Can the GSI Time Anomaly be due to Neutrino Mixing?

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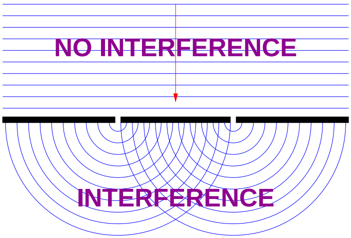
Can the GSI Time Anomaly

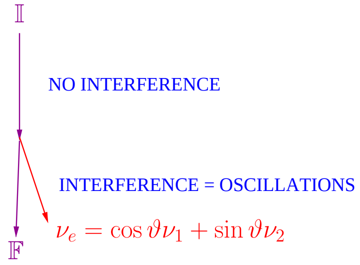
be due to

Neutrino Mixing?

NO

Interference: Double-Slit Analogy





- Decay rate of $\mathbb{I}$ corresponds to fraction of intensity of incoming wave which crosses the barrier
- Fraction of intensity of the incoming wave which crosses the barrier depends on the sizes of the holes
- It does not depend on interference effects which occur after the wave has passed through the barrier
- Analogy: decay rate of $\mathbb{I}$ cannot depend on interference of ν_1 and ν_2 which occurs after decay has happened

Causality

INTERFERENCE OF ν_1 AND ν_2

OCCURRING AFTER THE DECAY

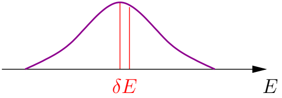
CANNOT AFFECT THE DECAY RATE

arXiv:0801.1465 and arXiv:0805.0435

H.J. Lipkin

- Causality is violated explicitly
- arXiv:0801.1465: The difference in momentum δp_ν between the two neutrino eigenstates with the same energy produces a small initial momentum change δP ...
- arXiv:0805.0435: Since the time dependence depends only on the propagation of the initial state, it is independent of the final state, which is created only at the decay point. Thus there is no violation of causality.
- But in calculation of effect: The phase difference at a time t between states produced by the neutrino mass difference on the motion of the initial ion in the laboratory frame with velocity $V = (P/E)$ is

$$\delta\phi \approx -\delta E \cdot t = \Delta m^2/2E$$



arXiv:0801.2121 – arXiv:0801.3262

A. N. Ivanov, R. Reda, P. Kienle – M. Faber

- $\mathbb{I} \to \mathbb{F} + \nu$ with final neutrino state $|\nu\rangle = \sum_k |\nu_k\rangle$
- Not even properly normalized to describe one particle:

$$\langle \nu_j | \nu_k \rangle = \delta_{jk} \implies \langle \nu | \nu \rangle = 3$$

- Standard QFT: $P_{\mathbb{I}\to\mathbb{F}+\nu} = |\langle\nu,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \left|\sum_k \langle\nu_k,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle\right|^2$

- S-matrix operator at first order in perturbation theory:

$$\mathsf{S} = 1 - i\int \mathrm{d}^4x\,\mathscr{H}_W(x)$$

- Effective four-fermion interaction Hamiltonian:

$$\mathscr{H}_W(x) = \frac{G_F}{\sqrt{2}}\cos\theta_C\bar{\nu}_e(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)$$
$$= \frac{G_F}{\sqrt{2}}\cos\theta_C\sum_k U_{ek}^*\bar{\nu}_k(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)$$

- $\langle\nu_k,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle = U_{ek}^*\mathcal{M}_k$ with

$$\mathcal{M}_k = -i\frac{G_F}{\sqrt{2}}\cos\theta_C\int \mathrm{d}^4x\langle\nu_k,\mathbb{F}|\bar{\nu}_k(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)|\mathbb{I}\rangle$$

- $P_{\mathbb{I}\to\mathbb{F}+\nu} = \left|\sum_k U_{ek}^*\mathcal{M}_k\right|^2$ different from standard $P = \sum_k |U_{ek}|^2\,|\mathcal{M}_k|^2$

- Check: in the limit of massless neutrinos decay probability should reduce to the Standard Model decay probability

$$P_{\mathrm{SM}} = |\mathcal{M}_{\mathrm{SM}}|^2$$

with

$$\mathcal{M}_{\mathrm{SM}} = -i \frac{G_F}{\sqrt{2}} \cos\theta_C \int \mathrm{d}^4x \langle \nu_e, \mathbb{F} | \bar{\nu}_e(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) | \mathbb{I} \rangle$$

where ν_e is the Standard Model massless electron neutrino

$$\mathcal{M}_k = -i \frac{G_F}{\sqrt{2}} \cos\theta_C \int \mathrm{d}^4x \langle \nu_k, \mathbb{F} | \bar{\nu}_k(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) | \mathbb{I} \rangle$$

$$\mathcal{M}_k \xrightarrow[m_k \to 0]{} \mathcal{M}_{\mathrm{SM}}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu} = \left| \sum_k U_{ek}^* \mathcal{M}_k \right|^2 \xrightarrow[m_k \to 0]{} |\mathcal{M}_{\mathrm{SM}}|^2 \left| \sum_k U_{ek}^* \right|^2 \neq P_{\mathrm{SM}}$$

- Correct normalized final neutrino state ($\langle \nu_e | \nu_e \rangle = 1$):

$$|\nu_e\rangle = \left(\sum_j |\langle \nu_j, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 \right)^{-1/2} \sum_k |\nu_k\rangle \langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle$$

$$= \left(\sum_j |U_{ej}|^2 |\mathcal{M}_j|^2 \right)^{-1/2} \sum_k U_{ek}^* \mathcal{M}_k |\nu_k\rangle$$

- Standard decay probability:

$$P_{\mathbb{I} \to \mathbb{F} + \nu_e} = |\langle \nu_e, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 = \sum_k |\langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 = \sum_k |U_{ek}|^2 |\mathcal{M}_k|^2$$

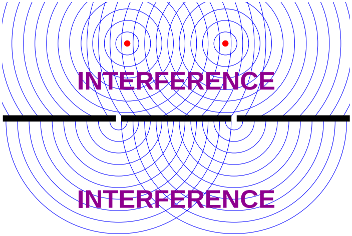
$$P_{\mathbb{I} \to \mathbb{F} + \nu_e} \xrightarrow[m_k \to 0]{} P_{\mathrm{SM}}$$

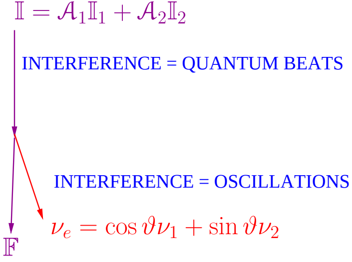
- In experiments which are not sensitive to the differences of the neutrino masses, as neutrino oscillation experiments,

$$\mathcal{M}_k \simeq \overline{\mathcal{M}} \quad \Longrightarrow \quad |\nu_e\rangle = \sum_k U_{ek}^* |\nu_k\rangle$$

Quantum Beats

- GSI time anomaly can be due to interference effects in initial state
- Two coherent energy states of the decaying ion $\Longrightarrow$ Quantum Beats





- Incoming waves interfere at holes in barrier
- Causality: interference due to different phases of incoming waves developed during propagation before reaching the barrier

- Quantum beats in GSI experiment can be due to interference of two coherent energy states of the decaying ion which develop different phases before the decay

- Coherence is preserved for a long time if measuring apparatus which monitors the ions with frequency ~ 2 MHz does not distinguish between the two states

- $|\mathbb{I}(t=0)\rangle = \mathcal{A}_1 \, |\mathbb{I}_1\rangle + \mathcal{A}_2 \, |\mathbb{I}_2\rangle \qquad\qquad (|\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 = 1)$

$$\Gamma = \Gamma_1 \simeq \Gamma_2 \quad \Longrightarrow \quad |\mathbb{I}(t)\rangle = \left(\mathcal{A}_1 \, e^{-iE_1 t} \, |\mathbb{I}_1\rangle + \mathcal{A}_2 \, e^{-iE_2 t} \, |\mathbb{I}_2\rangle\right) e^{-\Gamma t/2}$$

$$P_{\text{EC}}(t) = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}(t)\rangle|^2 = [1 + A \cos(\Delta E t + \varphi)] \, \overline{P}_{\text{EC}} \, e^{-\Gamma t}$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2| \,, \quad \Delta E \equiv E_2 - E_1 \,, \quad \overline{P}_{\text{EC}} = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_1\rangle|^2 \simeq |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_2\rangle|^2$$

$$\frac{\mathrm{d}N_{\text{EC}}(t)}{\mathrm{d}t} = N(0) \, [1 + A \cos(\Delta E t + \varphi)] \, \overline{\Gamma}_{\text{EC}} \, e^{-\Gamma t}$$

$$\frac{dN_{EC}(t)}{dt} = N(0)\left[1 + A\cos(\Delta E t + \varphi)\right]\overline{\Gamma}_{EC}\, e^{-\Gamma t}$$

$$\Delta E(^{140}\mathrm{Pr}^{58+}) = (5.86 \pm 0.07) \times 10^{-16}\,\mathrm{eV}\,, \quad A(^{140}\mathrm{Pr}^{58+}) = 0.18 \pm 0.03$$

$$\Delta E(^{142}\mathrm{Pm}^{60+}) = (5.82 \pm 0.18) \times 10^{-16}\,\mathrm{eV}\,, \quad A(^{142}\mathrm{Pm}^{60+}) = 0.23 \pm 0.04$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|$$

- Energy splitting is extremely small
- $|\mathcal{A}_1|^2/|\mathcal{A}_2|^2 \sim 1/99$ or $|\mathcal{A}_2|^2/|\mathcal{A}_1|^2 \sim 1/99$
- It is difficult to find an appropriate mechanism

Conclusions

- Interference: due to phase difference of two incoming waves
- Causality: there cannot be interference of waves before they exist
- The GSI ion lifetime anomaly cannot be due to interference of decay product before the decay product start to exist (neutrino mixing in the final state)
- The GSI ion lifetime anomaly can be due to interference of two energy states of the decaying ion: Quantum Beats
- No known mechanism, because
  - Energy splitting of the two energy states: $\Delta E \sim 6 \times 10^{-16}\,\text{eV}$
  - Ratio of probabilities of the two energy states: $1/99$