The GSI Time Anomaly: Facts and Fiction

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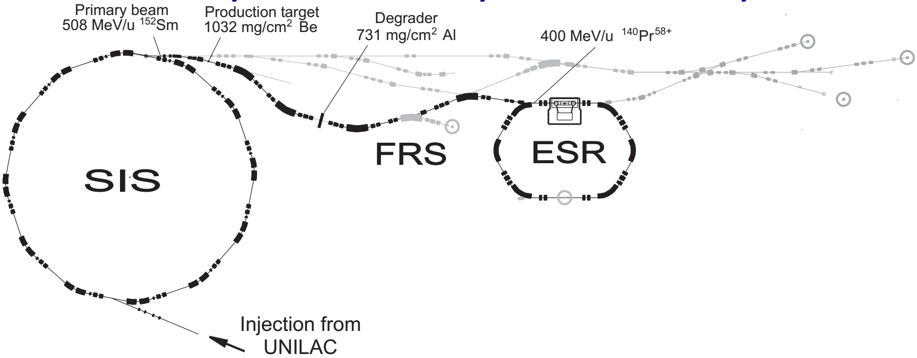
The GSI Experiment

Schematic layout of the secondary nuclear beam facility at GSI

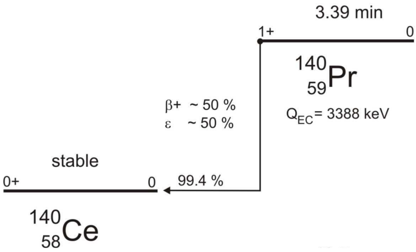


{Litvinov et al, nucl-ex/0509019}

SIS: Heavy Ion Synchrotron

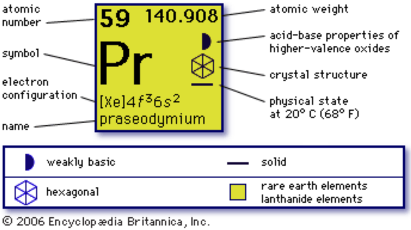
FRS: FRagment Separator

ESR: Experiment Storage Ring



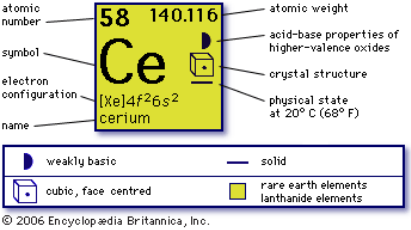
{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

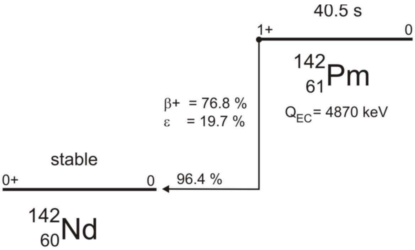
Praseodymium



$$ {}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_e $$

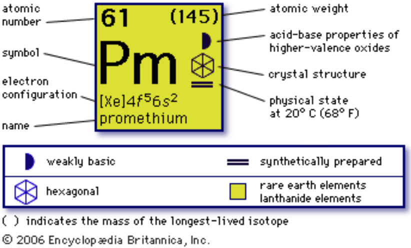
Cerium





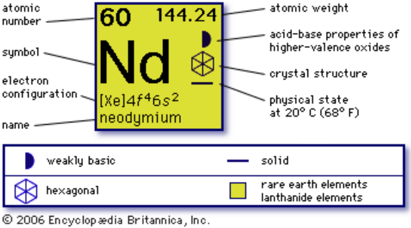
{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

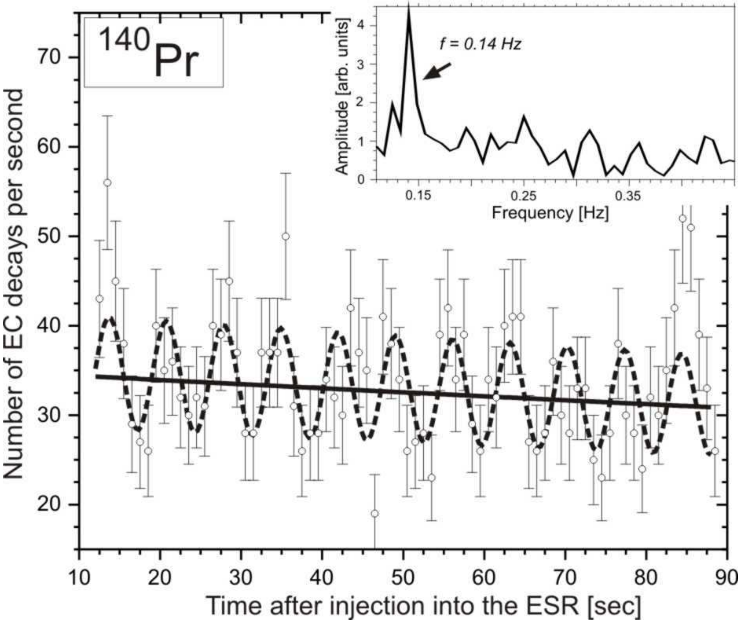
Promethium



$$ {}^{142}_{61}\mathrm{Pm}^{60+} \to {}^{142}_{60}\mathrm{Nd}^{60+} + \nu_e $$

Neodymium





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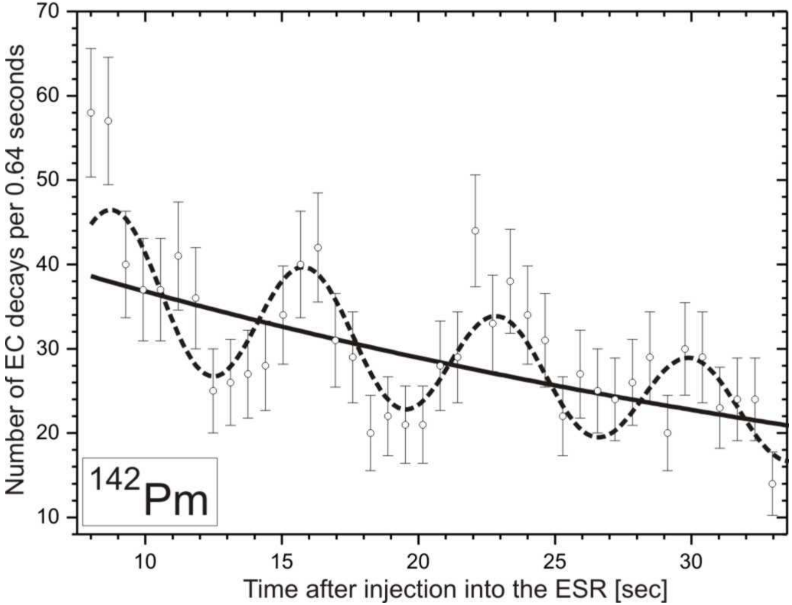


{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

$$(1) \qquad \frac{dN_{EC}(t)}{dt} = \lambda_{EC}\,N(t) = \lambda_{EC}\,N(0)\,e^{-\lambda t}$$

$$(2) \qquad \frac{dN_{EC}(t)}{dt} = \tilde{\lambda}_{EC}(t)\,N(t) = \tilde{\lambda}_{EC}(t)\,N(0)\,e^{-\lambda t}$$

$$\lambda = \lambda_{EC} + \lambda_{\beta^+} + \lambda_{loss} \qquad \tilde{\lambda}_{EC}(t) = \lambda_{EC}\,[1 + a\cos(\omega t + \phi)]$$

Fit parameters of ${}^{140}_{59}$Pr data					
Eq.	$N_0\lambda_{EC}$	λ	a	ω	χ^2/DoF
(1)	34.9(18)	0.00138(10)	-	-	107.2/73
(2)	35.4(18)	0.00147(10)	0.18(3)	0.89(1)	67.18/70
Fit parameters of ${}^{142}_{61}$Pm data					
Eq.	$N_0\lambda_{EC}$	λ	a	ω	χ^2/DoF
(1)	46.8(40)	0.0240(42)	-	-	63.77/38
(2)	46.0(39)	0.0224(41)	0.23(4)	0.89(3)	31.82/35

$$T({}^{140}_{59}\mathrm{Pr}^{58+}) = 7.06 \pm 0.08\,\mathrm{s} \qquad T({}^{142}_{61}\mathrm{Pm}^{60+}) = 7.10 \pm 0.22\,\mathrm{s}$$

$$\langle a \rangle = 0.20 \pm 0.02$$

{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

Neutrino Mixing?

{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

$I_i \to I_f + \nu_e$ $\qquad$ $|\nu_e\rangle = \cos\vartheta|\nu_1\rangle + \sin\vartheta|\nu_2\rangle$

Initial Ion at rest: Mass M_i

Massive ν_k: Momentum $\vec{p}_k$, Energy $E_k = \sqrt{p_k^2 + m_k^2}$

Final Ion: Momentum $-\vec{p}_k$, Energy $M_f + p_k^2/2M_f$

$E_1 + M_f + p_1^2/2M_f = M_i$ $\qquad$ $E_2 + M_f + p_2^2/2M_f = M_i$

$$\Delta E \equiv E_2 - E_1 \simeq \frac{\Delta m^2}{2M_f} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\Delta E \equiv E_2 - E_1 \simeq \frac{\Delta m^2}{2M_f}$$

$$\Delta m^2 = \Delta m^2_{\odot} \simeq 8 \times 10^{-5}\,\text{eV}^2 \qquad M_f \simeq 140\,\text{amu} \simeq 130\,\text{GeV}$$

$$\Delta E \simeq 3.1 \times 10^{-16}\,\text{eV}$$

$$T = \frac{2\pi}{\Delta E}\,\gamma \simeq 19.1\,\text{s} \qquad \gamma = 1.43$$

about 3 times larger than $T_{\text{GSI}} \simeq 7\,\text{s}$

ΔE is the massive neutrino energy difference!

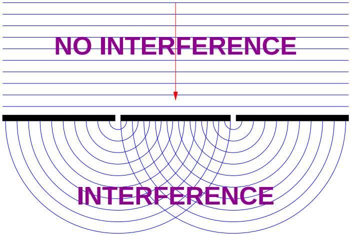
Can the GSI Time Anomaly

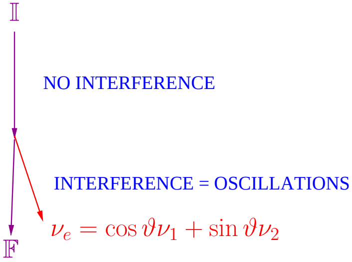
be due to

Neutrino Mixing?

NO

Interference: Double-Slit Analogy





- Decay rate of $\mathbb{I}$ corresponds to fraction of intensity of incoming wave which crosses the barrier
- Fraction of intensity of the incoming wave which crosses the barrier depends on the sizes of the holes
- It does not depend on interference effects which occur after the wave has passed through the barrier
- Analogy: decay rate of $\mathbb{I}$ cannot depend on interference of ν_1 and ν_2 which occurs after decay has happened

Causality

INTERFERENCE OF ν_1 AND ν_2

OCCURRING AFTER THE DECAY

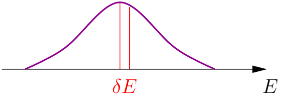
CANNOT AFFECT THE DECAY RATE

arXiv:0801.1465 and arXiv:0805.0435

H.J. Lipkin

- Causality is violated explicitly
- arXiv:0801.1465: The difference in momentum δp_ν between the two neutrino eigenstates with the same energy produces a small initial momentum change δP ...
- arXiv:0805.0435: Since the time dependence depends only on the propagation of the initial state, it is independent of the final state, which is created only at the decay point. Thus there is no violation of causality.
- But in calculation of effect: The phase difference at a time t between states produced by the neutrino mass difference on the motion of the initial ion in the laboratory frame with velocity $V = (P/E)$ is

$\delta\phi \approx -\delta E \cdot t = \Delta m^2/2E$



arXiv:0801.2121 – arXiv:0801.3262

A. N. Ivanov, R. Reda, P. Kienle – M. Faber

- $\mathbb{I} \to \mathbb{F} + \nu$ with final neutrino state $|\nu\rangle = \sum_k |\nu_k\rangle$
- Not even properly normalized to describe one particle:

$$\langle \nu_j | \nu_k \rangle = \delta_{jk} \quad \Longrightarrow \quad \langle \nu | \nu \rangle = 3$$

- Different from standard electron neutrino state $|\nu_e\rangle = \sum_k U_{ek}^* |\nu_k\rangle$

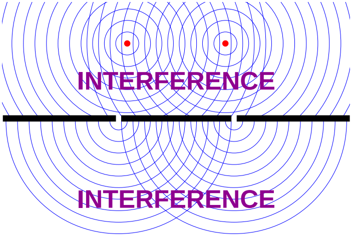
$$P_{\mathbb{I}\to\mathbb{F}+\nu} = \left| \sum_k U_{ek}^* \mathcal{M}_k \right|^2 \quad \neq \quad P_{\mathbb{I}\to\mathbb{F}+\nu_e} = \sum_k |U_{ek}|^2 \, |\mathcal{M}_k|^2$$

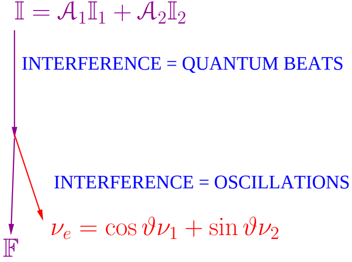
$$P_{\mathbb{I}\to\mathbb{F}+\nu_e} \xrightarrow[m_k \to 0]{} |\mathcal{M}_0|^2 \sum_k |U_{ek}|^2 = |\mathcal{M}_0|^2 = P_{\text{SM}}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu} \xrightarrow[m_k \to 0]{} |\mathcal{M}_0|^2 \left| \sum_k U_{ek}^* \right|^2 \neq P_{\text{SM}}$$

Quantum Beats?

- GSI time anomaly can be due to interference effects in initial state
- Two coherent energy states of the decaying ion $\Longrightarrow$ Quantum Beats





- Incoming waves interfere at holes in barrier
- Causality: interference due to different phases of incoming waves developed during propagation before reaching the barrier

- Quantum beats in GSI experiment can be due to interference of two coherent energy states of the decaying ion which develop different phases before the decay

- Coherence is preserved for a long time if measuring apparatus which monitors the ions with frequency ~ 2 MHz does not distinguish between the two states

- $|\mathbb{I}(t=0)\rangle = \mathcal{A}_1 \, |\mathbb{I}_1\rangle + \mathcal{A}_2 \, |\mathbb{I}_2\rangle \qquad\qquad (|\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 = 1)$

$$\Gamma = \Gamma_1 \simeq \Gamma_2 \quad \Longrightarrow \quad |\mathbb{I}(t)\rangle = \left(\mathcal{A}_1 \, e^{-iE_1 t} \, |\mathbb{I}_1\rangle + \mathcal{A}_2 \, e^{-iE_2 t} \, |\mathbb{I}_2\rangle \right) e^{-\Gamma t/2}$$

$$P_{\mathrm{EC}}(t) = |\langle \nu_e, \mathbb{F} | \mathsf{S} | \mathbb{I}(t) \rangle|^2 = [1 + A \cos(\Delta E t + \varphi)] \, \overline{P}_{\mathrm{EC}} \, e^{-\Gamma t}$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2| \,, \quad \Delta E \equiv E_2 - E_1 \,, \quad \overline{P}_{\mathrm{EC}} = |\langle \nu_e, \mathbb{F} | \mathsf{S} | \mathbb{I}_1 \rangle|^2 \simeq |\langle \nu_e, \mathbb{F} | \mathsf{S} | \mathbb{I}_2 \rangle|^2$$

$$\frac{\mathrm{d}N_{\mathrm{EC}}(t)}{\mathrm{d}t} = N(0) \, [1 + A \cos(\Delta E t + \varphi)] \, \overline{\Gamma}_{\mathrm{EC}} \, e^{-\Gamma t}$$

$$\frac{dN_{EC}(t)}{dt} = N(0)\,[1 + A\,\cos(\Delta E t + \varphi)]\,\overline{\Gamma}_{EC}\,e^{-\Gamma t}$$

$$\Delta E(^{140}_{59}\mathrm{Pr}^{58+}) = (5.86 \pm 0.07) \times 10^{-16}\,\mathrm{eV}\,, \qquad A(^{140}_{59}\mathrm{Pr}^{58+}) = 0.18 \pm 0.03$$

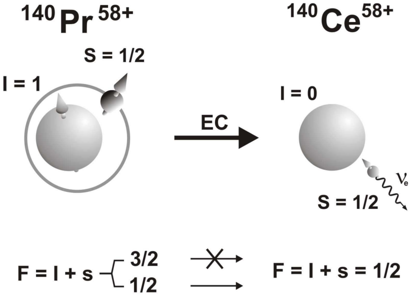
$$\Delta E(^{142}_{61}\mathrm{Pm}^{60+}) = (5.82 \pm 0.18) \times 10^{-16}\,\mathrm{eV}\,, \qquad A(^{142}_{61}\mathrm{Pm}^{60+}) = 0.23 \pm 0.04$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|$$

- Energy splitting is extremely small
- $|\mathcal{A}_1|^2/|\mathcal{A}_2|^2 \sim 1/99$ or $|\mathcal{A}_2|^2/|\mathcal{A}_1|^2 \sim 1/99$
- It is difficult to find an appropriate mechanism

Hyperfine Splitting

smallest known energy splitting



{Litvinov et al, PRL 99 (2007) 262501, arXiv:0711.3709}

$\Delta E \sim 10^{-6}\,\text{eV} \quad \Longrightarrow \quad T \sim 10^{-9}\,\text{s} \quad f \sim \text{GHz}$

too large to explain the GSI anomaly

$T_{\text{GSI}} \simeq 7\,\text{s} \qquad f_{\text{GSI}} \simeq 0.14\text{Hz} \qquad \Delta E_{\text{GSI}} = 2\pi / T_{\text{GSI}} \simeq 6 \times 10^{-16}\,\text{eV}$

Towards the Epilogue?

- Berkeley Group arXiv:0807.0649
  - $^{142}\text{Pm} \to {}^{142}\text{Nd} + \nu_e$ ^{142}Pm in an aluminum foil
    no oscillations at a level 31 times smaller than GSI
  - Reanalysis of old $^{142}\text{Eu} \to {}^{142}\text{Sm} + \nu_e$ data $\Longrightarrow$ no oscillations
  - "It might be argued that our experiment using neutral atoms would be insensitive to the proposed neutrino oscillation effect, since the participation of the remaining atomic electrons could provide a decoherence of the neutrino momentum states in the larger phase space of the final atomic states after the decay."
  - "If multiple electron effects destroy the coherence of the mixed neutrinos' momenta in the final state, this would be apparent in data from the GSI group comparing the decay time spectrum of hydrogen-like and helium-like stored ions."
  - "A further desirable confirmation of the data from the GSI group would be to examine the β^+ decays of the hydrogen-like ions, which should show no oscillations on the timescales available for examination."

- Comment of GSI Group arXiv:0807.2308
  - "It is argued that orbital electron-capture decays of neutral ^{142}Pm atoms implanted into the lattice of a solid do not fulfil the constraints of true two-body beta decays, since momentum as well as energy of the final state are distributed among three objects, namely the electron neutrino, the recoiling daughter atom and the lattice phonons."
  - "It is interesting to note that the observed modulation frequency, if indeed due to the interference of two neutrino mass eigenstates, corresponds to a very small neutrino and, thus, daughter recoil energy difference of about $8 \cdot 10^{-16}$ eV. This is much smaller than typical phonon energies excited by the recoiling daughter nuclei in an aluminum lattice which are in the order of meV. Thus, the modulations could be washed out in a solid environment."

    ! $Q_{\mathrm{EC}} \approx 3 - 5\,\mathrm{MeV}$!
  - "A measurement of the EC-decay of helium-like ^{142}Pm ions should reveal the (probably small) differences to the EC-decay of hydrogen-like ions (such time-resolved measurements are planned, but not yet performed). And, without doubt, the outcome of the three-body β^+ decay of ^{142}Pm is crucial for the interpretation of the GSI data. The–not simple–evaluation of this data is still in progress."

- Munich Group + F. Bosch (GSI) arXiv:0807.3297
  - $^{180}\text{Re} \to {}^{180}\text{W} + \nu_e$ ^{180}Re in a tantalum foil
    no oscillations at a level more than 10 times smaller than GSI
  - "The GSI oscillations are not observable in a conventional experiment with radioactive atoms in a solid environment but must have to do with the unique conditions in the GSI experiment where hydrogen-like ions are moving independently in a storage ring and decaying directly by a true two-body decay to a long-lived (ground-) state."
  - "The recoiling daughter atom has not a well defined momentum, because it moves in a lattice and can only assume momenta allowed by the phonon spectrum. Therefore the effects causing oscillations in the EC decay to bare ions coasting in a storage ring could be washed out in a standard experiment."
  - "However our non-observation of oscillations in the EC decay probability for a system of atoms in a solid decaying to a short lived excited state might restrict theoretical interpretations of the GSI oscillations."

Conclusions

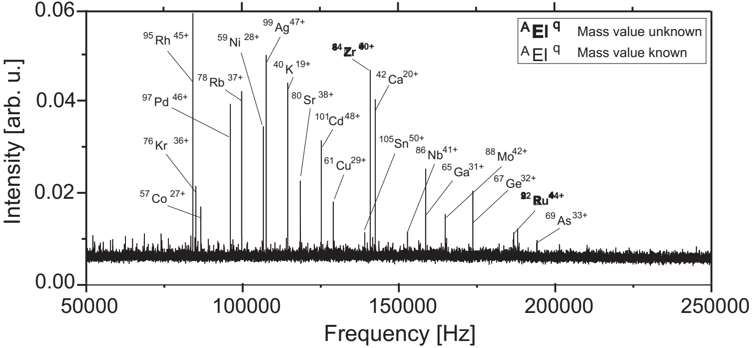
- Interference: due to phase difference of two incoming waves
- Causality: there cannot be interference of waves before they exist
- The GSI ion lifetime anomaly cannot be due to interference of decay product before the decay product start to exist (neutrino mixing in the final state)
- The GSI ion lifetime anomaly can be due to interference of two energy states of the decaying ion: Quantum Beats
- No known mechanism, because
  - Energy splitting of the two energy states: $\Delta E \sim 6 \times 10^{-16}\,\text{eV}$
  - Ratio of probabilities of the two energy states: $1/99$

Schottky Mass Spectrometry

- Stored ions circulate in ESR with revolution frequencies ~ 2 MHz
- At each turn they induce mirror charges on two electrodes
- Revolution frequency spectra provide information about q/m:

$$\text{f} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m\gamma}$$

- Area of each frequency peak is proportional to number of stored ions



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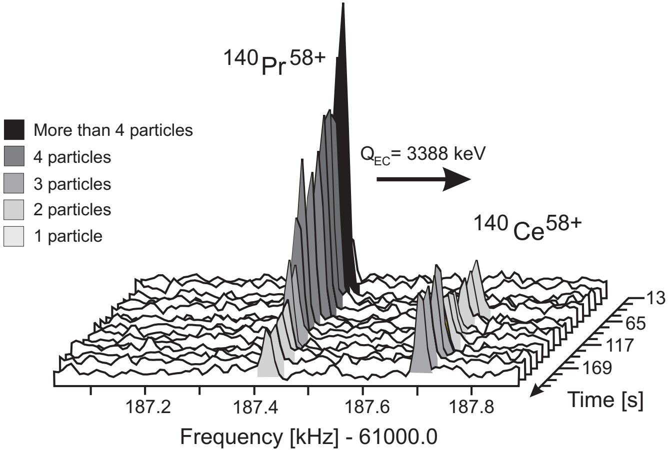
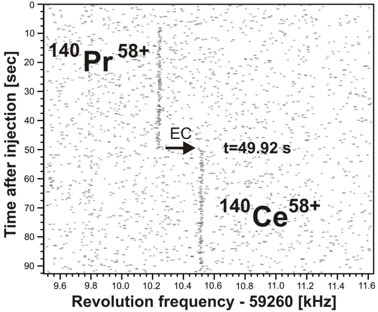
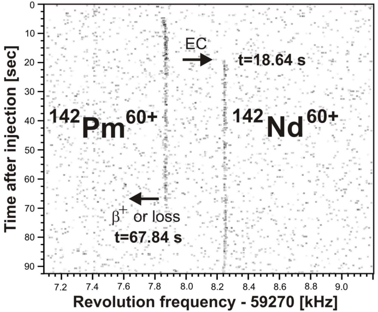


- About six initial $^{140}_{59}\mathrm{Pr}^{58+}$ ions ($\mathrm{f} = qB/2\pi m\gamma$)
- Two decayed via nuclear electron capture into $^{140}_{58}\mathrm{Ce}^{58+}$
- Seen because $\Delta q = 0 \Rightarrow \Delta\mathrm{f}/\mathrm{f} = -\Delta m/m$ (small)
- Other decayed via β^+ decay ($\Delta q = -1 \Rightarrow \Delta\mathrm{f} \sim -150\,\mathrm{kHz}$) or were lost (interactions with residual gas)

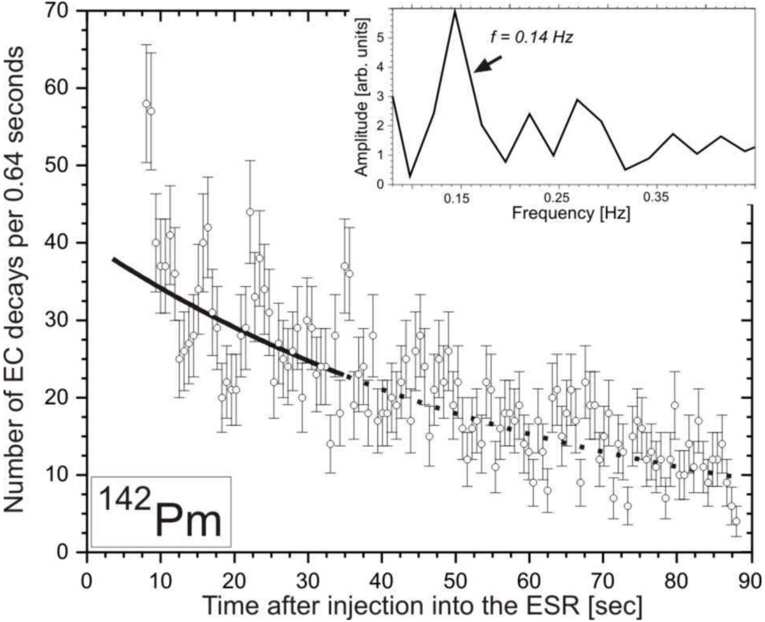


$$ {}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_e $$



$$ {}^{142}_{61}\mathrm{Pm}^{60+} \to {}^{142}_{60}\mathrm{Nd}^{60+} + \nu_e $$

{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}



{Litvinov et al, arXiv:0801.2079v1 [nucl-ex]}

arXiv:0801.2121 – arXiv:0801.3262

A. N. Ivanov, R. Reda, P. Kienle – M. Faber

- $\mathbb{I} \to \mathbb{F} + \nu$ decay rate in time-dependent perturbation theory

  with final neutrino state $|\nu\rangle = \sum_k |\nu_k\rangle$

- Not even properly normalized to describe one particle:

$$\langle \nu_j | \nu_k \rangle = \delta_{jk} \quad \Longrightarrow \quad \langle \nu | \nu \rangle = 3$$

- Different from standard electron neutrino state

$$|\nu_e\rangle = \sum_k U_{ek}^* |\nu_k\rangle$$

Time-Dependent Perturbation Theory

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) = \left| \int_0^t \mathrm{d}\tau \langle \nu, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle \right|^2 = \left| \sum_k \int_0^t \mathrm{d}\tau \langle \nu_k, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle \right|^2$$

$$\mathcal{H}_W(t) = \int \mathrm{d}^3x \, \mathscr{H}_W(x)$$

Effective Four-Fermion Interaction Hamiltonian

$$\mathscr{H}_W(x) = \frac{G_F}{\sqrt{2}} \cos\theta_C \bar{\nu}_e(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x)$$
$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \sum_k U_{ek}^* \bar{\nu}_k(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x)$$

$$\langle \nu_k, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle = U_{ek}^* \, e^{i\Delta E_k t} \, T_k \qquad \text{with} \qquad \Delta E_k = E_k + E_{\mathbb{F}} - E_{\mathbb{I}}$$

$$\int_0^t \mathrm{d}\tau \, e^{i\Delta E_k t} = e^{i\Delta E_k t/2} \, \frac{\sin(\Delta E_k t/2)}{\Delta E_k/2} \xrightarrow[\Delta E_k t \gg 1]{} 2\pi \, \delta(\Delta E_k) \, e^{i\Delta E_k t/2}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) = 4\pi^2 \left| \sum_k U_{ek}^* \, e^{i\Delta E_k t} \, \delta(\Delta E_k) \; T_k \right|^2$$

$T_k \simeq T_j$ $\delta(\Delta E_k)$ satisfied by wave packet

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) \propto \left| \sum_k U_{ek}^* \, e^{i\Delta E_k t} \right|^2$$

Two-Neutrino Mixing

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) \propto \left| \cos\vartheta \, e^{i\Delta E_1 t} + \sin\vartheta \, e^{i\Delta E_2 t} \right|^2 = 1 + \sin 2\vartheta \, \cos\left(\frac{\Delta E t}{2} \right)$$

$$= 1 + \sin 2\vartheta \, \cos\left(\frac{\Delta m^2 t}{4M} \right)$$

$$\Delta E = \Delta E_2 - \Delta E_1 = E_2 - E_1 = \frac{\Delta m^2}{2M}$$

- Standard QFT: $P_{\mathbb{I}\to\mathbb{F}+\nu} = |\langle\nu,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \left|\sum_k \langle\nu_k,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle\right|^2$
- S-matrix operator at first order in perturbation theory:

$$\mathsf{S} = 1 - i\int \mathrm{d}^4x\, \mathscr{H}_W(x)$$

- Effective four-fermion interaction Hamiltonian:

$$\begin{aligned}\mathscr{H}_W(x) &= \frac{G_F}{\sqrt{2}}\cos\theta_C\,\bar{\nu}_e(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)\\ &= \frac{G_F}{\sqrt{2}}\cos\theta_C\sum_k U_{ek}^*\bar{\nu}_k(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)\end{aligned}$$

- $\langle\nu_k,\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle = U_{ek}^*\mathcal{M}_k$ with

$$\mathcal{M}_k = -i\frac{G_F}{\sqrt{2}}\cos\theta_C\int \mathrm{d}^4x\langle\nu_k,\mathbb{F}|\bar{\nu}_k(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)|\mathbb{I}\rangle$$

- $P_{\mathbb{I}\to\mathbb{F}+\nu} = \left|\sum_k U_{ek}^*\mathcal{M}_k\right|^2$ different from standard $P = \sum_k |U_{ek}|^2\,|\mathcal{M}_k|^2$

- Check: in the limit of massless neutrinos decay probability should reduce to the Standard Model decay probability

$$P_{\text{SM}} = \left|\mathcal{M}_{\text{SM}}\right|^2$$

with

$$\mathcal{M}_{\text{SM}} = -i\frac{G_F}{\sqrt{2}}\cos\theta_{\text{C}}\int d^4x \langle \nu_e, \mathbb{F}|\bar{\nu}_e(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)|\mathbb{I}\rangle$$

where ν_e is the Standard Model massless electron neutrino

$$\mathcal{M}_k = -i\frac{G_F}{\sqrt{2}}\cos\theta_{\text{C}}\int d^4x \langle \nu_k, \mathbb{F}|\bar{\nu}_k(x)\gamma_\rho(1-\gamma^5)e(x)\bar{n}(x)\gamma^\rho(1-g_A\gamma^5)p(x)|\mathbb{I}\rangle$$

$$\mathcal{M}_k \xrightarrow[m_k\to 0]{} \mathcal{M}_{\text{SM}}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu} = \left|\sum_k U_{ek}^* \mathcal{M}_k\right|^2 \xrightarrow[m_k\to 0]{} \left|\mathcal{M}_{\text{SM}}\right|^2 \left|\sum_k U_{ek}^*\right|^2 \neq P_{\text{SM}}$$

WRONG!

- Correct normalized final neutrino state ($\langle \nu_e | \nu_e \rangle = 1$):

$$|\nu_e\rangle = \left(\sum_j |\langle \nu_j, \mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 \right)^{-1/2} \sum_k |\nu_k\rangle \langle \nu_k, \mathbb{F}|\mathsf{S}|\mathbb{I}\rangle$$

$$= \left(\sum_j |U_{ej}|^2 |\mathcal{M}_j|^2 \right)^{-1/2} \sum_k U_{ek}^* \mathcal{M}_k |\nu_k\rangle$$

- Standard decay probability:

$$P_{\mathbb{I}\to\mathbb{F}+\nu_e} = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \sum_k |\langle \nu_k, \mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \sum_k |U_{ek}|^2 |\mathcal{M}_k|^2$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu_e} \xrightarrow[m_k \to 0]{} P_{\mathsf{SM}}$$

- In experiments which are not sensitive to the differences of the neutrino masses, as neutrino oscillation experiments,

$$\mathcal{M}_k \simeq \overline{\mathcal{M}} \quad \Longrightarrow \quad |\nu_e\rangle = \sum_k U_{ek}^* |\nu_k\rangle$$

Time-Dependent Perturbation Theory

$$A_k(t) = \int_0^t \mathrm{d}\tau \, \langle \nu_k, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle$$

$$|\nu_e(t)\rangle = \left(\sum_j |A_j(t)|^2 \right)^{-1/2} \sum_k A_k(t) \, |\nu_k\rangle$$

$$P_{\mathbb{I} \to \mathbb{F} + \nu_e} = \left| \left(\sum_j |A_j(t)|^2 \right)^{-1/2} \sum_k A_k^*(t) \int_0^t \mathrm{d}\tau \, \langle I_f, \nu_k | H_W(\tau) | I_i \rangle \right|^2$$

$$\boxed{P_{\mathbb{I} \to \mathbb{F} + \nu_e} = \sum_k |A_k(t)|^2}$$

Quantum Beats

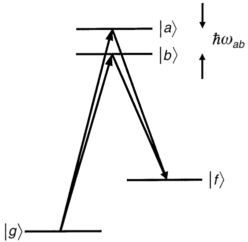


Fig. 1 Diagram of the four level system. A photon is absorbed by the ground state $|b\rangle$ and excites a superposition of states $|a\rangle$ and $|b\rangle$ whose energy separation is $\Delta E = \hbar\omega_{ab}$. Emission of a second photon leaves the system in the final state $|f\rangle$.

$$I_{\rm fl}(t) \propto |\mu_{ag}|^2|\mu_{fa}|^2 e^{-\gamma_a t} + |\mu_{bg}|^2|\mu_{fb}|^2 e^{-\gamma_b t} + |\mu_{ag}\mu_{bg}\mu_{fa}\mu_{fb}| e^{-(\gamma_a+\gamma_b)t/2}\cos(\omega_{ab}t + \theta). \qquad (4)$$

Examination of this expression shows that it consists of two parts, one incoherent term (first two terms) describing the independent decays of the two states $|a\rangle$ and $|b\rangle$ and one coherent or cross term (last term) which decays at the average rate of the two states and, most importantly, is modulated at the angular frequency ω_{ab}. The modulation frequency is the difference of the two angular frequencies in eqn. (2), *i.e.* $\omega_{ab} = |\omega_a - \omega_b|$, and the coherent term in eqn. (4) is therefore termed the quantum beat. The angle θ is included in eqn. (4) to describe the phase of the quantum beat, which depends on a number of factors such as the excitation and detection polarisations and transitions. When the transition moments and decay rates are equal, as is often the case, a particularly simple expression is derived for the four level system. In this case eqn. (4) becomes

$$I_{\rm fl}(t) \propto [1 + \cos(\omega_{ab}t + \theta)]e^{-\gamma t}, \qquad (5)$$

clearly illustrating the contributions of the incoherent and coherent terms to the fluorescence decay. In this special case the quantum beat is 100% modulated. It is important to point out

{Carter, Huber, Quantum beat spectroscopy in chemistry, Chem. Soc. Rev., 29 (2000) 305}

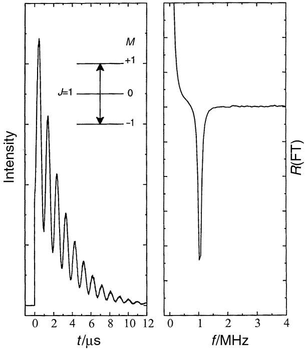


Fig. 2 Zeeman quantum beat recorded for the $R(0)$ line of the 17U transition in CS_2 in an external field of ~15 Gauss. The laser polarisation was perpendicular to the magnetic field direction and prepares a coherence between the $M = \pm 1$ sublevels as shown in the level diagram. This is manifested by a single quantum beat on the fluorescence decay; the real part of the Fourier transform is also shown. The less than 100% modulation, which is observed in virtually all quantum beat measurements in molecules, is due to incoherent emission from the excited states.

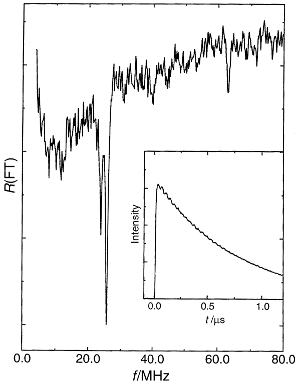
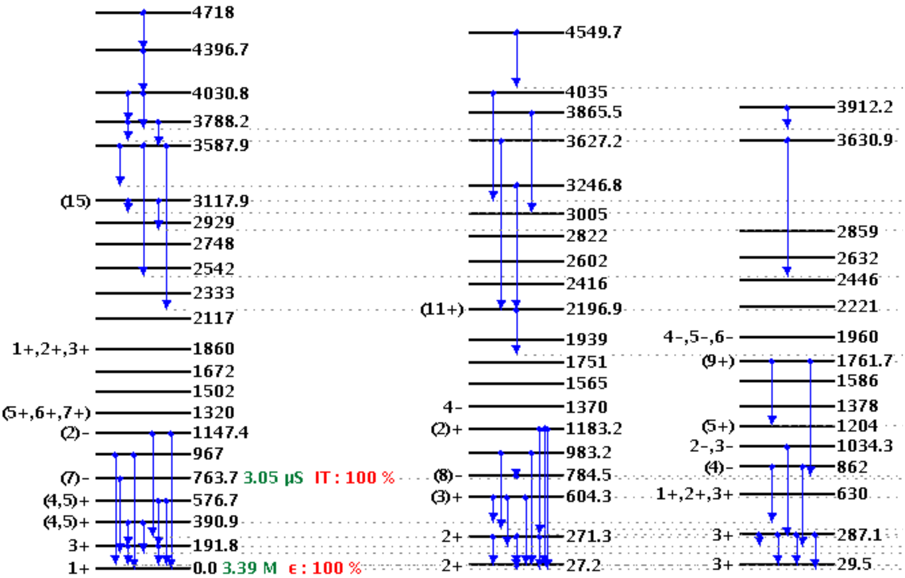


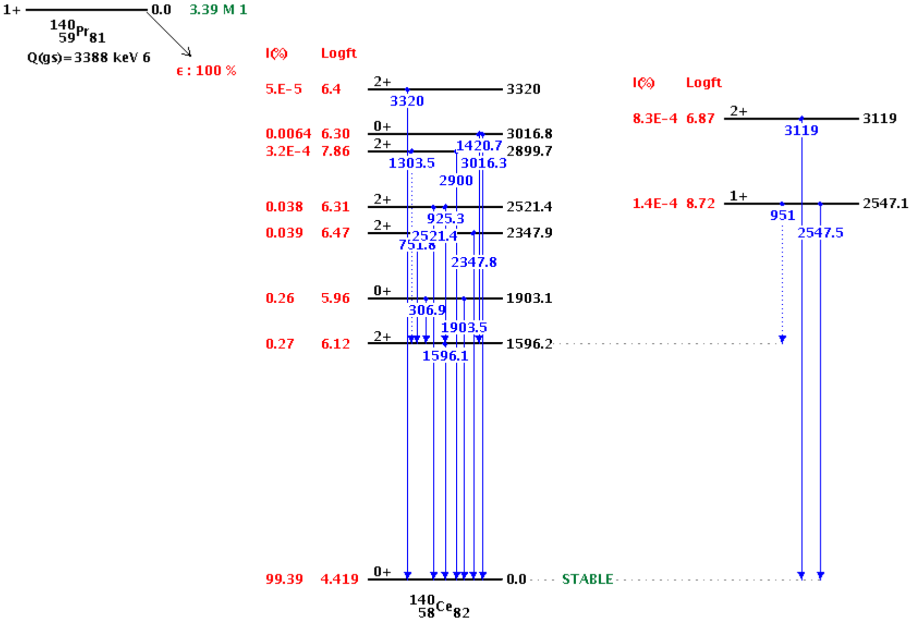
Fig. 6 Nuclear hyperfine quantum beats recorded for the $P_{21}/Q_1(3/2)$ line in a vibrational band of the $A^2\Sigma^+ - X^2\Pi$ transition in the Ar·OD van der Waals complex. The inset shows the fluorescence decay which exhibits weakly modulated quantum beats. Following Fourier transformation the beat frequencies between hyperfine levels in the $A^2\Sigma^+$ state are clearly visible.

{Carter, Huber, Quantum beat spectroscopy in chemistry, Chem. Soc. Rev., 29 (2000) 305}

Praseodymium ($^{140}_{59}Pr_{81}$)

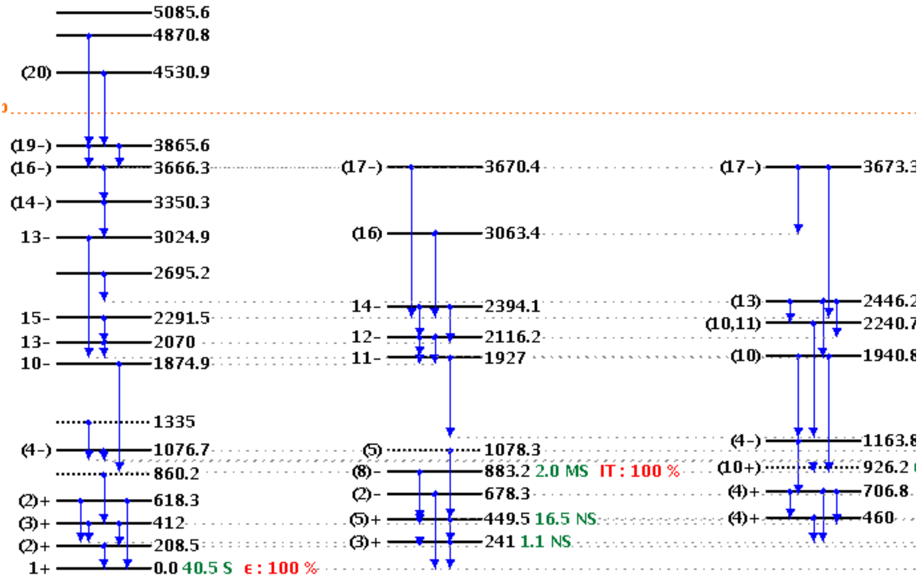


{NuDat, http://www.nndc.bnl.gov/nudat/}



{NuDat, http://www.nndc.bnl.gov/nudat/}

Promethium ($^{142}_{61}Pm_{81}$)



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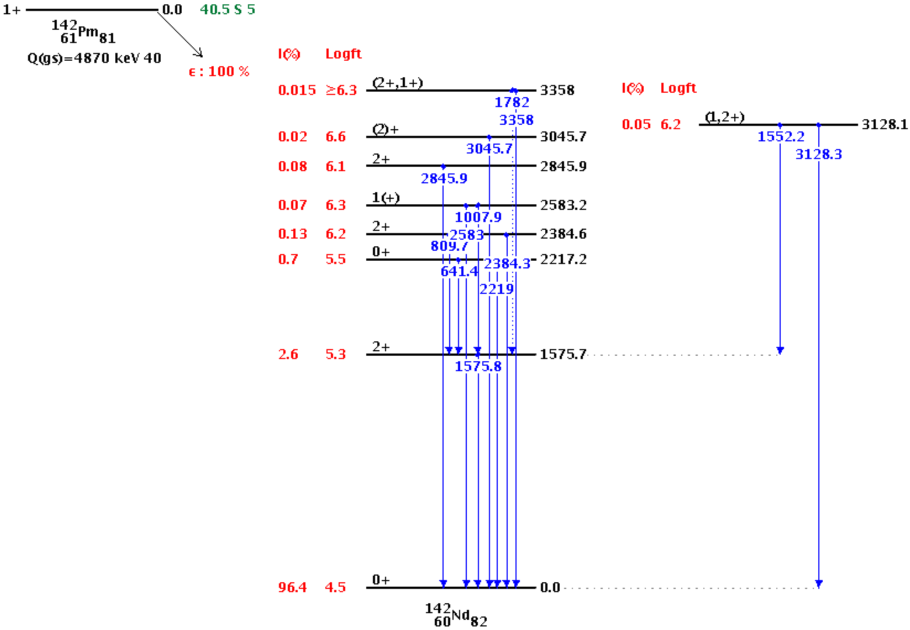

{NuDat, http://www.nndc.bnl.gov/nudat/}