Status of Neutrino Masses and Mixing

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Introduction to Neutrino Masses, Mixing and Oscillations
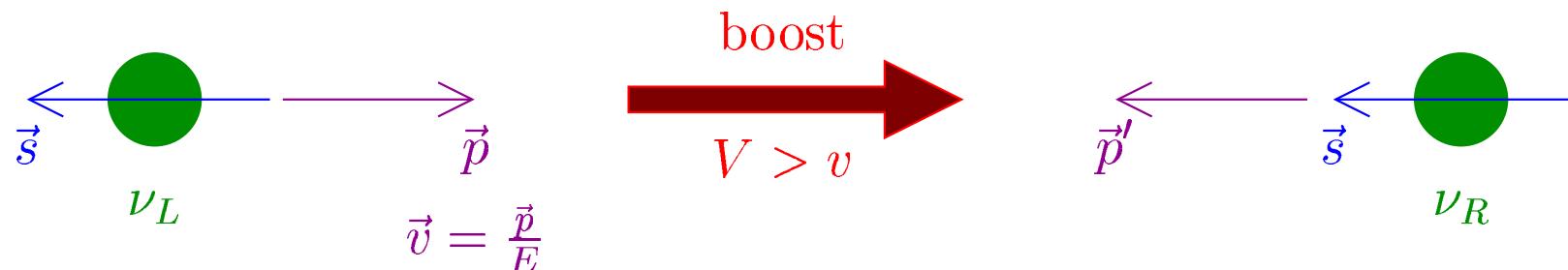
Solar $\nu_e \rightarrow \nu_\mu, \nu_\tau$ + Atmospheric $\nu_\mu \rightarrow \nu_\tau \Longrightarrow$ 3-ν Mixing

Absolute Scale of Neutrino Masses

Cosmological Bound on Neutrino Masses

Neutrinoless Double-β Decay $\Longleftrightarrow$ Majorana Mass

Neutrino Mass

Standard Model: $\nu_L, \nu_R^c \implies$ no Dirac mass term $\mathcal{L}^D \sim m^D \overline{\nu_L} \nu_R$

(no ν_R, ν_L^c)

Majorana Neutrino: $\nu \equiv \nu^c$

$\nu_R^c \equiv \nu_R \implies$ Majorana mass term $\mathcal{L}^M \sim m^M \overline{\nu_L} \nu_R^c$

Standard Model: Majorana mass term not allowed by $\text{SU}(2)_L \times \text{U}(1)_Y$

(no Higgs triplet)

Standard Model can be extended with ν_R (e_L, e_R; u_L, u_R; d_L, d_R; ...)

$\nu_L + \nu_R \Rightarrow$ Dirac neutrino mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \overline{\nu_L} \nu_R \Rightarrow m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$
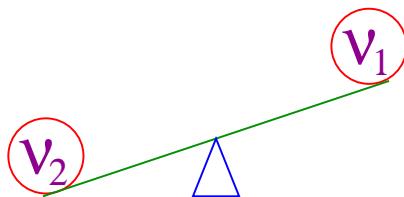
surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}^{\mathrm{M}}_R \sim m^{\mathrm{M}}_R \overline{\nu^c_L} \nu_R$

total neutrino mass term $\mathcal{L}^{\mathrm{D+M}} \sim \begin{pmatrix} \overline{\nu_L} & \overline{\nu^c_L} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \nu_R \end{pmatrix}$

m^{M}_R can be arbitrarily large (not protected by SM symmetries)

$m^{\mathrm{M}}_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m^{\mathrm{M}}_R \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix}$ $\Rightarrow$ $m_1 \simeq \dfrac{(m^{\mathrm{D}})^2}{m^{\mathrm{M}}_R}$, $\quad m_2 \simeq m^{\mathrm{M}}_R$

natural explanation of
smallness of neutrino masses

see-saw mechanism   massive neutrinos are Majorana!

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e,\, e^-)$	$+1$	0	0
$(\nu_\mu,\, \mu^-)$	0	$+1$	0
$(\nu_\tau,\, \tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c,\, e^+)$	-1	0	0
$(\nu_\mu^c,\, \mu^+)$	0	-1	0
$(\nu_\tau^c,\, \tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Dirac mass term $m^{\mathrm{D}}\overline{\nu_L}\nu_R \Rightarrow \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\mathrm{D}} & m_{e\mu}^{\mathrm{D}} & m_{e\tau}^{\mathrm{D}} \\ m_{\mu e}^{\mathrm{D}} & m_{\mu\mu}^{\mathrm{D}} & m_{\mu\tau}^{\mathrm{D}} \\ m_{\tau e}^{\mathrm{D}} & m_{\tau\mu}^{\mathrm{D}} & m_{\tau\tau}^{\mathrm{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^{\mathrm{M}}\overline{\nu_L}\nu_R^c \Rightarrow \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\mathrm{M}} & m_{e\mu}^{\mathrm{M}} & m_{e\tau}^{\mathrm{M}} \\ m_{\mu e}^{\mathrm{M}} & m_{\mu\mu}^{\mathrm{M}} & m_{\mu\tau}^{\mathrm{M}} \\ m_{\tau e}^{\mathrm{M}} & m_{\tau\mu}^{\mathrm{M}} & m_{\tau\tau}^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \nu_{eR}^c \\ \nu_{\mu R}^c \\ \nu_{\tau R}^c \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu_{\alpha R}^c) = -L(\nu_{\beta L}) \Rightarrow |\Delta L| = 2$

Diagonalization of Mass Matrix $\Rightarrow$ Mixing

Dirac Mass Term: $\quad \mathcal{L}^{\mathrm{D}} \sim \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\mathrm{D}} & m_{e\mu}^{\mathrm{D}} & m_{e\tau}^{\mathrm{D}} \\ m_{\mu e}^{\mathrm{D}} & m_{\mu\mu}^{\mathrm{D}} & m_{\mu\tau}^{\mathrm{D}} \\ m_{\tau e}^{\mathrm{D}} & m_{\tau\mu}^{\mathrm{D}} & m_{\tau\tau}^{\mathrm{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

$$\mathcal{L}^{\mathrm{D}} \sim \sum_{\alpha,\beta} \overline{\nu_{\alpha L}} m_{\alpha\beta}^{\mathrm{D}} \nu_{\beta R} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL} \qquad \nu_{\beta R} = \sum_{j=1}^{3} V_{\beta j} \nu_{jR} \qquad U^{\dagger} m^{\mathrm{D}} V = \mathrm{diag}(m_1, m_2, m_3)$$

$$\mathcal{L}^{\mathrm{D}} \sim \sum_{k=1}^{3} m_k \overline{\nu_{kL}} \nu_{kR}$$

weak charged current: neutrino production and detection

$$j_{\rho}^{\mathrm{CC}} \sim \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha L}} \, \gamma_{\rho} \, \nu_{\alpha L} = \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma_{\rho} \, U_{\alpha k} \, \nu_{kL}$$

U = unitary 3×3 mixing matrix

Neutrino Oscillations

[Pontecorvo, SPJETP 6 (1957) 429]   [Pontecorvo, SPJETP 7 (1958) 172]   [Gribov, Pontecorvo, PLB 28 (1969) 49]

[Eliezer,Swift, NPB 105 (1976) 45]   [Fritzsch, Minkowski, PLB 62 (1976) 72]   [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

[Bilenky, Pontecorvo, NCimL 17 (1976) 56]   [Bilenky, Pontecorvo, PRep 41 (1978) 225]

Neutrino Mixing:
$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

$$\alpha = e, \mu, \tau$$
$$\nu_k \rightarrow m_k$$

$$|\nu_k(x,t)\rangle = e^{-iE_k t + ip_k x}|\nu_k\rangle \implies |\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k}\, e^{-iE_k t + ip_k x}\, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k}^* |\nu_\beta\rangle$$

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k}^* \right)}_{\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(x,t)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha \to \nu_\beta}(x, t) = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(x, t) \right|^2 = \left| \sum_k U_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k}^* \right|^2$$

ultra-relativistic neutrinos $\implies$ $t \simeq x = L$ source-detector distance
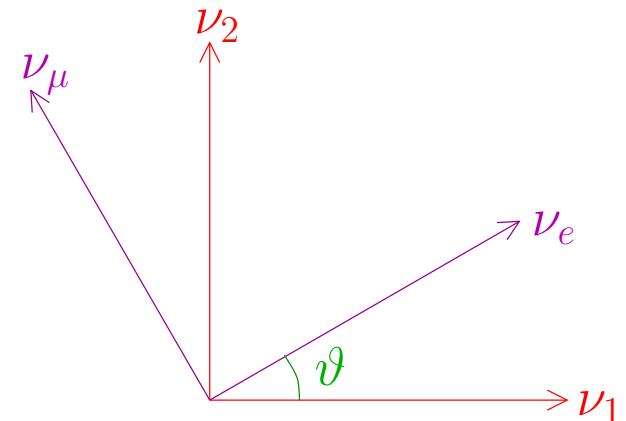
$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_k U_{\alpha k} e^{-im_k^2 L/2E} U_{\beta k}^* \right|^2$$

$$= \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Two-Neutrino Mixing

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k} |\nu_k\rangle \qquad (\alpha = e, \mu)$$

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta \, |\nu_1\rangle + \sin\vartheta \, |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta \, |\nu_1\rangle + \cos\vartheta \, |\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

Transition Probability: $\qquad P_{\nu_e \to \nu_\mu} = P_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta \, \sin^2\left(\dfrac{\Delta m^2 L}{4E}\right)$

Survival Probabilities: $\qquad P_{\nu_e \to \nu_e} = P_{\nu_\mu \to \nu_\mu} = 1 - P_{\nu_e \to \nu_\mu}$

Types of Experiments

Two-Neutrino
Mixing

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

observable if
$\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 1\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}, E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}, E \gtrsim 1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}, E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}, E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}, E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO

SUN
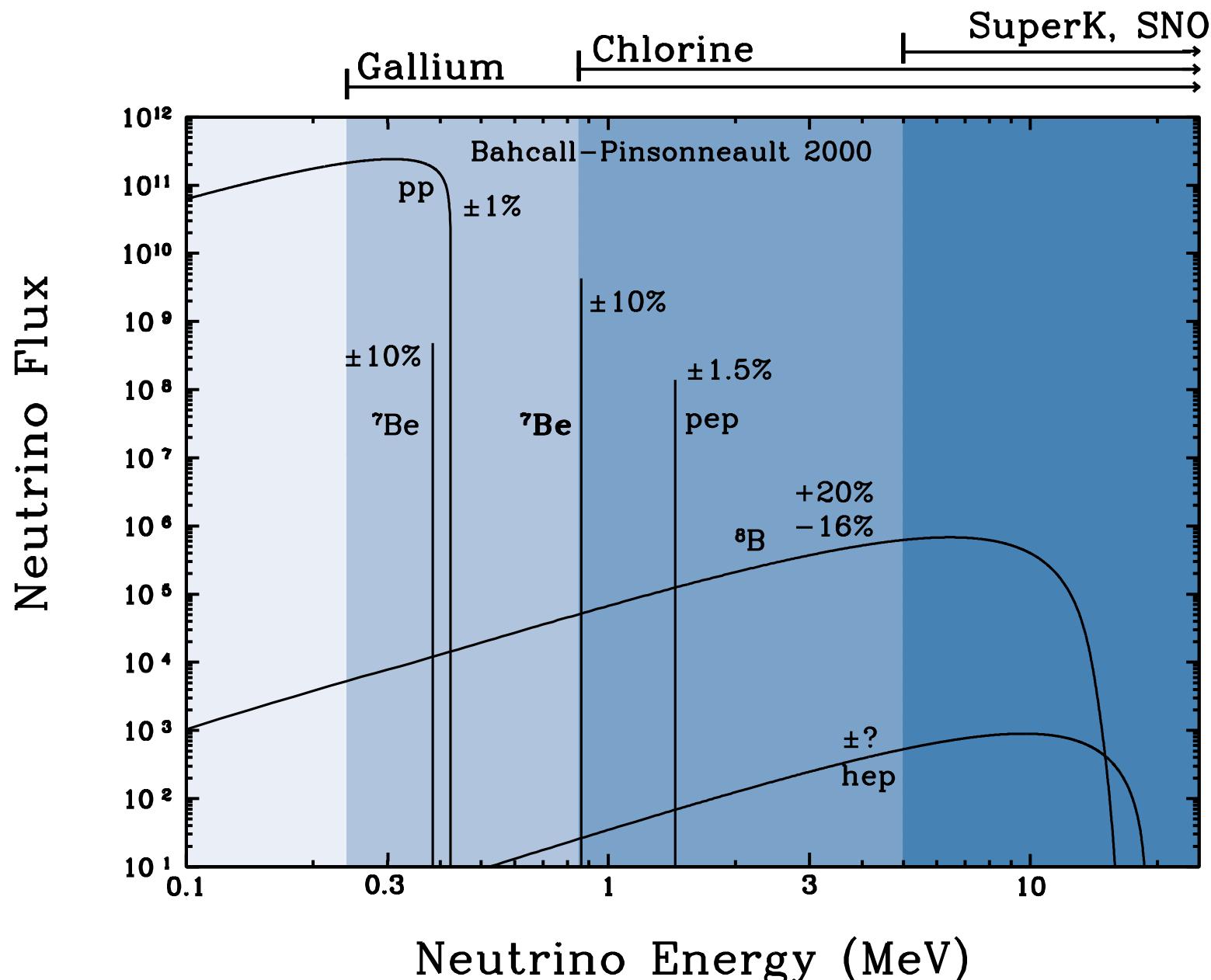
$L \sim 10^8\,\text{km}, \quad E \sim 0.1 - 10\,\text{MeV}$

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

Homestake, Kamiokande, GALLEX, SAGE

Super-Kamiokande, GNO, SNO

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1, \ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

Solar Neutrinos

[J.N. Bahcall, http://www.sns.ias.edu/~jnb]

SNO

$$\Phi_{\nu_e}^{SNO} = 1.76 \pm 0.11 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1}$$

$$\Phi_{\nu_\mu, \nu_\tau}^{SNO} = 5.41 \pm 0.66 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1}$$

SNO solved

solar neutrino problem

$\Downarrow$

Neutrino Physics

(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]
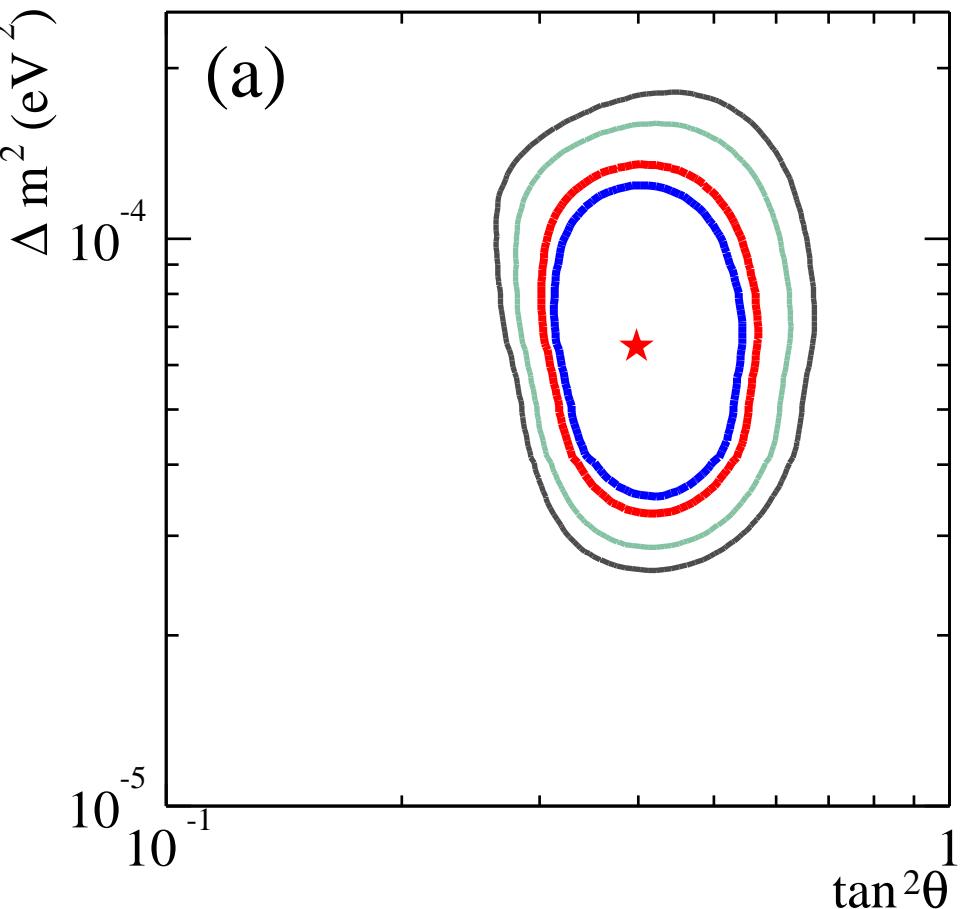
$\nu_e \rightarrow \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 5 \times 10^{-5} \, \text{eV}^2$$

$$\tan^2 \vartheta \simeq 0.4$$
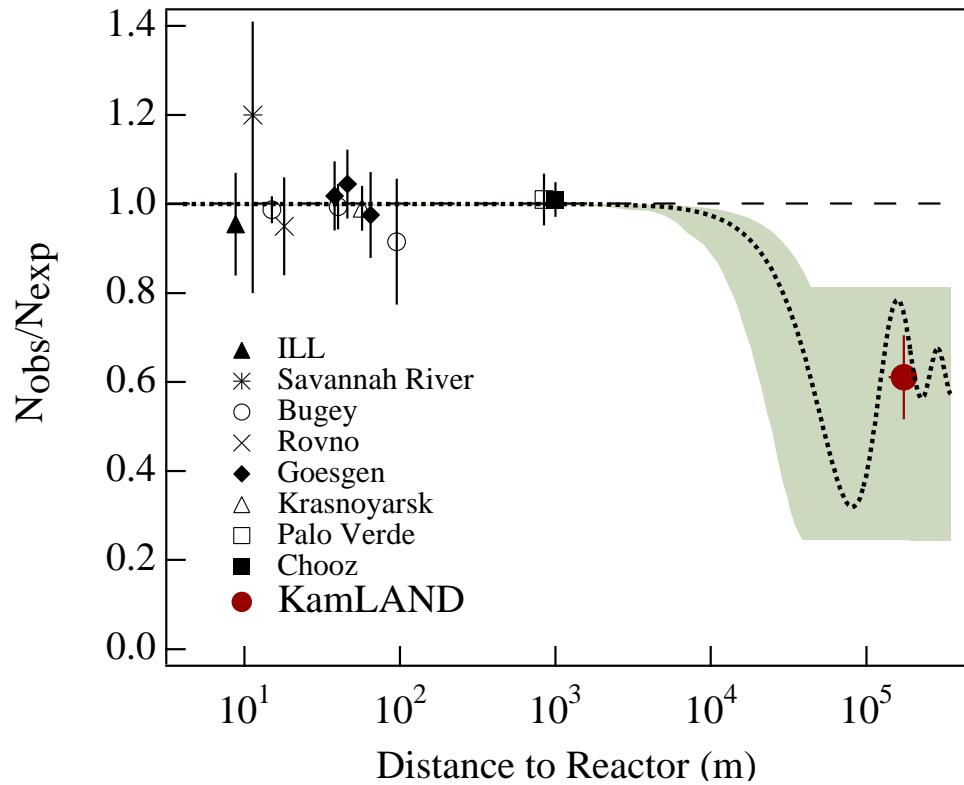
90%, 95%, 99%, 99.73% C.L.

[SNO, PRL 92 (2004) 181301, nucl-ex/0309004]
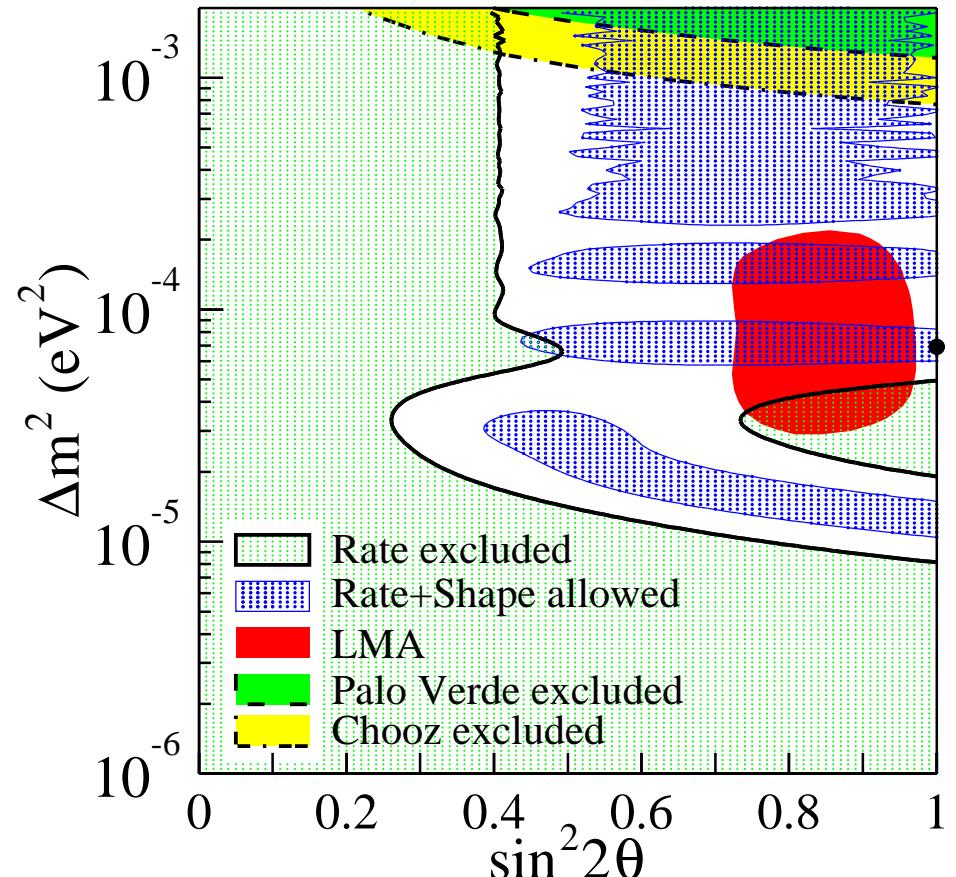
KamLAND

confirmation of LMA (December 2002)

Shade: 95% C.L. LMA

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$
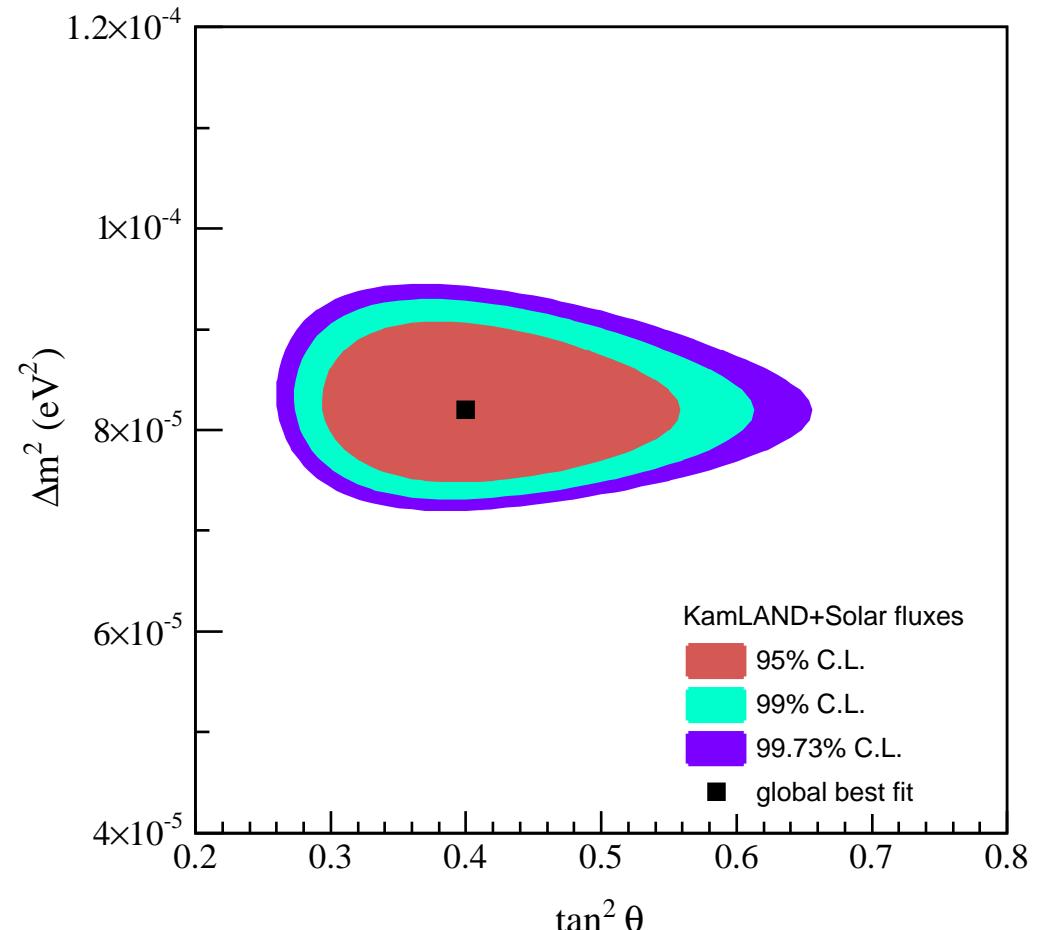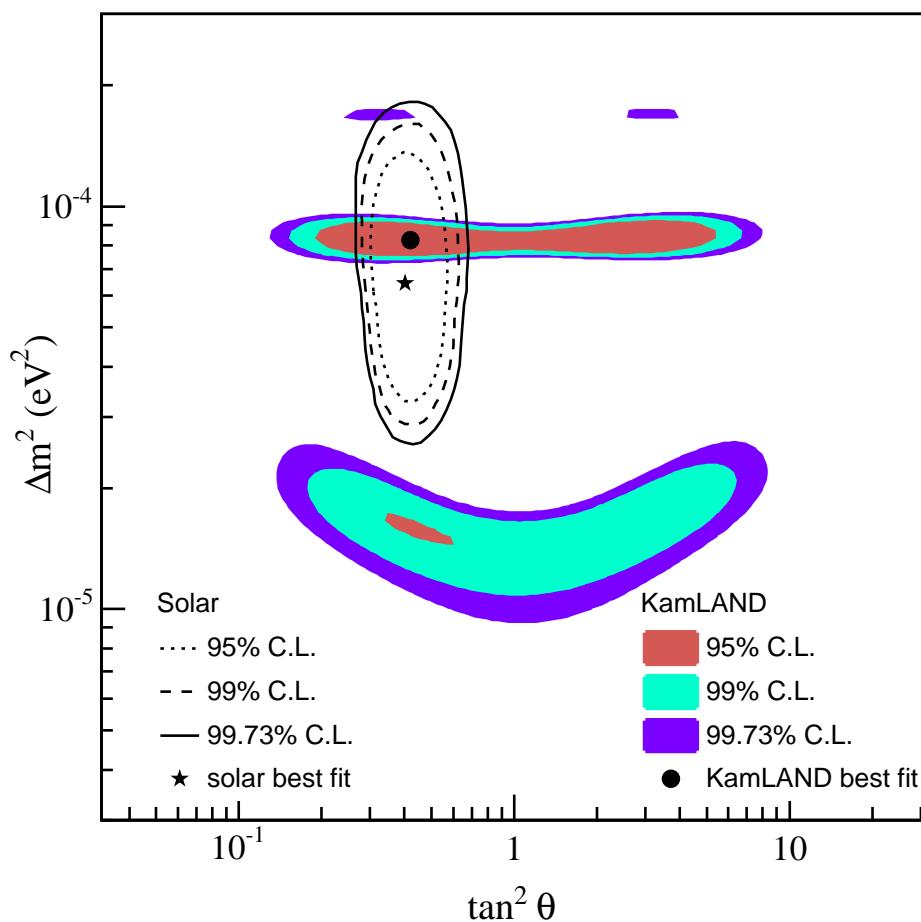
95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

Combined Fit of Solar + Reactor Neutrino Data

Best Fit: $\Delta m^2 = 0.82^{+0.6}_{-0.5} \times 10^{-5} \, \text{eV}^2$ $\qquad \tan^2 \vartheta = 0.40^{+0.09}_{-0.07}$

Atmospheric Neutrinos

$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios
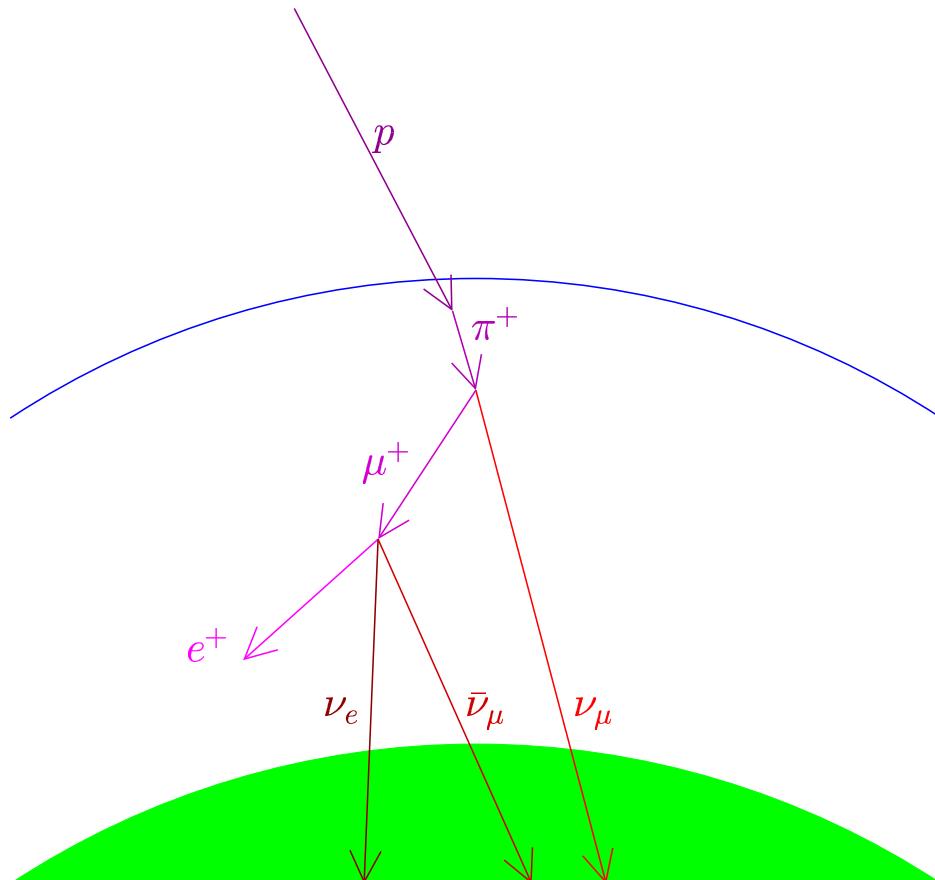
$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$
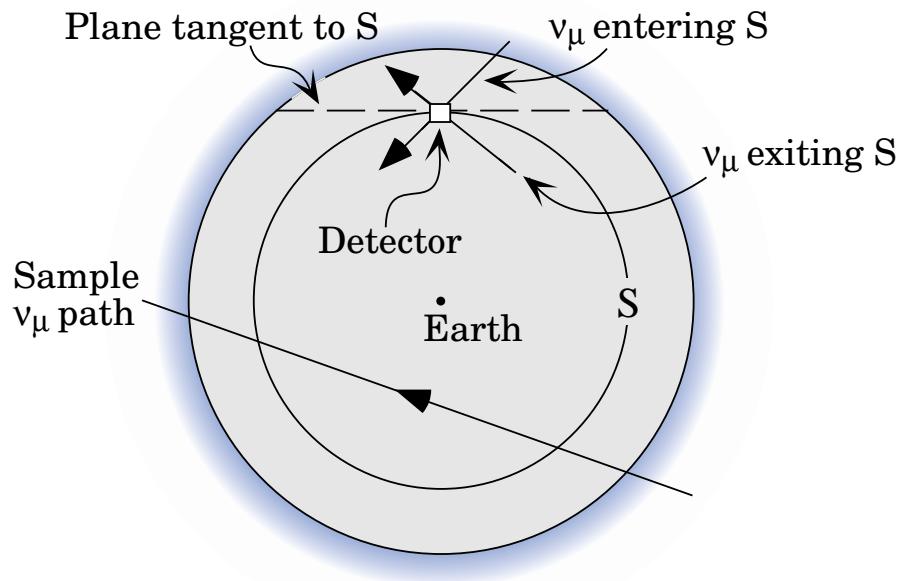
[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry

- any path entering the sphere S later exits
- steady state $\Rightarrow \Phi^{\text{in}}(S) = \Phi^{\text{out}}(S)$
- $E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays
- homogeneity $\Rightarrow \Phi^{\text{in}}(s) = \Phi^{\text{out}}(s),\ \forall s \in S$
- $D \in S \Rightarrow \Phi^{\text{up}}(D) = \Phi^{\text{down}}(D),$

[B. Kayser, Rev. Part. Prop., PRD 66 (2002) 010001]
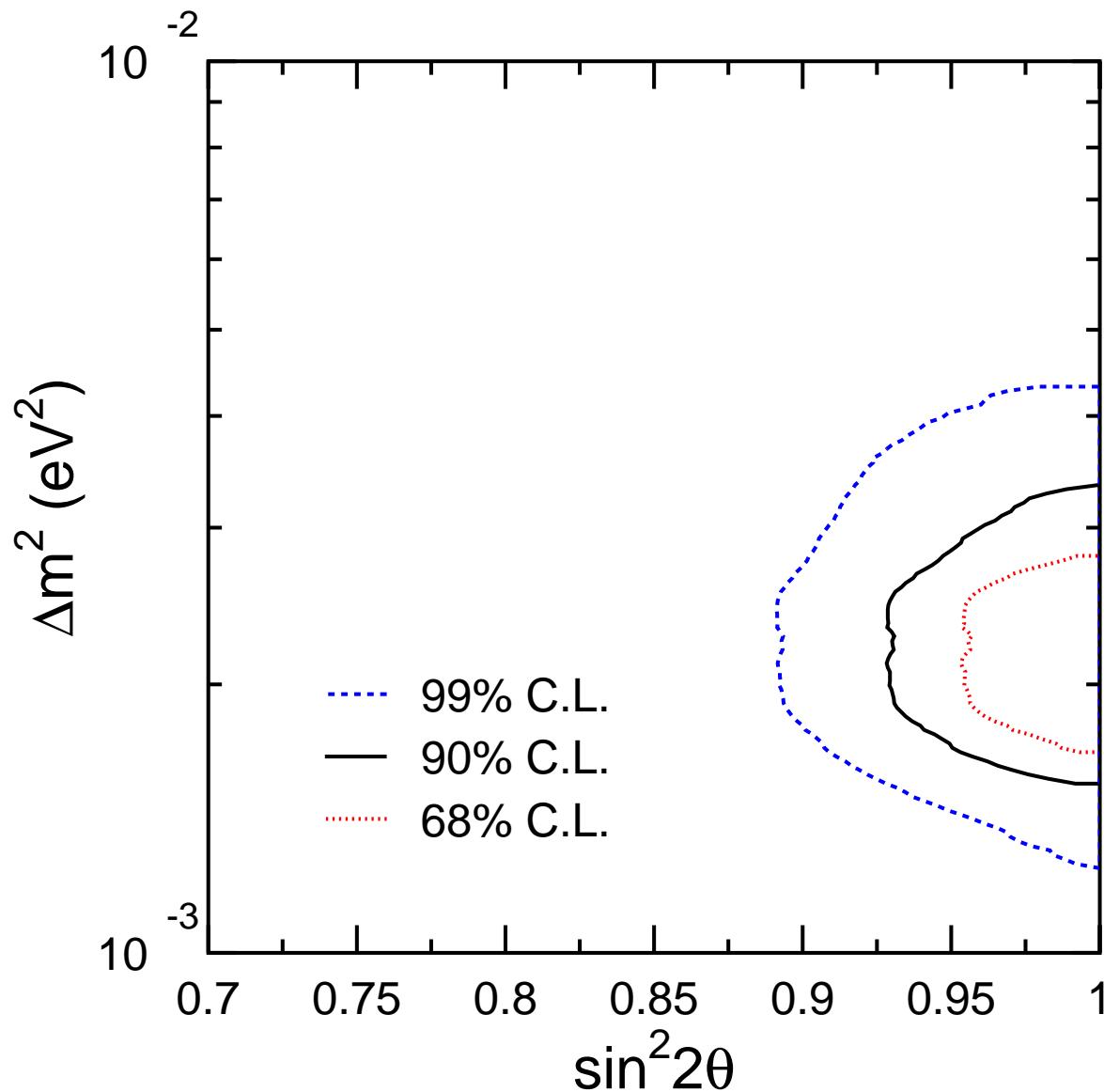
(December 1998)

$$A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}} \right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

$\nu_\mu \to \nu_\tau$ Fit of Super-Kamiokande Atmospheric Data
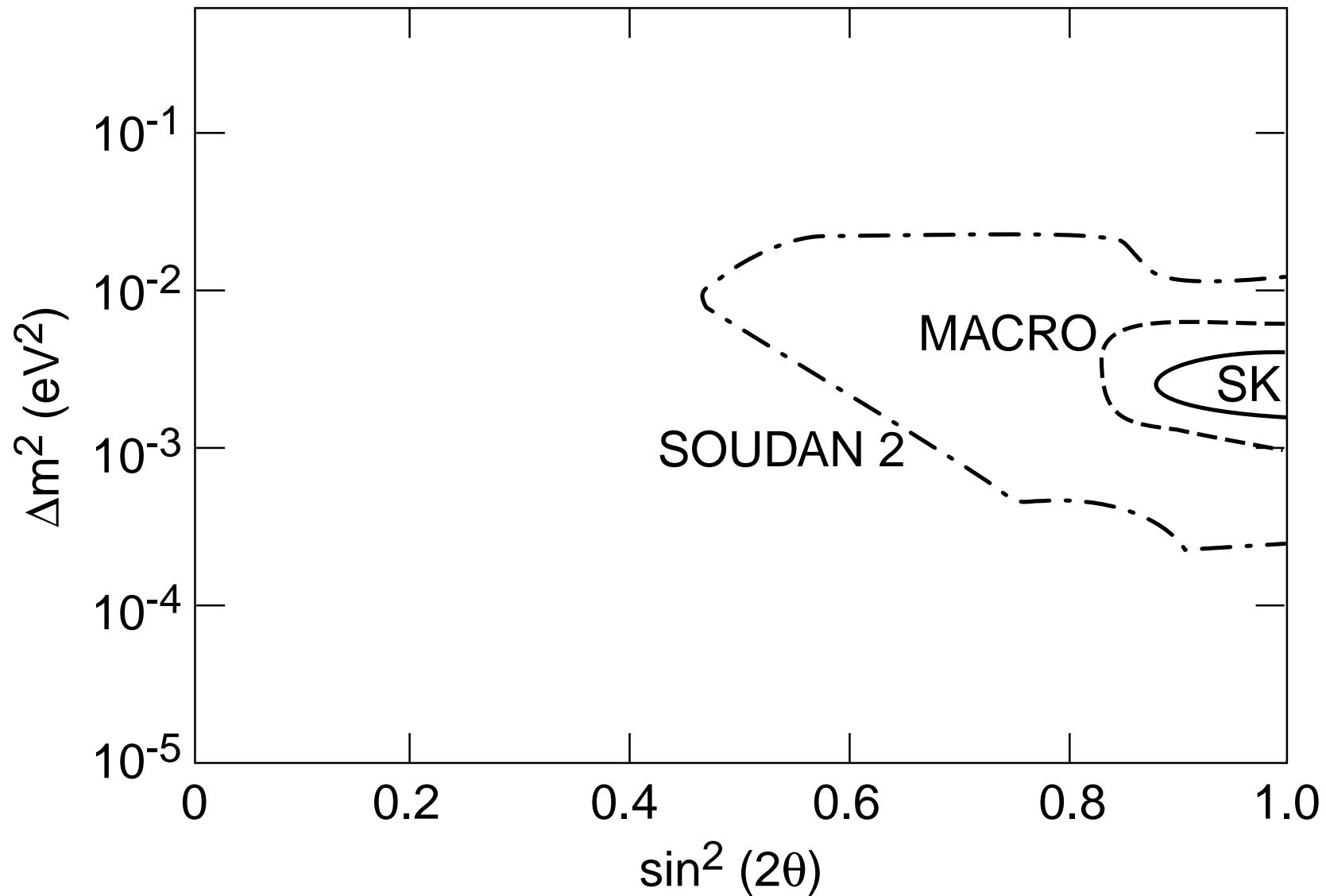
Best Fit
$\Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2$
$\sin^2 2\theta = 1.0$

1489 live-days
April 1996 − July 2001

[Super-Kamiokande, hep-ex/0501064]
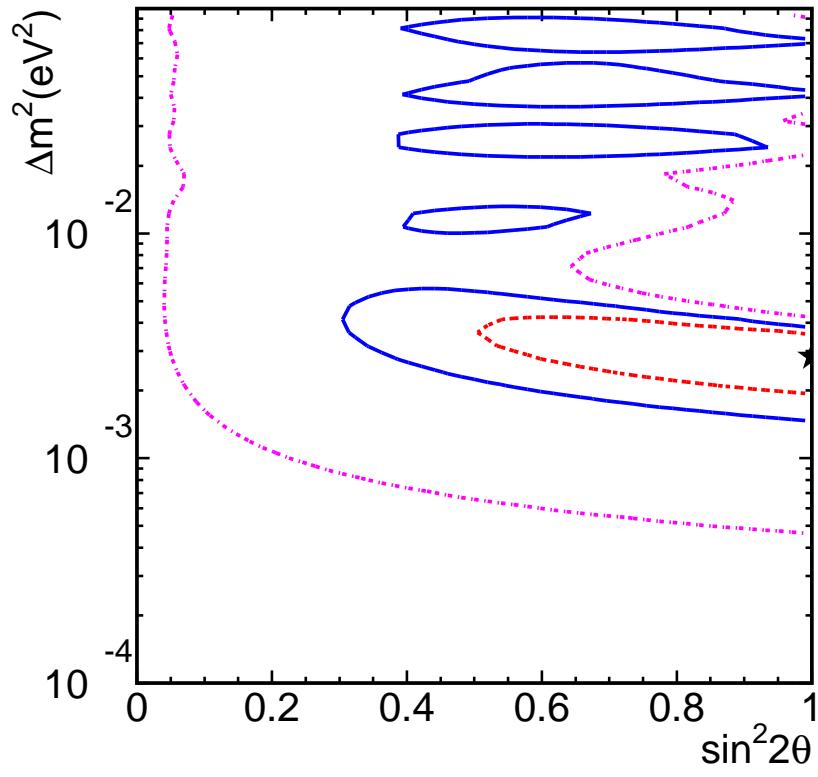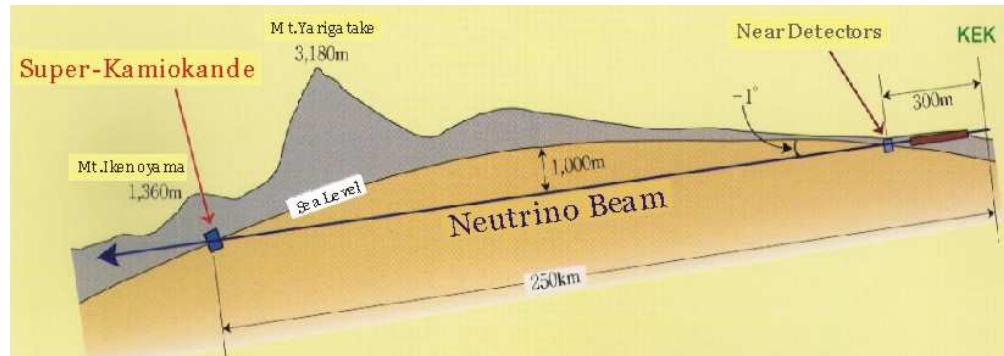
Soudan-2 & MACRO
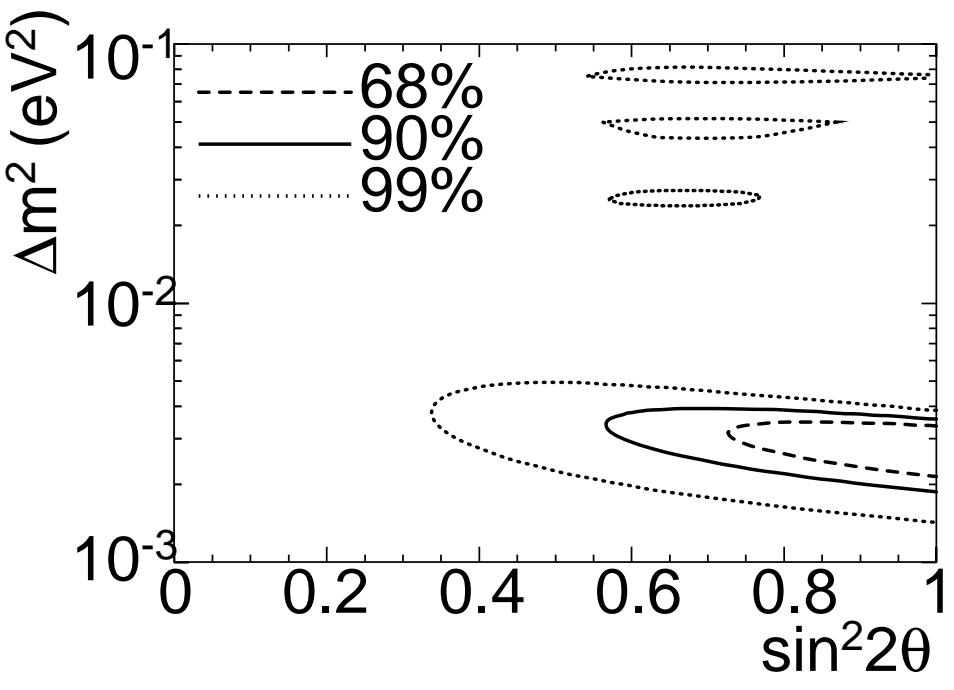

[Giacomelli, Giorgini, Spurio, hep-ex/0201032]

K2K

confirmation of atmospheric allowed region   (June 2002)

[K2K, hep-ex/0411038]

[K2K, Phys. Rev. Lett. 90 (2003) 041801]

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$
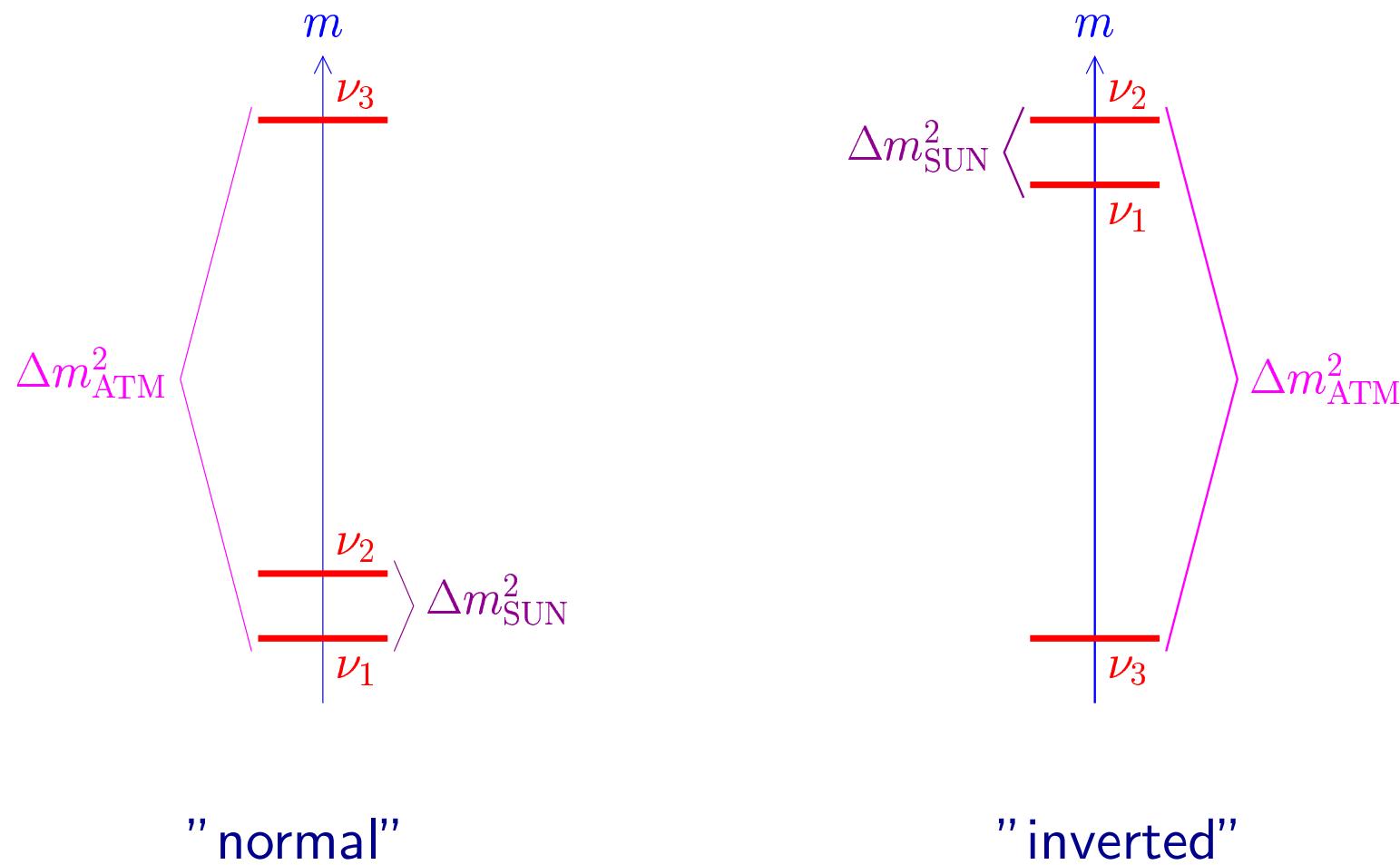
three flavor fields ν_e, ν_μ, ν_τ

three massive fields ν_1, ν_2, ν_3

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \simeq 8.2 \times 10^{-5}\,\text{eV}^2$$
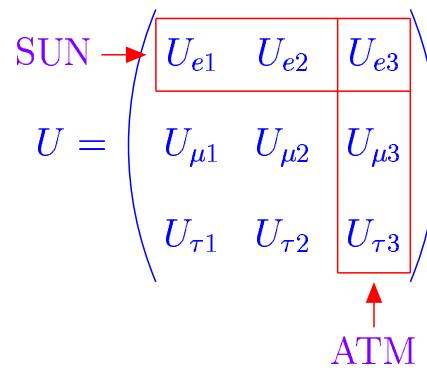
$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes

absolute scale is not determined by neutrino oscillation data

$$\Delta m^2_{21} \ll |\Delta m^2_{31}|$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

SUN $\rightarrow$ (column $U_{e1}, U_{\mu 1}, U_{\tau 1}$)

ATM $\uparrow$ (column $U_{e3}, U_{\mu 3}, U_{\tau 3}$)

Analysis A

$\nu_e \rightarrow \nu_x$

90% CL Kamiokande (multi-GeV)

90% CL Kamiokande (sub+multi-GeV)

95% CL

90% CL
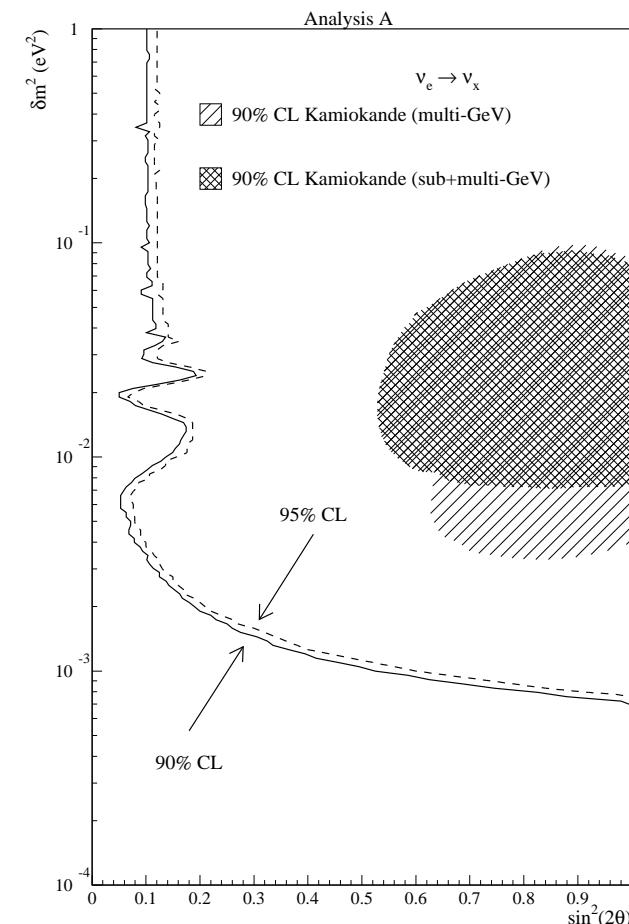
CHOOZ:
$$\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2 (1 - |U_{e3}|^2) \end{cases}$$

$\Downarrow$

$$|U_{e3}|^2 < 5 \times 10^{-2} \ (99.73\% \ \text{C.L.})$$

[Fogli et al., PRD 66 (2002) 093008]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!

[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2 \vartheta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2 \vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, Giunti, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = R_{23}\, W_{13}\, R_{12}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\qquad\quad \vartheta_{23} \simeq \vartheta_{\text{ATM}} \qquad\qquad\qquad \vartheta_{13} = \vartheta_{\text{CHOOZ}} \qquad\qquad \vartheta_{12} = \vartheta_{\text{SUN}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Global Fit of Oscillation Data $\Rightarrow$ Bilarge Mixing

$$\Delta m^2_{\text{SUN}} = 7.92 \, (1 \pm 0.09) \times 10^{-5} \, \text{eV}^2 \qquad \sin^2 \vartheta_{\text{SUN}} = 0.314 \left(1^{+0.18}_{-0.15}\right)$$

$$\Delta m^2_{\text{ATM}} = 2.4 \left(1^{+0.21}_{-0.26}\right) \times 10^{-3} \, \text{eV}^2 \qquad \sin^2 \vartheta_{\text{ATM}} = 0.44 \left(1^{+0.41}_{-0.22}\right)$$

$$\sin^2 \vartheta_{\text{CHOOZ}} = 0.9^{+2.3}_{-0.9} \times 10^{-2}$$
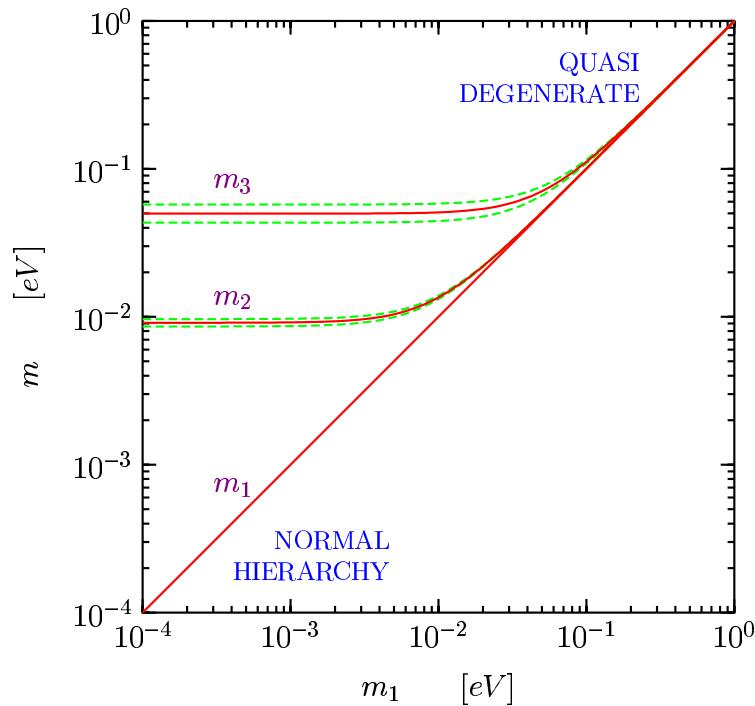
[Fogli, Lisi, Marrone, Palazzo. hep-ph/0506083]

$$|U|_{\text{bf}} \simeq \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.31 - 0.43 & 0.51 - 0.59 & 0.75 \\ 0.37 - 0.47 & 0.59 - 0.66 & 0.66 \end{pmatrix}$$

$$|U|_{3\sigma} \simeq \begin{pmatrix} 0.78 - 0.86 & 0.51 - 0.61 & 0.00 - 0.18 \\ 0.19 - 0.57 & 0.39 - 0.73 & 0.61 - 0.80 \\ 0.20 - 0.57 & 0.40 - 0.74 & 0.59 - 0.79 \end{pmatrix}$$

Absolute Scale of Neutrino Masses
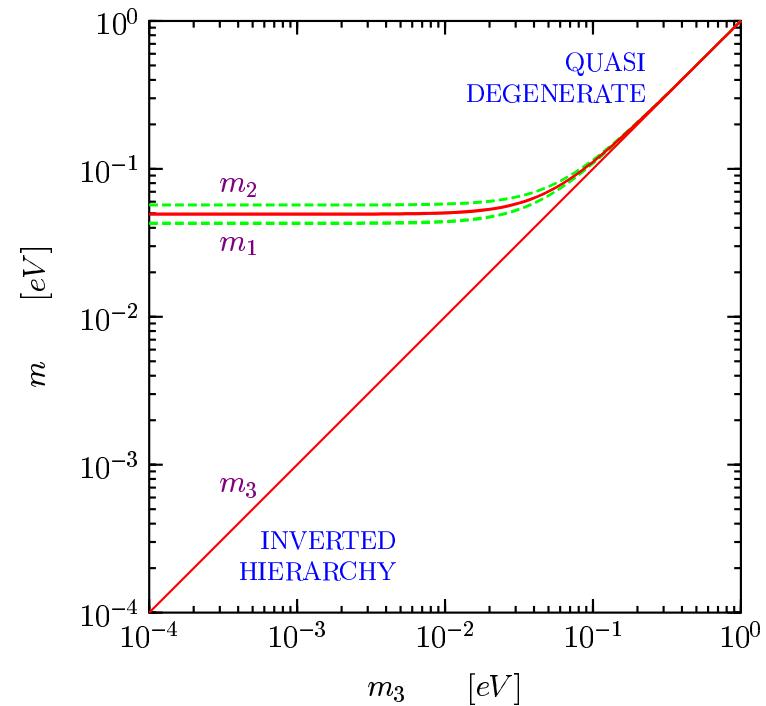
normal scheme

inverted scheme

$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium β Decay

$$^3\text{H} \rightarrow \, ^3\text{He} + e^- + \bar{\nu}_e$$
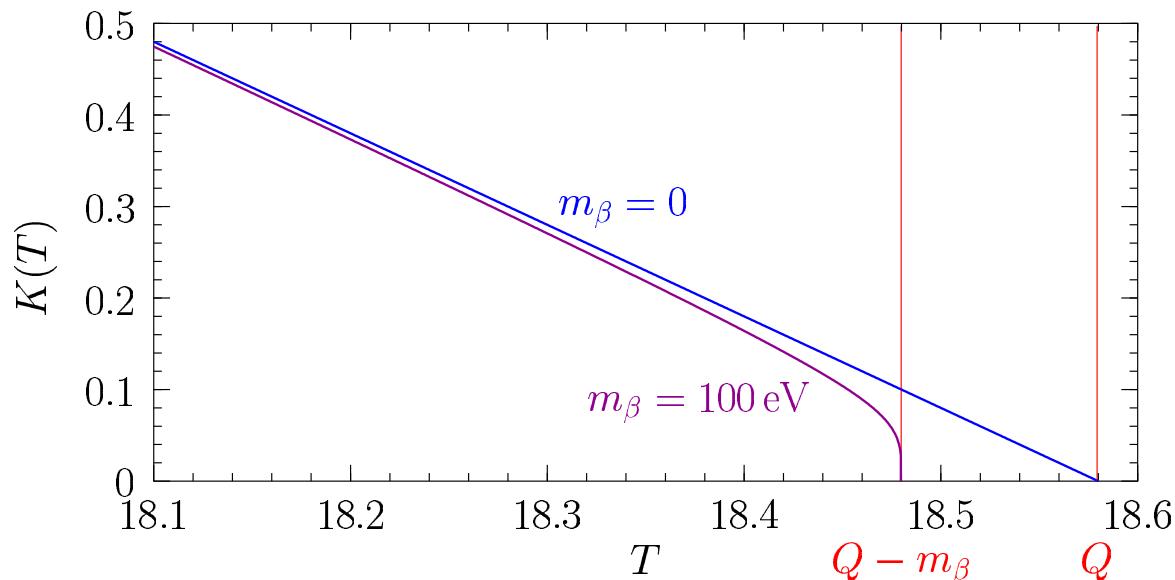
$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C\, G_F)^2}{2\pi^3}\, |\mathcal{M}|^2\, F(E)\, pE\, (Q-T)\sqrt{(Q-T)^2 - m_\beta^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

$$\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$$

Kurie plot:
$$K(T) = \sqrt{\frac{d\Gamma/dT}{\dfrac{(\cos\vartheta_C\, G_F)^2}{2\pi^3}\, |\mathcal{M}|^2\, F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_\beta^2}\right]^{1/2}$$
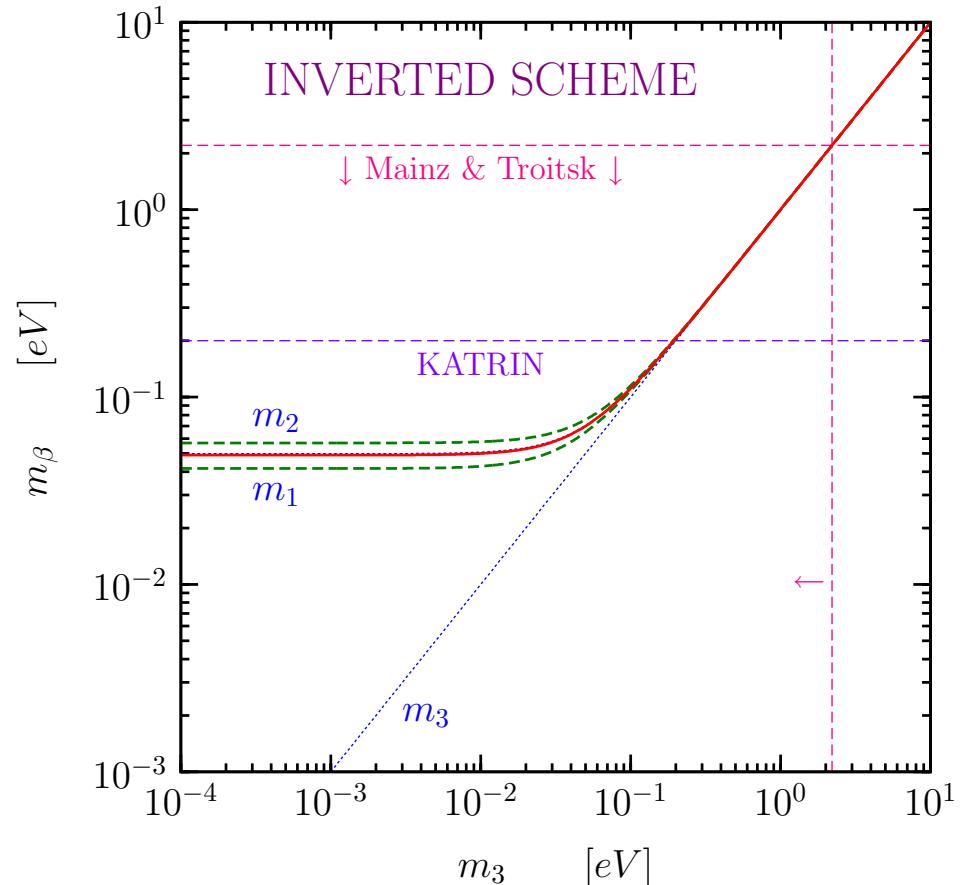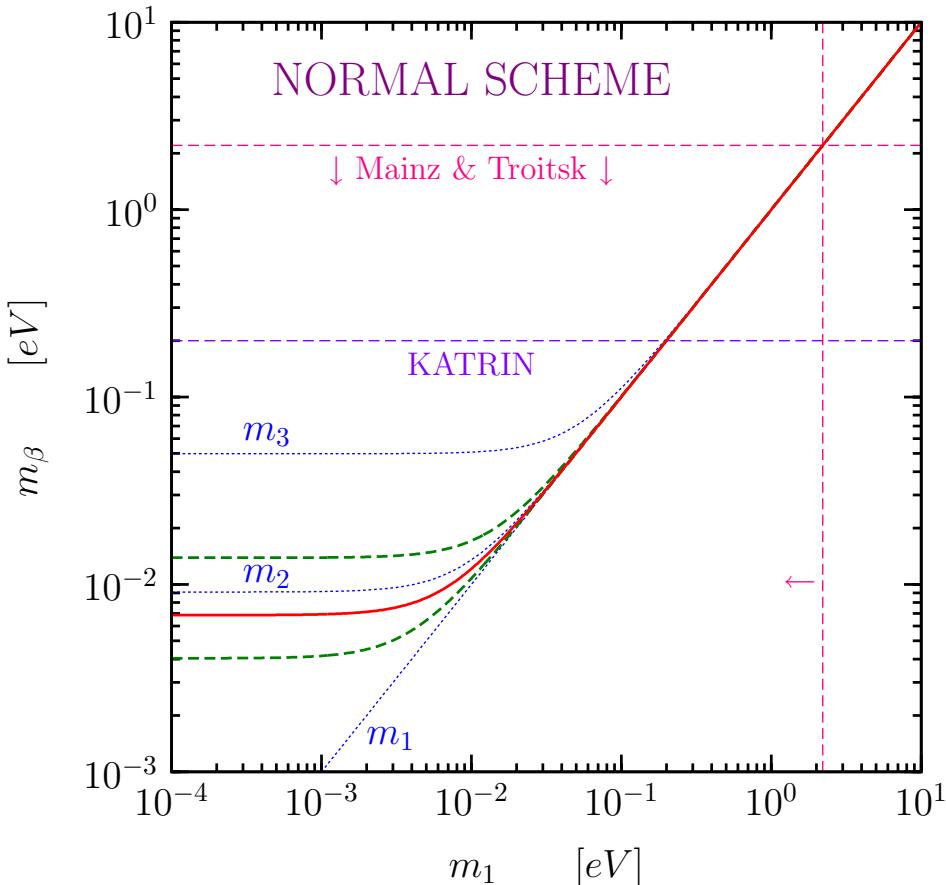
$$\boxed{m_\beta < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$$

Mainz & Troitsk
[Weinheimer, hep-ex/0210050]

future: KATRIN
[hep-ex/0109033]   [hep-ex/0309007]

sensitivity: $m_\beta \simeq 0.2 - 0.3\,\text{eV}$

$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$

Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\mathrm{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+ e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

Relic Neutrinos: $\quad T_\nu = \left(\dfrac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \simeq 1.676 \times 10^{-4}\,\text{eV}$

$(T_\gamma = 2.725 \pm 0.001\,\text{K})$

number density: $\quad n_f = \dfrac{3}{4}\dfrac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k, \bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$
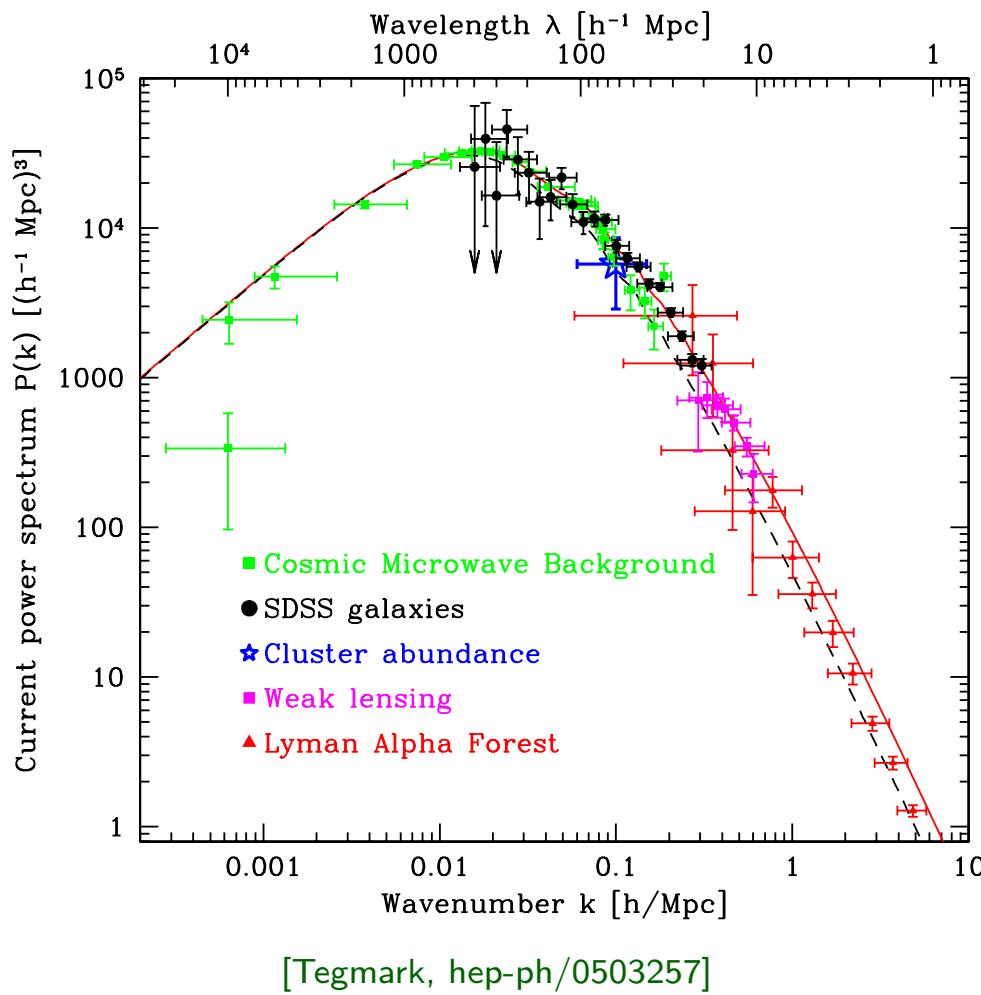
density contribution: $\quad \Omega_k = \dfrac{n_{\nu_k, \bar{\nu}_k}\, m_k}{\rho_c} \simeq \dfrac{1}{h^2}\dfrac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\, h^2 = \dfrac{\sum_k m_k}{94.14\,\text{eV}}}$

$\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations

[Tegmark, hep-ph/0503257]

Solid Curve: flat scalar scale-invariant ΛCDM model
$$(\Omega_M^0 = 0.28, \ h = 0.72, \ \Omega_B^0/\Omega_M^0 = 0.16)$$

Dashed Curve: $\sum_k m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m}$$

$$\approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m\,h^2}\right)$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\,\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

CMB (WMAP, CBI, ACBAR) + LSS (2dFGRS) + Lyα + HST + SN-Ia

ΛCDM

$$T_0 = 13.7 \pm 0.1 \, \text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_{\text{tot}} = 1.02 \pm 0.02 \qquad \Omega_b h^2 = 0.0224 \pm 0.0009 \qquad \Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_\nu h^2 < 0.0076 \quad (95\% \text{ conf.}) \implies \sum_k m_k < 0.71 \, \text{eV}$$

$\sum_k m_k < 1.01 \, \text{eV}$ (95% conf.) WMAP+CBI+2dFGRS+HST+SN-Ia

$\sum_k m_k < 1.20 \, \text{eV}$ (95% conf.) WMAP+CBI+2dFGRS

$\sum_k m_k < 2.12 \, \text{eV}$ (95% conf.) WMAP+2dFGRS

$\sum_k m_k < 1.1 \, \text{eV}$ (95% conf.) WMAP+2dFGRS+HST

CMB(WMAP)+LSS(SDSS)+SN-Ia

$$h = 0.70^{+0.04}_{-0.03} \quad \Omega_m = 0.30 \pm 0.04 \quad \sum_k m_k < 1.7\,\text{eV} \quad (95\% \text{ conf.})$$

CMB(WMAP)+LSS(SDSS)+bias(SDSS) $\qquad P_g(k) = b^2\,P_m(k)$

$$\Omega_m = 0.25 \pm 0.03 \qquad \sum_k m_k < 0.54\,\text{eV} \quad (95\% \text{ conf.})$$

CMB(WMAP)+LSS(SDSS)+bias(SDSS)+Lyα(SDSS)+SN-Ia

$$\Omega_\Lambda = 0.72 \pm 0.02 \qquad \sum_k m_k < 0.42\,\text{eV} \quad (95\% \text{ conf.})$$
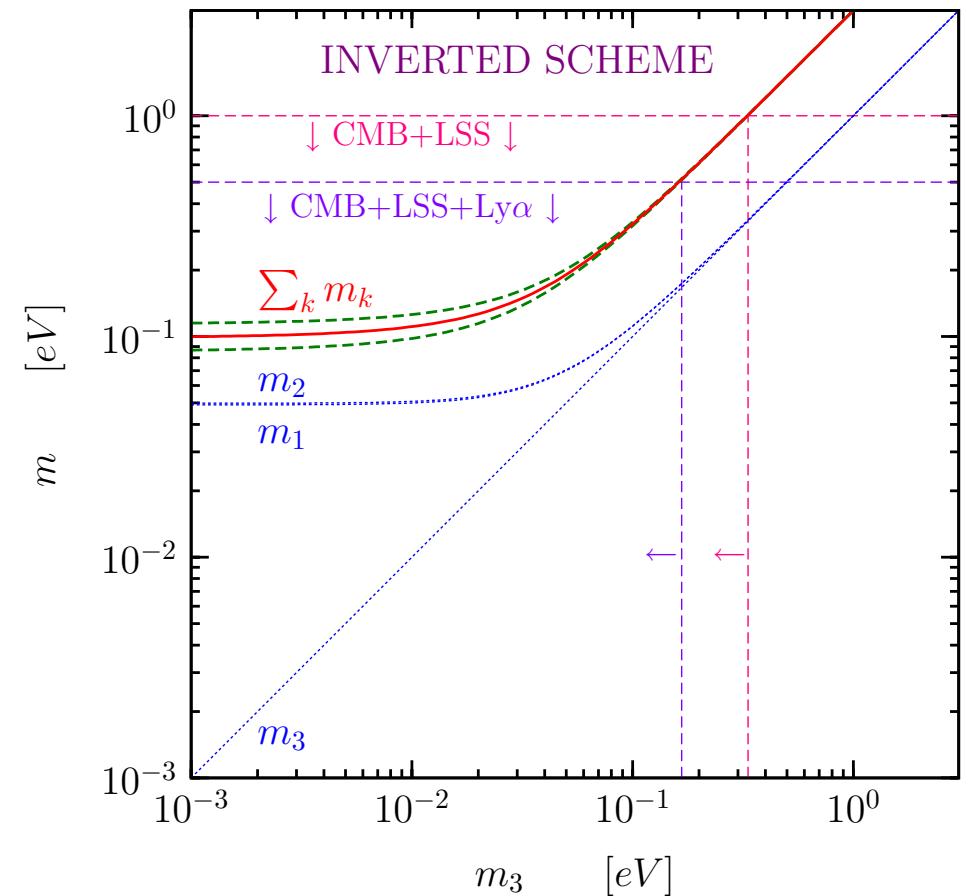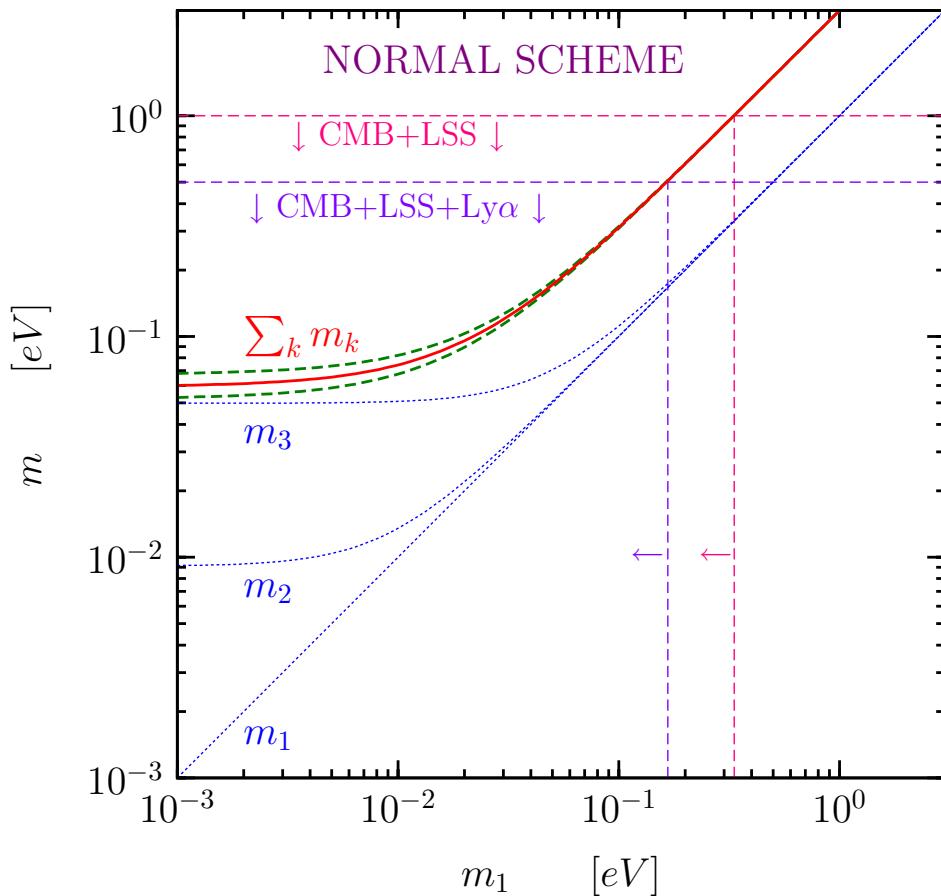
$$\sum_k m_k < 1.4\,\text{eV} \qquad (2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia}$$
$$\sum_k m_k < 0.47\,\text{eV} \qquad (2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia+Ly}\alpha\text{(SDSS)}$$

$$\sum_k m_k \lesssim 1\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia}$$
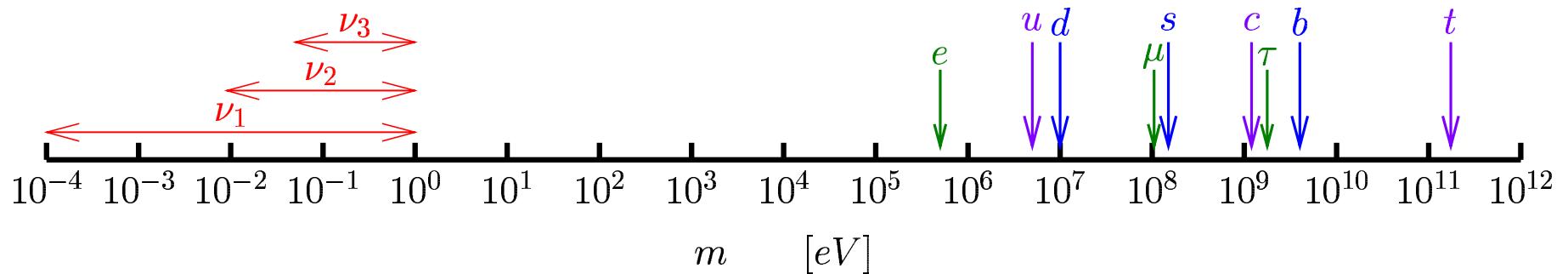
$$\sum_k m_k \lesssim 0.5\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia+Ly}\alpha$$

$$\text{FUTURE: IF } \sum_k m_k \lesssim 8 \times 10^{-2}\,\text{eV} \implies \text{NORMAL HIERARCHY}$$

Majorana Neutrino Mass?

known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M}$ $\Rightarrow$
$\begin{cases} \text{See-Saw Mechanism (if } \nu_R\text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{cases}$

both imply
$\begin{cases} \text{Majorana } \nu \text{ masses} \Longleftrightarrow |\Delta L| = 2 \Longleftrightarrow \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \dfrac{\mathcal{M}^2_{\text{EW}}}{\mathcal{M}} \end{cases}$
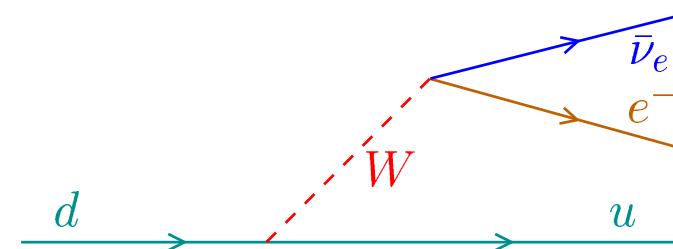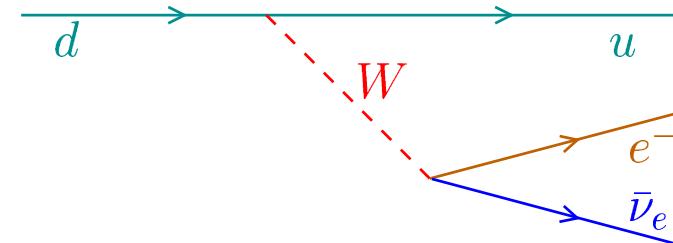
Majorana neutrino masses provide the most accessible
window on New Physics Beyond the Standard Model

Two-Neutrino Double-β Decay: $\Delta L = 0$

$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$

$$\left(T_{1/2}^{2\nu}\right)^{-1} = G_{2\nu} \, |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model
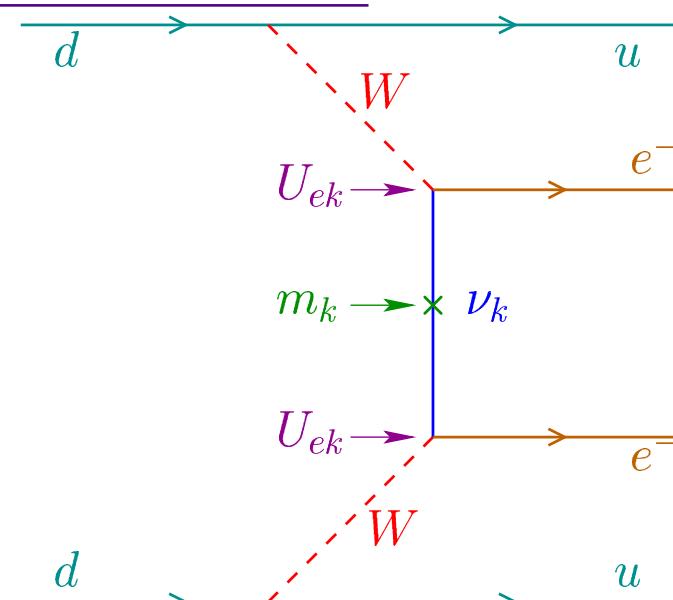
Neutrinoless Double-β Decay: $\Delta L = 2$

$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z + 2) + e^- + e^-$

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, |m_{\beta\beta}|^2$$

effective
Majorana
mass
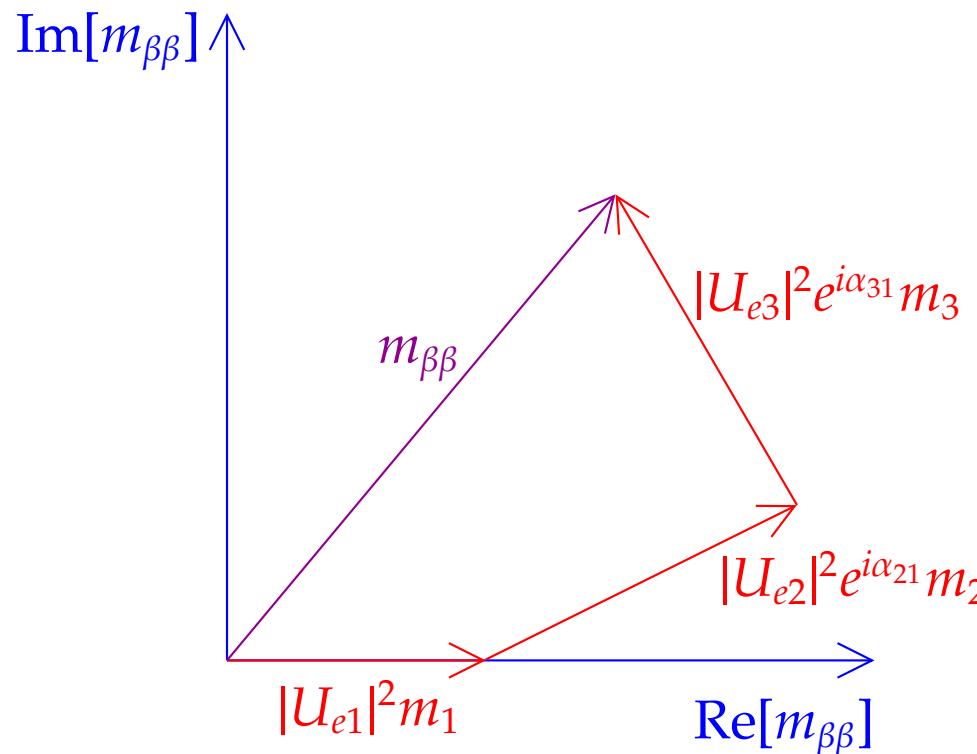
$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

Effective Majorana Neutrino Mass in $\beta\beta_{0\nu}$ Decay

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

complex $U_{ek} \Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$

Best limits for $\beta\beta_{0\nu}$ Decay

Heidelberg-Moscow $\qquad$ ^{76}Ge $\qquad$ [EPJA 12 (2001) 147]

$$T_{1/2}^{0\nu} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}$$

IGEX $\qquad$ ^{76}Ge $\qquad$ [PRD 65 (2002) 092007]

$$T_{1/2}^{0\nu} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.35 - 1.1\,\text{eV}$$

FUTURE EXPERIMENTS

NEMO3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES
$$|m_{\beta\beta}| \sim \text{few}\, 10^{-1}\,\text{eV}$$
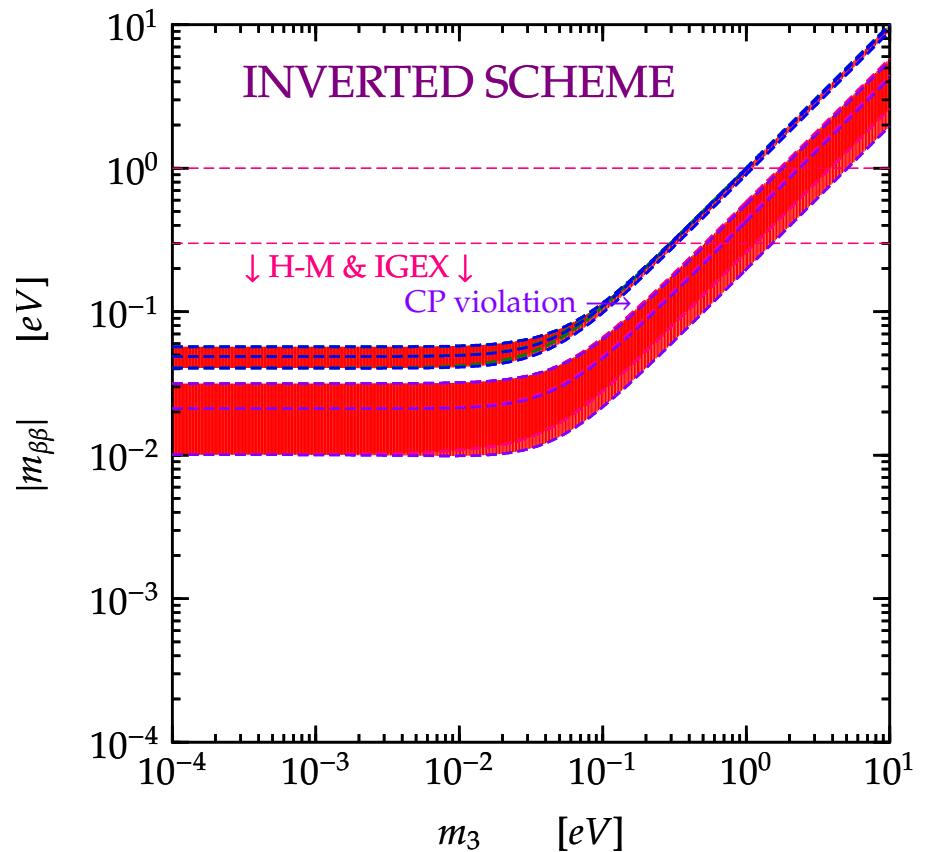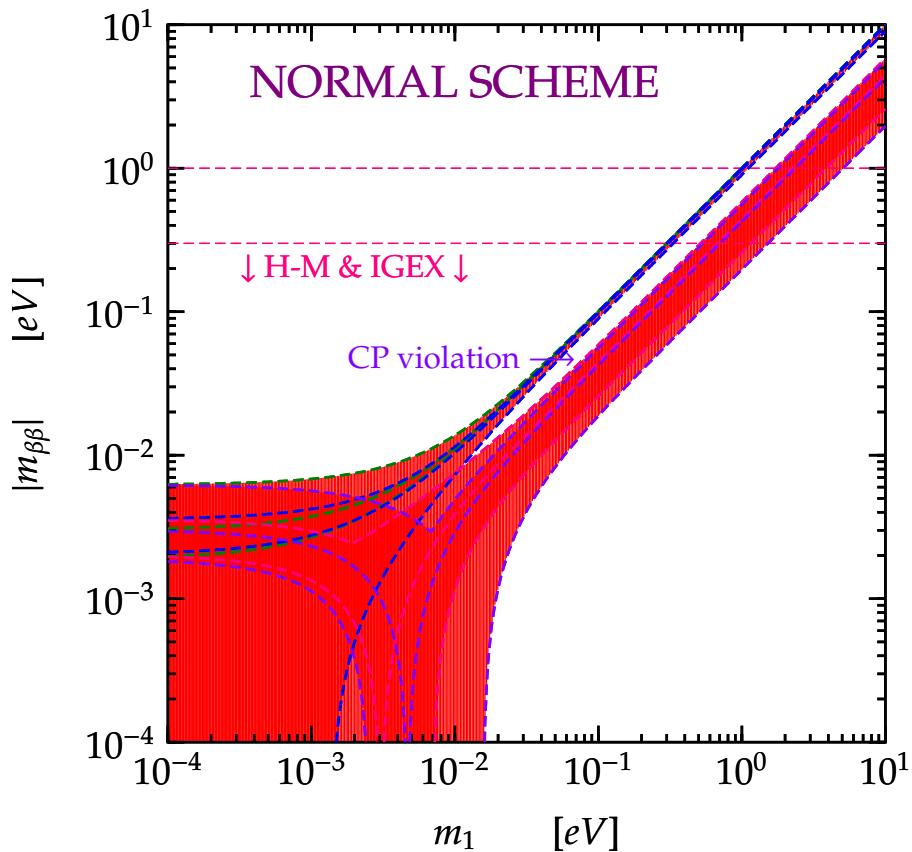
EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA
$$|m_{\beta\beta}| \sim \text{few}\, 10^{-2}\,\text{eV}$$

[Zdesenko, RMP 74 (2002) 663] [Elliott,Vogel, ARNPS 52 (2002) 115] [Elliott, Engel, JPG 30 (2004) R183]

General Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_{21}}\, m_2 + |U_{e3}|^2\, e^{i\alpha_{31}}\, m_3$$
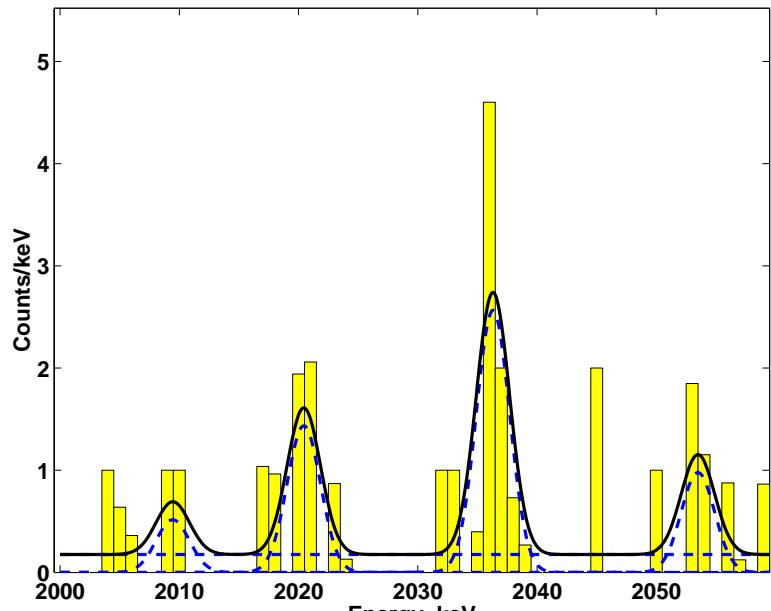

FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}$ eV $\Longrightarrow$ NORMAL HIERARCHY
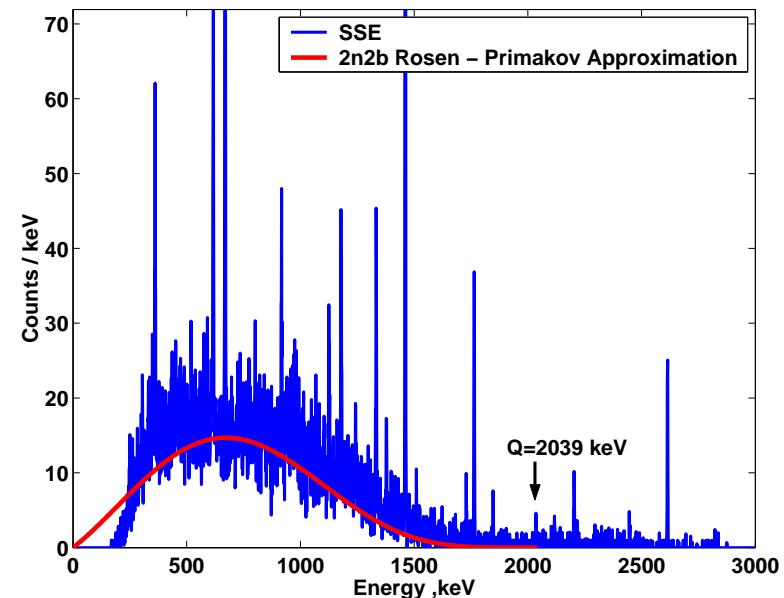
Indication of $\beta\beta_{0\nu}$ Decay

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$$T_{1/2}^{0\nu\,\text{bf}} = 1.19 \times 10^{25}\,\text{y} \quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\text{y}\,(3\sigma) \quad 4.2\sigma\ \text{evidence}$$

pulse-shape selected spectrum
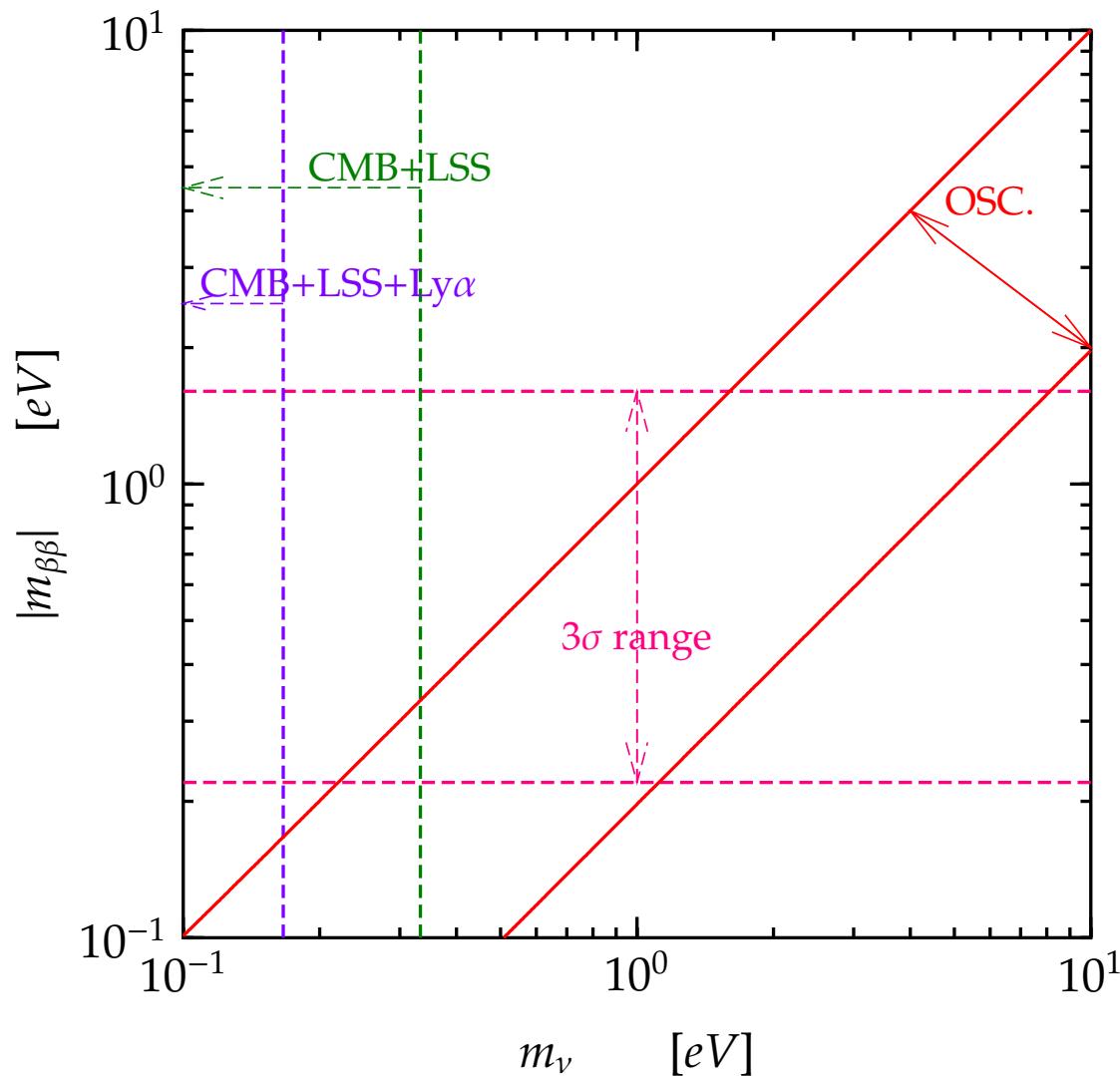
3.8σ evidence   [PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \implies 0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$$

if confirmed, very exciting (Majorana ν and large mass scale)

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ (3σ range)
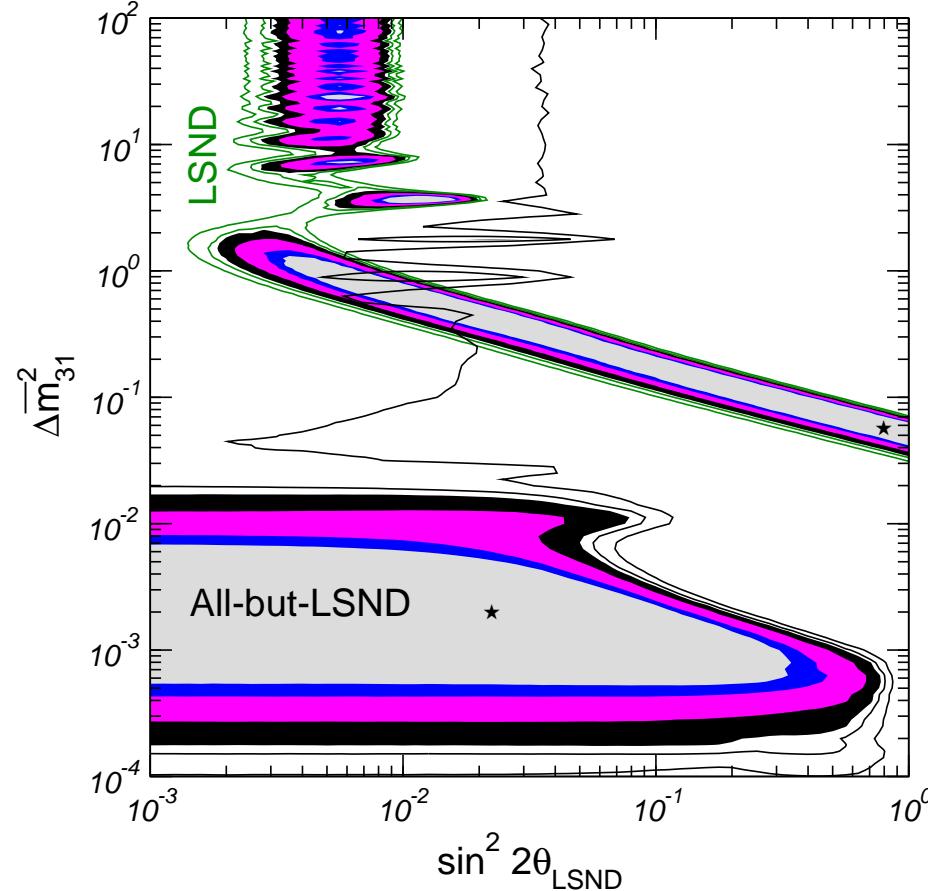
tension among oscillation data, CMB+LSS+Lyα and $\beta\beta_{0\nu}$ signal

? LSND ?

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\Delta m^2_{\text{LSND}} \gtrsim 0.1 \, \text{eV}^2 \; (\gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SUN}})$
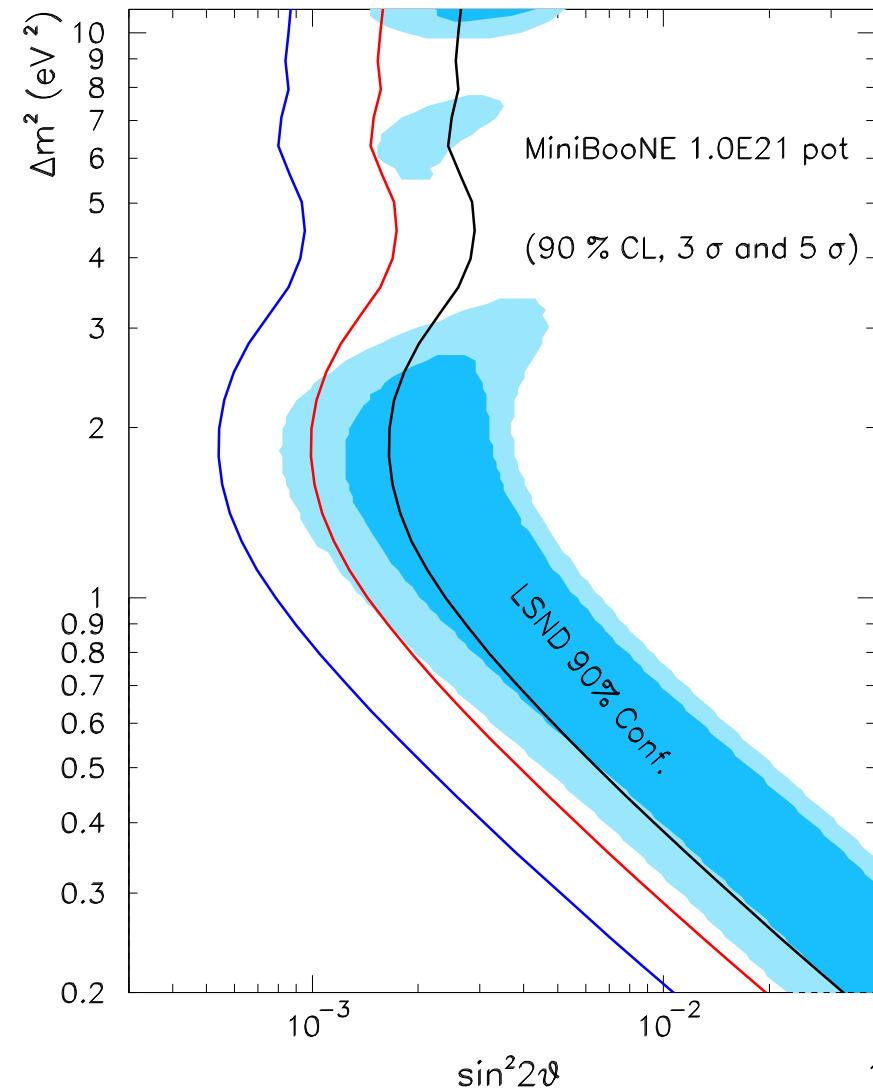
[Gonzalez-Garcia, Maltoni, Schwetz PRD 68 (2003) 053007]

even allowing $\Delta\overline{m}^2_{31} \neq \Delta m^2_{31}$

(CPT violation) GoF $= 7.5 \times 10^{-4}$

MiniBooNE $\nu_\mu \to \nu_e$ [hep-ex/0406048]

Summary

$\nu_e \rightarrow \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SUN}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \rightarrow \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (atmospheric ν, K2K)

$$\Downarrow$$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?
Why only $|U_{e3}|^2 \ll 1$?
Improve calculation of $\mathcal{M}_{0\nu}$!

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP violation
Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale
Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements