Neutrino Oscillations

Carlo Giunti

INFN, Sezione di Torino, and
Dipartimento di Fisica Teorica, Università di Torino
giunti@to.infn.it

ISAPP 2005, Belgirate, Lago Maggiore, Italy, 9 July 2005

Introduction to Neutrino Masses, Mixing and Oscillations

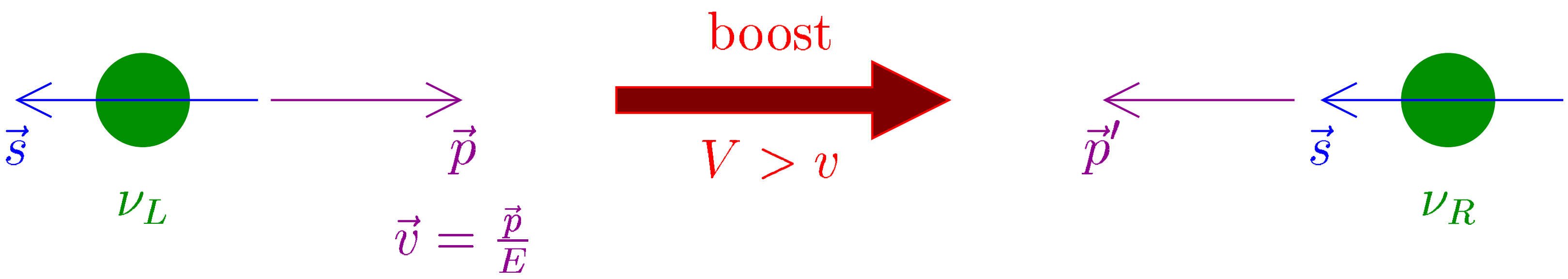
Solar $\nu_e \to \nu_\mu, \nu_\tau$ + Atmospheric $\nu_\mu \to \nu_\tau \Longrightarrow$ 3-ν Mixing

Absolute Scale of Neutrino Masses

Cosmological Bound on Neutrino Masses

Neutrinoless Double-β Decay $\Longleftrightarrow$ Majorana Mass

Neutrino Mass



Standard Model: $\nu_L, \nu_R^c \implies$ no Dirac mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R$

(no ν_R, ν_L^c)

Majorana Neutrino: $\nu \equiv \nu^c$

$\nu_R^c \equiv \nu_R \implies$ Majorana mass term $\mathcal{L}^{\mathrm{M}} \sim m^{\mathrm{M}} \overline{\nu_L} \nu_R^c$

Standard Model: Majorana mass term not allowed by $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(no Higgs triplet)

Standard Model can be extended with ν_R $(e_L, e_R;\ u_L, u_R;\ d_L, d_R;\ \ldots)$

$\nu_L + \nu_R \Rightarrow$ Dirac neutrino mass term $\mathcal{L}^{\text{D}} \sim m^{\text{D}} \overline{\nu_L} \nu_R \Rightarrow m^{\text{D}} \lesssim 100\,\text{GeV}$

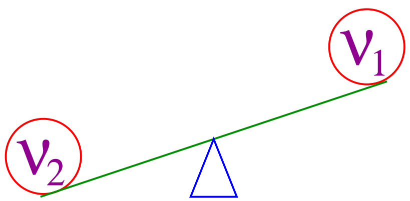
surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}^{\text{M}}_R \sim m^{\text{M}}_R \overline{\nu^c_L} \nu_R$

total neutrino mass term $\mathcal{L}^{\text{D+M}} \sim \begin{pmatrix} \overline{\nu_L} & \overline{\nu^c_L} \end{pmatrix} \begin{pmatrix} 0 & m^{\text{D}} \\ m^{\text{D}} & m^{\text{M}}_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix}$

m^{M}_R can be arbitrarily large (not protected by SM symmetries)

$m^{\text{M}}_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m^{\text{M}}_R \gg m^{\text{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\text{D}} \\ m^{\text{D}} & m^{\text{M}}_R \end{pmatrix} \Rightarrow m_1 \simeq \dfrac{(m^{\text{D}})^2}{m^{\text{M}}_R}, \quad m_2 \simeq m^{\text{M}}_R$



see-saw mechanism

natural explanation of smallness of neutrino masses

massive neutrinos are Majorana!

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Dirac mass term $m^{\mathrm{D}}\overline{\nu_L}\nu_R \Rightarrow \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^{\mathrm{M}}\overline{\nu_L}\nu_R^c \Rightarrow \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{M}}_{ee} & m^{\mathrm{M}}_{e\mu} & m^{\mathrm{M}}_{e\tau} \\ m^{\mathrm{M}}_{\mu e} & m^{\mathrm{M}}_{\mu\mu} & m^{\mathrm{M}}_{\mu\tau} \\ m^{\mathrm{M}}_{\tau e} & m^{\mathrm{M}}_{\tau\mu} & m^{\mathrm{M}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu^c_{eR} \\ \nu^c_{\mu R} \\ \nu^c_{\tau R} \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu^c_{\alpha R}) = -L(\nu_{\beta L}) \Rightarrow |\Delta L| = 2$

Diagonalization of Mass Matrix $\Rightarrow$ Mixing

Dirac Mass Term: $\mathcal{L}^{\text{D}} \sim \left(\overline{\nu_{eL}} \quad \overline{\nu_{\mu L}} \quad \overline{\nu_{\tau L}}\right) \begin{pmatrix} m_{ee}^{\text{D}} & m_{e\mu}^{\text{D}} & m_{e\tau}^{\text{D}} \\ m_{\mu e}^{\text{D}} & m_{\mu\mu}^{\text{D}} & m_{\mu\tau}^{\text{D}} \\ m_{\tau e}^{\text{D}} & m_{\tau\mu}^{\text{D}} & m_{\tau\tau}^{\text{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

$$\mathcal{L}^{\text{D}} \sim \sum_{\alpha,\beta} \overline{\nu_{\alpha L}} m_{\alpha\beta}^{\text{D}} \nu_{\beta R} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL} \qquad \nu_{\beta R} = \sum_{j=1}^{3} V_{\beta j} \nu_{jR} \qquad U^{\dagger} m^{\text{D}} V = \text{diag}(m_1, m_2, m_3)$$

$$\mathcal{L}^{\text{D}} \sim \sum_{k=1}^{3} m_k \overline{\nu_{kL}} \nu_{kR}$$

weak charged current: neutrino production and detection

$$j_{\rho}^{\text{CC}} \sim \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha L}} \gamma_{\rho} \nu_{\alpha L} = \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \gamma_{\rho} U_{\alpha k} \nu_{kL}$$

U = unitary 3×3 mixing matrix

Neutrino Oscillations

[Pontecorvo, SPJETP 6 (1957) 429] [Pontecorvo, SPJETP 7 (1958) 172] [Gribov, Pontecorvo, PLB 28 (1969) 49]
[Eliezer,Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]
[Bilenky, Pontecorvo, NCimL 17 (1976) 56] [Bilenky, Pontecorvo, PRep 41 (1978) 225]

Neutrino Mixing:

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

$$\alpha = e, \mu, \tau$$

$$\nu_k \to m_k$$

$$|\nu_k(x,t)\rangle = e^{-iE_k t + ip_k x} |\nu_k\rangle \quad \Longrightarrow \quad |\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k}\, e^{-iE_k t + ip_k x}\, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U^*_{\beta k} |\nu_\beta\rangle$$

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k} e^{-iE_k t + ip_k x} U^*_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(x,t)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha\to\nu_\beta}(x,t) = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(x,t)\right|^2 = \left|\sum_k U_{\alpha k}\, e^{-iE_k t + i p_k x}\, U_{\beta k}^*\right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

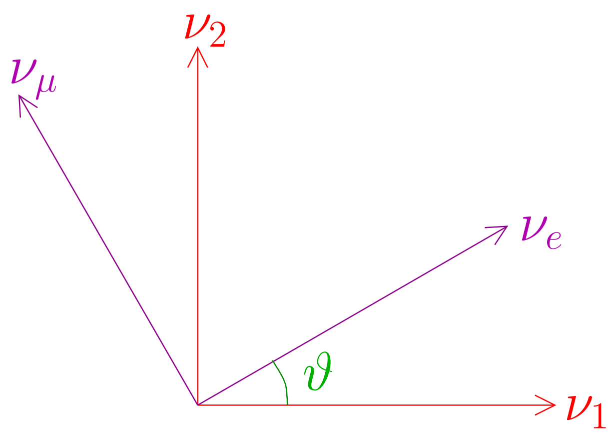
$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}\, e^{-i m_k^2 L/2E}\, U_{\beta k}^*\right|^2 \\ &= \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Two-Neutrino Mixing

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$|\nu_e\rangle = \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

Types of Experiments

Two-Neutrino Mixing

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 1\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}$, $E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}$, $E \gtrsim 1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}$, $E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}$, $E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}$, $E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}$, $E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE

Super-Kamiokande, GNO, SNO

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1$, $10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

MSW effect (resonant transitions in matter)

a flavor neutrino ν_α with momentum p is described by

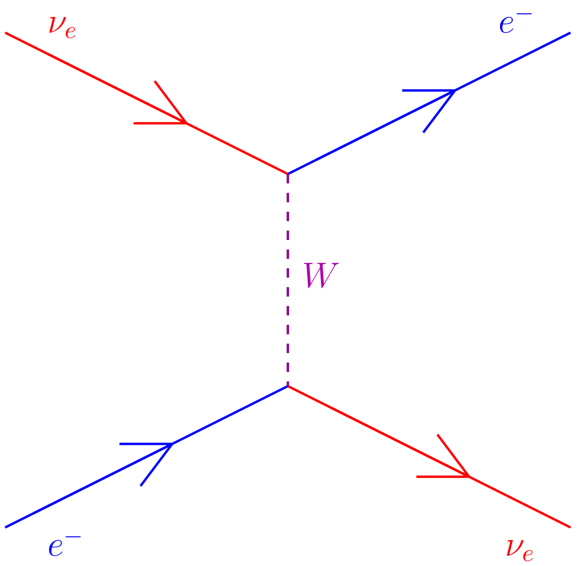
$$|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$$

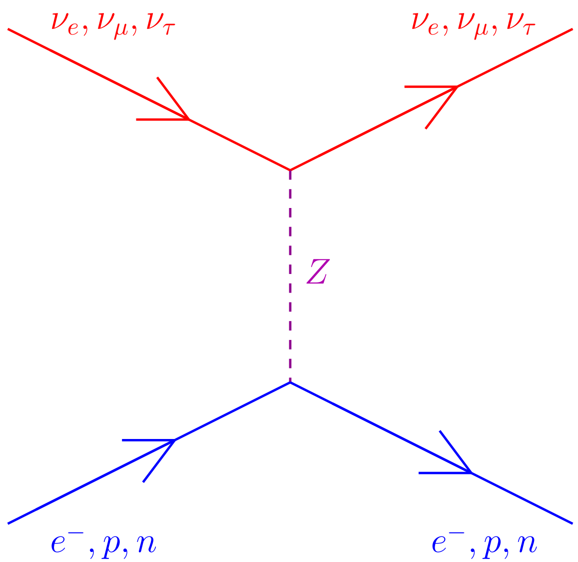
$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

in matter $\qquad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$

$V_\alpha =$ effective potential due to coherent interactions with medium
forward elastic CC and NC scattering

Effective Potential in Matter





$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}}\,, \quad V_\mu = V_\tau = V_{\text{NC}} \quad \text{(common phase)} \quad \Longrightarrow \quad V_e - V_\mu = V_{\text{CC}}$$

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}}$ $\qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Flavor Transition Amplitudes

$$i \frac{d}{dx} \nu = \frac{1}{2E} \left(U \mathbb{M}^2 U^\dagger + \mathbb{A} \right) \nu$$

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} A_{CC} &= 2EV_{CC} \\ &= 2\sqrt{2} E G_F N_e \end{aligned}$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\text{VAC}} = U \mathbb{M}^2 U^\dagger \xrightarrow{\text{matter}} U \mathbb{M}^2 U^\dagger + 2\,E\,\mathbb{V} = \mathbb{M}^2_{\text{MAT}}$$

effective mass-squared matrix in matter

$\uparrow$ potential due to coherent forward elastic scattering

simplest case: $\nu_e \to \nu_\mu$ with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$ $(2\nu$ mixing$)$

$$U \mathbb{M}^2 U^\dagger = \frac{1}{2} \Sigma m^2 + \frac{1}{2} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

$\uparrow$ irrelevant common phase (of $\frac{1}{2}\Sigma m^2$)

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2 \cos2\vartheta + 2A_{CC} & \Delta m^2 \sin2\vartheta \\ \Delta m^2 \sin2\vartheta & \Delta m^2 \cos2\vartheta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

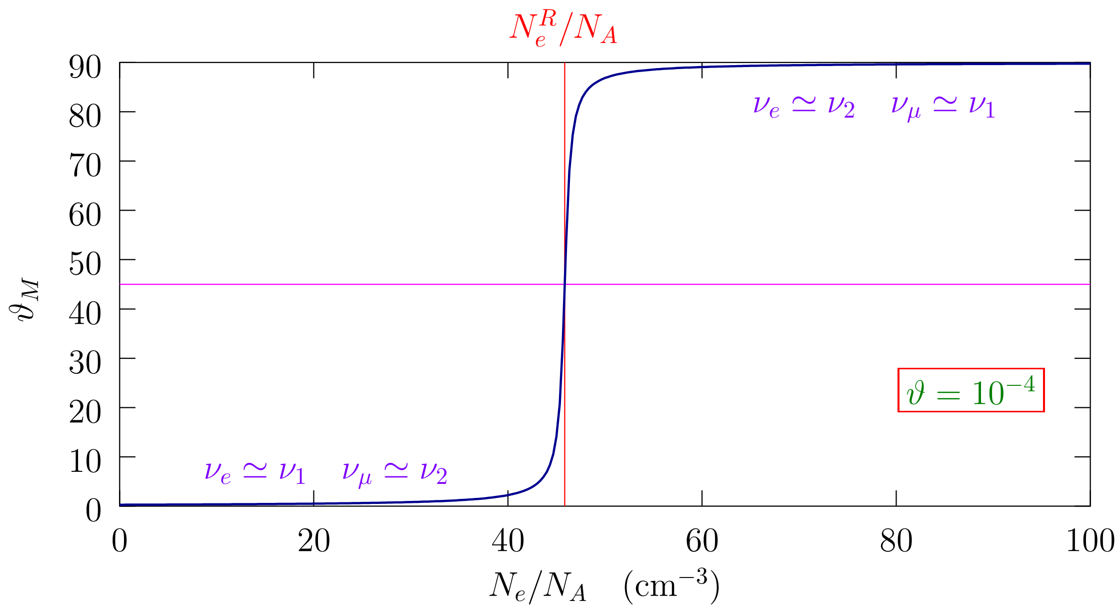
initial ν_e $\Longrightarrow$ $\begin{pmatrix}\nu_e(0)\\ \nu_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

$$P_{\nu_e\to\nu_\mu}(x) = |\nu_\mu(x)|^2$$
$$P_{\nu_e\to\nu_e}(x) = |\nu_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x)$$

Diagonalization $\Longrightarrow$ Effective Mixing Angle in Matter $\tan 2\vartheta_M = \dfrac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$

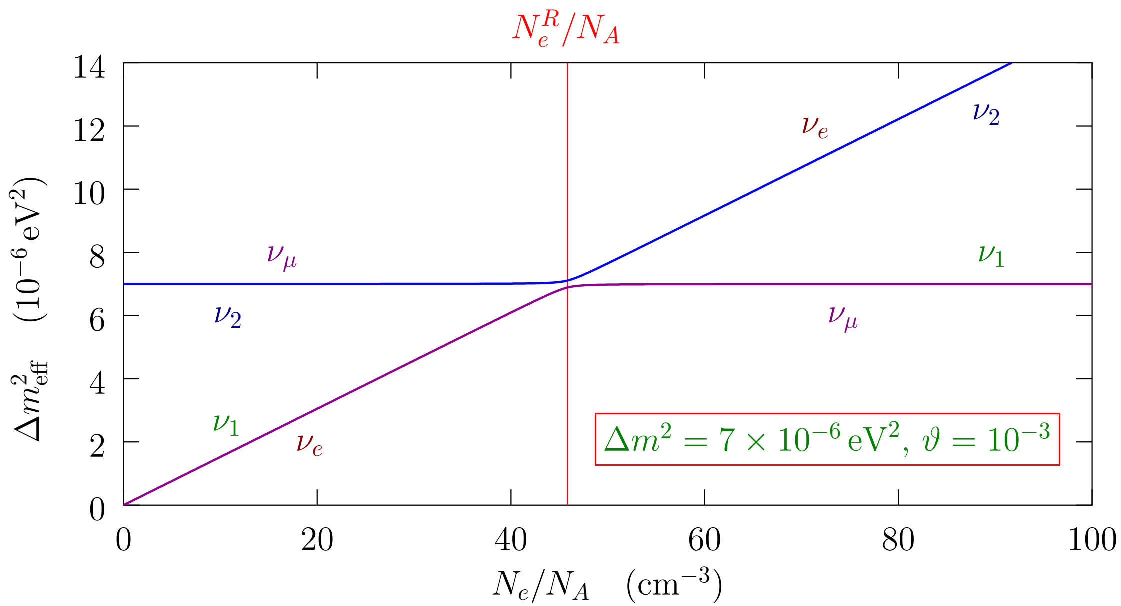
Resonance ($\vartheta_M = \pi/4$): $A_{CC}^R = \Delta m^2 \cos 2\vartheta$ $\Longrightarrow$ $N_e^R = \dfrac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_F}$

Effective Squared-Mass Difference $\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2}$



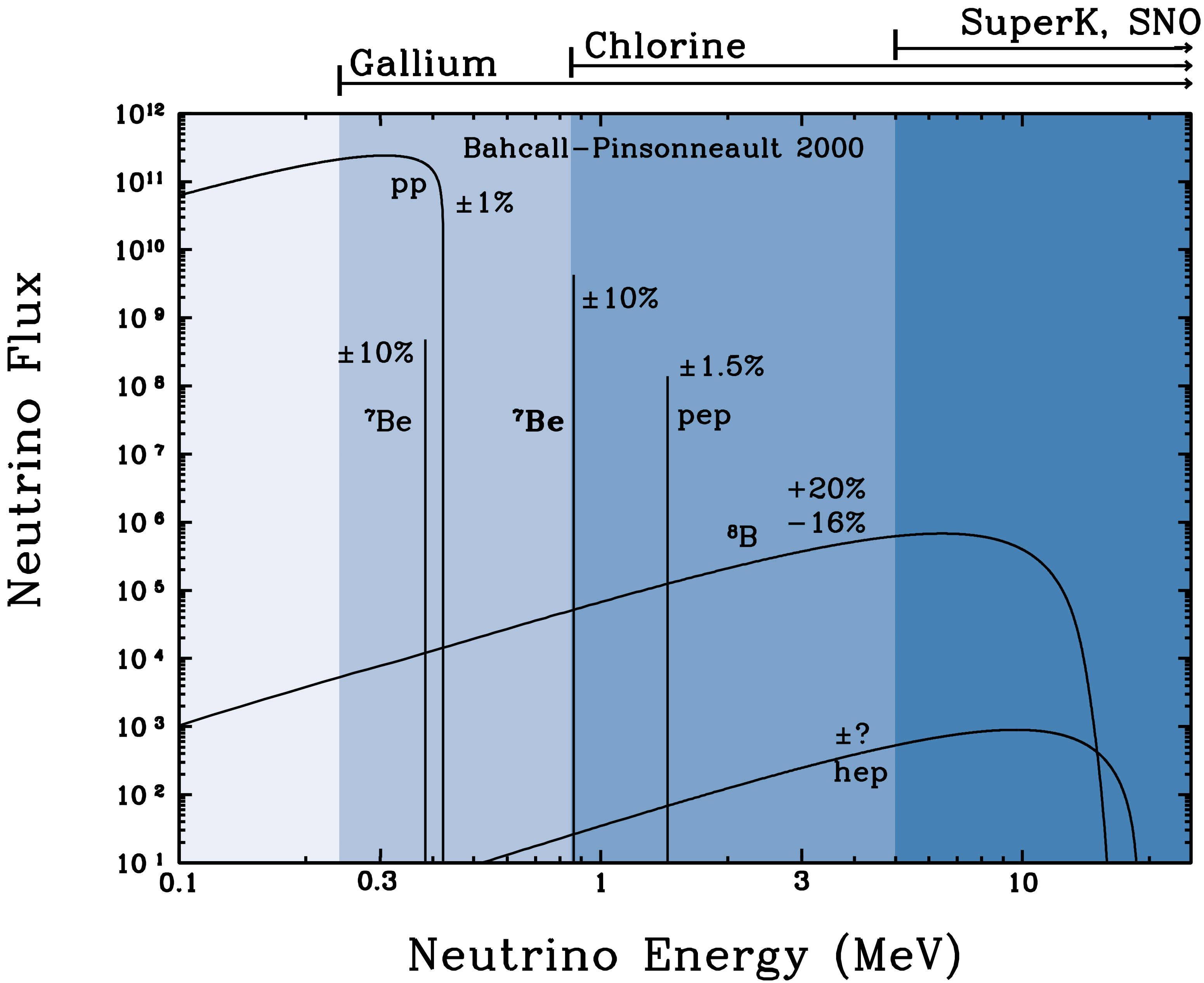
$$\nu_e = \cos\vartheta_\mathrm{M}\,\nu_1^\mathrm{M} + \sin\vartheta_\mathrm{M}\,\nu_2^\mathrm{M}$$
$$\nu_\mu = -\sin\vartheta_\mathrm{M}\,\nu_1^\mathrm{M} + \cos\vartheta_\mathrm{M}\,\nu_2^\mathrm{M}$$

$$\tan 2\vartheta_\mathrm{M} = \frac{\tan 2\vartheta}{1 - \dfrac{A_\mathrm{CC}}{\Delta m^2 \cos 2\vartheta}}$$



$$\Delta m^2_\mathrm{M} = \left[\left(\Delta m^2 \cos 2\vartheta - A_\mathrm{CC} \right)^2 + \left(\Delta m^2 \sin 2\vartheta \right)^2 \right]^{1/2}$$

Solar Neutrinos



[J.N. Bahcall, http://www.sns.ias.edu/~jnb]

SNO

$$\Phi_{\nu_e}^{\mathrm{SNO}} = 1.76 \pm 0.11 \times 10^6 \,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

$$\Phi_{\nu_\mu,\nu_\tau}^{\mathrm{SNO}} = 5.41 \pm 0.66 \times 10^6 \,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

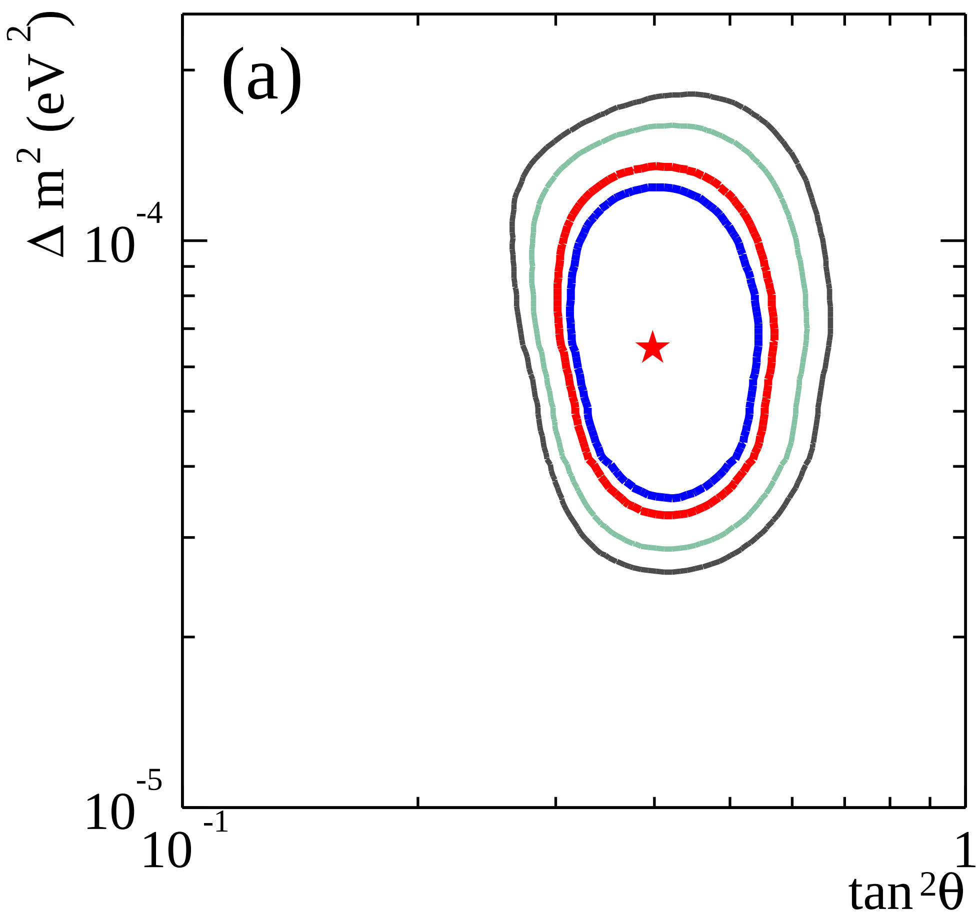
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 5 \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\vartheta \simeq 0.4$$

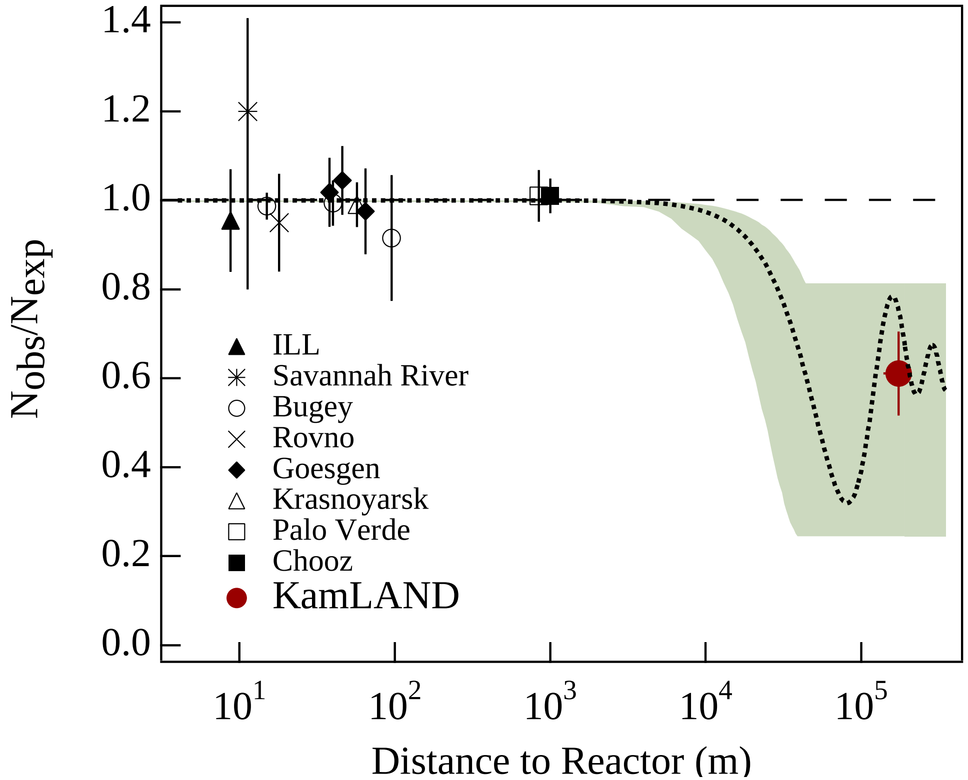


90%, 95%, 99%, 99.73% C.L.

[SNO, PRL 92 (2004) 181301, nucl-ex/0309004]

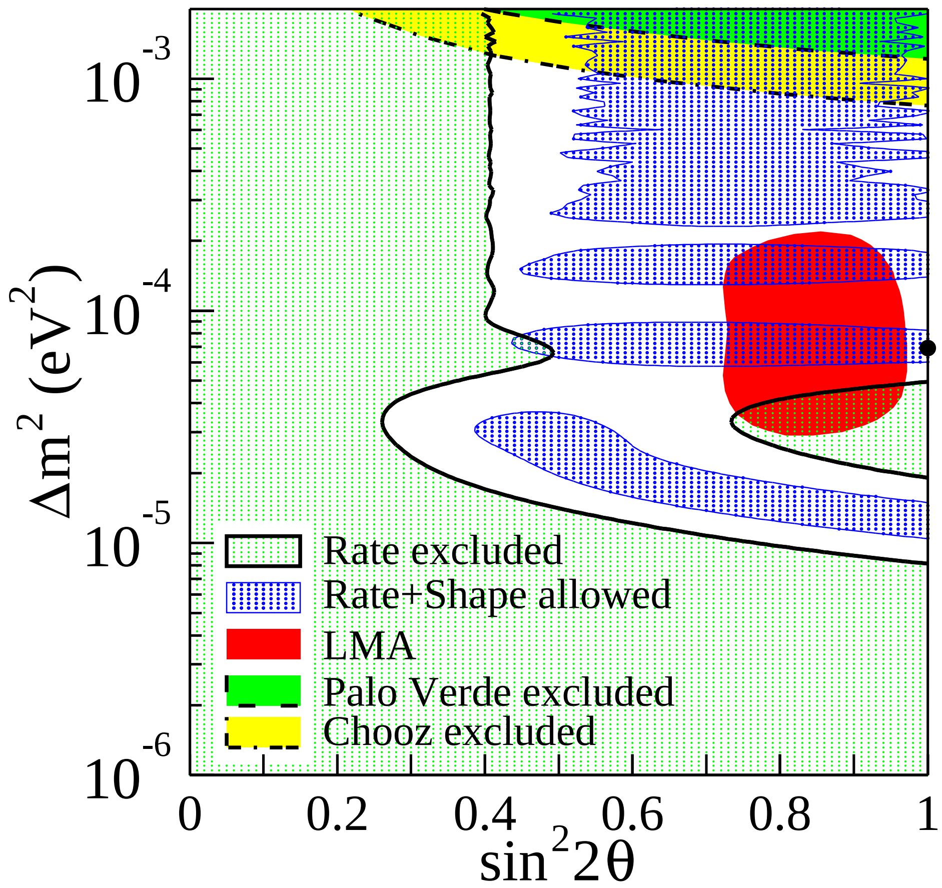
KamLAND

confirmation of LMA (December 2002)



Shade: 95% C.L. LMA

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$

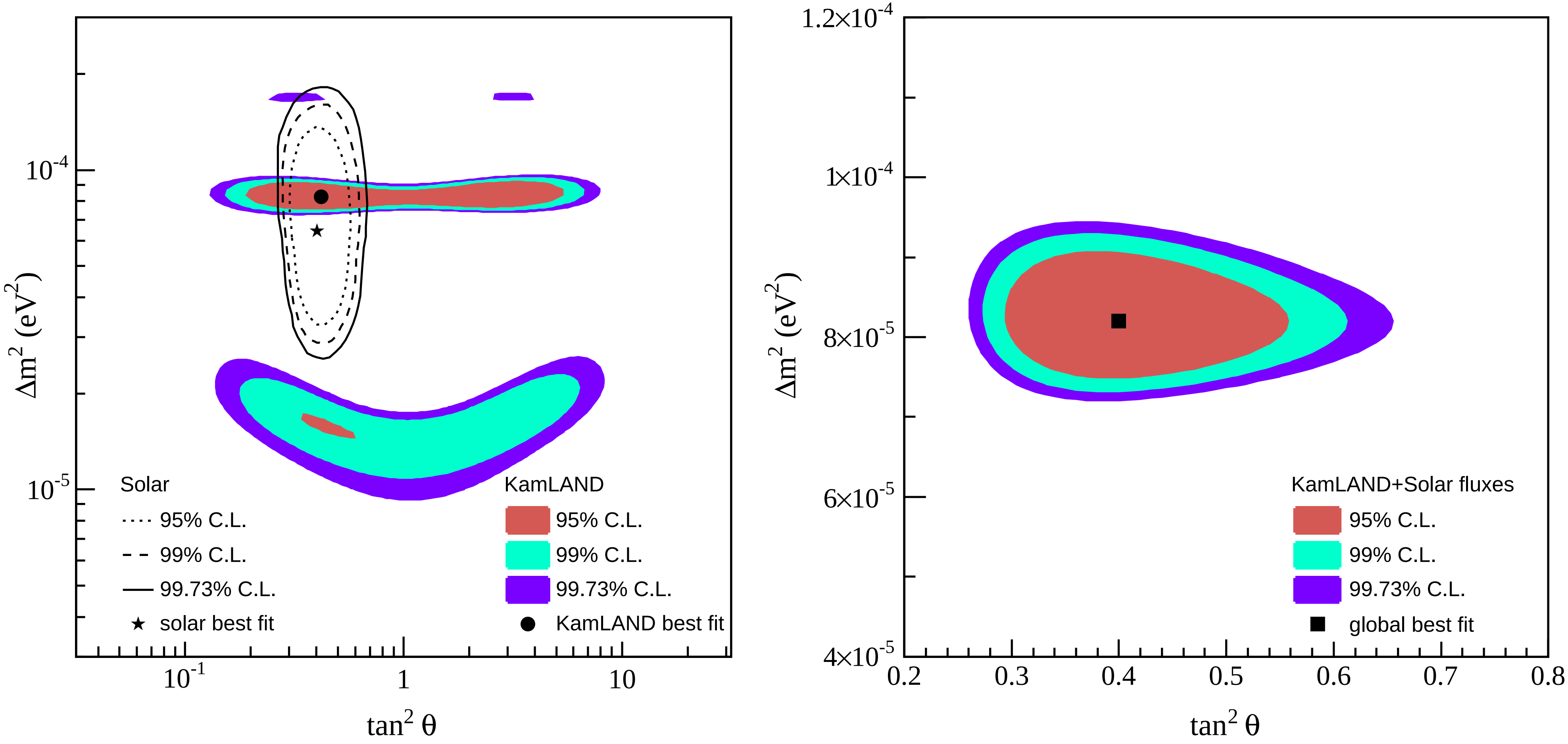


95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

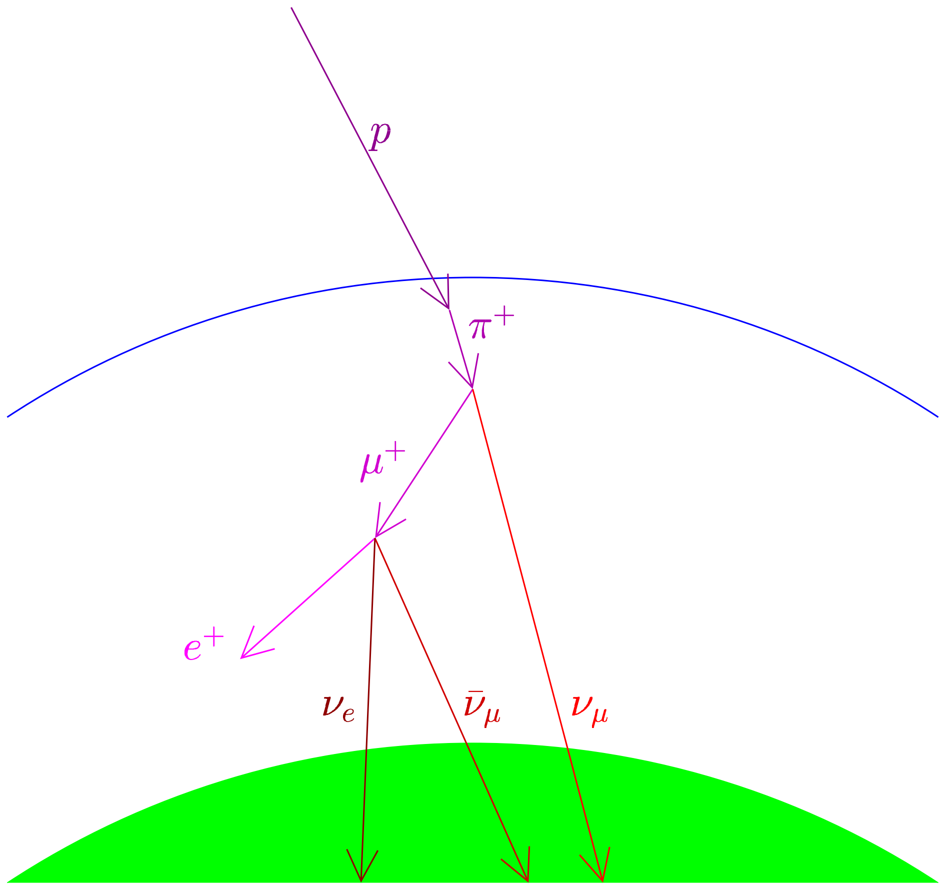
Combined Fit of Solar + Reactor Neutrino Data

[KamLAND, hep-ex/0406035]



Best Fit: $\Delta m^2 = 0.82^{+0.6}_{-0.5} \times 10^{-5}\,\text{eV}^2$ $\quad \tan^2\vartheta = 0.40^{+0.09}_{-0.07}$

Atmospheric Neutrinos



$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

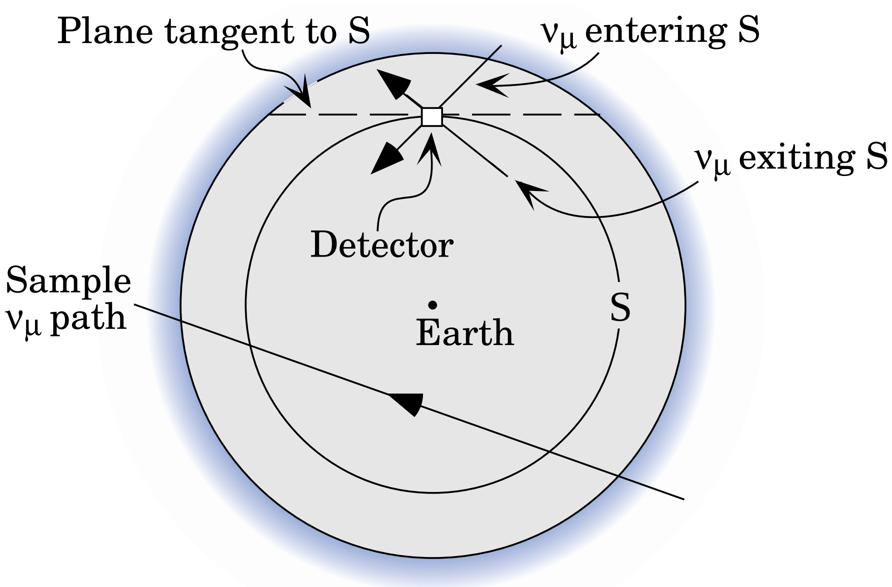
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



– any path entering the sphere S later exits

– steady state $\Rightarrow$ $\Phi^{\text{in}}(S) = \Phi^{\text{out}}(S)$

– $E_\nu \gtrsim 1\,\text{GeV}$ $\Rightarrow$ isotropic flux of cosmic rays

– homogeneity $\Rightarrow$ $\Phi^{\text{in}}(s) = \Phi^{\text{out}}(s)$, $\forall s \in S$

– $\text{D} \in S \Rightarrow \Phi^{\text{up}}(\text{D}) = \Phi^{\text{down}}(\text{D})$,

[B. Kayser, Rev. Part. Prop., PRD 66 (2002) 010001]

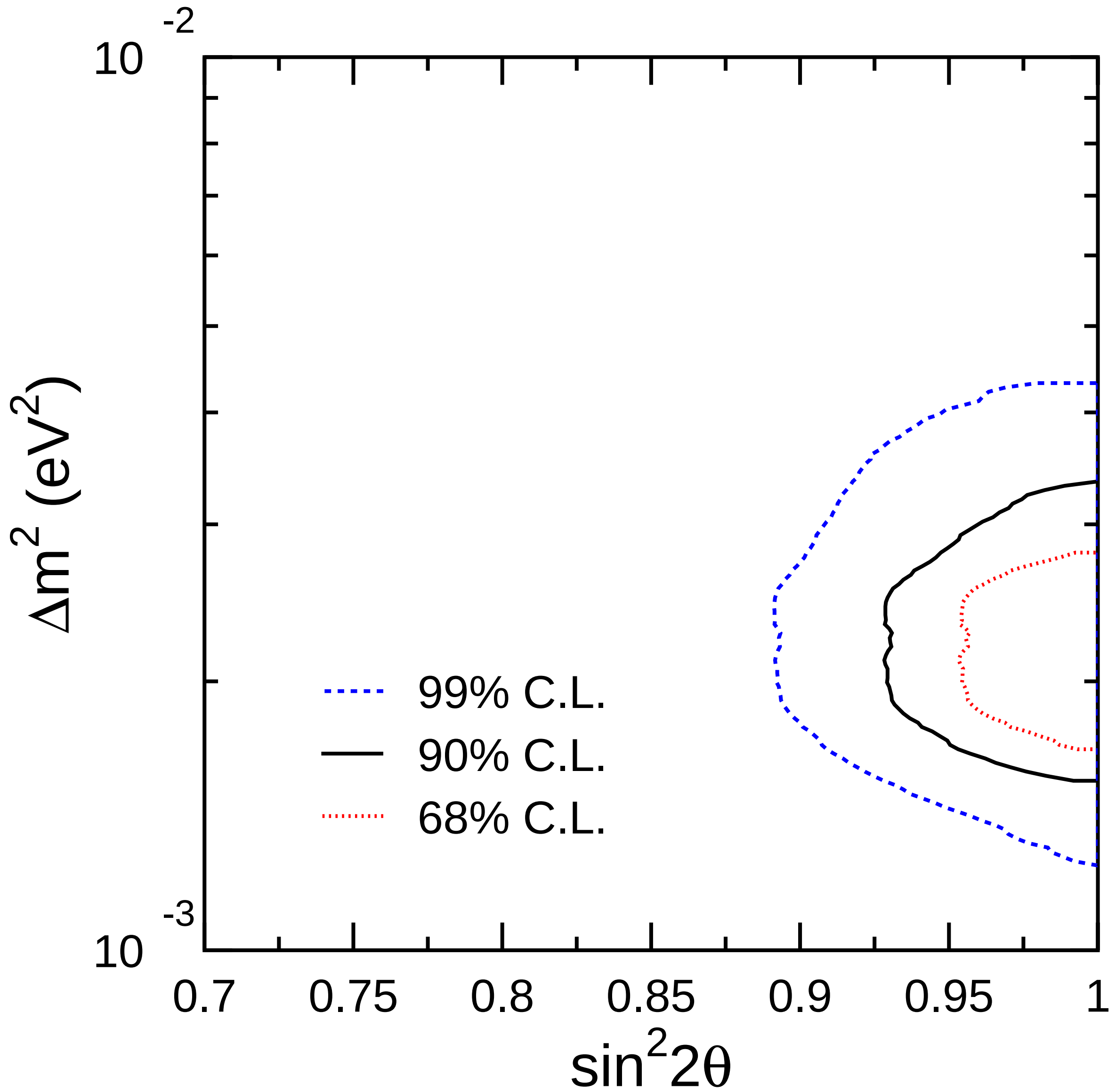
(December 1998)

$$A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}} \right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

$\nu_\mu \to \nu_\tau$ Fit of Super-Kamiokande Atmospheric Data



Best Fit

$\Delta m^2 = 2.1 \times 10^{-3}\, eV^2$

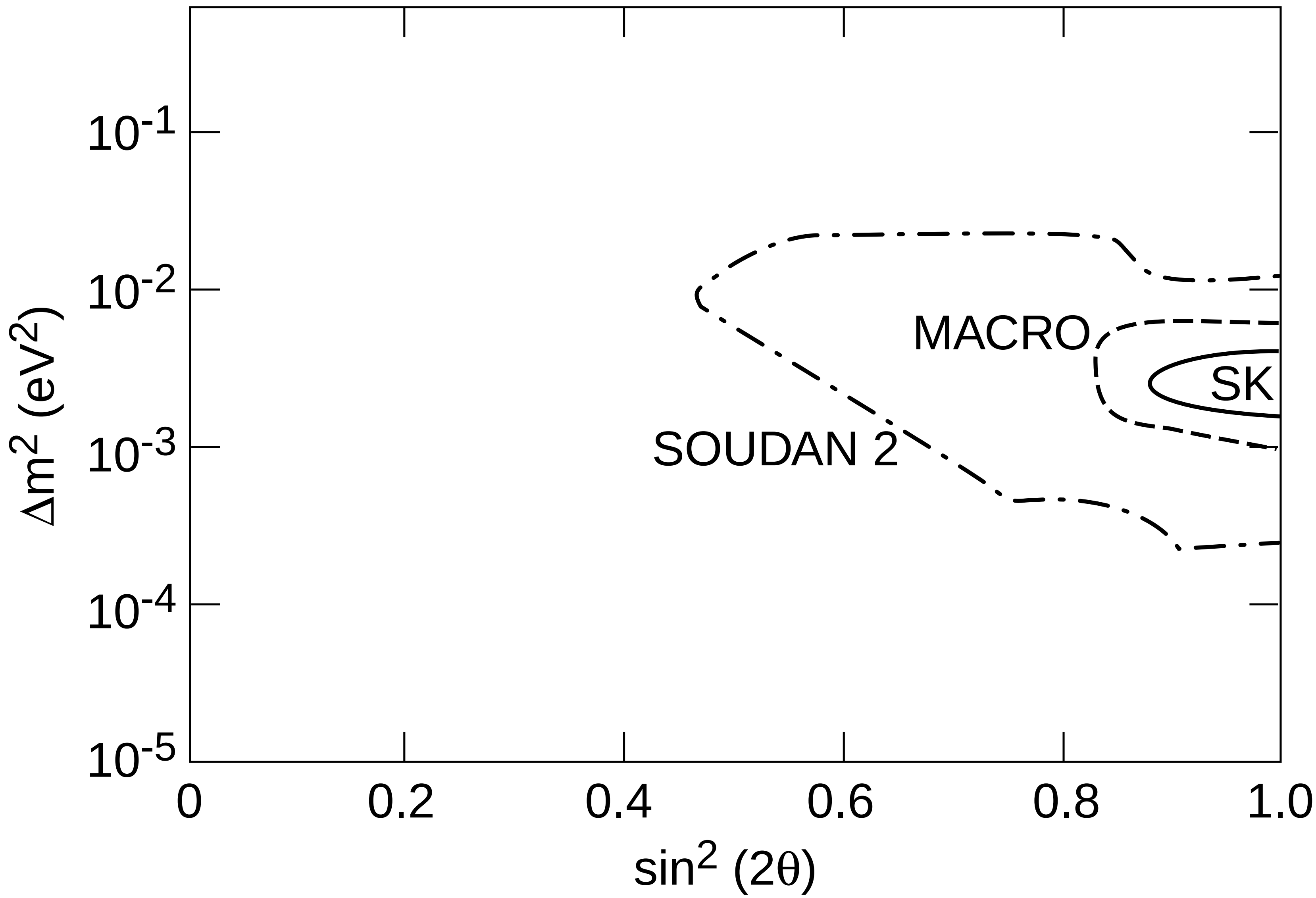
$\sin^2 2\theta = 1.0$

1489 live-days

April 1996 – July 2001

[Super-Kamiokande, hep-ex/0501064]

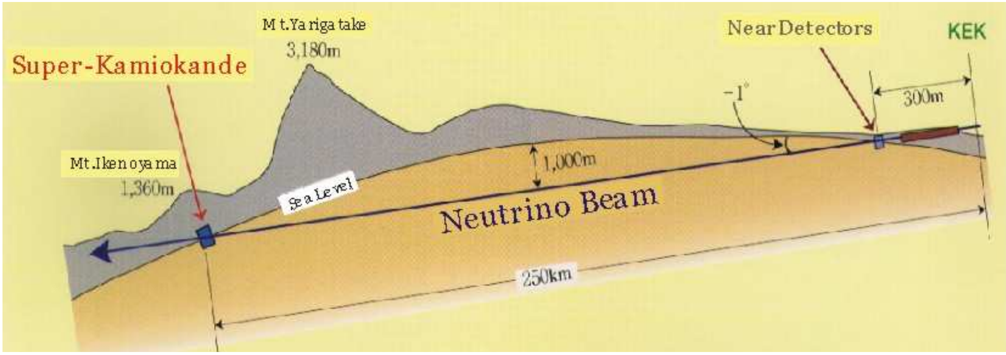
Soudan-2 & MACRO

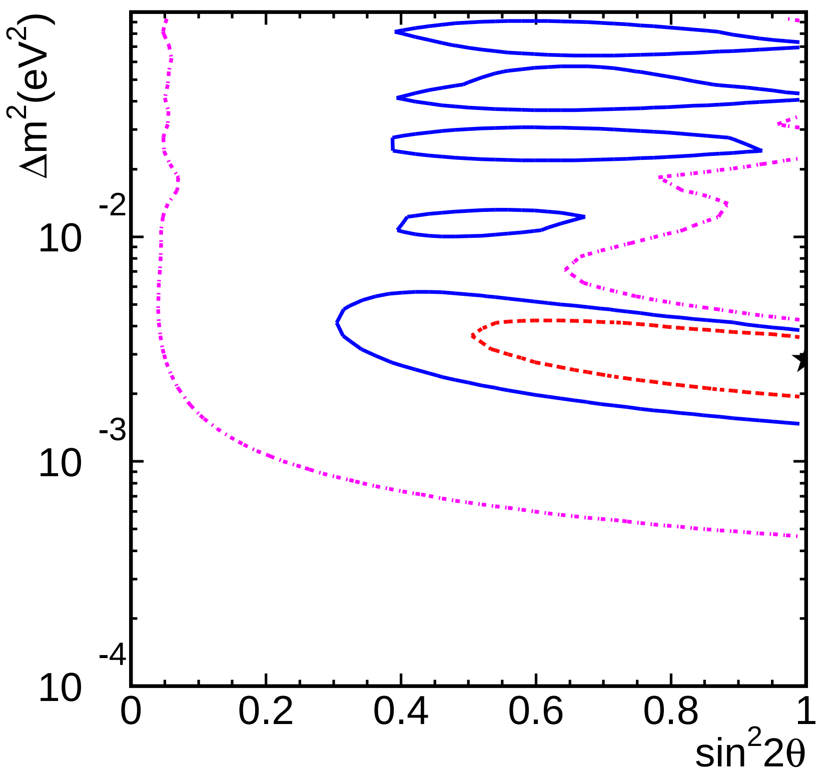


[Giacomelli, Giorgini, Spurio, hep-ex/0201032]

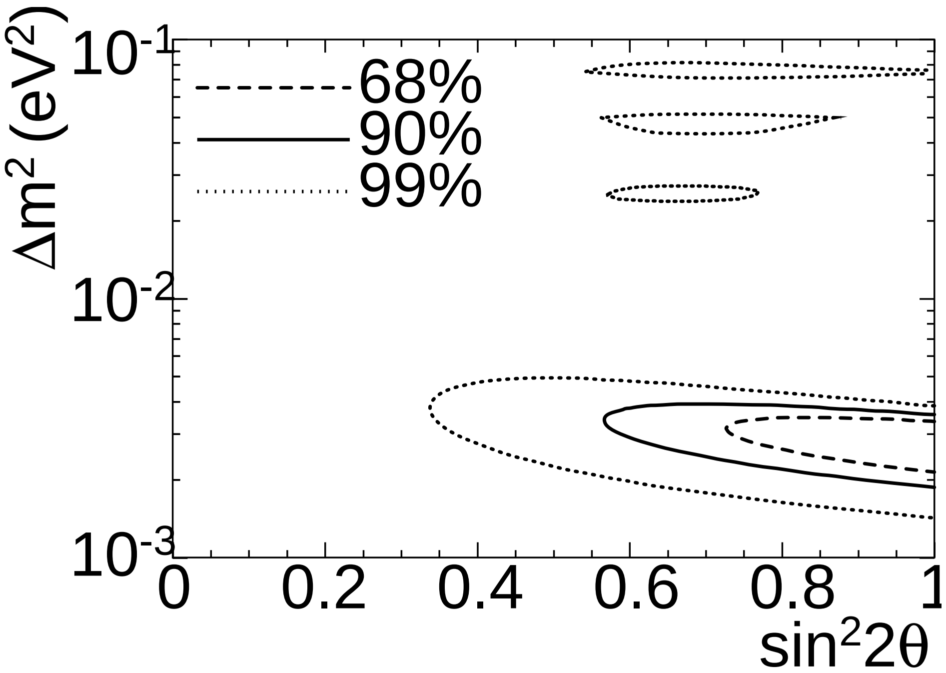
K2K

confirmation of atmospheric allowed region (June 2002)





[K2K, Phys. Rev. Lett. 90 (2003) 041801]



[K2K, hep-ex/0411038]

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

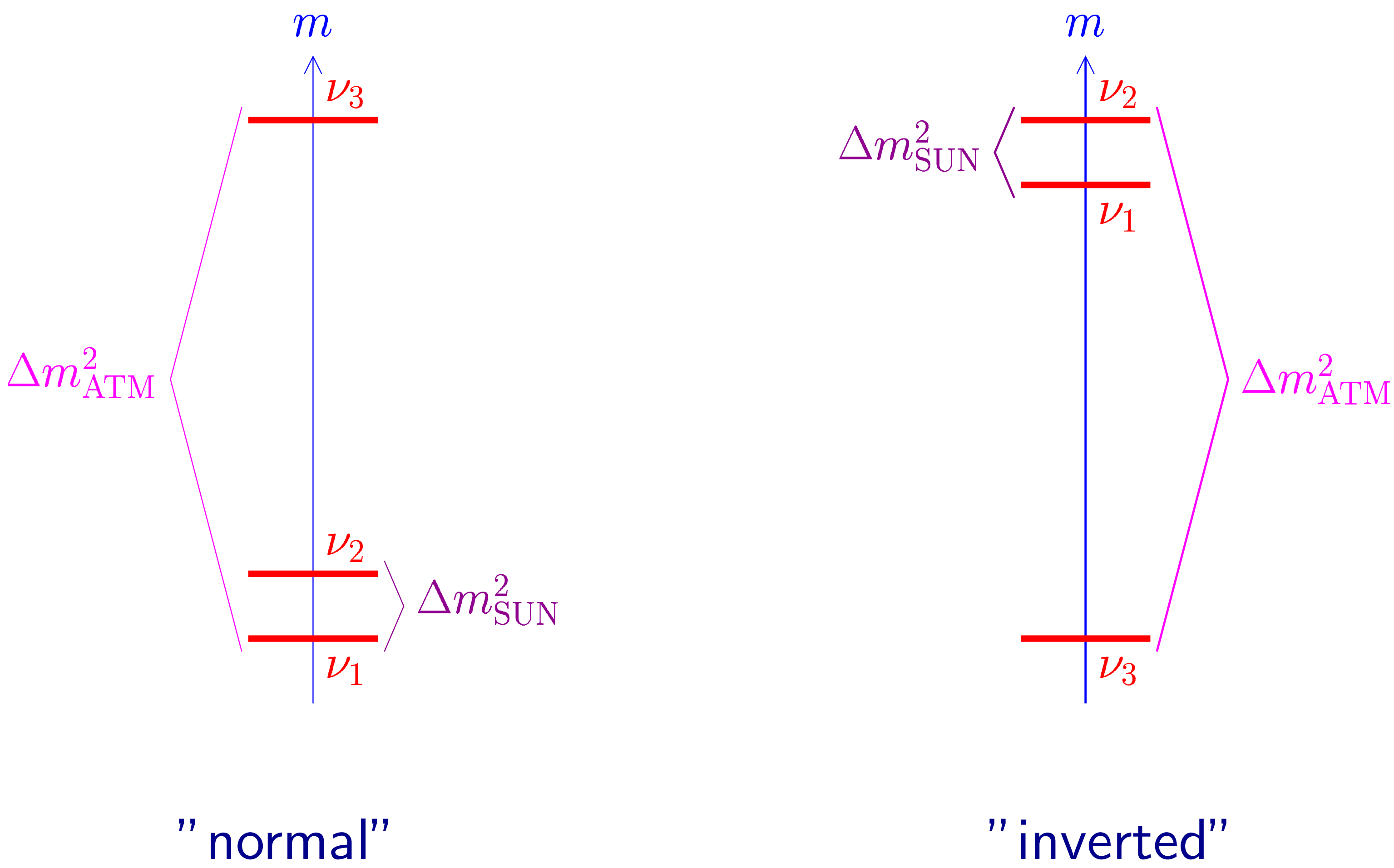
three flavor fields ν_e, ν_μ, ν_τ

three massive fields ν_1, ν_2, ν_3

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \simeq 8.2 \times 10^{-5} \, \text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3} \, \text{eV}^2$$

Allowed Three-Neutrino Schemes



absolute scale is not determined by neutrino oscillation data

$$\Delta m^2_{21} \ll |\Delta m^2_{31}|$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

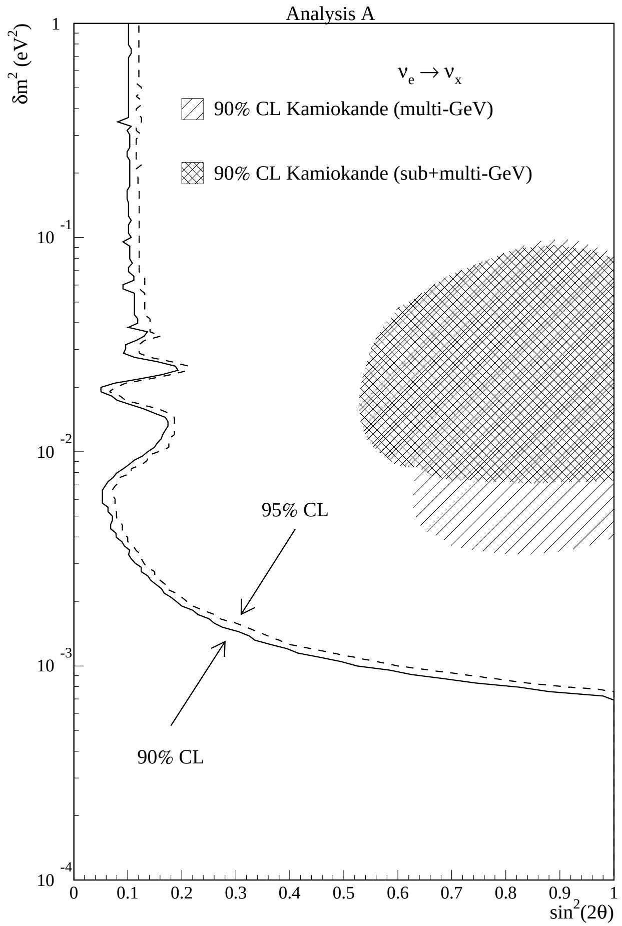
SUN → (first row: U_{e1}, U_{e2}) ; ATM → (third column: U_{e3}, $U_{\mu 3}$, $U_{\tau 3}$)

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$|U_{e3}|^2 < 5 \times 10^{-2} \; (99.73\% \text{ C.L.})$$

[Fogli et al., PRD 66 (2002) 093008]



[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2\vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, Giunti, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = R_{23}\, W_{13}\, R_{12}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} = \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} = \vartheta_{\text{SUN}}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13}\, e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13}\, e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13}\, e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13}\, e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

Global Fit of Oscillation Data ⇒ Bilarge Mixing

$$\Delta m^2_{\text{SUN}} = 7.92\,(1 \pm 0.09) \times 10^{-5}\,\text{eV}^2 \qquad \sin^2\vartheta_{\text{SUN}} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\Delta m^2_{\text{ATM}} = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\,\text{eV}^2 \qquad \sin^2\vartheta_{\text{ATM}} = 0.44\,(1^{+0.41}_{-0.22})$$

$$\sin^2\vartheta_{\text{CHOOZ}} = 0.9^{+2.3}_{-0.9} \times 10^{-2}$$

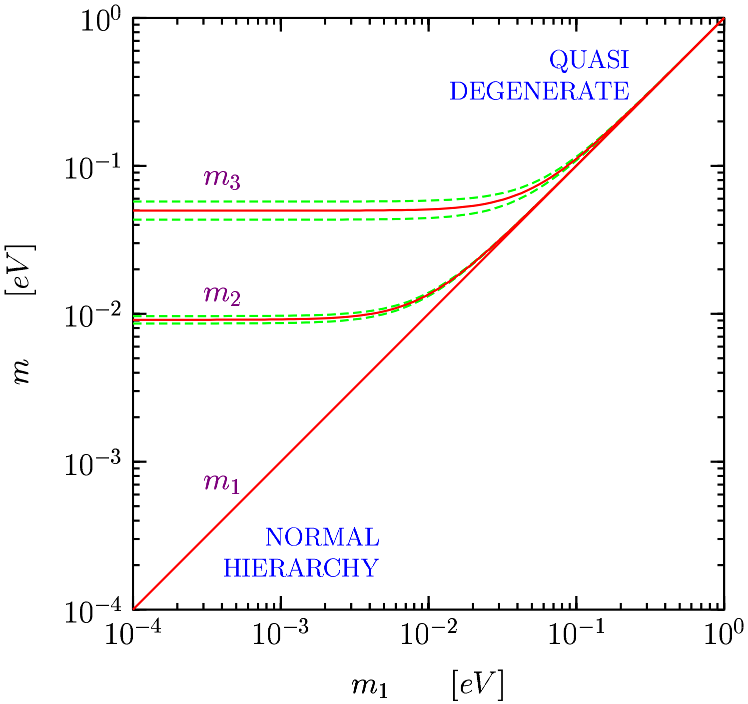
[Fogli, Lisi, Marrone, Palazzo. hep-ph/0506083]

$$|U|_{\text{bf}} \simeq \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.31-0.43 & 0.51-0.59 & 0.75 \\ 0.37-0.47 & 0.59-0.66 & 0.66 \end{pmatrix}$$

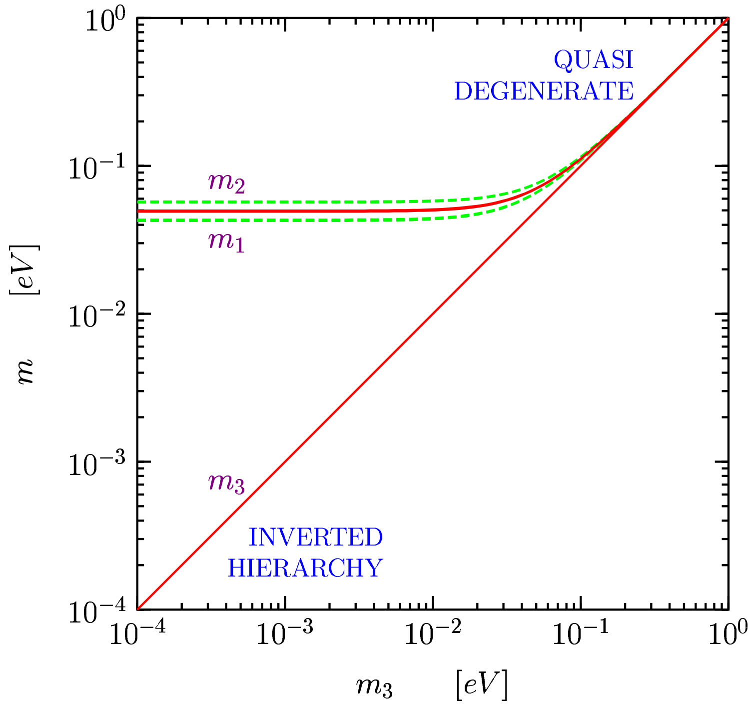
$$|U|_{3\sigma} \simeq \begin{pmatrix} 0.78-0.86 & 0.51-0.61 & 0.00-0.18 \\ 0.19-0.57 & 0.39-0.73 & 0.61-0.80 \\ 0.20-0.57 & 0.40-0.74 & 0.59-0.79 \end{pmatrix}$$

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\rm SUN}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\rm ATM}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\rm ATM}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\rm ATM}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\rm ATM}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium β Decay

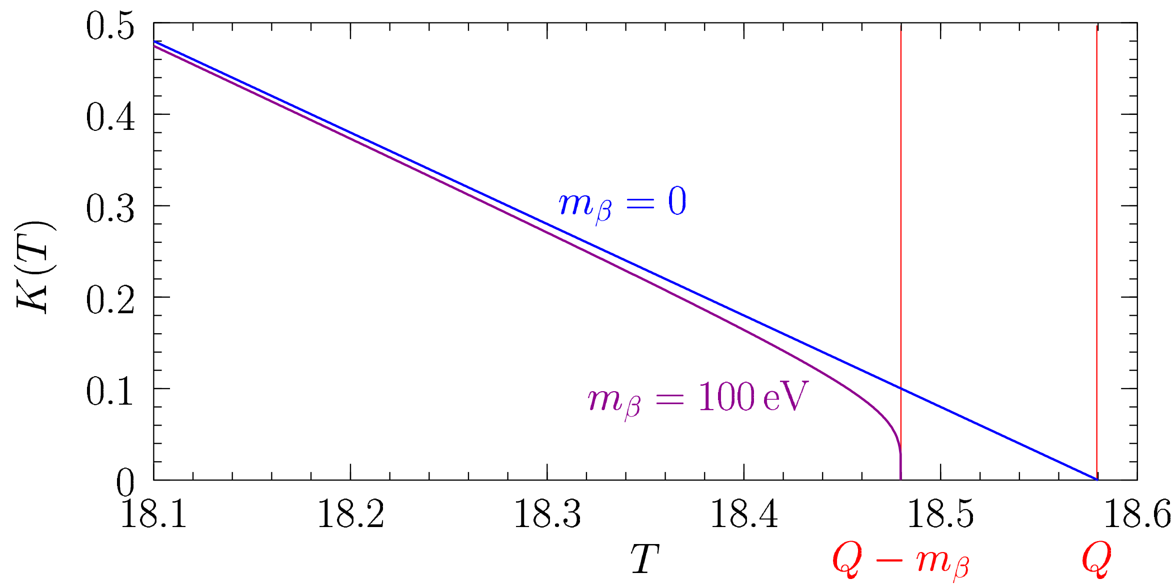
$^3H \to {}^3He + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q - T) \sqrt{(Q - T)^2 - m_\beta^2}$$

$$Q = M_{^3H} - M_{^3He} - m_e = 18.58\,\text{keV}$$

$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

Kurie plot: $$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \right]^{1/2}$$



$m_\beta < 2.2\,\text{eV}$ (95% C.L.)

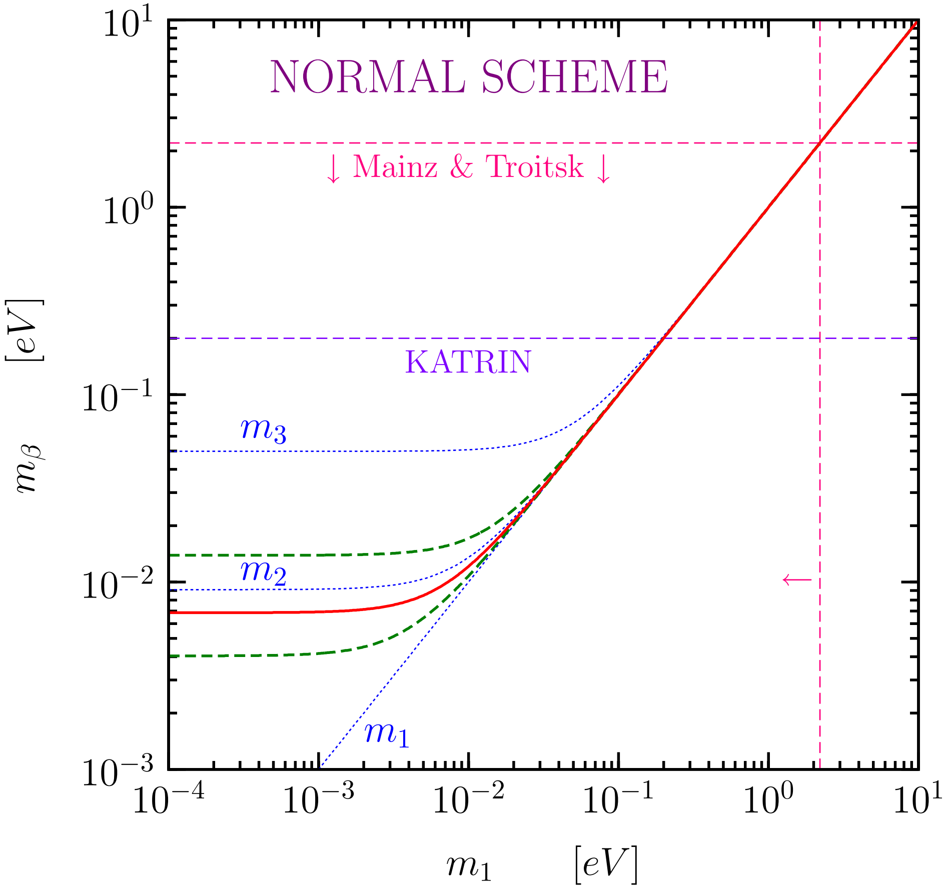
Mainz & Troitsk

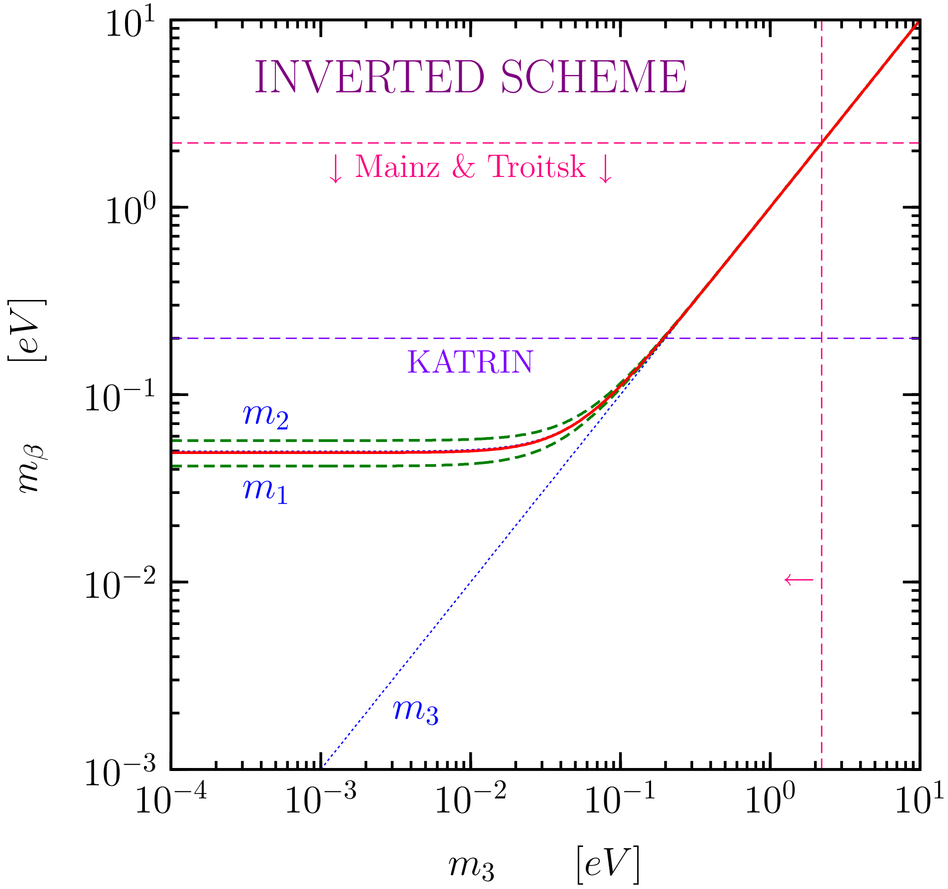
[Weinheimer, hep-ex/0210050]

future: KATRIN

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_\beta \simeq 0.2 - 0.3\,\text{eV}$





Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

$$\text{Relic Neutrinos:} \quad T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$$

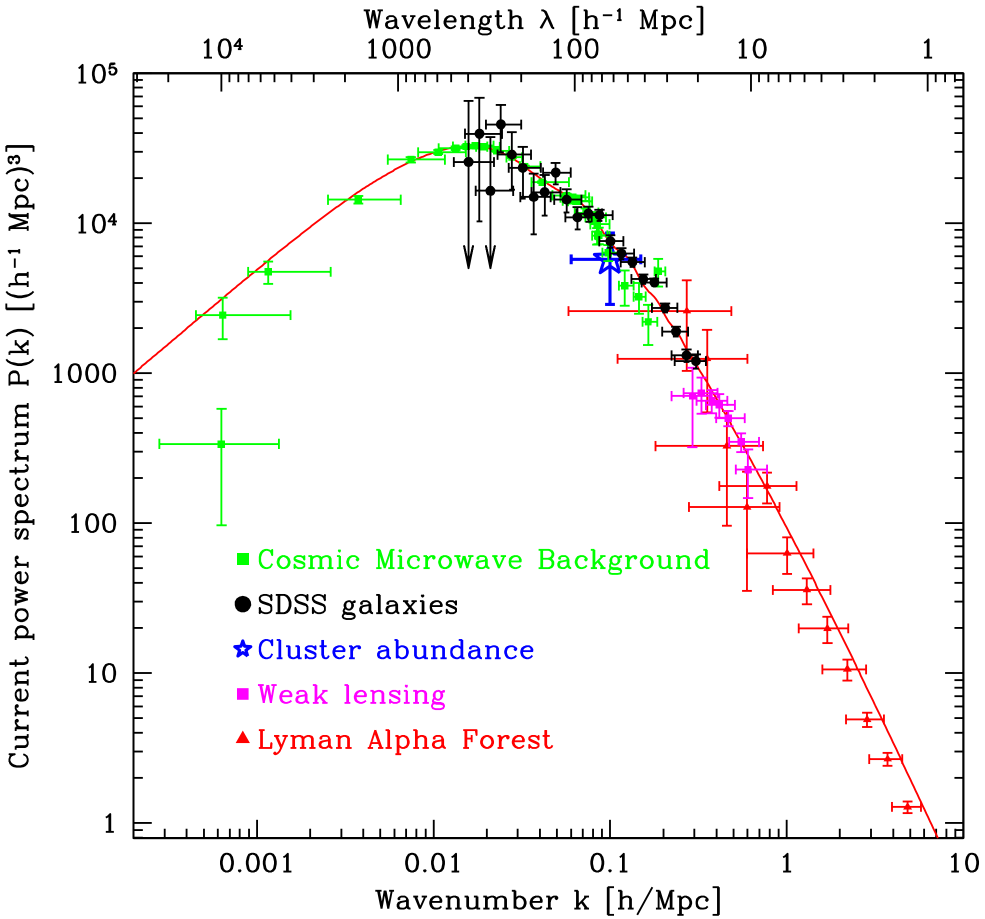
$$\text{number density:} \quad n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$$

$$\underset{\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)}{\text{density contribution:}} \quad \Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[SDSS, astro-ph/0310725]

hot dark matter
prevents early galaxy formation

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026\sqrt{\frac{m_\nu}{1\,\text{eV}}}\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

[Dolgov, PRep 370 (2002) 33] [Kainulainen, Olive, hep-ph/0206163] [Sarkar, hep-ph/0302175] [Hannestad, NJP 6 (2004) 108]

WMAP, AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, CBI, ACBAR) + LSS (2dFGRS) + Lyα + HST + SN-Ia

ΛCDM

$T_0 = 13.7 \pm 0.1\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$

$\Omega_{\text{tot}} = 1.02 \pm 0.02 \qquad \Omega_b h^2 = 0.0224 \pm 0.0009 \qquad \Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$$\Omega_\nu h^2 < 0.0076 \quad (95\%\ \text{conf.}) \quad \Longrightarrow \quad \sum_k m_k < 0.71\,\text{eV}$$

Hannestad, JCAP 0305 (2003) 004, astro-ph/0303076

$\sum_k m_k < 1.01\,\text{eV}$ (95% conf.) WMAP+CBI+2dFGRS+HST+SN-Ia

$\sum_k m_k < 1.20\,\text{eV}$ (95% conf.) WMAP+CBI+2dFGRS

$\sum_k m_k < 2.12\,\text{eV}$ (95% conf.) WMAP+2dFGRS

Elgaroy and Lahav, JCAP 04 (2003) 004, astro-ph/0303089

$\sum_k m_k < 1.1\,\text{eV}$ (95% conf.) WMAP+2dFGRS+HST

SDSS, PRD 69 (2004) 103501, astro-ph/0310723

CMB(WMAP)+LSS(SDSS)+SN-Ia

$h = 0.70^{+0.04}_{-0.03}$ $\Omega_m = 0.30 \pm 0.04$ $\sum_k m_k < 1.7\,\text{eV}$ (95% conf.)

SDSS, PRD 71 (2005) 043511, astro-ph/0406594

CMB(WMAP)+LSS(SDSS)+bias(SDSS) $P_g(k) = b^2\, P_m(k)$

$\Omega_m = 0.25 \pm 0.03$ $\sum_k m_k < 0.54\,\text{eV}$ (95% conf.)

SDSS, astro-ph/0407372

CMB(WMAP)+LSS(SDSS)+bias(SDSS)+Lyα(SDSS)+SN-Ia

$\Omega_\Lambda = 0.72 \pm 0.02$ $\sum_k m_k < 0.42\,\text{eV}$ (95% conf.)

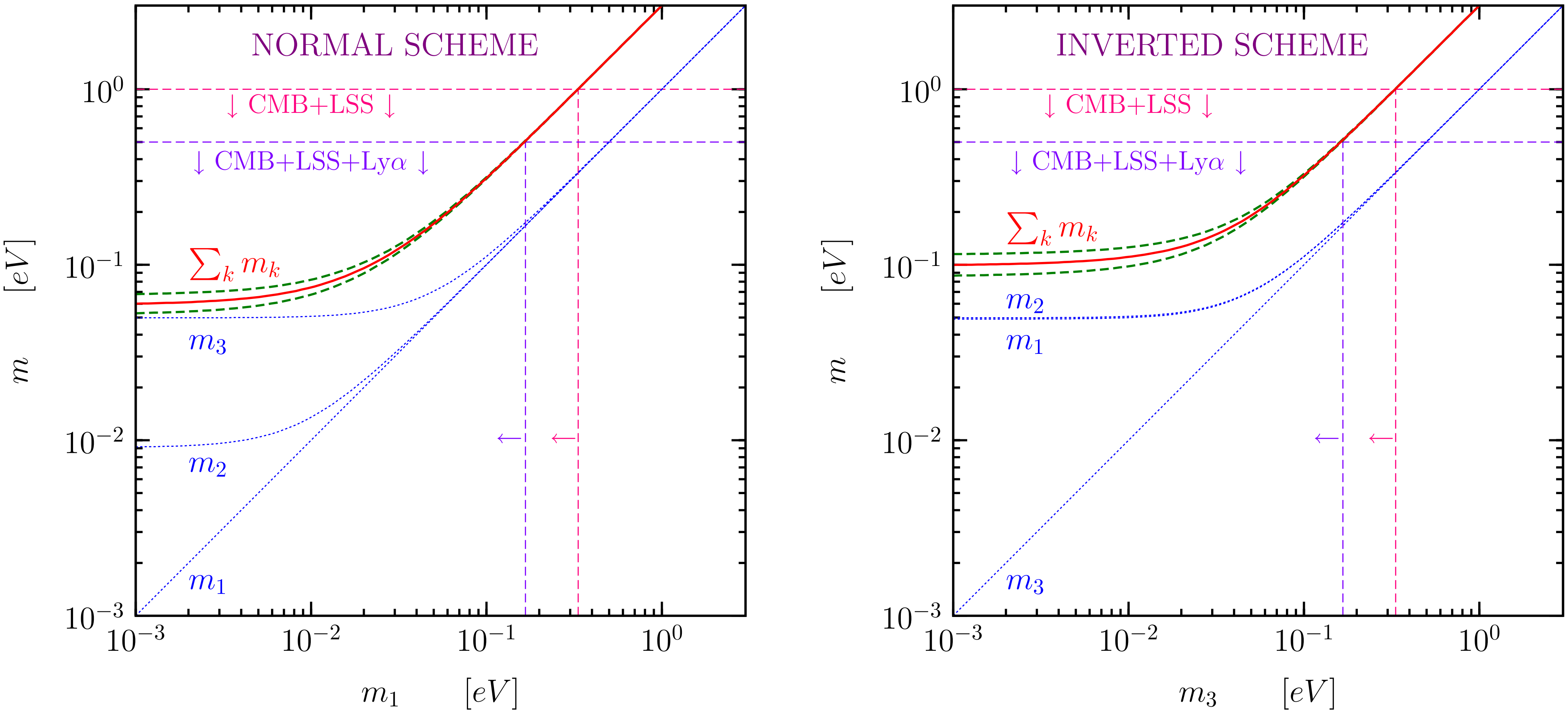
Fogli et al., PRD 70 (2004) 113003, hep-ph/0408045

$\sum_k m_k < 1.4\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia

$\sum_k m_k < 0.47\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia+Lyα(SDSS)

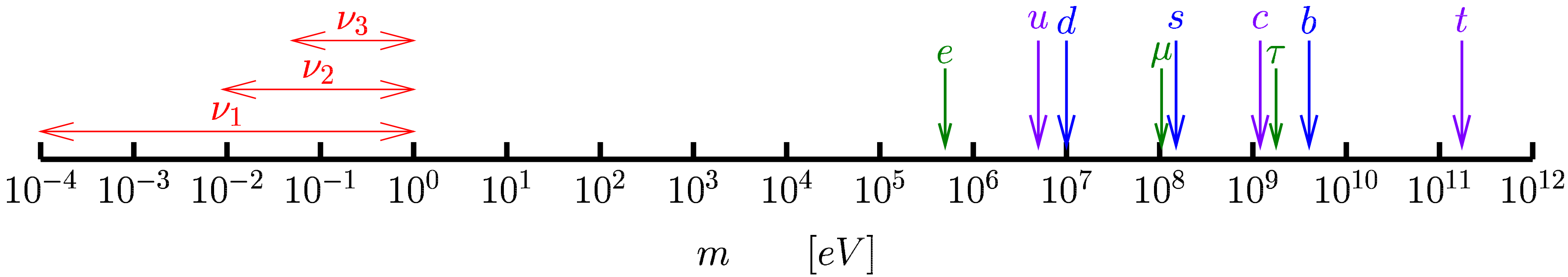
$$\sum_k m_k \lesssim 1\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia}$$

$$\sum_k m_k \lesssim 0.5\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia+Ly}\alpha$$



FUTURE: IF $\sum_k m_k \lesssim 8 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Majorana Neutrino Mass?



known natural explanations of smallness of ν masses $\begin{cases} \text{See-Saw Mechanism} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{cases}$

both imply
- Majorana ν masses $\Longleftrightarrow$ $|\Delta L| = 2$ $\Longleftrightarrow$ $\beta\beta_{0\nu}$ decay
- see-saw type relation $m_\nu \sim \dfrac{\mathcal{M}^2_{\mathrm{EW}}}{\mathcal{M}}$
- new high energy scale $\mathcal{M}$

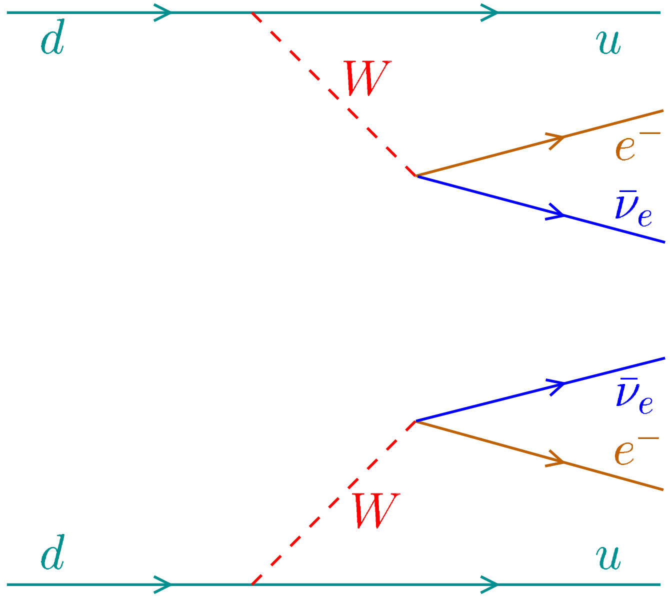
Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

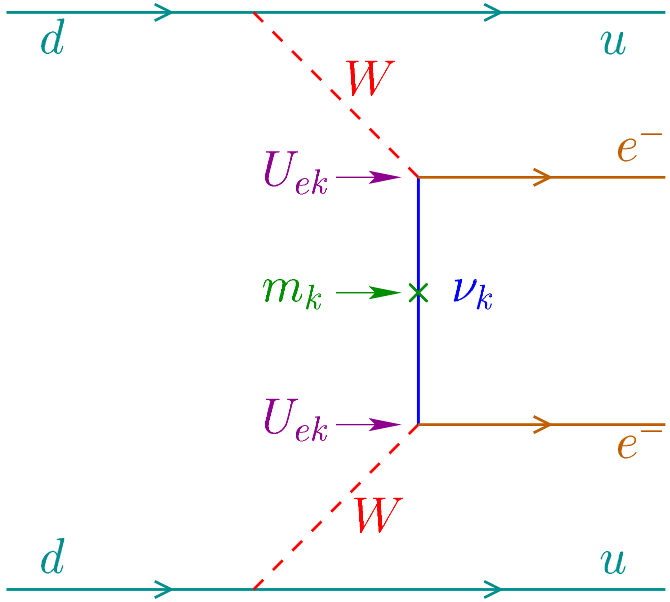


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$

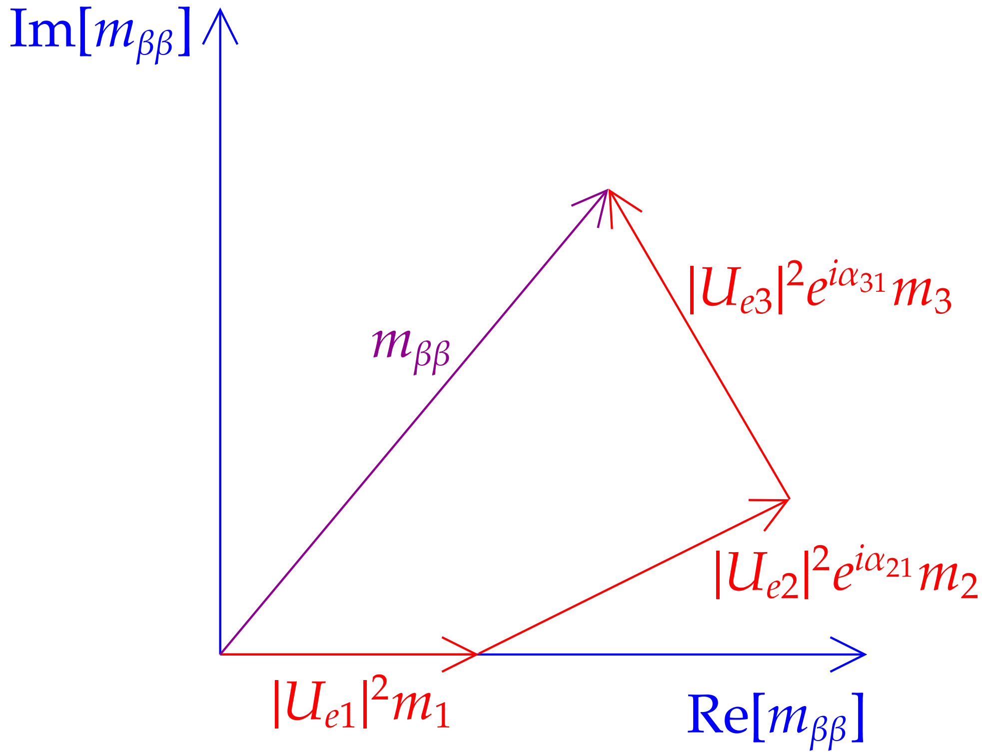


Effective Majorana Neutrino Mass in $\beta\beta_{0\nu}$ Decay

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$



Best limits for $\beta\beta_{0\nu}$ Decay

Heidelberg-Moscow ^{76}Ge [EPJA 12 (2001) 147]

$$T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}$$

IGEX ^{76}Ge [PRD 65 (2002) 092007]

$$T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.35 - 1.1\,\text{eV}$$

FUTURE EXPERIMENTS

NEMO3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES

$|m_{\beta\beta}| \sim$ few 10^{-1} eV

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

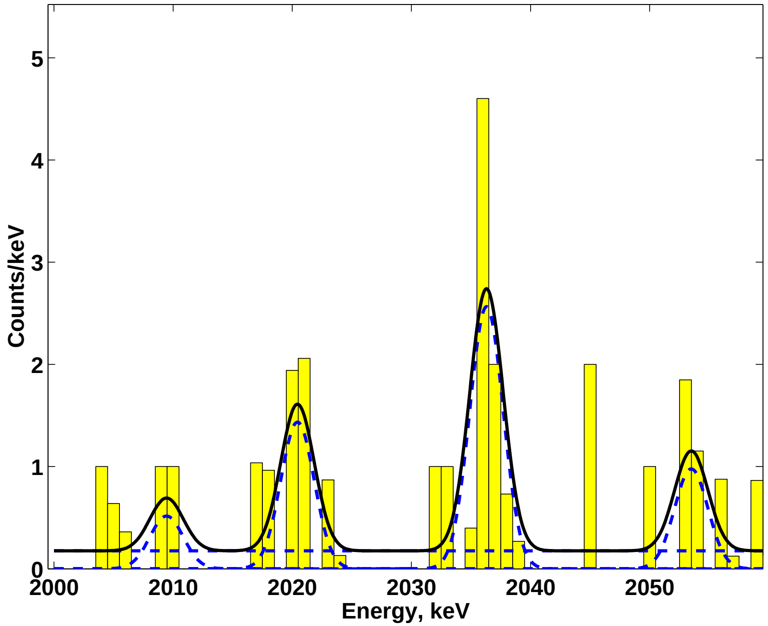
$|m_{\beta\beta}| \sim$ few 10^{-2} eV

[Zdesenko, RMP 74 (2002) 663] [Elliott,Vogel, ARNPS 52 (2002) 115] [Elliott, Engel, JPG 30 (2004) R183]

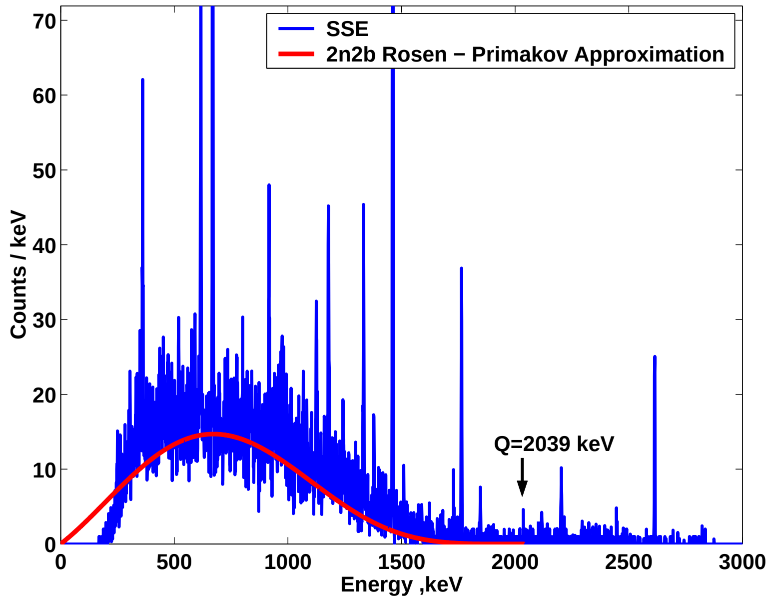
Indication of $\beta\beta_{0\nu}$ Decay

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$T_{1/2}^{0\nu\,\text{bf}} = 1.19 \times 10^{25}\,\text{y}$ $\quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\text{y}\,(3\sigma)$ $\quad 4.2\sigma$ evidence



pulse-shape selected spectrum



3.8σ evidence [PLB 586 (2004) 198]

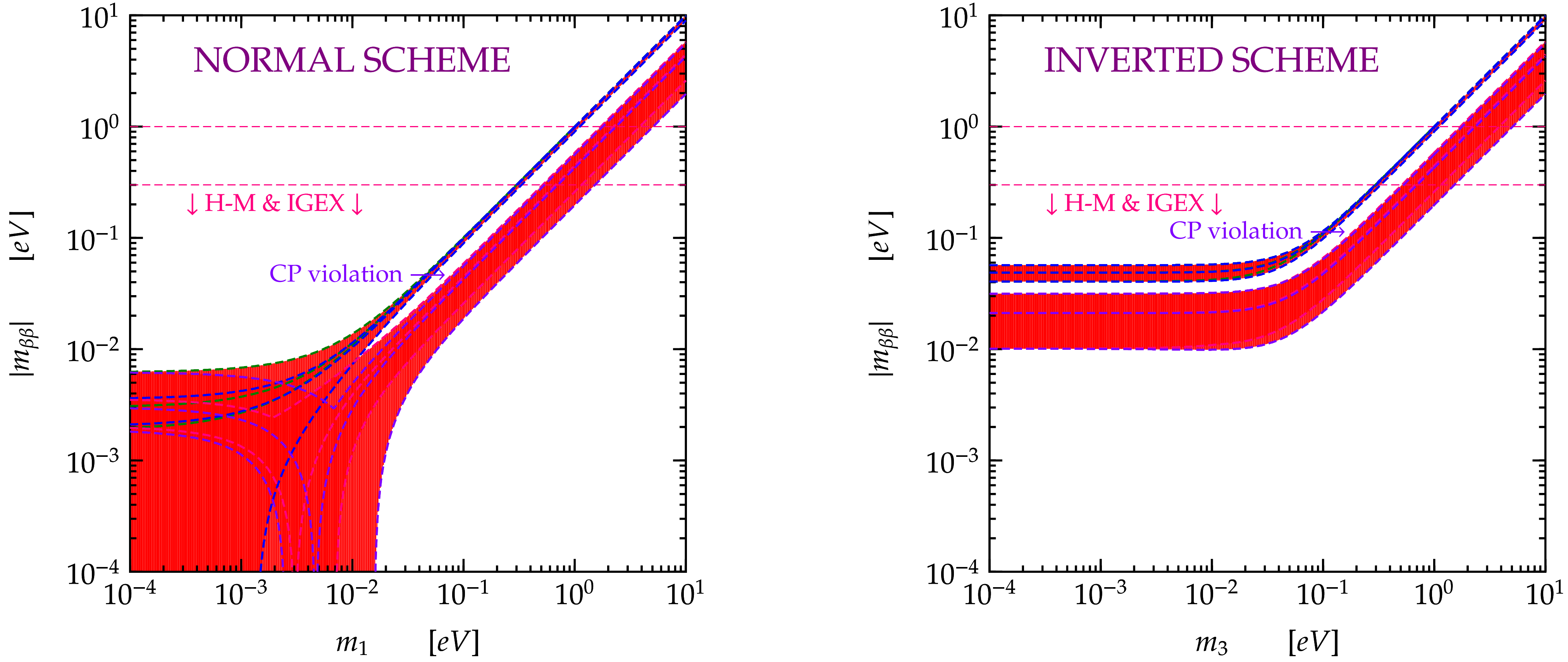
the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Longrightarrow \quad 0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$$

if confirmed very exciting (Majorana ν and large mass scale)

General Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

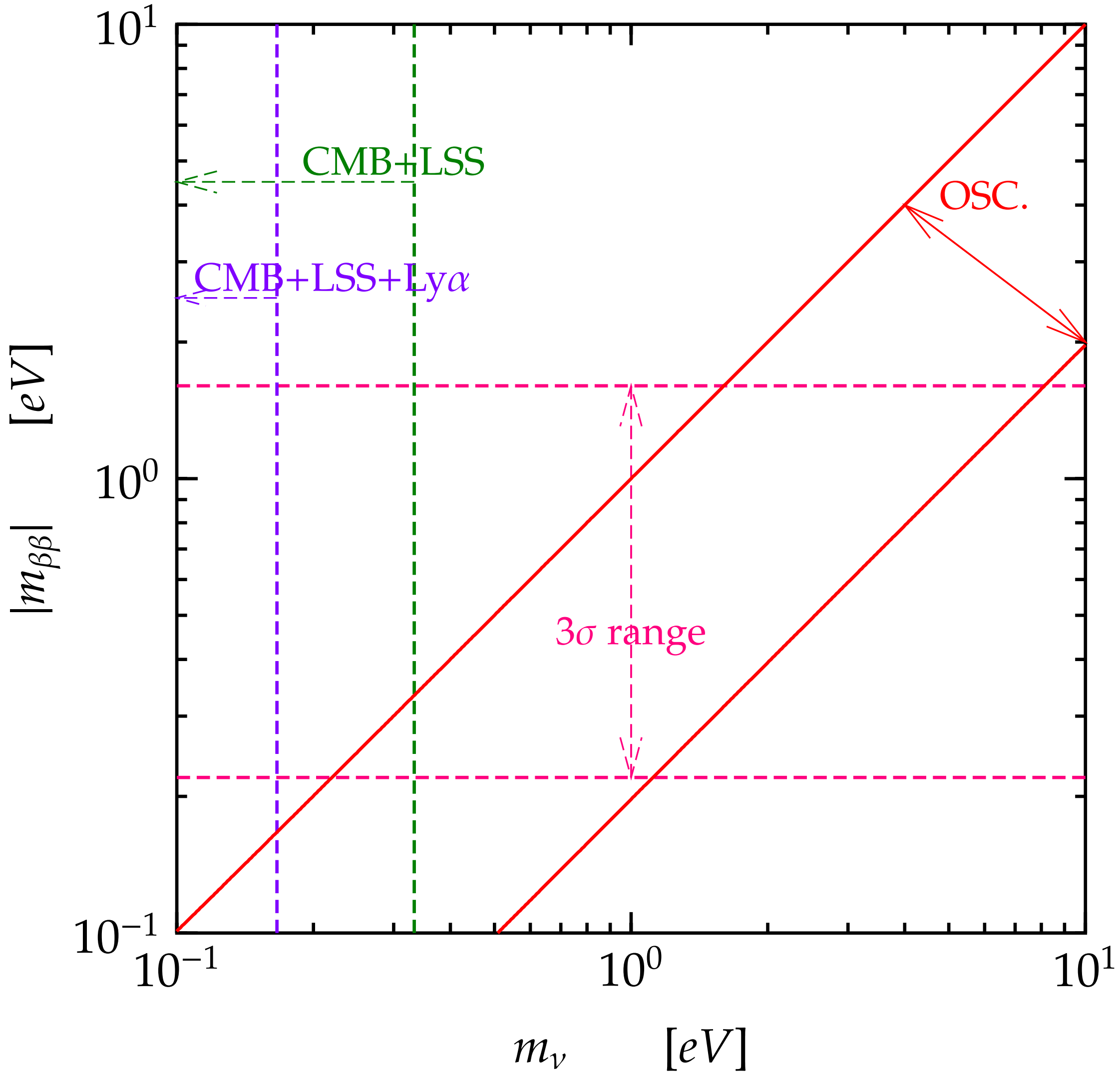
$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$



FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay

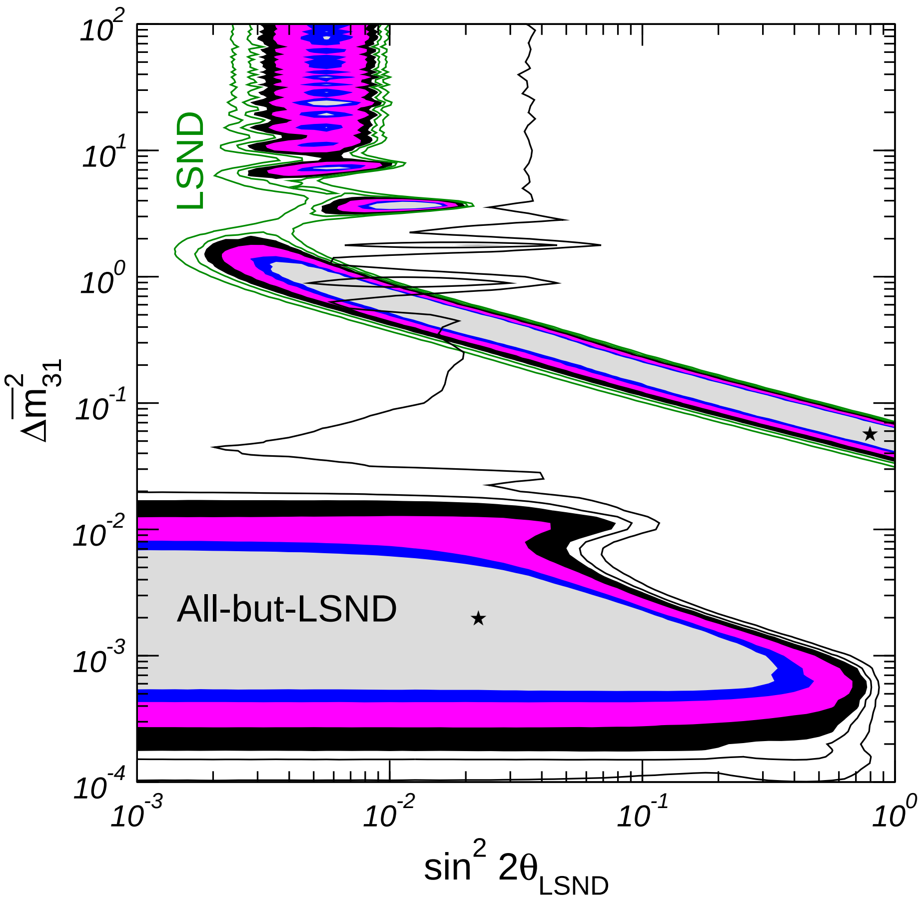
$0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ (3σ range)



tension among oscillation data, CMB+LSS+Lyα and $\beta\beta_{0\nu}$ signal

? LSND ?

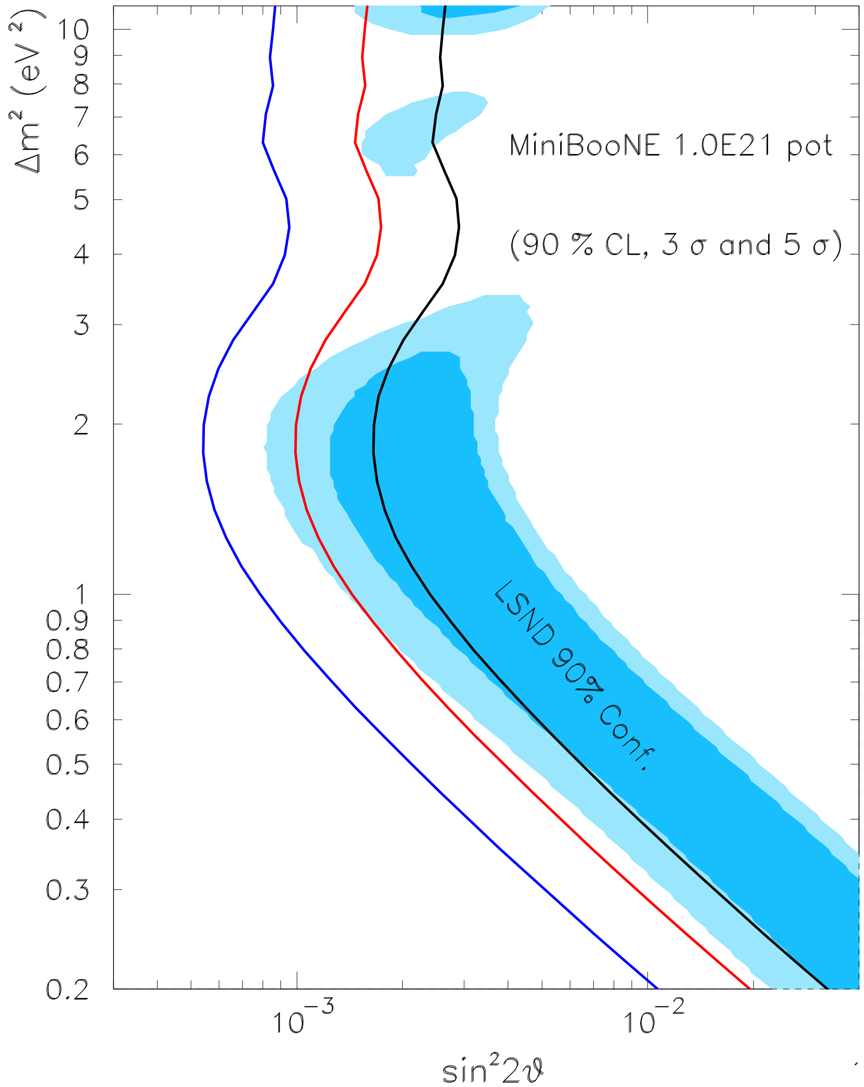
$\bar{\nu}_\mu \to \bar{\nu}_e$ $\Delta m^2_{\mathrm{LSND}} \gtrsim 0.1\,\mathrm{eV}^2$ $(\gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SUN}})$



[Gonzalez-Garcia, Maltoni, Schwetz PRD 68 (2003) 053007]

even allowing $\Delta\overline{m}^2_{31} \neq \Delta m^2_{31}$

(CPT violation) GoF $= 7.5 \times 10^{-4}$



MiniBooNE $\nu_\mu \to \nu_e$ [hep-ex/0406048]

Summary

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SUN}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (atmospheric ν, K2K)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?
Why only $|U_{e3}|^2 \ll 1$?

Exp.: Measure $|U_{e3}| > 0$ $\Rightarrow$ CP violation
Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale
Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements