta^2}{(Q-T)^2}\right]$$

$$\simeq (Q-T)^2 \sqrt{1 - \frac{m_\beta^2}{(Q-T)^2}} = (Q-T)\sqrt{(Q-T)^2 - m_\beta^2}$$

$m_{\nu_e} < 2.2\,\mathrm{eV}$ (95% C.L.) $\Longrightarrow$ $m_\beta < 2.2\,\mathrm{eV}$ (95% C.L.)

Three-Neutrino Mixing $\Longrightarrow$ $\boxed{m_\beta^2 = c_{12}^2\, c_{13}^2\, m_1^2 + s_{12}^2\, c_{13}^2\, m_2^2 + s_{13}^2\, m_3^2}$



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$
$$m_\beta^2 \simeq m_1^2 + |U_{e2}|^2 \, \Delta m_{\text{SUN}}^2 + |U_{e3}|^2 \, \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_\beta^2 \simeq m_3^2 + \left(|U_{e1}|^2 + |U_{e2}|^2\right) \Delta m_{\text{ATM}}^2$$

Quasi Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in the primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \qquad \nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_{\text{F}}^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \qquad \Longrightarrow \quad T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \simeq 1.676 \times 10^{-4}\,\text{eV}$ $\quad (T_\gamma = 2.725 \pm 0.001\,\text{K})$

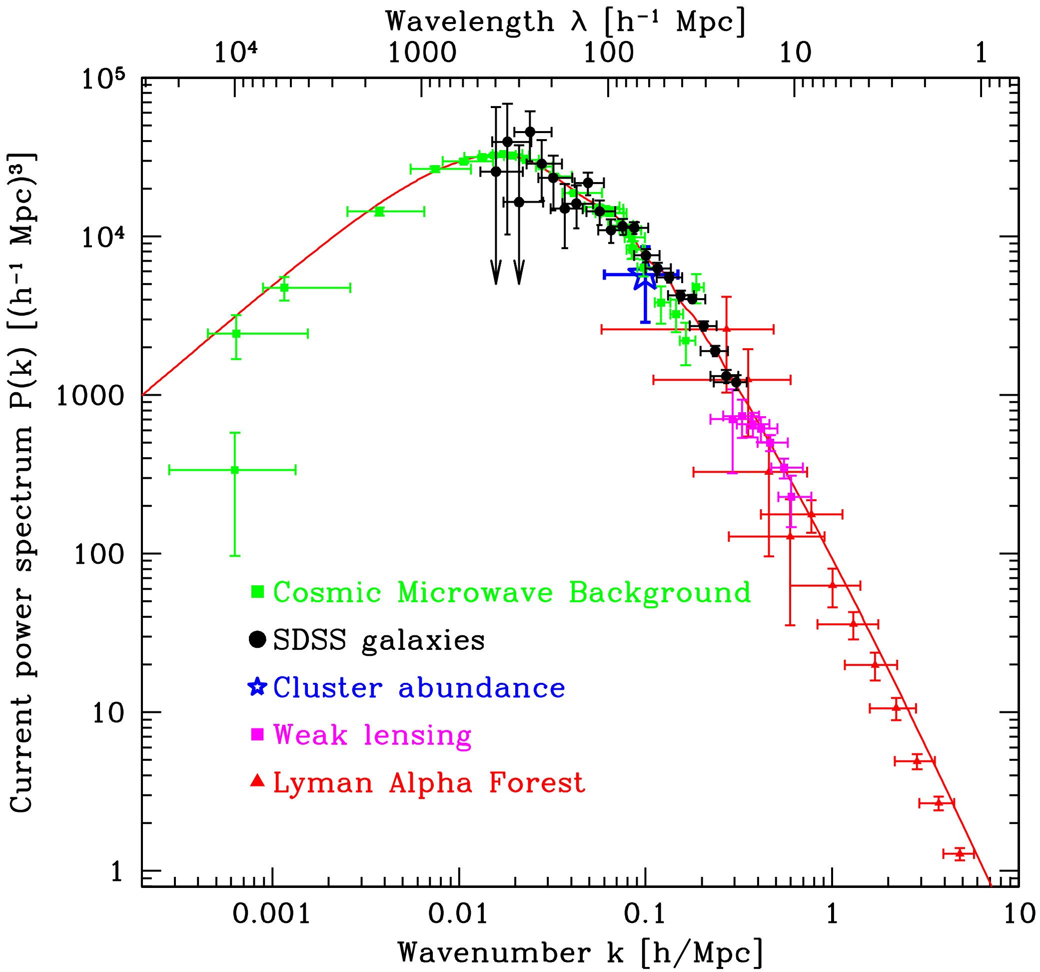
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\, T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$ $\quad \left(\rho_c = \frac{3H^2}{8\pi G_{\text{N}}}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 1 \qquad \Longrightarrow \qquad \sum_k m_k \lesssim 46\,\text{eV}$$

Power Spectrum of Density Fluctuations



[SDSS, astro-ph/0310725]

hot dark matter prevents early galaxy formation

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right)\left(\frac{0.1}{\Omega_m\,h^2}\right)$$

for $k \gtrsim k_{\mathrm{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\,\sqrt{\Omega_m}\,h\,\mathrm{Mpc}^{-1}$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

Reviews: [Dolgov, Phys. Rept. 370 (2002) 33], [Kainulainen, Olive, hep-ph/0206163], [Sarkar, hep-ph/0302175], [Hannestad, NJP 6 (2004) 108]

WMAP, AJ SS 148 (2003) 175

CMB (WMAP, CBI, ACBAR) + LSS (2dFGRS) + Lyα + HST + SN-Ia

$$\Lambda\text{CDM:} \begin{cases} T_0 = 13.7 \pm 0.1\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}, \\ \Omega_{\text{tot}} = 1.02 \pm 0.02 \qquad \Omega_b h^2 = 0.0224 \pm 0.0009 \qquad \Omega_m h^2 = 0.135^{+0.008}_{-0.009} \end{cases}$$

$\Omega_\nu h^2 < 0.0076$ (95% confidence) $\Longrightarrow$ $\displaystyle\sum_k m_k < 0.71\,\text{eV}$

Hannestad, JCAP 0305 (2003) 004

$\sum_k m_k < 1.01\,\text{eV}$ (95% confidence) WMAP+CBI+2dFGRS+HST+SN-Ia

$\sum_k m_k < 1.20\,\text{eV}$ (95% confidence) WMAP+CBI+2dFGRS

$\sum_k m_k < 2.12\,\text{eV}$ (95% confidence) WMAP+2dFGRS

Elgaroy and Lahav, JCAP 04 (2003) 004

$\sum_k m_k < 1.1\,\text{eV}$ (95% confidence) WMAP+2dFGRS+HST

SDSS, PRD 69 (2004) 103501

CMB(WMAP)+LSS(SDSS)+SN-Ia

$h = 0.70^{+0.04}_{-0.03}$ $\Omega_m = 0.30 \pm 0.04$ $\sum_k m_k < 1.7\,\text{eV}$ (95% confidence)

SDSS, astro-ph/0406594

CMB(WMAP)+LSS(SDSS)+bias(SDSS) $P_g(k) = b^2\, P_m(k)$

$\Omega_m = 0.25 \pm 0.03$ $\sum_k m_k < 0.54\,\text{eV}$ (95% confidence)

SDSS, astro-ph/0407372

CMB(WMAP)+LSS(SDSS)+bias(SDSS)+Lyα(SDSS)+SN-Ia

$\Omega_\Lambda = 0.72 \pm 0.02$ $\sum_k m_k < 0.42\,\text{eV}$ (95% confidence)

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045

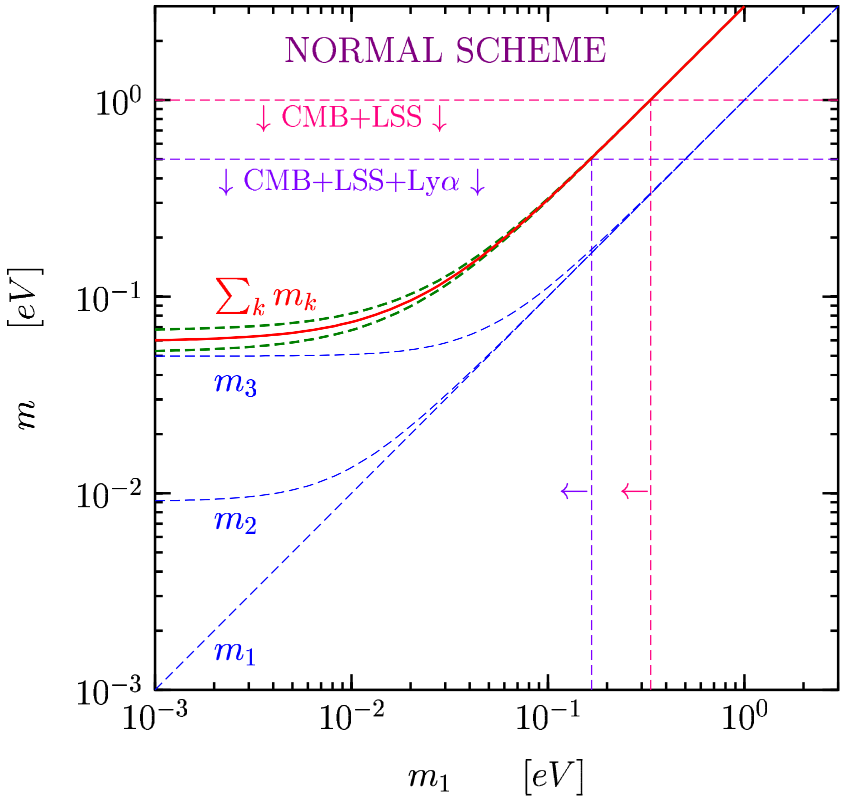
$\sum_k m_k < 1.4\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia

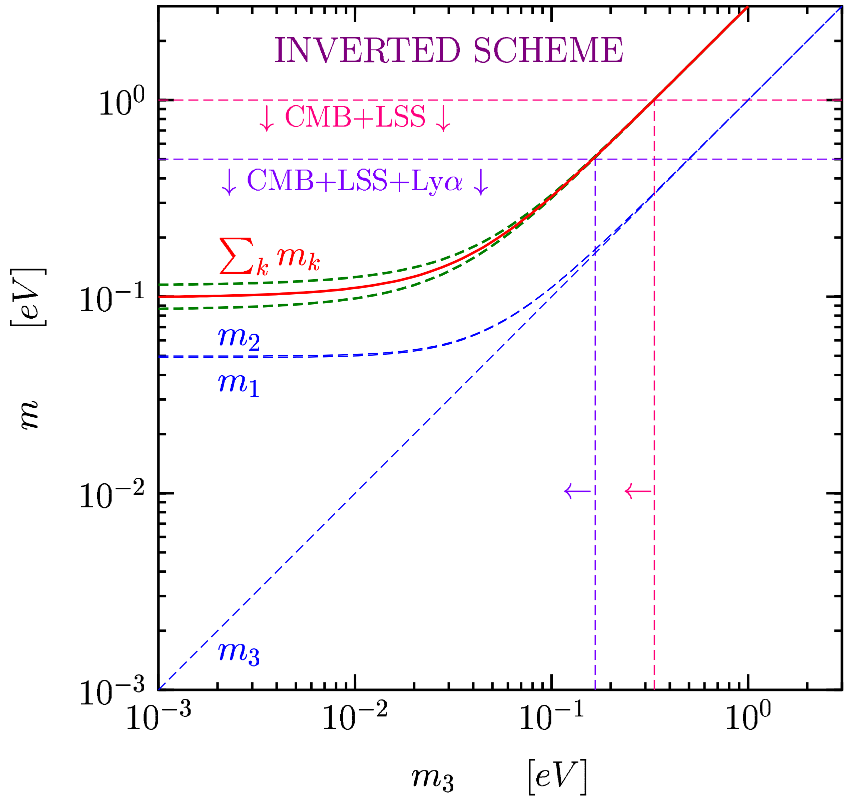
$\sum_k m_k < 0.47\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia+Lyα(SDSS)

Approximate Estimate

$$\sum_k m_k \lesssim 1\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia}$$

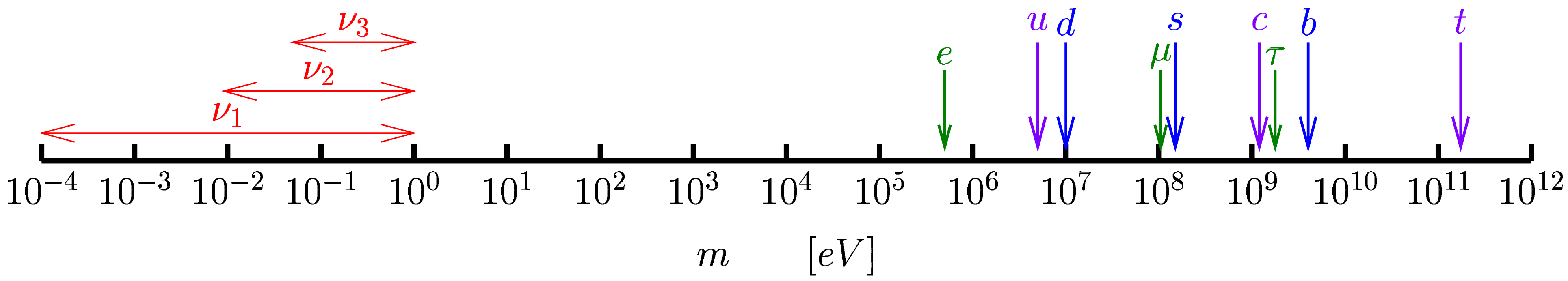
$$\sum_k m_k \lesssim 0.5\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia+Ly}\alpha$$





FUTURE: IF $\sum_k m_k \lesssim 8 \times 10^{-2}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

Majorana Neutrino Mass?



known natural explanations of smallness of ν masses:

- ★ See-Saw Mechanism
- ★ 5-D Non-Renormalizable Effective Operator

both imply

- ★ Majorana ν masses $\Longleftrightarrow$ $|\Delta L| = 2$ $\Longleftrightarrow$ $\beta\beta_{0\nu}$ decay
- ★ see-saw type relation $m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}}$
- ★ new high energy scale $\mathcal{M}$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

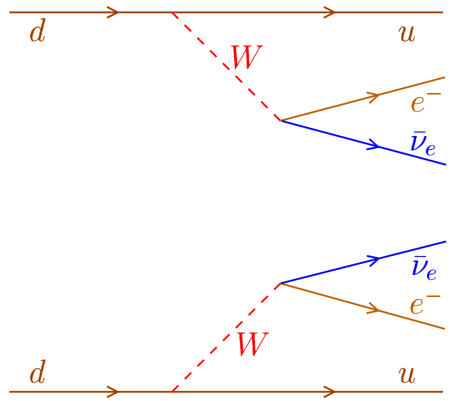
Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process

in the Standard Model

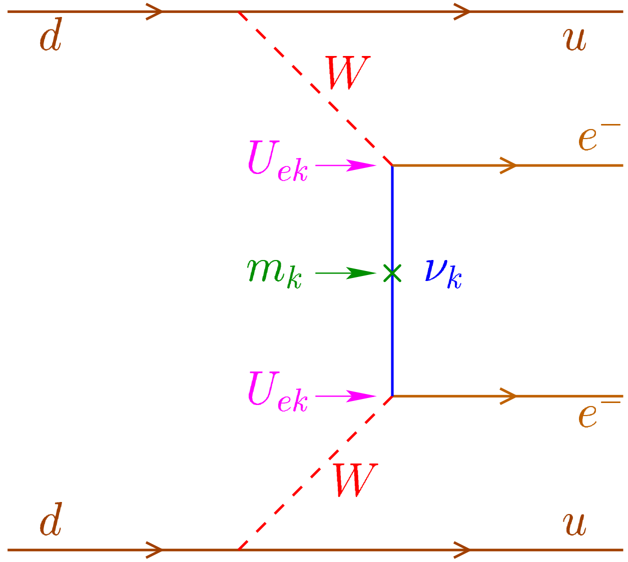


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

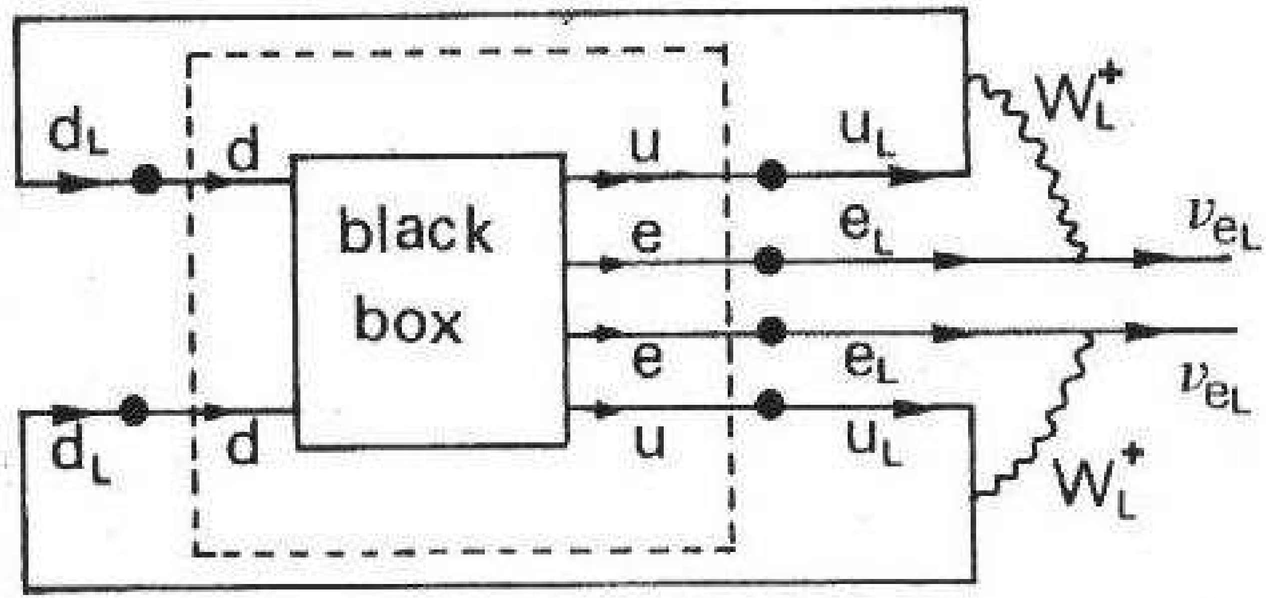
$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$



Majorana Neutrino Mass $\Leftrightarrow \beta\beta_{0\nu}$ Decay

[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]



Majorana Mass Term: $\boxed{\mathcal{L}^{M}_{eL} = -\frac{1}{2}\, m \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right) = \frac{1}{2}\, m \left(\nu^T_{eL}\, \mathcal{C}^\dagger\, \nu_{eL} + \nu^\dagger_{eL}\, \mathcal{C}\, \nu^*_{eL} \right)}$

two conditions: $\left\{ \begin{array}{l} u, d, e \text{ are massive} \\ \text{standard left-handed weak interaction exists} \end{array} \right.$

cancellations with other diagrams are very unlikely (unstable under perturbations)

The Problem of Calculation of Nuclear Matrix Elements

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2 \qquad\qquad m_{\beta\beta} = \sum_k U_{ek}^2\,m_k$$

Theoretically evaluated $\beta\beta(0\nu)$ half-lives (units of 10^{28} years for $\langle m_\nu\rangle = 10$ meV).

Isotope	[10]	[11]	[12]	[13]	[14]	[15]
^{48}Ca	3.18	8.83	-	-	-	2.5
^{76}Ge	1.7	17.7	14.0	2.33	3.2	3.6
^{82}Se	0.58	2.4	5.6	0.6	0.8	1.5
^{100}Mo	-	-	1.0	1.28	0.3	3.9
^{116}Cd	-	-	-	0.48	0.78	4.7
^{130}Te	0.15	5.8	0.7	0.5	0.9	0.85
^{136}Xe	-	12.1	3.3	2.2	5.3	1.8
^{150}Nd	-	-	-	0.025	0.05	-
^{160}Gd	-	-	-	0.85	-	-

10. W.C. Haxton and G.J. Stephenson Jr., Progr. Part. Nucl. Phys. 12(1984) 409.
11. E. Caurier et al., Nucl. Phys. A 654 (1999) 973.
12. J. Engel et al., Phys. Rev. C 37 (1988) 731.
13. A. Staudt et al., Europhys. Lett 13 (1990) 31.
14. A. Faessler and F. Simkovic, J. Phys. G 24 (1998) 2139.
15. G. Pantis et al., Phys. Rev. C 53 (1996) 695.

[Cremonesi, NPB PS 118 (2003) 287]

traditional estimated range for $|\mathcal{M}_{0\nu}|^2$: about one order of magnitude

$\Downarrow$

about factor of 3 range for $|\mathcal{M}_{0\nu}|$ and $|m_{\beta\beta}|$

^{76}Ge

$\|\mathcal{M}_{0\nu}\|$	$\|m_{\beta\beta}\|$ [eV]	Method	Reference
1.35	1.24	QRPA with np pairing	Pantis et al. (1999)
1.56	1.07	Large-scale shell model	Caurier et al. (1996)
1.52 – 1.68	1.0 – 1.1	QRPA	Bobyk et al. (2001)
1.87	0.89	Full RQRPA	Simkovic et al. (1997)
2.15	0.78	RQRPA with forbidden	Rodin et al. (2003)
1.68 – 3.36	0.50 – 1.00	RQRPA	Stoica and Klapdor-K. (2001)
2.40	0.70	QRPA with forbidden	Rodin et al. (2003)
1.54 – 3.99	0.42 – 1.09	QRPA	Stoica and Klapdor-K. (2001)
2.07 – 3.30	0.51 – 0.81	Full RQRPA	Stoica and Klapdor-K. (2001)
2.65 – 2.84	0.59 – 0.63	RQRPA	Bobyk et al. (2001)
2.81	0.60	RQRPA with forbidden	Simkovic et al. (1999)
2.98	0.56	QRPA	Suhonen et al. (1992)
3.06	0.55	QRPA	Pantis et al. (1996)
3.10	0.54	Number-projected QRPA	Suhonen et al. (1992)
2.88 – 3.43	0.49 – 0.58	Second QRPA	Stoica and Klapdor-K. (2001)
3.25	0.52	QRPA	Barbero et al. (1999)
3.25	0.51	RQRPA	Faessler and Simkovic (1998)
3.27	0.51	QRPA	Civitarese and Suhonen (2003)
3.63	0.46	RQRPA	Simkovic et al. (1999)
3.78	0.44	QRPA	Muto et al. (1989), Staudt et al. (1990)
3.88	0.43	QRPA	Tomoda (1991)
4.12	0.41	QRPA	Aunola and Suhonen (1998)

[Elliott, Engel, J. Phys. G 30 (2004) R183]

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

$$G_{0\nu} = 0.30 \times 10^{25}\ \mathrm{y\,eV^{-2}}$$

unrealistic calculations excluded

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12$$

about factor of 3 range

for $|\mathcal{M}_{0\nu}|$ and $|m_{\beta\beta}|$

other estimates

[Bahcall et al, PRD 70 (2004) 033012]

[Fogli et al, hep-ph/0408045]

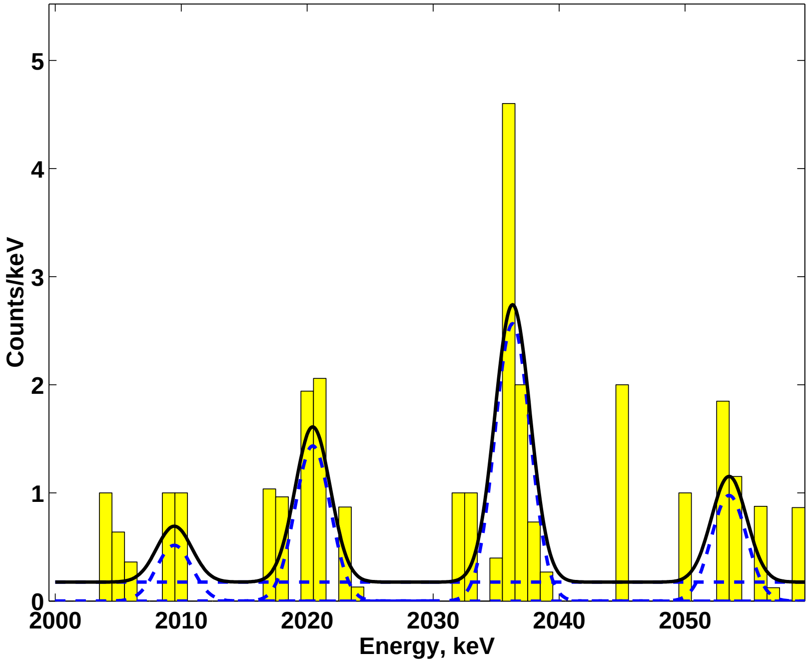
[Deppisch, Paes, Suhonen, hep-ph/0409306]

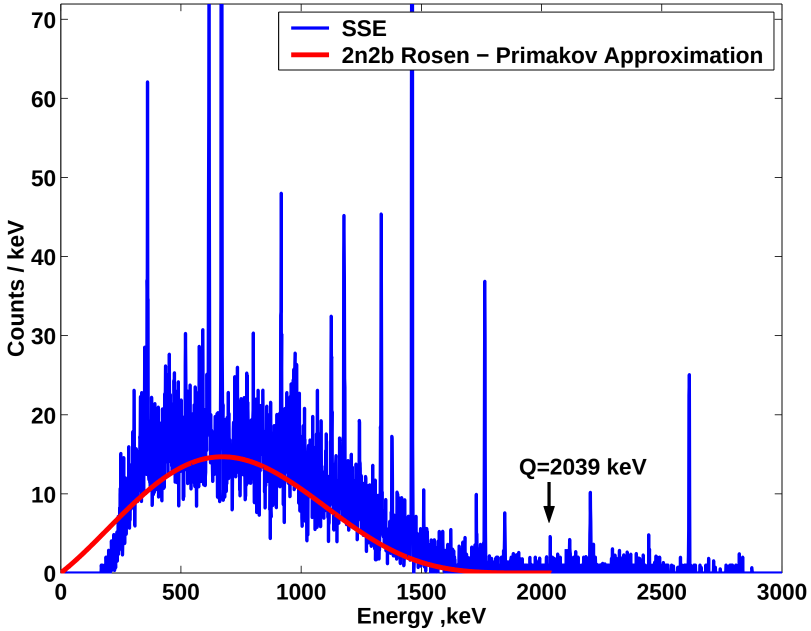
Indication of $\beta\beta_{0\nu}$ Decay at Quasi Degenerate Mass Scale

[Klapdor-Kleingrothaus, Dietz, Harney, Krivosheina, Mod. Phys. Lett. A16 (2001) 2409] [Klapdor-Kleingrothaus, Dietz, Krivosheina, Found. Phys. 32 (2002) 1181]

[Klapdor-Kleingrothaus, Dietz, Chkvorez, Krivosheina, NIMA 522 (2004) 371] [Klapdor-Kleingrothaus, Krivosheina, Dietz, Chkvorets, PLB 586 (2004) 198]

$T^{0\nu\,\mathrm{bf}}_{1/2} = 1.19 \times 10^{25}\,\mathrm{y}$ $\quad T^{0\nu}_{1/2} = (0.69 - 4.18) \times 10^{25}\,\mathrm{y} \quad (3\sigma)$ $\quad 4.2\sigma$ evidence





pulse-shape selected spectrum $\quad 3.8\sigma$ evidence $\quad$ [PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Longrightarrow \quad 0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$$

if confirmed very exciting (Majorana ν and large mass scale)

Best limits for $\beta\beta_{0\nu}$ Decay

Heidelberg-Moscow $^{76}\mathrm{Ge}$ [EPJA 12 (2001) 147]

$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\mathrm{y} \quad \text{(90\% C.L.)}} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\mathrm{eV} \quad \text{(90\% C.L.)}}$

IGEX $^{76}\mathrm{Ge}$ [PRD 65 (2002) 092007]

$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\mathrm{y} \quad \text{(90\% C.L.)}} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.35 - 1.1\,\mathrm{eV} \quad \text{(90\% C.L.)}}$

FUTURE EXPERIMENTS

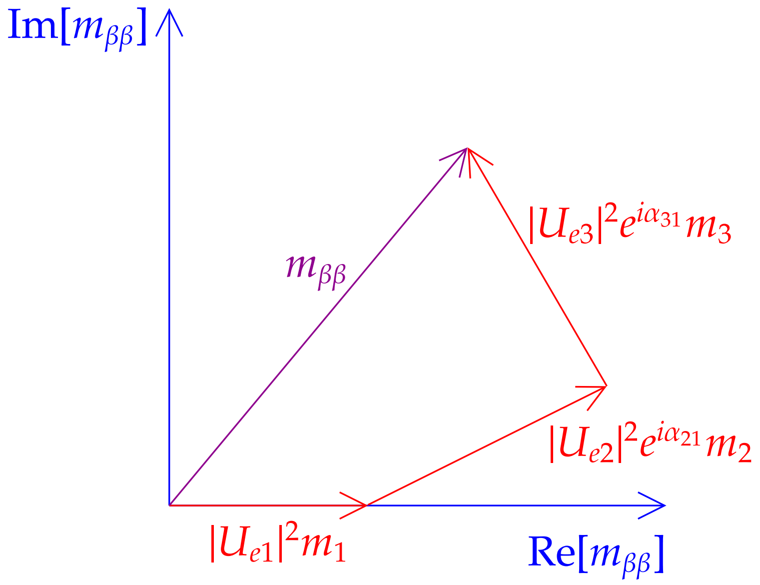
NEMO3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES $|m_{\beta\beta}| \sim$ few $10^{-1}\,\mathrm{eV}$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA $|m_{\beta\beta}| \sim$ few $10^{-2}\,\mathrm{eV}$

see [Zdesenko, RMP 74 (2002) 663], [Elliott,Vogel, Ann. Rev. Nucl. Part. Sci. 52 (2002) 115], [Elliott, Engel, J. Phys. G 30 (2004) R183]

Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$



complex U_{ek} $\Rightarrow$ possible cancellations

$$\begin{aligned} m_{\beta\beta} &= |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{2i\lambda_{21}} \, m_2 + |U_{e3}|^2 \, e^{2i(\lambda_{31}-\delta)} \, m_3 \\ &= |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3 \end{aligned}$$

conserved CP $\Longrightarrow$ $\delta = 0 \quad \lambda_{kj} = \dfrac{\alpha_{kj}}{2} = 0 \, , \, \dfrac{\pi}{2}$ $\Longrightarrow$ $e^{2i\lambda_{kj}} = e^{i\alpha_{kj}} = \pm 1$

$\delta \neq 0$ $\Longrightarrow$ calling "Majorana phases" λ_{kj} or $\alpha_{kj}/2$ is a convention

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ \ldots & \ldots & s_{23}c_{13} \\ \ldots & \ldots & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ \ldots & \ldots & s_{23}c_{13}e^{i\delta} \\ \ldots & \ldots & c_{23}c_{13}e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Normal Hierarchy $m_1 \ll m_2 \ll m_3$

$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2 \, m_k\right| \simeq \left|U_{e2}^2 \, m_2 + U_{e3}^2 \, m_3\right| = \left||U_{e2}|^2 \, m_2 + |U_{e3}|^2 \, e^{i(\alpha_{31}-\alpha_{21})} \, m_3\right| \leq |U_{e2}|^2 \, m_2 + |U_{e3}|^2 \, m_3$$

$$m_2 \simeq \sqrt{\Delta m^2_{21}} \simeq \sqrt{\Delta m^2_{\mathrm{SUN}}} \qquad m_3 \simeq \sqrt{\Delta m^2_{31}} \simeq \sqrt{\Delta m^2_{\mathrm{ATM}}}$$

$$1.8 \times 10^{-3} \lesssim |U_{e2}|^2 \, m_2 \lesssim 3.6 \times 10^{-3} \qquad |U_{e3}|^2 \, m_3 \lesssim 2.9 \times 10^{-3}$$

ν_2 contribution $|U_{e2}|^2 \, m_2$ may be dominant! (no cancellation $\Longrightarrow$ lower limit for $|m_{\beta\beta}|$)

[Giunti, PRD 61 (2000) 036002]

overlap of allowed ranges for $|U_{e2}|^2 \, m_2$ and $|U_{e3}|^2 \, m_3$ $\Longrightarrow$ strong cancellation is possible

in any case: $|m_{\beta\beta}| \lesssim 6 \times 10^{-3} \, \mathrm{eV}$

[Vissani, JHEP 06 (1999) 022]

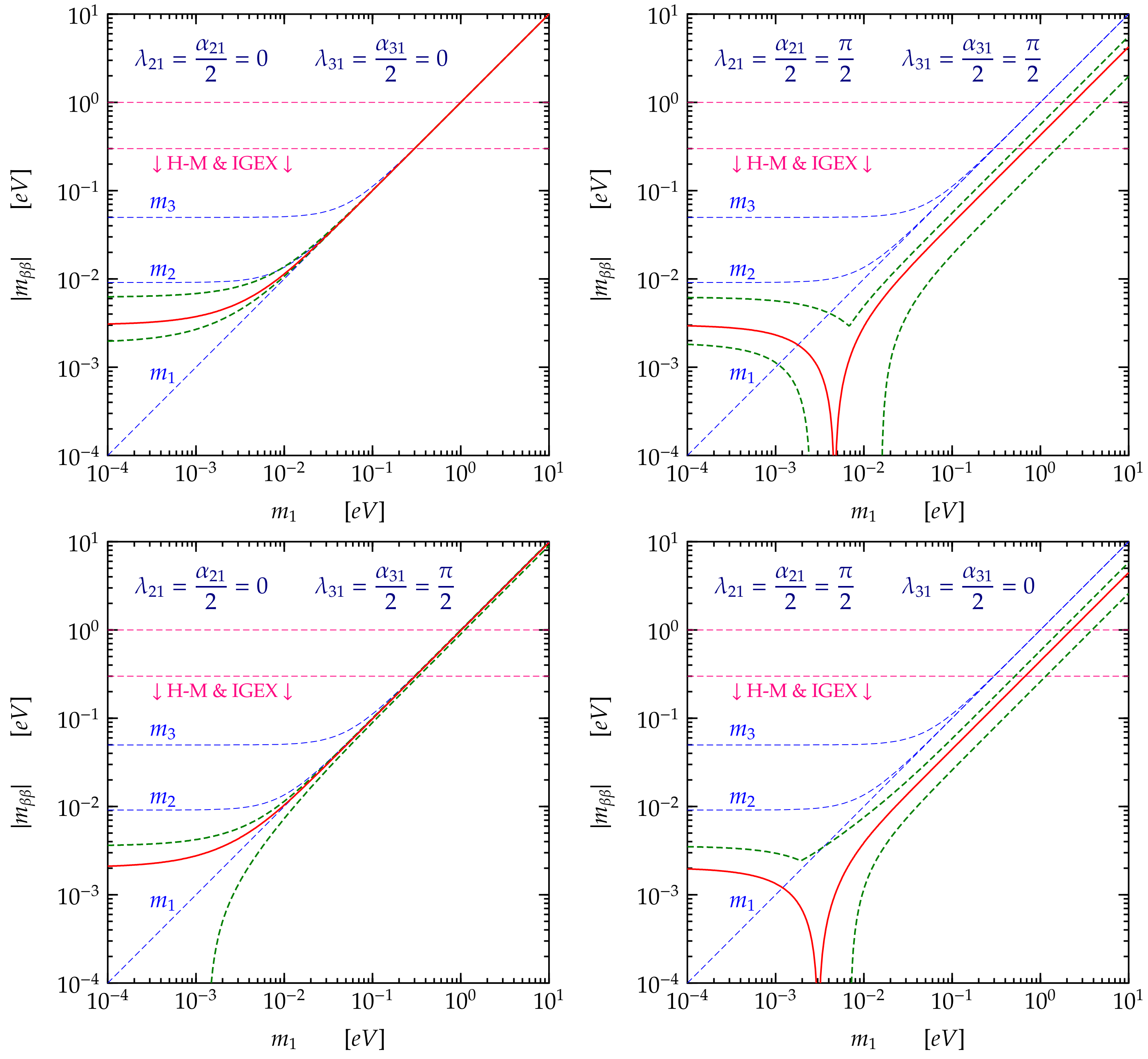
[Bilenky et al, PLB 465 (1999) 193]

the difference $\alpha_{31} - \alpha_{21}$ of the two Majorana phases is potentially measurable

$$|m_{\beta\beta}|^2 \simeq |U_{e2}|^4 \, \Delta m^2_{\mathrm{SUN}} + |U_{e3}|^4 \, \Delta m^2_{\mathrm{ATM}} + 2 \, |U_{e2}|^2 \, |U_{e3}|^2 \, \sqrt{\Delta m^2_{\mathrm{SUN}}} \, \sqrt{\Delta m^2_{\mathrm{ATM}}} \, \cos(\alpha_{31} - \alpha_{21})$$

[Bilenky, Pascoli, Petcov, PRD 64 (2001) 053010]

CP Conservation: Normal Scheme



$$m_{\beta\beta} \simeq |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} \sqrt{m_1^2 + \Delta m^2_{\rm SUN}} + |U_{e3}|^2 e^{i\alpha_{31}} \sqrt{m_1^2 + \Delta m^2_{\rm ATM}}$$

$$\Delta m^2_{\rm SUN} \simeq (7.4 - 9.3) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm ATM} \simeq (1.8 - 3.2) \times 10^{-3}\,{\rm eV}^2$$

$$|U_{e1}|^2 \simeq 0.59 - 0.77$$

$$|U_{e2}|^2 \simeq 0.21 - 0.37$$

$$|U_{e3}|^2 \simeq 0.00 - 0.05$$

Inverted Hierarchy $m_1 \simeq m_2 \gg m_3$

$$
\begin{aligned}
|m_{\beta\beta}| &= \left| \sum_k U_{ek}^2 \, m_k \right| \simeq \left| U_{e1}^2 \, m_1 + U_{e2}^2 \, m_2 \right| \\
&\simeq \left| U_{e1}^2 + U_{e2}^2 \right| \sqrt{\Delta m^2_{\mathrm{ATM}}} \\
&= \left| |U_{e1}|^2 + |U_{e2}|^2 \, e^{i\alpha_{21}} \right| \sqrt{\Delta m^2_{\mathrm{ATM}}}
\end{aligned}
$$

$$
0.59 \lesssim |U_{e1}|^2 \lesssim 0.77 \qquad 0.21 \lesssim |U_{e2}|^2 \lesssim 0.37
$$

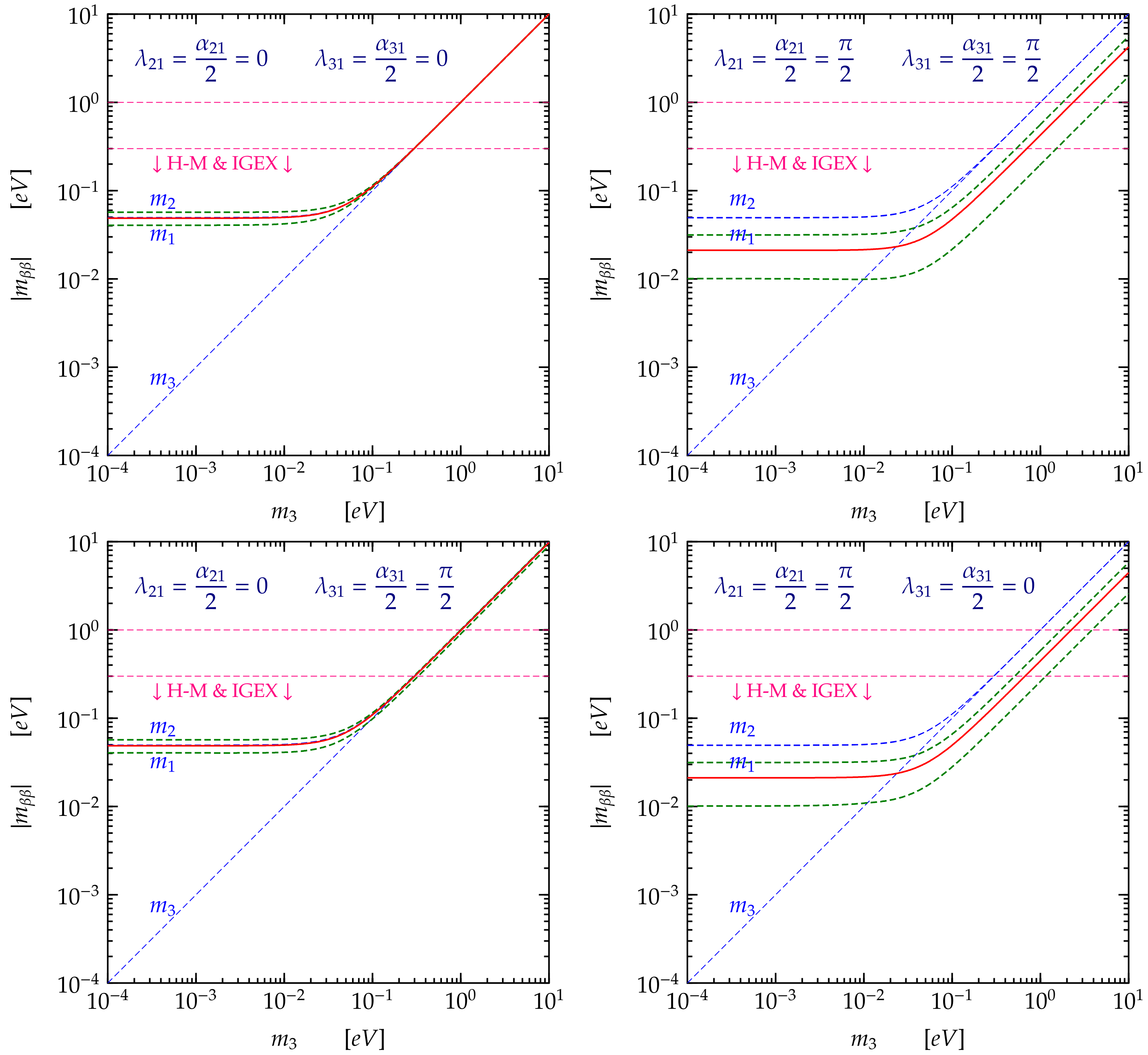
no overlap of allowed ranges for $|U_{e1}|^2$ and $|U_{e2}|^2$ $\Longrightarrow$ complete cancellation is not possible!

$$
|m_{\beta\beta}| \sim \sqrt{\Delta m^2_{\mathrm{ATM}}}
$$

the Majorana phase α_{21} is potentially measurable [Bilenky et al, PRD 54 (1996) 4432]

$$
\frac{|m_{\beta\beta}|^2}{\Delta m^2_{\mathrm{ATM}}} \simeq |U_{e1}|^4 + |U_{e2}|^4 + 2 \, |U_{e1}|^2 \, |U_{e2}|^2 \, \cos \alpha_{21}
$$

CP Conservation: Inverted Scheme



$$m_{\beta\beta} \simeq \left(|U_{e1}|^2 + |U_{e2}|^2 e^{i\alpha_{21}} \right) \times \sqrt{m_3^2 + \Delta m^2_{\rm ATM}} + |U_{e3}|^2 e^{i\alpha_{31}} \, m_3$$

$$\Delta m^2_{\rm ATM} \simeq (1.8 - 3.2) \times 10^{-3} \, {\rm eV}^2$$

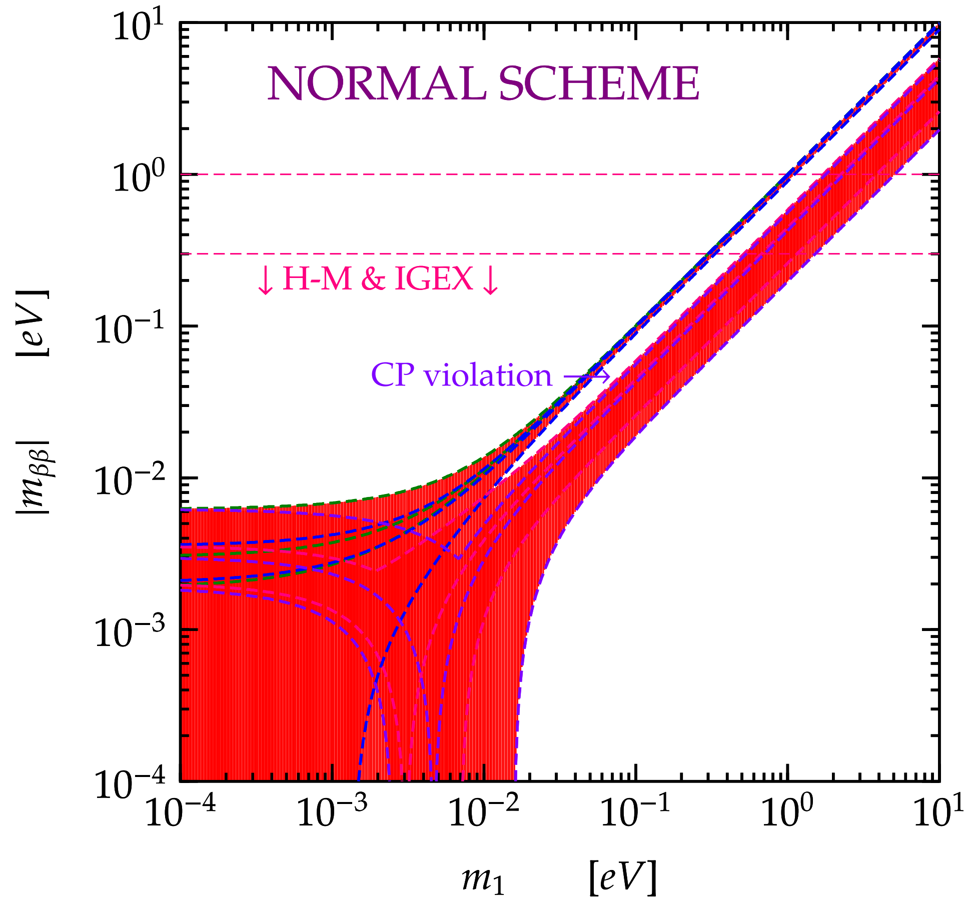
$$|U_{e1}|^2 \simeq 0.59 - 0.77$$

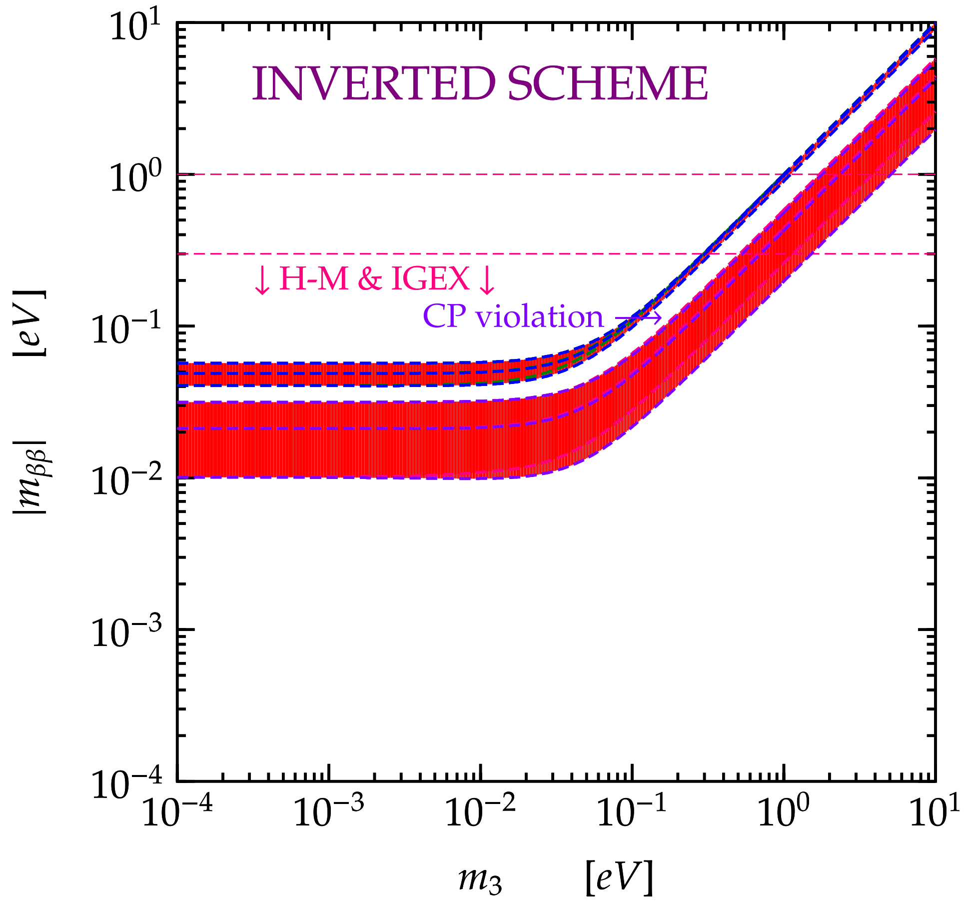
$$|U_{e2}|^2 \simeq 0.21 - 0.37$$

$$|U_{e3}|^2 \simeq 0.00 - 0.05$$

General Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

[Vissani, JHEP 06 (1999) 022] [Pascoli et al, PLB 549 (2002) 177] [Czakon et al, PRD65 (2002) 053008] [Elliott, Vogel, ARNPS 52 (2002) 115] [Joaquim, PRD68 (2003) 033019] [Giunti, Laveder, hep-ph/0310238] [Feruglio et al, NPB659 (2003) 359] [Pascoli, Petcov, PLB 580 (2004) 280] [Bilenky et al, PRD70 (2004) 033003] [Bahcall et al, PRD 70 (2004) 033012] [Petcov, NJP 6 (2004) 109]



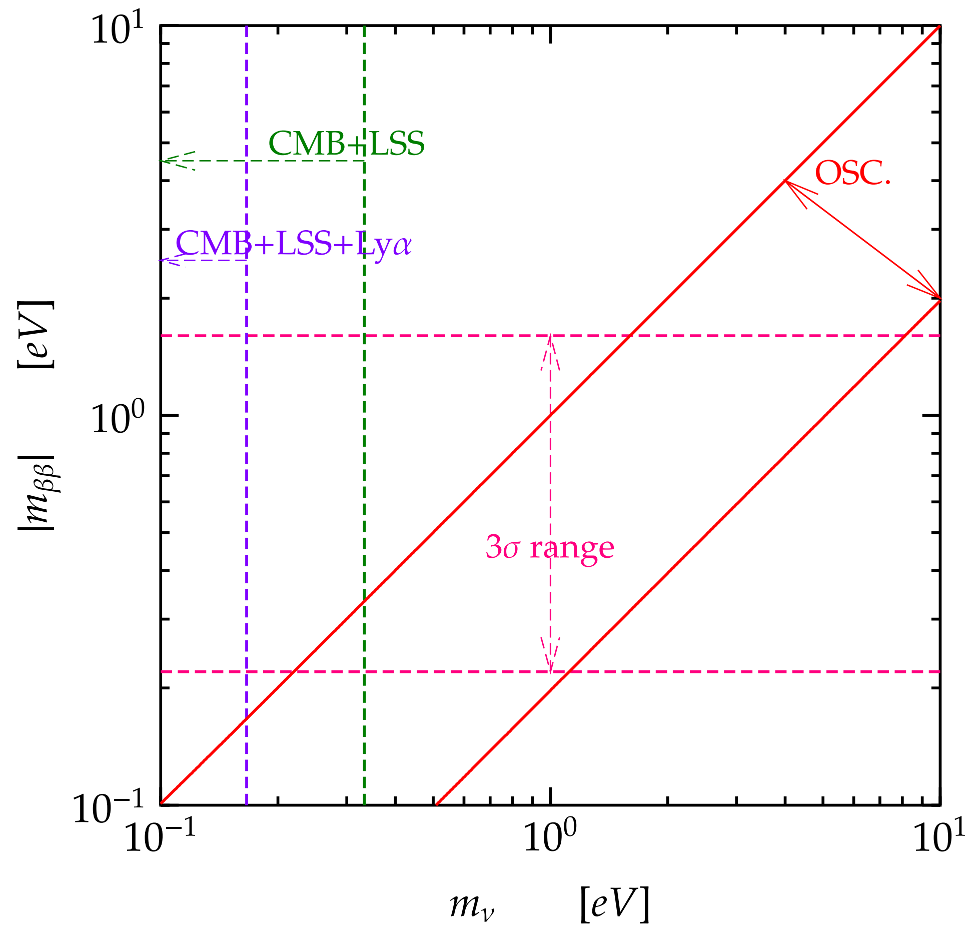


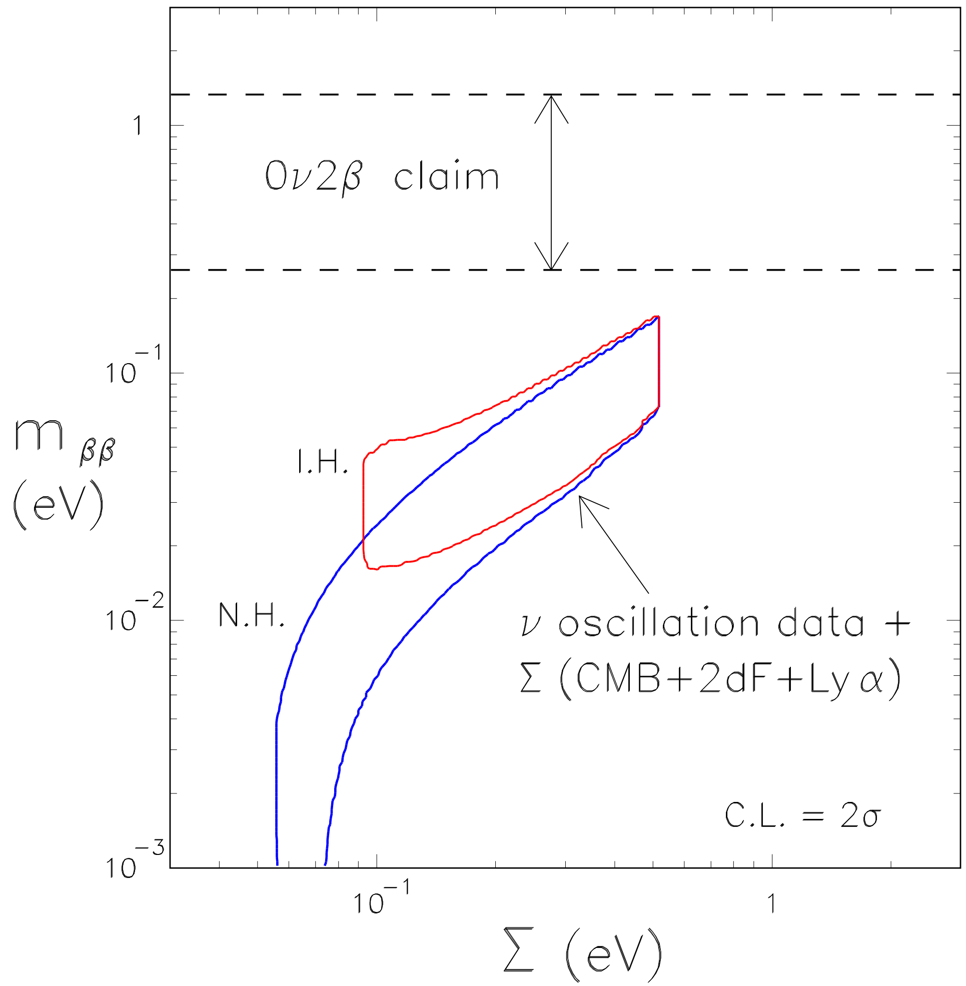
Quasi Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \Longrightarrow |m_{\beta\beta}| \simeq \left| |U_{e1}|^2 + |U_{e2}|^2 e^{i\alpha_{21}} + |U_{e3}|^2 e^{i\alpha_{31}} \right| m_\nu$

FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay at Quasi Degenerate Mass Scale

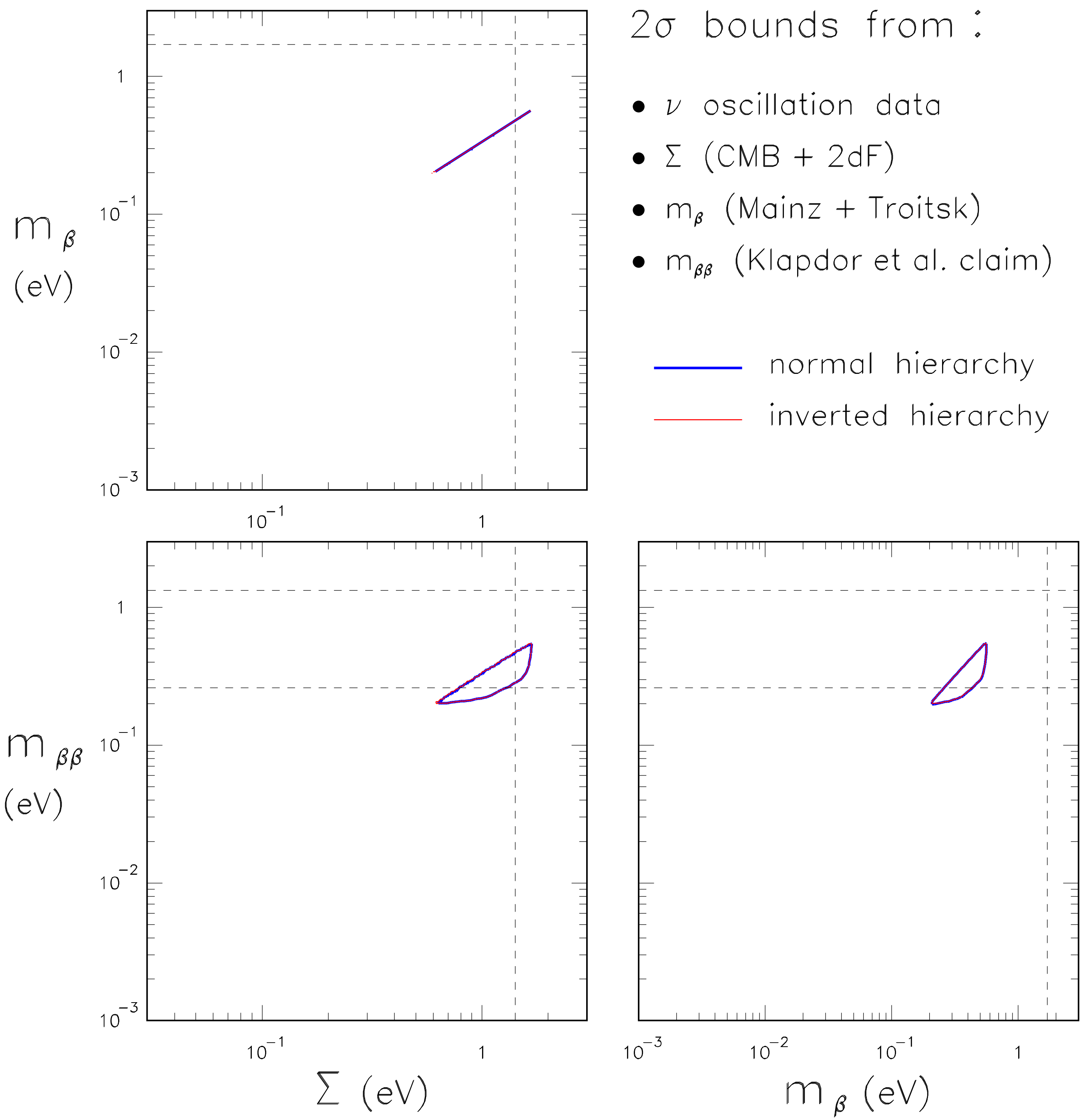
$$0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV} \qquad \text{(}3\sigma\text{ range)}$$





[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

tension among oscillation data, CMB+LSS+Lyα and $\beta\beta_{0\nu}$ signal



[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

Summary

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\rm SUN} \simeq 8.3 \times 10^{-5}\,{\rm eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\rm ATM} \simeq 2.4 \times 10^{-3}\,{\rm eV}^2$ (atmospheric ν, K2K)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,{\rm eV}$

FUTURE

Theory: Why $m_\nu \ll m_e$? Why only $|U_{e3}|^2 \ll 1$?

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ normal or inverted scheme and CP violation in ν osc.

Check $\beta\beta_{0\nu}$ signal at Quasi Degenerate mass scale

Improve bounds from β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay

Conclusions

Neutrino Physics is a very active and interesting field of research

next years will hopefully bring new interesting results

Open Fundamental Questions

Absolute Scale of Neutrino Masses?
Nature of Neutrinos (Dirac or Majorana)?
Are There Sterile Neutrinos?
Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ (LSND)? $\Longleftarrow$ MiniBooNE
Electromagnetic Properties of Neutrinos?

Neutrino Unbound

http://www.nu.to.infn.it

Carlo Giunti & Marco Laveder