A critical view of the GSI anomaly

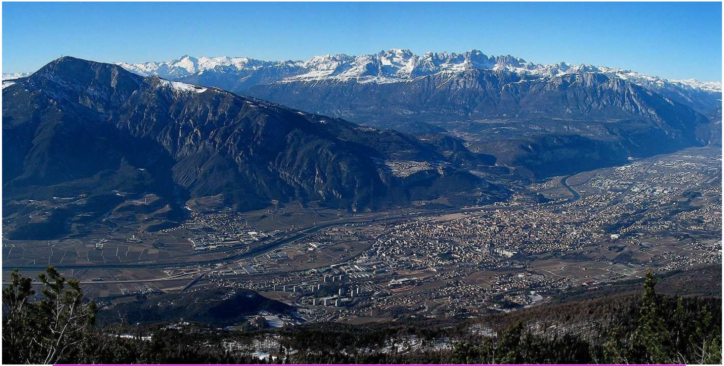
Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

mailto://giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

ECT*, Trento, 18 November 2008



The GSI Anomaly

$${}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_e \qquad {}^{142}_{61}\mathrm{Pm}^{60+} \to {}^{142}_{60}\mathrm{Nd}^{60+} + \nu_e$$

$$\frac{dN_{EC}(t)}{dt} = \widetilde{\lambda}_{EC}(t)\, N(t) \qquad \widetilde{\lambda}_{EC}(t) = \lambda_{EC}\left[1 + A\cos\left(2\pi\,\frac{t}{T} + \varphi\right)\right]$$

$$T({}^{140}_{59}\mathrm{Pr}^{58+}) = 7.06 \pm 0.08\,\mathrm{s} \qquad T({}^{142}_{61}\mathrm{Pm}^{60+}) = 7.10 \pm 0.22\,\mathrm{s}$$

$$\langle A \rangle = 0.20 \pm 0.02$$

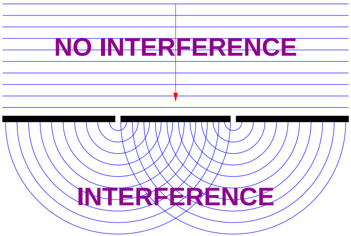
[Litvinov et al, PLB 664 (2008) 162, nucl-ex/0801.2079]

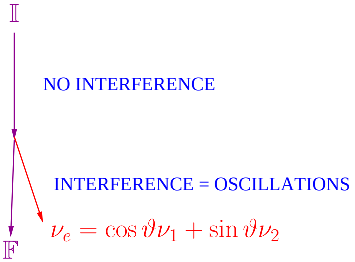
$$\nu_e = \cos\vartheta_{SOL}\,\nu_1 + \sin\vartheta_{SOL}\,\nu_2$$

PROPOSED EXPLANATION: INTERFERENCE OF ν_1 AND ν_2

CAN INTERFERENCE IN FINAL STATE AFFECT DECAY RATE?

Interference: Double-Slit Analogy





- Decay rate of $\mathbb{I}$ corresponds to fraction of intensity of incoming wave which crosses the barrier
- Fraction of intensity of the incoming wave which crosses the barrier depends on the sizes of the holes
- It does not depend on interference effects which occur after the wave has passed through the barrier
- Analogy: decay rate of $\mathbb{I}$ cannot depend on interference of ν_1 and ν_2 which occurs after decay has happened

Causality

INTERFERENCE OF

COHERENT ENERGY STATES

(ν_1 AND ν_2)

OCCURRING AFTER THE DECAY

(flavor neutrino oscillations)

CANNOT AFFECT THE DECAY RATE

Cross Sections and Decay Rates are always summed incoherently over different final channels:

$$\mathbb{I} \to \mathbb{F}_1 \,, \qquad \mathbb{I} \to \mathbb{F}_2 \,, \qquad \ldots \qquad \Longrightarrow \qquad P_{\mathbb{I}\to\mathbb{F}} = \sum_k P_{\mathbb{I}\to\mathbb{F}_k}$$

entangled final state: $$|\mathbb{F}\rangle = \sum_k A_k |\mathbb{F}_k\rangle$$

$$|\mathbb{F}\rangle \propto (\mathsf{S} - \mathbf{1})\,|\mathbb{I}\rangle \qquad \Longrightarrow \qquad A_k \propto \langle\mathbb{F}_k|\,(\mathsf{S} - \mathbf{1})\,|\mathbb{I}\rangle = \langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle$$

$$\langle\mathbb{F}|\mathbb{F}\rangle = 1 \qquad \Longleftrightarrow \qquad A_k = \frac{\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle}{\left(\sum_j |\langle\mathbb{F}_j|\mathsf{S}|\mathbb{I}\rangle|^2\right)^{1/2}}$$

$$P_{\mathbb{I}\to\mathbb{F}} = |\langle\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \left|\sum_k A_k^* \langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle\right|^2 = \left(\frac{\sum_k |\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle|^2}{\left(\sum_j |\langle\mathbb{F}_j|\mathsf{S}|\mathbb{I}\rangle|^2\right)^{1/2}}\right)^2$$

$$= \sum_k |\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle|^2 = \sum_k P_{\mathbb{I}\to\mathbb{F}_k}$$

coherent character of final state is irrelevant for interaction probability!

Time-Dependent Perturbation Theory?

not necessary because electron capture and decay are interrupted by

Schottky Mass Spectrometry

with ESR revolution frequency ~ 2 MHz, i.e. every

$$\sim 5 \times 10^{-7}\,\mathrm{s}$$

much smaller than ion lifetime

$T_{1/2}({}^{140}_{59}\mathrm{Pr}) \simeq 3.39\,\mathrm{m}$ $\quad$ $T_{1/2}({}^{142}_{61}\mathrm{Pm}) \simeq 40.5\,\mathrm{s}$

and period of anomalous oscillations $T \simeq 7\,\mathrm{s}$

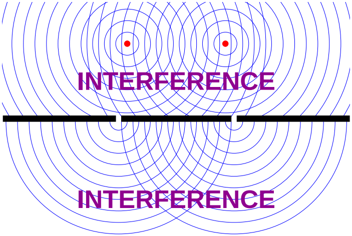
$t \to \infty$ in Time-Dependent Perturbation Theory

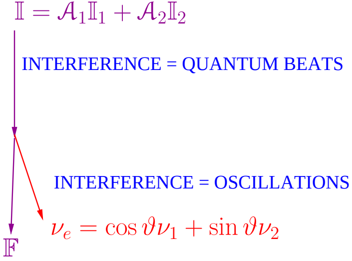
$\Downarrow$

Quantum Field Theory result

Quantum Beats?

- GSI time anomaly can be due to interference effects in initial state
- Two coherent energy states of the decaying ion $\Longrightarrow$ Quantum Beats





- Incoming waves interfere at holes in barrier
- Causality: interference due to different phases of incoming waves developed during propagation before reaching the barrier

Causality

INTERFERENCE OF

COHERENT ENERGY STATES

OCCURRING BEFORE THE DECAY

CAN AFFECT THE DECAY RATE

Quantum Beats

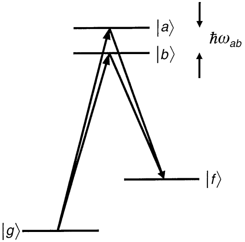


Fig. 1 Diagram of the four level system. A photon is absorbed by the ground state $|b\rangle$ and excites a superposition of states $|a\rangle$ and $|b\rangle$ whose energy separation is $\Delta E = \hbar\omega_{ab}$. Emission of a second photon leaves the system in the final state $|f\rangle$.

$$I_{\rm fl}(t) \propto |\mu_{ag}|^2 |\mu_{fa}|^2 e^{-\gamma_a t} + |\mu_{bg}|^2 |\mu_{fb}|^2 e^{-\gamma_b t} + |\mu_{ag}\mu_{bg}\mu_{fa}\mu_{fb}| e^{-(\gamma_a+\gamma_b)t/2} \cos(\omega_{ab} t + \theta). \quad (4)$$

Examination of this expression shows that it consists of two parts, one incoherent term (first two terms) describing the independent decays of the two states $|a\rangle$ and $|b\rangle$ and one coherent or cross term (last term) which decays at the average rate of the two states and, most importantly, is modulated at the angular frequency ω_{ab}. The modulation frequency is the difference of the two angular frequencies in eqn. (2), *i.e.* $\omega_{ab} = |\omega_a - \omega_b|$, and the coherent term in eqn. (4) is therefore termed the quantum beat. The angle θ is included in eqn. (4) to describe the phase of the quantum beat, which depends on a number of factors such as the excitation and detection polarisations and transitions. When the transition moments and decay rates are equal, as is often the case, a particularly simple expression is derived for the four level system. In this case eqn. (4) becomes

$$I_{\rm fl}(t) \propto [1 + \cos(\omega_{ab} t + \theta)] e^{-\gamma t}, \quad (5)$$

clearly illustrating the contributions of the incoherent and coherent terms to the fluorescence decay. In this special case the quantum beat is 100% modulated. It is important to point out

[Carter, Huber, Quantum beat spectroscopy in chemistry, Chem. Soc. Rev., 29 (2000) 305]

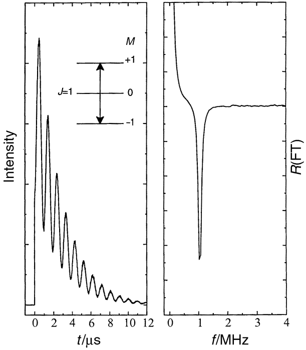


Fig. 2 Zeeman quantum beat recorded for the $R(0)$ line of the 17U transition in CS_2 in an external field of ~15 Gauss. The laser polarisation was perpendicular to the magnetic field direction and prepares a coherence between the $M = \pm 1$ sublevels as shown in the level diagram. This is manifested by a single quantum beat on the fluorescence decay; the real part of the Fourier transform is also shown. The less than 100% modulation, which is observed in virtually all quantum beat measurements in molecules, is due to incoherent emission from the excited states.

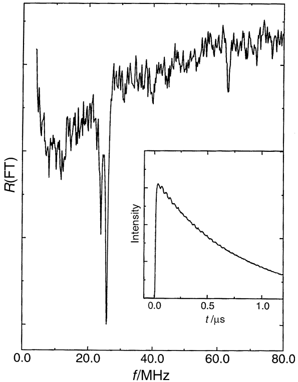


Fig. 6 Nuclear hyperfine quantum beats recorded for the $P_{21}/Q_1(3/2)$ line in a vibrational band of the $A^2\Sigma^+ - X^2\Pi$ transition in the Ar·OD van der Waals complex. The inset shows the fluorescence decay which exhibits weakly modulated quantum beats. Following Fourier transformation the beat frequencies between hyperfine levels in the $A^2\Sigma^+$ state are clearly visible.

[Carter, Huber, Quantum beat spectroscopy in chemistry, Chem. Soc. Rev., 29 (2000) 305]

- Quantum beats in GSI experiment can be due to interference of two coherent energy states of the decaying ion which develop different phases before the decay

- Coherence is preserved for a long time if measuring apparatus which monitors the ions with frequency ~ 2 MHz does not distinguish between the two states

- $|\mathbb{I}(t=0)\rangle = \mathcal{A}_1\,|\mathbb{I}_1\rangle + \mathcal{A}_2\,|\mathbb{I}_2\rangle \qquad (|\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 = 1)$

$$\Gamma = \Gamma_1 \simeq \Gamma_2 \quad \Longrightarrow \quad |\mathbb{I}(t)\rangle = \left(\mathcal{A}_1\,e^{-iE_1 t}\,|\mathbb{I}_1\rangle + \mathcal{A}_2\,e^{-iE_2 t}\,|\mathbb{I}_2\rangle\right) e^{-\Gamma t/2}$$

$$P_{\mathrm{EC}}(t) = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}(t)\rangle|^2 = [1 + A\cos(\Delta E t + \varphi)]\,\overline{P}_{\mathrm{EC}}\,e^{-\Gamma t}$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|\,, \quad \Delta E \equiv E_2 - E_1\,, \quad \overline{P}_{\mathrm{EC}} = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_1\rangle|^2 \simeq |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_2\rangle|^2$$

$$\frac{\mathrm{d}N_{\mathrm{EC}}(t)}{\mathrm{d}t} = N(0)\,[1 + A\cos(\Delta E t + \varphi)]\,\overline{\Gamma}_{\mathrm{EC}}\,e^{-\Gamma t}$$

$$\frac{\mathrm{d}N_{\mathrm{EC}}(t)}{\mathrm{d}t} = N(0)\,[1 + A\,\cos(\Delta E t + \varphi)]\,\overline{\Gamma}_{\mathrm{EC}}\,e^{-\Gamma t}$$

$$\Delta E({}^{140}_{59}\mathrm{Pr}^{58+}) = (5.86 \pm 0.07) \times 10^{-16}\,\mathrm{eV}\,, \qquad A({}^{140}_{59}\mathrm{Pr}^{58+}) = 0.18 \pm 0.03$$

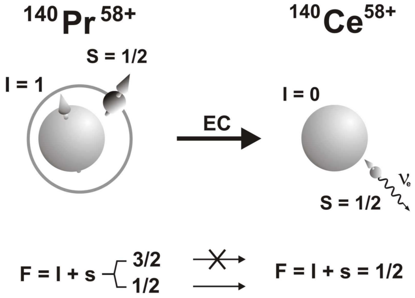
$$\Delta E({}^{142}_{61}\mathrm{Pm}^{60+}) = (5.82 \pm 0.18) \times 10^{-16}\,\mathrm{eV}\,, \quad A({}^{142}_{61}\mathrm{Pm}^{60+}) = 0.23 \pm 0.04$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|$$

- Energy splitting is extremely small
- $|\mathcal{A}_1|^2/|\mathcal{A}_2|^2 \sim 1/99$ or $|\mathcal{A}_2|^2/|\mathcal{A}_1|^2 \sim 1/99$
- It is difficult to find an appropriate mechanism

Hyperfine Splitting

smallest known energy splitting



[Litvinov et al, PRL 99 (2007) 262501, nucl-ex/0711.3709]

$\Delta E \sim 10^{-6}\,\text{eV} \quad \Longrightarrow \quad T \sim 10^{-9}\,\text{s} \quad \text{f} \sim \text{GHz}$

too large to explain the GSI anomaly

$T_{\text{GSI}} \simeq 7\,\text{s} \quad \text{f}_{\text{GSI}} \simeq 0.14\text{Hz} \quad \Delta E_{\text{GSI}} = 2\pi/T_{\text{GSI}} \simeq 6 \times 10^{-16}\,\text{eV}$

feeling of smallness of $\Delta E_{\mathrm{GSI}} \simeq 6 \times 10^{-16}\,\mathrm{eV}$

$$\mu_{\mathrm{N}} B_{\oplus} \simeq \left(3 \times 10^{-12}\,\mathrm{eV\,G^{-1}}\right)(0.5\,\mathrm{G}) = 1.5 \times 10^{-12}\,\mathrm{eV}$$

$$\Delta E_{\mathrm{GSI}} \simeq 4 \times 10^{-4}\,\mu_{\mathrm{N}} B_{\oplus}$$

Conclusions

- Interference: due to phase difference of two incoming waves
- Causality: there cannot be interference of waves before they exist
- The GSI ion lifetime anomaly cannot be due to interference of decay product before the decay product start to exist (neutrino mixing in the final state, which generates flavor neutrino oscillations)
- The GSI ion lifetime anomaly can be due to interference of two energy states of the decaying ion: Quantum Beats
- No known mechanism, which would require
  - Energy splitting of the two energy states: $\Delta E \sim 6 \times 10^{-16}\,\text{eV}$
  - Ratio of probabilities of the two energy states: $1/99$