Neutrino Flavor States and the Quantum Theory of Neutrino Oscillations

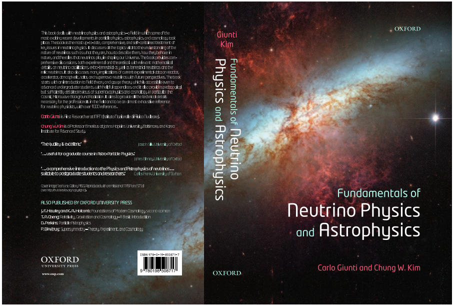
Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

mailto://giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

MPI Heidelberg, 22 October 2007



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Outline

- Standard Theory of Neutrino Oscillations
- Flavor Neutrino States
- Neutrino Production and Detection
- Covariant Plane-Wave Theory of NuOsc
- Wave-Packet Theory of NuOsc

Ultrarelativistic Approximation

Laboratory and Astrophysical Limits $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

Only neutrinos with energy $E \gtrsim 0.2\,\text{MeV}$ are detectable!

Charged-Current Processes: Threshold

$$\nu + A \to B + C + \ldots$$
$$\Downarrow$$
$$s = 2Em_A + m_A^2 \geq (m_B + m_C + \ldots)^2$$
$$\Downarrow$$
$$E_{\text{th}} = \frac{(m_B + m_C + \ldots)^2}{2m_A} - \frac{m_A}{2}$$

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$	$E_{\text{th}} = 0.233\,\text{MeV}$
$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$	$E_{\text{th}} = 0.81\,\text{MeV}$
$\bar{\nu}_e + p \to n + e^+$	$E_{\text{th}} = 1.8\,\text{MeV}$
$\nu_\mu + n \to p + \mu^-$	$E_{\text{th}} = 110\,\text{MeV}$
$\nu_\mu + e^- \to \nu_e + \mu^-$	$E_{\text{th}} = 10.9\,\text{GeV}$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$\nu + e^- \to \nu + e^-$ $\qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO)

Example of Neutrino Production: Pion Decay

$$\pi^+ \to \mu^+ + \nu_\mu (\text{superposition of } \nu_1, \nu_2, \nu_3)$$

two-body decay $\Longrightarrow$ fixed kinematics $\quad \boxed{E_k^2 = p_k^2 + m_k^2} \quad k = 1, 2, 3$

π at rest:

$$\begin{cases} p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\, m_\pi^2} \\ E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\, m_\pi^2} \end{cases}$$

0^{th} order: $m_k = 0 \Rightarrow p_k = E_k = E = \dfrac{m_\pi}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1^{st} order: $\boxed{E_k \simeq E + \xi \dfrac{m_k^2}{2E}} \qquad \boxed{p_k \simeq E - (1 - \xi)\dfrac{m_k^2}{2E}}$

$$\xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

Standard Derivation of Neutrino Oscillations

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

[Bilenky, Pontecorvo, Nuovo Cim. Lett. 17 (1976) 569] [Bilenky, Pontecorvo, Phys. Rep. 41 (1978) 225]

Flavor Neutrino Production: $j_{\rho}^{\mathrm{CC}} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \, \gamma_{\rho} \, \ell_{\alpha L}$

$$\boxed{\nu_{\alpha L} = \sum_k U_{\alpha k} \, \nu_{kL}}$$

Fields $\overline{\nu_{\alpha L}} = \sum_k U_{\alpha k}^{*} \, \overline{\nu_{kL}} \quad \Longrightarrow \quad |\nu_{\alpha}\rangle = \sum_k U_{\alpha k}^{*} \, |\nu_k\rangle$ States

$$\mathcal{H} |\nu_k\rangle = E_k |\nu_k\rangle \quad \Rightarrow \quad |\nu_k(t)\rangle = e^{-iE_k t} \, |\nu_k\rangle \quad \Rightarrow \quad |\nu_{\alpha}(t)\rangle = \sum_k U_{\alpha k}^{*} \, e^{-iE_k t} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} \, |\nu_{\beta}\rangle \quad \Rightarrow \quad |\nu_{\alpha}(t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^{*} e^{-iE_k t} U_{\beta k} \right)}_{\mathcal{A}_{\nu_{\alpha} \to \nu_{\beta}}(t)} |\nu_{\beta}\rangle$$

Transition Probability

$$P_{\nu_{\alpha} \to \nu_{\beta}}(t) = \left| \langle \nu_{\beta} | \nu_{\alpha}(t) \rangle \right|^2 = \left| \mathcal{A}_{\nu_{\alpha} \to \nu_{\beta}}(t) \right|^2 = \left| \sum_k U_{\alpha k}^{*} e^{-iE_k t} U_{\beta k} \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\,(E_k - E_j)\,t\right]$$

Ultrarelativistic Approximation + Assumption $p_k = p = E$

(neutrinos with the same momentum propagate in the same direction)

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} = E + \frac{m_k^2}{2E} \quad \Longrightarrow \quad E_k - E_j \simeq \frac{\Delta m_{kj}^2}{2E}$$

$$\boxed{\Delta m_{kj}^2 \equiv m_k^2 - m_j^2}$$

$t \simeq L \quad \Longrightarrow$

$$\boxed{P_{\nu_\alpha \to \nu_\beta}(L/E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\,\frac{\Delta m_{kj}^2 L}{2E}\right]}$$

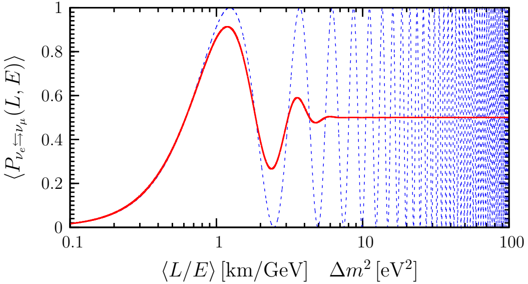
$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L/E) \quad = \quad & \delta_{\alpha\beta} - 4 \sum_{k>j} \mathrm{Re}\left[U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j}\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) \\ & + 2 \sum_{k>j} \mathrm{Im}\left[U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j}\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu) \qquad U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$P_{\nu_e \leftrightarrows \nu_\mu}(L/E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

$$\langle P_{\nu_e \leftrightarrows \nu_\mu}\rangle = \int \mathrm{d}\left(\frac{L}{E}\right)\, \phi\left(\frac{L}{E}\right)\, P_{\nu_e \leftrightarrows \nu_\mu}(L/E)$$

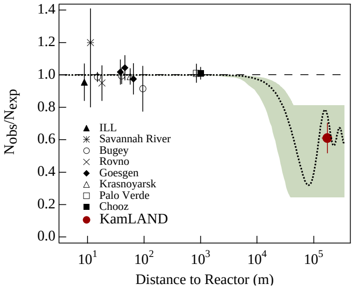


$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma}\,\exp\left(\frac{(x - \langle x\rangle)^2}{2\sigma^2}\right)$$

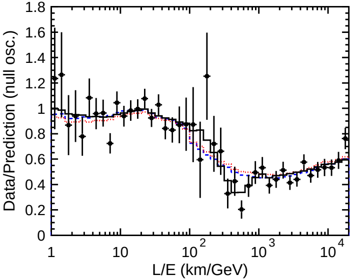
$$\sin^2 2\vartheta = 1$$

$$\sigma_{L/E} = 0.2\,\langle L/E\rangle$$

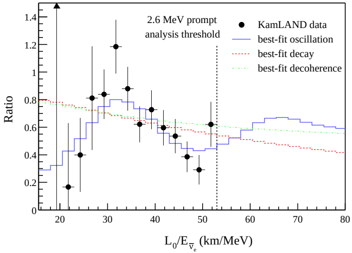
Observations of Neutrino Oscillations



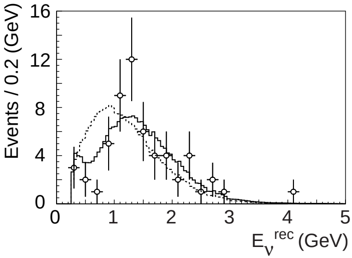
[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]



[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



[KamLAND, PRL 94 (2005) 081801, hep-ex/0406035]



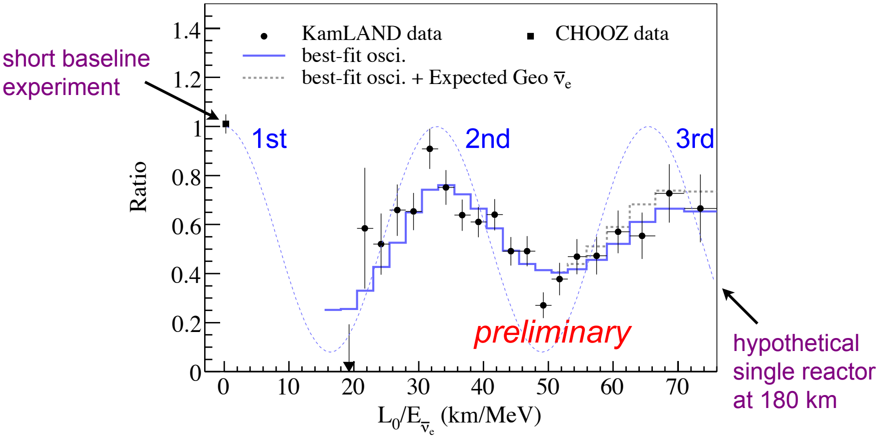
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

KamLAND

Reactor $\bar{\nu}_e \to \bar{\nu}_e$

53 nuclear power reactors in Japan and Korea $\to$ Kamioka Mine

$\langle L \rangle \simeq 180\,\text{km}$ $\langle E \rangle \simeq 4\,\text{MeV}$



[I. Shimizu (KamLAND), TAUP 2007]

Main Assumptions of Standard Theory

(A1)

Flavor neutrinos produced by CC weak interactions are described by the flavor states $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$

Approximation for ultrarelativistic ν's [CG, Kim, Lee, PRD 45 (1992) 2414]

(A2)

Massive neutrino states $|\nu_k\rangle$ have the same momentum $p_k = p = E$ ("Equal Momentum Assumption") and different energies $E_k \simeq E + \frac{m_k^2}{2E}$

Unrealistic assumption (energy-momentum conservation and special relativity), but standard oscillation probability is correct

[Winter, LNC 30 (1981) 101] [CG, Kim, FPL 14 (2001) 213]

[CG, MPLA 16 (2001) 2363, FPL 17 (2004) 103] [Burkhardt et al., PLB 566 (2003) 137]

(A3)

Propagation Time $T = L$ Source-Detector Distance

OK! $\Longleftrightarrow$ Wave Packets

[Nussinov, PLB 63 (1976) 201] [Kayser, PRD 24 (1981) 110] [CG, Kim, Lee, PRD 44 (1991) 3635]

[Kiers, Nussinov, Weiss, PRD 53 (1996) 537] [Beuthe, Phys. Rep. 375 (2003) 105]

[CG, Kim, FPL 14 (2001) 213] [CG, FPL 17 (2004) 103, JPG 34 (2007) R93]

Flavor Neutrino States

Quantum Field Theory: $|f\rangle = \mathsf{S}\,|i\rangle$

If different final channels: $|f\rangle = \sum_k \mathcal{A}_k\,|f_k\rangle$

Production amplitudes: $\mathcal{A}_k = \langle f_k|f\rangle = \langle f_k|\mathsf{S}|i\rangle$

Pion decay: $\pi^+ \to \begin{cases} \mu^+ + \nu_\mu(\text{superposition of } \nu_1, \nu_2, \nu_3) & (99.988\%) \\ e^+ + \nu_e(\text{superposition of } \nu_1, \nu_2, \nu_3) & (0.012\%) \end{cases}$

$$\mathsf{S}\,|\pi^+\rangle = |f\rangle = \sum_{k=1}^{3} \mathcal{A}_{\mu k}\,|\mu^+, \nu_k\rangle + \sum_{k=1}^{3} \mathcal{A}_{ek}\,|e^+, \nu_k\rangle$$

$$\mathcal{A}_{\mu k} = \langle \mu^+, \nu_k | f \rangle = \langle \mu^+, \nu_k | \mathsf{S} | \pi^+ \rangle$$

$$\mathcal{A}_{ek} = \langle e^+, \nu_k | f \rangle = \langle e^+, \nu_k | \mathsf{S} | \pi^+ \rangle$$

μ^+ and e^+ have different interactions with the environment

$\Downarrow$

collapse to $|\mu^+\rangle|\nu_\mu\rangle$ or $|e^+\rangle|\nu_e\rangle$

$|\nu_\mu\rangle$ is the neutrino part of the final state associated with μ^+

$$|\nu_\mu\rangle \propto \langle \mu^+ | f \rangle = \sum_{k=1}^{3} \mathcal{A}_{\mu k}\,|\nu_k\rangle$$

normalized state: $|\nu_\mu\rangle = \left(\sum_j |\mathcal{A}_{\mu j}|^2 \right)^{-1/2} \sum_k \mathcal{A}_{\mu k} \, |\nu_k\rangle$

is this state different from the standard state $|\nu_\mu\rangle = \sum_{k=1}^{3} U_{\mu k}^* \, |\nu_k\rangle$?

in principle yes, but in practice no

S-matrix operator: $\quad \mathsf{S} \simeq 1 - i \int \mathrm{d}^4x \, \mathscr{H}_{\mathrm{CC}}(x)$

$$\mathscr{H}_{\mathrm{CC}}(x) = \frac{G_{\mathrm{F}}}{\sqrt{2}} \, j_\rho^\dagger(x) \, j^\rho(x)$$

$$\begin{aligned} j^\rho(x) &= 2 \, \overline{\nu_{\mu L}}(x) \, \gamma^\rho \, \mu_L(x) + \ldots \\ &= 2 \sum_k U_{\mu k}^* \, \overline{\nu_{kL}}(x) \, \gamma^\rho \, \mu_L(x) + \ldots \end{aligned}$$

$\mathcal{A}_{\mu k} = \langle \mu^+, \nu_k | \mathsf{S} | \pi^+ \rangle = U_{\mu k}^* \, \mathcal{M}_{\mu k}$ $\qquad$ $\mathcal{M}_{\mu k}$ depends on m_k!

$$|\nu_\mu\rangle = \left(\sum_j |\mathcal{A}_{\mu j}|^2 \right)^{-1/2} \sum_k \mathcal{A}_{\mu k} \, |\nu_k\rangle$$

$$= \sum_k \frac{\mathcal{M}_{\mu k}}{\sqrt{\sum_j |U_{\mu j}|^2 \, |\mathcal{M}_{\mu j}|^2}} \, U_{\mu k}^* \, |\nu_k\rangle \neq \sum_k U_{\mu k}^* \, |\nu_k\rangle$$

neutrino oscillation experiments are not sensitive to the dependence of $\mathcal{M}_{\mu k}$ on m_k

$$\left. \begin{array}{l} \mathcal{M}_{\mu k} \simeq \mathcal{M}_\mu \\ \sum_j |U_{\mu j}|^2 = 1 \end{array} \right\} \Longrightarrow \boxed{|\nu_\mu\rangle \simeq \sum_k U_{\mu k}^* \, |\nu_k\rangle}$$

standard ν_μ state!

Neutrino Production and Detection

example: $\nu_\mu \to \nu_e$ oscillation experiment

production process: $\pi^+ \to \mu^+ + \nu_\mu$

event rate: $R(L,E) \propto \underbrace{\Gamma_{\pi^+\to\mu^++\nu_\mu}}_{\text{production}} \, \underbrace{P_{\nu_\mu\to\nu_e}(L/E)}_{\text{oscillations}} \, \underbrace{\sigma_{\nu_e}(E)}_{\text{detection}}$

$$|\nu_\mu\rangle = \left(\sum_j |\mathcal{A}_{\mu j}|^2\right)^{-1/2} \sum_k \mathcal{A}_{\mu k}\,|\nu_k\rangle \qquad \mathcal{A}_{\mu k} = \langle \mu^+, \nu_k|\mathsf{S}|\pi^+\rangle$$

$$\begin{aligned}
\Gamma_{\pi^+\to\mu^++\nu_\mu} &\sim |\langle \mu^+, \nu_\mu|\,\mathsf{S}\,|\pi^+\rangle|^2 \\
&= \left(\sum_j |\mathcal{A}_{\mu j}|^2\right)^{-1} \left|\sum_k \mathcal{A}^*_{\mu k}\,\langle \mu^+, \nu_k|\mathsf{S}|\pi^+\rangle\right|^2 \\
&= \left(\sum_j |\mathcal{A}_{\mu j}|^2\right)^{-1} \left|\sum_k |\mathcal{A}_{\mu k}|^2\right|^2 = \sum_k |\mathcal{A}_{\mu k}|^2 \qquad \text{OK!}
\end{aligned}$$

coherent character of flavor state is irrelevant for decay probability!

Covariant Plane-Wave Theory of NuOsc

$$p_k \neq p_j \quad \Longleftrightarrow \quad \text{space-time evolution of states}$$

Neutrino Mixing:

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle \qquad \alpha = e, \mu, \tau \quad k = 1, 2, 3, \ldots$$

$$|\nu_k(x,t)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Longrightarrow \quad |\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k} \, e^{-iE_k t + i p_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta = e,\mu,\tau} U_{\beta k}^* |\nu_\beta\rangle$$

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta = e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k} e^{-iE_k t + i p_k x} U_{\beta k}^* \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(x,t)} |\nu_\beta\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(x,t) = \left|\mathcal{A}_{\nu_\alpha \to \nu_\beta}(x,t)\right|^2 = \left|\sum_k U_{\alpha k} e^{-iE_k t + i p_k x} U^*_{\beta k}\right|^2$$

Lorentz-invariant transition probability

[Dolgov et al., NPB 502 (1997) 3] [CG, Kim, FPL 14 (2001) 213] [Bilenky, CG, IJMPA 16 (2001) 3931]

[Dolgov, Phys. Rep. 370 (2002) 333] [Beuthe, Phys. Rep. 375 (2003) 105] [CG, hep-ph/0402217, JPG 34 (2007) R93]

flavor is Lorentz invariant $\Longrightarrow$ oscillation probability is Lorentz invariant

ultrarelativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L/E) &= \left|\sum_k U_{\alpha k}\, e^{-i m_k^2 L/2E}\, U^*_{\beta k}\right|^2 \\ &= \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left[-i\,\frac{\Delta m^2_{kj} L}{2E}\right] \end{aligned}$$

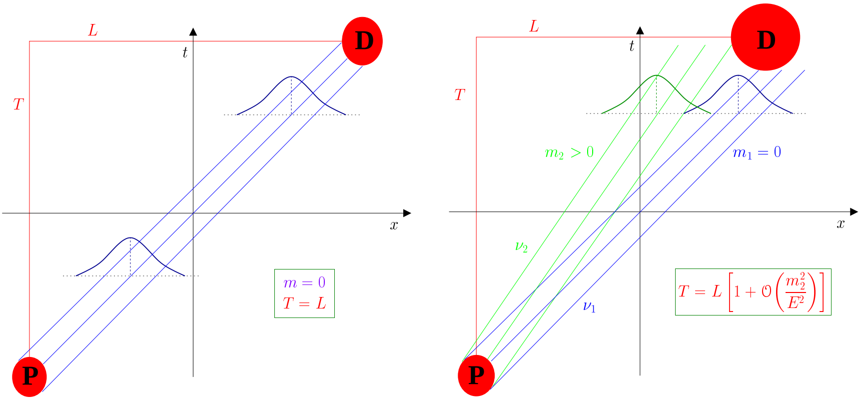
standard oscillation probability!

Wave-Packet Theory of NuOsc

$t \simeq x = L \quad \Longleftrightarrow \quad$ Wave Packets

Space-Time uncertainty	$\Longleftrightarrow$	Localization of production and detection processes
Energy-Momentum uncertainty	$\Longleftrightarrow$	Coherent creation and detection of different massive neutrinos

[Kayser, PRD 24 (1981) 110] [CG, FPL 17 (2004) 103]



The size of the massive neutrino wave packets is determined by the coherence time δt_{P} of the Production Process

($\delta t_{\mathrm{P}} \gtrsim \delta x_{\mathrm{P}}$, because the coherence region must be causally connected)

velocity of neutrino wave packets: $\quad v_k = \frac{p_k}{E_k} \simeq 1 - \frac{m_k^2}{2E^2}$

Coherence Length

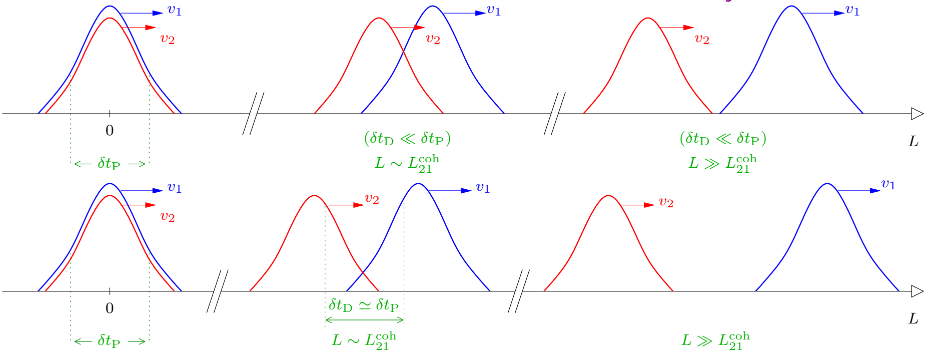
[Nussinov, PLB 63 (1976) 201] [Kiers, Nussinov, Weiss, PRD 53 (1996) 537]

Wave Packets have different velocities and separate

different massive neutrinos can interfere
if and only if wave packets arrive with $\delta t_{kj} < \delta t_{\mathrm{D}}$

$\Longrightarrow \quad L \lesssim L^{\mathrm{coh}}_{kj}$

$$|\delta t_{kj}| \simeq |v_k - v_j|\, T \simeq \frac{|\Delta m^2_{kj}|}{2E^2} L \quad \Longrightarrow \quad L^{\mathrm{coh}}_{kj} \sim \frac{2E^2}{|\Delta m^2_{kj}|} \sqrt{\delta t_{\mathrm{P}}^2 + \delta t_{\mathrm{D}}^2}$$



Quantum Mechanical Wave Packet Model

[CG, Kim, Lee, PRD 44 (1991) 3635] [CG, Kim, PRD 58 (1998) 017301]

neglecting mass effects in amplitudes of production and detection processes

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* \int \mathrm{d}p\, \psi_k^P(p)\, |\nu_k(p)\rangle \qquad |\nu_\beta\rangle = \sum_k U_{\beta k}^* \int \mathrm{d}p\, \psi_k^D(p)\, |\nu_k(p)\rangle$$

$$\begin{aligned} \mathcal{A}_{\alpha\beta}(x,t) &= \langle \nu_\beta |\, e^{-i\widehat{E}t + i\widehat{P}x}\, | \nu_\alpha \rangle \\ &= \sum_k U_{\alpha k}^*\, U_{\beta k} \int \mathrm{d}p\, \psi_k^P(p)\, \psi_k^{D*}(p)\, e^{-iE_k(p)t + ipx} \end{aligned}$$

Gaussian Approximation of Wave Packets

$$\psi_k^P(p) = \left(2\pi\sigma_{pP}^2\right)^{-1/4} \exp\left[-\frac{(p-p_k)^2}{4\sigma_{pP}^2}\right]$$

$$\psi_k^D(p) = \left(2\pi\sigma_{pD}^2\right)^{-1/4} \exp\left[-\frac{(p-p_k)^2}{4\sigma_{pD}^2}\right]$$

the value of p_k is determined by the production process (causality)

$$\mathcal{A}_{\alpha\beta}(x,t) \propto \sum_k U_{\alpha k}^* \, U_{\beta k} \int \mathrm{d}p \, \exp\left[-iE_k(p)t + ipx - \frac{(p-p_k)^2}{4\sigma_p^2} \right]$$

global energy-momentum uncertainty: $\boxed{\dfrac{1}{\sigma_p^2} = \dfrac{1}{\sigma_{pP}^2} + \dfrac{1}{\sigma_{pD}^2}}$

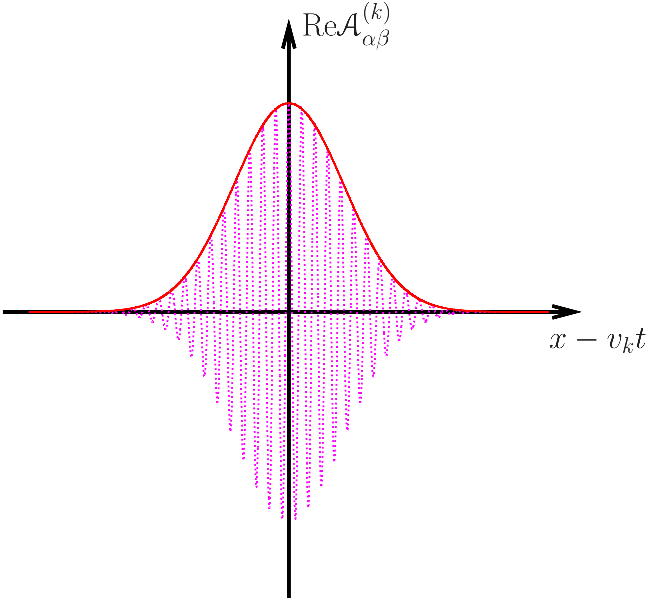
sharply peaked wave packets

$$\sigma_p \ll E_k^2(p_k)/m_k \quad \Longrightarrow \quad E_k(p) = \sqrt{p^2 + m_k^2} \simeq E_k + v_k \, (p - p_k)$$

$$E_k = E_k(p_k) = \sqrt{p_k^2 + m_k^2} \qquad v_k = \left. \frac{\partial E_k(p)}{\partial p} \right|_{p=p_k} = \frac{p_k}{E_k} \quad \text{group velocity}$$

$$\mathcal{A}_{\alpha\beta}(x,t) \propto \sum_k U_{\alpha k}^* \, U_{\beta k} \, \exp\left[-iE_k t + ip_k x - \underbrace{\frac{(x - v_k t)^2}{4\sigma_x^2}}_{\substack{\text{suppression factor} \\ \text{for } |x - v_k t| \gtrsim \sigma_x}} \right]$$

$\boxed{\sigma_x \, \sigma_p = \dfrac{1}{2}}$ global space-time uncertainty: $\boxed{\sigma_x^2 = \sigma_{xP}^2 + \sigma_{xD}^2}$



ν_k–ν_j interferene only if $\mathcal{A}_{\alpha\beta}^{(k)}$ and $\mathcal{A}_{\alpha\beta}^{(k)}$ overlap at detection

$$
\begin{aligned}
-E_k t + p_k x &= -(E_k - p_k)\, x + E_k\,(x - t) = -\frac{E_k^2 - p_k^2}{E_k + p_k}\, x + E_k\,(x - t) \\
&= -\frac{m_k^2}{E_k + p_k}\, x + E_k\,(x - t) \simeq -\frac{m_k^2}{2E}\, x + E_k\,(x - t)
\end{aligned}
$$

$$
\mathcal{A}_{\alpha\beta}(x,t) \propto \sum_k U_{\alpha k}^{*}\, U_{\beta k}\, \exp\left[\underbrace{-i\,\frac{m_k^2}{2E}\,x}_{\substack{\text{standard}\\ \text{phase}\\ \text{for } t = x}} \underbrace{+\, iE_k\,(x - t)}_{\substack{\text{additional}\\ \text{phase}\\ \text{for } t \neq x}} - \frac{(x - v_k t)^2}{4\sigma_x^2} \right]
$$

Space-Time Flavor Transition Probability

$$P_{\alpha\beta}(x,t) \propto \sum_{k,j} U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*} \exp\Bigg[\underbrace{-i\,\frac{\Delta m^2_{kj} x}{2E}}_{\substack{\text{standard}\\ \text{phase}\\ \text{for } t = x}} \underbrace{+i\,(E_k - E_j)\,(x-t)}_{\substack{\text{additional}\\ \text{phase}\\ \text{for } t \neq x}}\Bigg]$$

$$\times \exp\Bigg[\underbrace{-\frac{(x-\overline{v}_{kj}t)^2}{4\sigma_x^2}}_{\substack{\text{suppression}\\ \text{factor for}\\ |x-\overline{v}_{kj}t| \gtrsim \sigma_x}} \underbrace{-\frac{(v_k - v_j)^2\, t^2}{8\sigma_x^2}}_{\substack{\text{suppression}\\ \text{factor}\\ \text{due to}\\ \text{separation of}\\ \text{wave packets}}}\Bigg]$$

$$v_k = \frac{p_k}{E_k} \simeq 1 - \frac{m_k^2}{2E^2} \qquad \overline{v}_{kj} = \frac{v_k + v_j}{2} \simeq 1 - \frac{m_k^2 + m_j^2}{4E^2}$$

Oscillations in Space: $P_{\alpha\beta}(L) \propto \int \mathrm{d}t\, P_{\alpha\beta}(L,t)$

Gaussian integration over $\mathrm{d}t$

$$
\begin{aligned}
P_{\alpha\beta}(L) \propto \sum_{k,j} U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*} \exp\left[-i\,\frac{\Delta m_{kj}^{2} L}{2E}\right] \\
\times \underbrace{\sqrt{\frac{2}{v_k^2+v_j^2}}}_{\simeq 1} \exp\left[-\underbrace{\frac{(v_k-v_j)^2}{v_k^2+v_j^2}}_{\simeq\left(\Delta m_{kj}^2\right)^2/8E^4}\frac{L^2}{4\sigma_x^2} - \underbrace{\frac{(E_k-E_j)^2}{v_k^2+v_j^2}}_{\simeq\xi^2\left(\Delta m_{kj}^2\right)^2/8E^2}\sigma_x^2\right] \\
\times \exp\left[\underbrace{i\,(E_k-E_j)\left(1-\frac{2\overline{v}_{kj}^2}{v_k^2+v_j^2}\right)}_{\ll \Delta m_{kj}^2/2E} L\right]
\end{aligned}
$$

Ultrarelativistic Neutrinos: $p_k \simeq E - (1-\xi)\,\dfrac{m_k^2}{2E}$ $\qquad E_k \simeq E + \xi\,\dfrac{m_k^2}{2E}$

$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*} \, \exp\left[-i \, \frac{\Delta m_{kj}^2 L}{2E}\right]$$
$$\times \exp\left[-\left(\frac{\Delta m_{kj}^2 L}{4\sqrt{2} E^2 \sigma_x}\right)^2 - 2\xi^2 \left(\frac{\Delta m_{kj}^2 \sigma_x}{4E}\right)^2\right]$$

Oscillation Lengths $L_{kj}^{\text{osc}} = \frac{4\pi E}{\Delta m_{kj}^2}$

Coherence Lengths $L_{kj}^{\text{coh}} = \frac{4\sqrt{2} E^2}{|\Delta m_{kj}^2|} \sigma_x$

$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*} \, \exp\left[-2\pi i \, \frac{L}{L_{kj}^{\text{osc}}}\right]$$
$$\times \exp\left[-\left(\frac{L}{L_{kj}^{\text{coh}}}\right)^2 - 2\pi^2 \xi^2 \left(\frac{\sigma_x}{L_{kj}^{\text{osc}}}\right)^2\right]$$

new localization term: $\exp\left[-2\pi^2\xi^2\left(\frac{\sigma_x}{L^{\text{osc}}_{kj}}\right)^2\right]$

interference is suppressed for $\sigma_x \gtrsim L^{\text{osc}}_{kj}$

equivalent to neutrino mass measurement

uncertainty of neutrino mass measurement:

$$m_k^2 = E_k^2 - p_k^2 \quad \Longrightarrow \quad \delta m_k^2 \simeq \sqrt{(2\,E_k\,\delta E_k)^2 + (2\,p_k\,\delta p_k)^2} \sim 4\,E\,\sigma_p$$

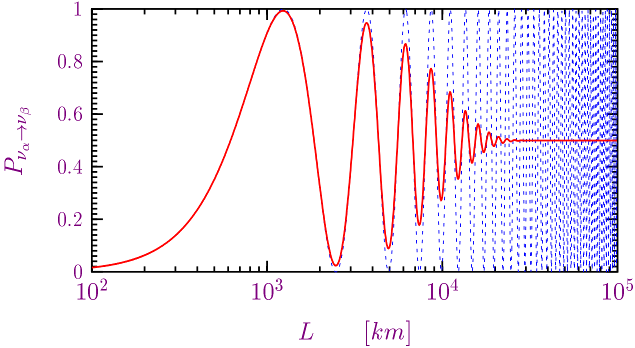
$$\sigma_p = \frac{1}{2\,\sigma_x} \qquad E = \frac{|\Delta m^2_{kj}|\,L^{\text{osc}}_{kj}}{4\pi} \quad \Longrightarrow \quad \delta m_k^2 \sim \frac{|\Delta m^2_{kj}|\,L^{\text{osc}}_{kj}}{\sigma_x}$$

$$\boxed{\sigma_x \gtrsim L^{\text{osc}}_{kj} \quad \Longrightarrow \quad \delta m_k^2 \lesssim |\Delta m^2_{kj}|} \quad \Longrightarrow \quad \text{only one massive neutrino!}$$

Decoherence in Two-Neutrino Mixing

$\Delta m^2 = 10^{-3}\,\mathrm{eV}^2$ $\sin^2 2\vartheta = 1$ $E = 1\,\mathrm{GeV}$ $\sigma_p = 50\,\mathrm{MeV}$

$$L^{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2} = 2480\,\mathrm{km} \qquad L^{\mathrm{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m^2|}\,\sigma_x = 11163\,\mathrm{km}$$



Decoherence for $L \gtrsim L^{\mathrm{coh}} \sim 10^4\,\mathrm{km}$

Achievements of the QM Wave Packet Model

- Confirmed Standard Oscillation Length: $L_{kj}^{\text{osc}} = 4\pi E / \Delta m_{kj}^2$
- Derived Coherence Length: $L_{kj}^{\text{coh}} = 4\sqrt{2} E^2 \sigma_x / |\Delta m_{kj}^2|$
- The localization term quantifies the conditions for coherence

problem

flavor states in production and detection processes have to be assumed

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* \int \mathrm{d}p \, \psi_k^P(p) \, |\nu_k(p)\rangle \qquad |\nu_\beta\rangle = \sum_k U_{\beta k}^* \int \mathrm{d}p \, \psi_k^D(p) \, |\nu_k(p)\rangle$$

calculation of neutrino production and detection?

$\Uparrow$

Quantum Field Theoretical Wave Packet Model

[CG, Kim, Lee, Lee, PRD 48 (1993) 4310] [CG, Kim, Lee, PLB 421 (1998) 237] [Kiers, Weiss, PRD 57 (1998) 3091]
[Zralek, Acta Phys. Polon. B29 (1998) 3925] [Cardall, PRD 61 (2000) 07300]
[Beuthe, PRD 66 (2002) 013003] [Beuthe, Phys. Rep. 375 (2003) 105] [CG, JHEP 11 (2002) 017]

Estimates of Coherence Length

$$L^{\rm osc} = \frac{4\pi E}{\Delta m^2} = 2.5 \, \frac{(E/\mathrm{MeV})}{(\Delta m^2/\mathrm{eV}^2)} \, \mathrm{m}$$

$$L^{\rm coh} \sim \frac{4\sqrt{2}E^2}{|\Delta m^2|}\sigma_x = 10^{12} \, \frac{(E^2/\mathrm{MeV}^2)}{(|\Delta m^2|/\mathrm{eV}^2)} \left(\frac{\sigma_x}{\mathrm{m}}\right) \mathrm{m}$$

Process	$\lvert\Delta m^2\rvert$	$L^{\rm osc}$	σ_x	$L^{\rm coh}$
$\pi \to \mu + \nu$ at rest in vacuum: $E \simeq 30\,\mathrm{MeV}$ natural linewidth	$2.5 \times 10^{-3}\,\mathrm{eV}^2$	$30\,\mathrm{km}$	$\tau_\pi \sim 10\,\mathrm{m}$	$\sim 10^{16}\,\mathrm{km}$
$\pi \to \mu + \nu$ at rest in matter: $E \simeq 30\,\mathrm{MeV}$ collision broadening	$2.5 \times 10^{-3}\,\mathrm{eV}^2$	$30\,\mathrm{km}$	$\tau_{\rm col} \sim 10^{-5}\,\mathrm{m}$	$\sim 10^{10}\,\mathrm{km}$
$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ at rest in matter: $E \lesssim 50\,\mathrm{MeV}$ collision broadening	$1\,\mathrm{eV}^2$	$\lesssim 125\,\mathrm{m}$	$\tau_{\rm col} \sim 10^{-10}\,\mathrm{m}$	$\lesssim 10^2\,\mathrm{km}$
${}^7\mathrm{Be} + e^- \to {}^7\mathrm{Li} + \nu_e$ in solar core: $E \simeq 0.86\,\mathrm{MeV}$ collision broadening	$7 \times 10^{-5}\,\mathrm{eV}^2$	$31\,\mathrm{km}$	$\tau_{\rm col} \sim 10^{-9}\,\mathrm{m}$	$\sim 10^4\,\mathrm{km}$

Conclusions

- The standard expression of the oscillation probability of ultrarelativistic neutrinos is robust.
- The definition of the flavor states as appropriate superpositions of massive states gives a consistent framework for the description of neutrino oscillations and interactions in neutrino oscillation experiments.
- Taking into account the space-time evolution of neutrino states we obtain a Lorentz-invariant oscillation probability which reduces to the standard one for $t \simeq x$.
- $t \simeq x$ is justified by a wave packet description, which is connected with the localization of the production and detection processes.
- A Quantum Mechanical Wave-Packet Model confirms the standard oscillation length and allows to estimate the coherence length.
- A complete description of neutrino oscillations is achieved with a Quantum Field Theoretical Wave Packet Model, which includes the calculation of neutrino production and detection.

Common Question: Do Charged Leptons Oscillate?

- Mass is the only property which distinguishes e, μ, τ.
- The flavor of a charged lepton is defined by its mass!
- By definition, the flavor of a charged lepton cannot change.

THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

a misleading argument

[Sassaroli, Srivastava, Widom, hep-ph/9509261, EPJC 2 (1998) 769] [Srivastava, Widom, hep-ph/9707268]

in $\pi^+ \to \mu^+ + \nu_\mu$ the final state of the antimuon and neutrino is entangled

$\Downarrow$

if the probability to detect the neutrino oscillates as a function of distance, also the probability to detect the muon must oscillate

WRONG!

the probability to detect the neutrino (as ν_μ or ν_τ or ν_e) does not oscillate as a function of distance, because

$$\sum_{\beta=e,\mu,\tau} P_{\nu_\mu \to \nu_\beta} = 1$$

conservation of probability (unitarity)

[Dolgov, Morozov, Okun, Shchepkin, NPB 502 (1997) 3] [CG, Kim, FPL 14 (2001) 213]

Λ oscillations from $\pi^- + p \to \Lambda + K^0$

[Widom, Srivastava, hep-ph/9605399] [Srivastava, Widom, Sassaroli, PLB 344 (1995) 436]

refuted in [Lowe et al., PLB 384 (1996) 288] [Burkhardt, Lowe, Stephenson, Goldman, PRD 59 (1999) 054018]

Correct definition of Charged Lepton Oscillations

[Pakvasa, Nuovo Cim. Lett. 31 (1981) 497]

Analogy

- Neutrino Oscillations: massive neutrinos propagate unchanged between production and detection, with a difference of mass (flavor) of the charged leptons involved in the production and detection processes.
- Charged-Lepton Oscillations: massive charged leptons propagate unchanged between production and detection, with a difference of mass of the neutrinos involved in the production and detection processes.

NO FLAVOR CONVERSION!

The propagating charged leptons must be ultrarelativistic, in order to be produced and detected coherently (if τ is not ultrarelativistic, only e and μ contribute to the phase).

Practical Problems

- The initial and final neutrinos must be massive neutrinos of known type: precise neutrino mass measurements.

- The energy of the propagating charged leptons must be extremely high, in order to have a measurable oscillation length

$$\frac{4\pi E}{(m_\mu^2 - m_e^2)} \simeq \frac{4\pi E}{m_\mu^2} \simeq 2 \times 10^{-11} \left(\frac{E}{\text{GeV}} \right) \text{cm}$$

detailed discussion: [Akhmedov, JHEP 09 (2007) 116, arXiv:0706.1216]

Mistake: Oscillation Phase Larger by a Factor of 2

[Field, hep-ph/0110064, hep-ph/0110066, EPJC 30 (2003) 305, EPJC 37 (2004) 359, Annals Phys. 321 (2006) 627]

$K^0 - \bar{K}^0$: [Srivastava, Widom, Sassaroli, ZPC 66 (1995) 601, PLB 344 (1995) 436] [Widom, Srivastava, hep-ph/9605399]

massive neutrinos: $v_k = \frac{p_k}{E_k} \implies t_k = \frac{L}{v_k} = \frac{E_k}{p_k} L$

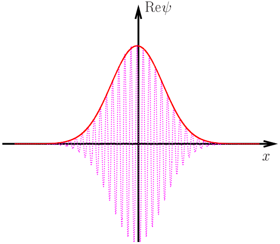
$$\widetilde{\Phi}_k = p_k L - E_k t_k = p_k L - \frac{E_k^2}{p_k} L = \frac{p_k^2 - E_k^2}{p_k} L = \frac{m_k^2}{p_k} L \simeq \frac{m_k^2}{E} L$$

$\Delta\widetilde{\Phi}_{kj} = -\frac{\Delta m_{kj}^2 L}{E}$ twice the standard phase $\Delta\Phi_{kj} = -\frac{\Delta m_{kj}^2 L}{2E}$

WRONG!

group velocities are irrelevant for the phase!

the group velocity is the velocity of the factor which modulates the amplitude of the wave packet



in the plane wave approximation the interference
of different massive neutrino contribution must be calculated
at a definite space distance L and after a definite time interval T

[Nieto, hep-ph/9509370] [Kayser, Stodolsky, PLB 359 (1995) 343] [Lowe et al., PLB 384 (1996) 288] [Kayser, hep-ph/9702327]
[CG, Kim, FPL 14 (2001) 213] [CG, Physica Scripta 67 (2003) 29] [Burkhardt et al., PLB 566 (2003) 137]

$$\Delta\widetilde{\Phi}_{kj} = (p_k - p_j)\,L - (E_k - E_j)\,t_k$$ WRONG!

$$\Delta\Phi_{kj} = (p_k - p_j)\,L - (E_k - E_j)\,T$$ CORRECT!

no factor of 2 ambiguity claimed in

[Lipkin, PLB 348 (1995) 604, hep-ph/9901399] [Grossman, Lipkin, PRD 55 (1997) 2760]
[De Leo, Ducati, Rotelli, MPLA 15 (2000) 2057]
[De Leo, Nishi, Rotelli, hep-ph/0208086, hep-ph/0303224, IJMPA 19 (2004) 677]