The GSI Time Anomaly: Facts and Fiction

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Results and Perspectives in Particle Physics
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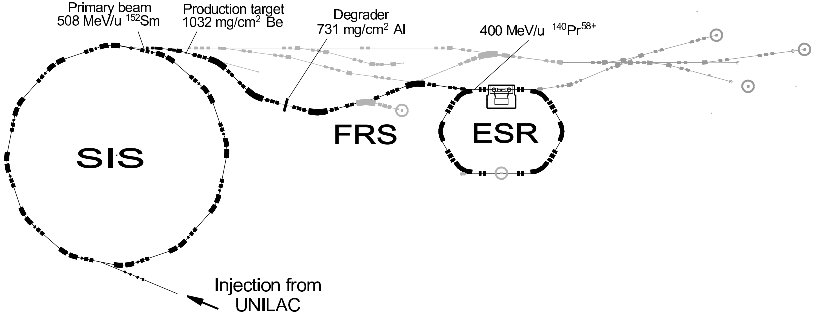
The GSI Experiment

Schematic layout of the secondary nuclear beam facility at GSI



[Litvinov et al, nucl-ex/0509019]

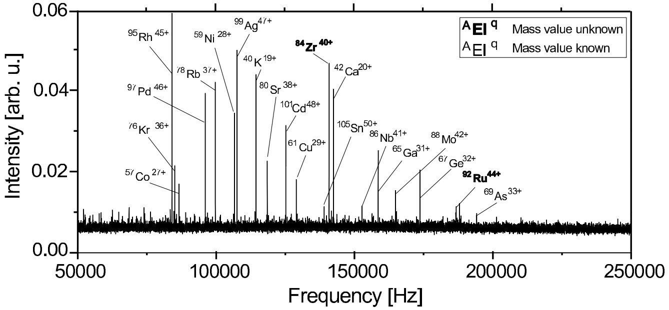
SIS: Heavy Ion Synchrotron
FRS: FRagment Separator
ESR: Experiment Storage Ring

Schottky Mass Spectrometry

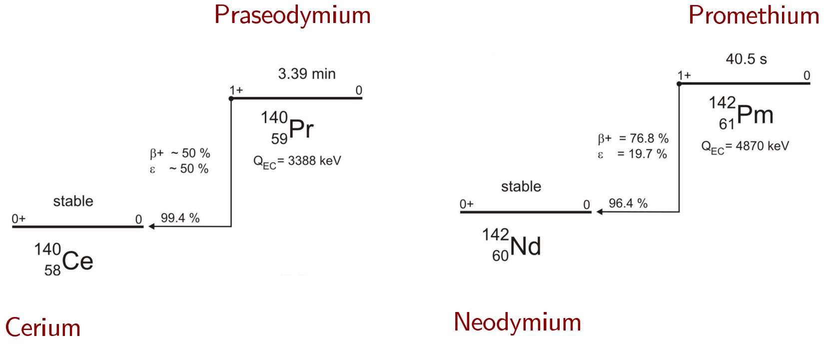
- Stored ions circulate in ESR with revolution frequencies $\sim$ 2 MHz
- At each turn they induce mirror charges on two electrodes
- Revolution frequency spectra provide information about q/m:

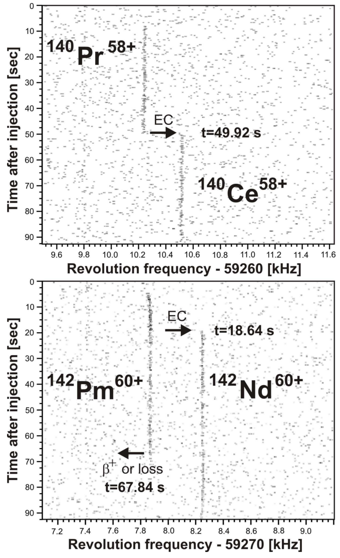
$$\mathrm{f} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m\gamma}$$

- Area of each frequency peak is proportional to number of stored ions



[Litvinov et al, nucl-ex/0509019]





[Litvinov et al, PLB 664 (2008) 168]

Electron Capture

$^{140}_{59}Pr^{58+} \to ^{140}_{58}Ce^{58+} + \nu_e$

$^{142}_{61}Pm^{60+} \to ^{142}_{60}Nd^{60+} + \nu_e$

seen because $\Delta q = 0$

$\Delta f/f = -\Delta m/m$ (small)

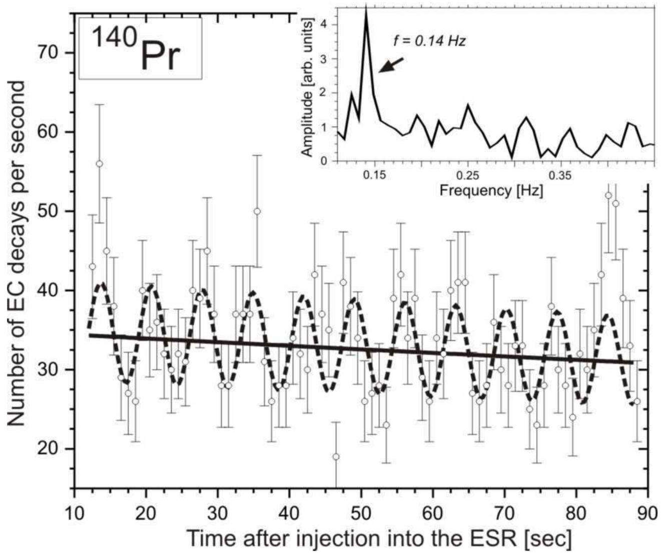
β^+ decay

$^{140}_{59}Pr^{58+} \to ^{140}_{58}Ce^{57+} + e^+ + \nu_e$

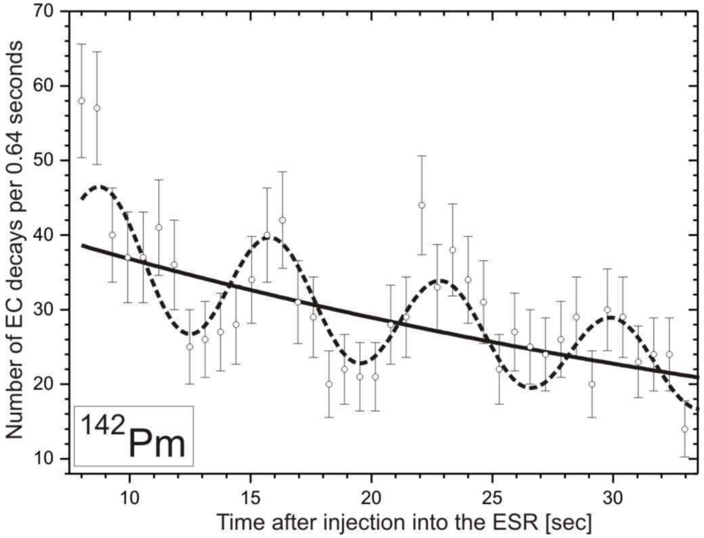
$^{142}_{61}Pm^{60+} \to ^{142}_{60}Nd^{59+} + e^+ + \nu_e$

not seen because $\Delta q = -1$

$\Delta f \sim -150\,\text{kHz}$



[Litvinov et al, PLB 664 (2008) 168]



[Litvinov et al, PLB 664 (2008) 168]

$$(1) \qquad \frac{dN_{EC}(t)}{dt} = \lambda_{EC}\, N(t) = \lambda_{EC}\, N(0)\, e^{-\lambda t}$$

$$(2) \qquad \frac{dN_{EC}(t)}{dt} = \widetilde{\lambda}_{EC}(t)\, N(t) = \widetilde{\lambda}_{EC}(t)\, N(0)\, e^{-\lambda t}$$

$$\lambda = \lambda_{EC} + \lambda_{\beta^+} + \lambda_{loss} \qquad \widetilde{\lambda}_{EC}(t) = \lambda_{EC}\,[1 + a\cos(\omega t + \phi)]$$

Fit parameters of $^{140}_{59}Pr$ data					
Eq.	$N_0\lambda_{EC}$	λ	a	ω	χ^2/DoF
(1)	34.9(18)	0.00138(10)	-	-	107.2/73
(2)	35.4(18)	0.00147(10)	0.18(3)	0.89(1)	67.18/70
Fit parameters of $^{142}_{61}Pm$ data					
Eq.	$N_0\lambda_{EC}$	λ	a	ω	χ^2/DoF
(1)	46.8(40)	0.0240(42)	-	-	63.77/38
(2)	46.0(39)	0.0224(41)	0.23(4)	0.89(3)	31.82/35

$T(^{140}_{59}Pr^{58+}) = 7.06 \pm 0.08\,\text{s} \qquad T(^{142}_{61}Pm^{60+}) = 7.10 \pm 0.22\,\text{s}$

$\langle a \rangle = 0.20 \pm 0.02$

[Litvinov et al, PLB 664 (2008) 168]

Neutrino Mixing?

[Litvinov et al, PLB 664 (2008) 168]

$I_i \to I_f + \nu_e \qquad \nu_e = \cos\vartheta_{\text{SOL}}\,\nu_1 + \sin\vartheta_{\text{SOL}}\,\nu_2$

PROPOSED EXPLANATION: INTERFERENCE OF ν_1 AND ν_2

Initial Ion: Momentum $\vec{P} = 0$, Energy E

Massive ν_k: Momentum $\vec{p}_k$, Energy $E_k = \sqrt{p_k^2 + m_k^2}$

Final Ion: Momentum $-\vec{p}_k$, Energy $M + p_k^2/2M$

$$E_1 + M + p_1^2/2M = E \qquad E_2 + M + p_2^2/2M = E$$

$$\Delta E \equiv E_2 - E_1 \simeq \frac{\Delta m^2}{2M} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

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massive neutrino energy difference: $\Delta E \equiv E_2 - E_1 \simeq \dfrac{\Delta m^2}{2M}$

$$\Delta m^2 = \Delta m^2_{\text{SOL}} \simeq 8 \times 10^{-5}\,\text{eV}^2 \qquad M \simeq 140\,\text{amu} \simeq 130\,\text{GeV}$$

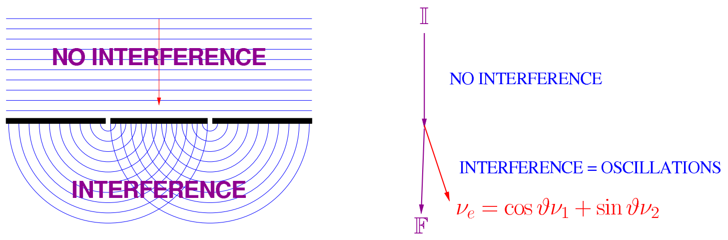
$$\Delta E \simeq 3.1 \times 10^{-16}\,\text{eV}$$

$$T = \frac{2\pi}{\Delta E}\,\gamma \simeq 19.1\,\text{s} \qquad \gamma = 1.43$$

about 3 times larger than $T_{\text{GSI}} \simeq 7\,\text{s}$

CAN INTERFERENCE IN FINAL STATE AFFECT DECAY RATE?

Interference: Double-Slit Analogy



- Decay rate of $\mathbb{I}$ corresponds to fraction of intensity of incoming wave which crosses the barrier
- Fraction of intensity of the incoming wave which crosses the barrier depends on the sizes of the holes
- It does not depend on interference effects which occur after the wave has passed through the barrier
- Analogy: decay rate of $\mathbb{I}$ cannot depend on interference of ν_1 and ν_2 which occurs after decay has happened $\Longleftrightarrow$ CAUSALITY!

Causality

INTERFERENCE OF
COHERENT ENERGY STATES
(ν_1 AND ν_2)
OCCURRING AFTER THE DECAY
(flavor neutrino oscillations)
CANNOT AFFECT THE DECAY RATE

Cross Sections and Decay Rates are always summed incoherently over different final channels:

$$\mathbb{I} \to \mathbb{F}_1 \,, \qquad \mathbb{I} \to \mathbb{F}_2 \,, \qquad \ldots \qquad \Longrightarrow \qquad P_{\mathbb{I}\to\mathbb{F}} = \sum_k P_{\mathbb{I}\to\mathbb{F}_k}$$

entangled final state: $|\mathbb{F}\rangle = \sum_k \mathcal{A}_k |\mathbb{F}_k\rangle$

$$|\mathbb{F}\rangle \propto (\mathsf{S}-\mathbf{1})\,|\mathbb{I}\rangle \qquad \Longrightarrow \qquad \mathcal{A}_k \propto \langle\mathbb{F}_k|\,(\mathsf{S}-\mathbf{1})\,|\mathbb{I}\rangle = \langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle$$

$$\langle\mathbb{F}|\mathbb{F}\rangle = 1 \qquad \Longleftrightarrow \qquad \mathcal{A}_k = \frac{\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle}{\left(\sum_j |\langle\mathbb{F}_j|\mathsf{S}|\mathbb{I}\rangle|^2\right)^{1/2}}$$

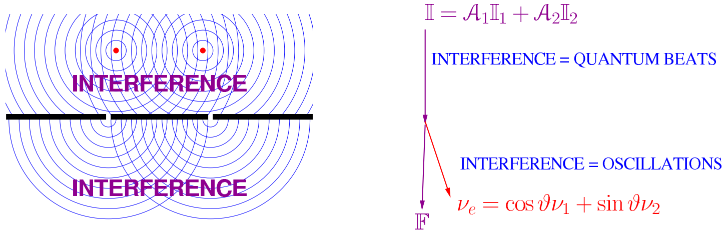
$$P_{\mathbb{I}\to\mathbb{F}} = |\langle\mathbb{F}|\mathsf{S}|\mathbb{I}\rangle|^2 = \left|\sum_k \mathcal{A}_k^* \langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle\right|^2 = \left(\frac{\sum_k |\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle|^2}{\left(\sum_j |\langle\mathbb{F}_j|\mathsf{S}|\mathbb{I}\rangle|^2\right)^{1/2}}\right)^2$$

$$= \sum_k |\langle\mathbb{F}_k|\mathsf{S}|\mathbb{I}\rangle|^2 = \sum_k P_{\mathbb{I}\to\mathbb{F}_k}$$

coherent character of final state is irrelevant for interaction probability!

Quantum Beats?

- GSI time anomaly can be due to interference effects in initial state
- Two coherent energy states of the decaying ion $\Longrightarrow$ Quantum Beats



- Incoming waves interfere at holes in barrier
- Causality: interference due to different phases of incoming waves developed during propagation before reaching the barrier

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Causality

INTERFERENCE OF

COHERENT ENERGY STATES

OCCURRING BEFORE THE DECAY

CAN AFFECT THE DECAY RATE

- Quantum beats in GSI experiment can be due to interference of two coherent energy states of the decaying ion which develop different phases before the decay
- Coherence is preserved for a long time if measuring apparatus which monitors the ions with frequency ∼ 2 MHz does not distinguish between the two states
- $|\mathbb{I}(t=0)\rangle = \mathcal{A}_1\,|\mathbb{I}_1\rangle + \mathcal{A}_2\,|\mathbb{I}_2\rangle \qquad (|\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 = 1)$

$$\Gamma = \Gamma_1 \simeq \Gamma_2 \qquad \Longrightarrow \qquad |\mathbb{I}(t)\rangle = \left(\mathcal{A}_1\, e^{-iE_1 t}\,|\mathbb{I}_1\rangle + \mathcal{A}_2\, e^{-iE_2 t}\,|\mathbb{I}_2\rangle\right) e^{-\Gamma t/2}$$

$$P_{\mathrm{EC}}(t) = |\langle\nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}(t)\rangle|^2 = [1 + A\cos(\Delta E t + \varphi)]\,\overline{P}_{\mathrm{EC}}\, e^{-\Gamma t}$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2| \,, \qquad \Delta E \equiv E_2 - E_1 \,, \qquad \overline{P}_{\mathrm{EC}} = |\langle\nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_1\rangle|^2 \simeq |\langle\nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_2\rangle|^2$$

$$\frac{\mathrm{d}N_{\mathrm{EC}}(t)}{\mathrm{d}t} = N(0)\,[1 + A\cos(\Delta E t + \varphi)]\,\overline{\Gamma}_{\mathrm{EC}}\, e^{-\Gamma t}$$

$$\frac{dN_{\text{EC}}(t)}{dt} = N(0)\,[1 + A\cos(\Delta E t + \varphi)]\,\overline{\Gamma}_{\text{EC}}\,e^{-\Gamma t}$$

$$\Delta E({}^{140}_{59}\text{Pr}^{58+}) = (5.86 \pm 0.07) \times 10^{-16}\,\text{eV}\,, \qquad A({}^{140}_{59}\text{Pr}^{58+}) = 0.18 \pm 0.03$$

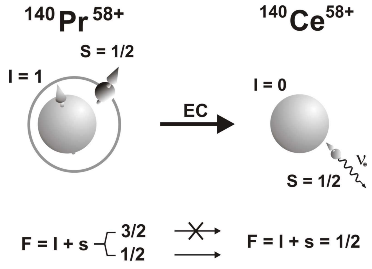
$$\Delta E({}^{142}_{61}\text{Pm}^{60+}) = (5.82 \pm 0.18) \times 10^{-16}\,\text{eV}\,, \quad A({}^{142}_{61}\text{Pm}^{60+}) = 0.23 \pm 0.04$$

$$A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|$$

- Energy splitting is extremely small
- $|\mathcal{A}_1|^2/|\mathcal{A}_2|^2 \sim 1/99$ or $|\mathcal{A}_2|^2/|\mathcal{A}_1|^2 \sim 1/99$
- It is difficult to find an appropriate mechanism

Hyperfine Splitting

smallest known energy splitting



[Litvinov et al, PRL 99 (2007) 262501]

$$\Delta E \sim 10^{-6}\,\text{eV} \quad \Longrightarrow \quad T \sim 10^{-9}\,\text{s} \qquad f \sim \text{GHz}$$

too large to explain the GSI anomaly

$$T_{\text{GSI}} \simeq 7\,\text{s} \qquad f_{\text{GSI}} \simeq 0.14\text{Hz} \qquad \Delta E_{\text{GSI}} = 2\pi/T_{\text{GSI}} \simeq 6 \times 10^{-16}\,\text{eV}$$

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feeling of smallness of $\Delta E_{\text{GSI}} \simeq 6 \times 10^{-16}\,\text{eV}$

$$\mu_{\text{N}} B_{\oplus} \simeq \left(3 \times 10^{-12}\,\text{eV}\,\text{G}^{-1}\right)(0.5\,\text{G}) = 1.5 \times 10^{-12}\,\text{eV}$$

$$\Delta E_{\text{GSI}} \simeq 4 \times 10^{-4}\,\mu_{\text{N}} B_{\oplus}$$

Further Developments

Berkeley Experiment – arXiv:0807.0649

- EC: $^{142}\text{Pm} \to {}^{142}\text{Nd} + \nu_e$ $\qquad$ ^{142}Pm in an aluminum foil

  no oscillations at a level 31 times smaller than GSI
- Reanalysis of old $^{142}\text{Eu} \to {}^{142}\text{Sm} + \nu_e$ EC data $\Longrightarrow$ no oscillations
- Differences with GSI Experiment: neutral and stopped atoms

Munich Group + F. Bosch (GSI) – arXiv:0807.3297

- $^{180}\text{Re} \to {}^{180}\text{W} + \nu_e$ $\qquad$ ^{180}Re in a tantalum foil
  no oscillations at a level more than 10 times smaller than GSI

Conclusions

- Interference: due to phase difference of two incoming waves
- Causality: there cannot be interference of waves before they exist
- The GSI ion lifetime anomaly cannot be due to interference of decay product before the decay product start to exist (neutrino mixing in the final state)
- The GSI ion lifetime anomaly can be due to interference of two energy states of the decaying ion: Quantum Beats
- No known mechanism, because
  - Energy splitting of the two energy states: $\Delta E \sim 6 \times 10^{-16}\,\text{eV}$
  - Ratio of probabilities of the two energy states: $1/99$
- GSI group is trying to measure EC of different hydrogen-like ions, EC of helium-like ions and β^{+} decay of hydrogen-like ions

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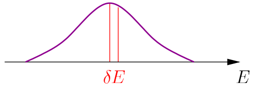
arXiv:0801.1465 and arXiv:0805.0435

H.J. Lipkin

- Causality is violated explicitly
- arXiv:0801.1465: The difference in momentum δp_ν between the two neutrino eigenstates with the same energy produces a small initial momentum change δP . . .
- arXiv:0805.0435: Since the time dependence depends only on the propagation of the initial state, it is independent of the final state, which is created only at the decay point. Thus there is no violation of causality.
- But in calculation of effect: The phase difference at a time t between states produced by the neutrino mass difference on the motion of the initial ion in the laboratory frame with velocity $V = (P/E)$ is

$$\delta\phi \approx -\delta E \cdot t = \Delta m^2/2E$$



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arXiv:0801.2121 – arXiv:0801.3262

A. N. Ivanov, R. Reda, P. Kienle – M. Faber

- $\mathbb{I} \to \mathbb{F} + \nu$ decay rate in time-dependent perturbation theory

  with final neutrino state $|\nu\rangle = \sum_k |\nu_k\rangle$
- Not even properly normalized to describe one particle:

$$\langle \nu_j | \nu_k \rangle = \delta_{jk} \quad \Longrightarrow \quad \langle \nu | \nu \rangle = 3$$

- Different from standard electron neutrino state

$$|\nu_e\rangle = \sum_k U_{ek}^{*} |\nu_k\rangle$$

Time-Dependent Perturbation Theory

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) = \left| \int_0^t d\tau \langle \nu, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle \right|^2 = \left| \sum_k \int_0^t d\tau \langle \nu_k, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle \right|^2$$

$$\mathcal{H}_W(t) = \int d^3x \, \mathscr{H}_W(x)$$

Effective Four-Fermion Interaction Hamiltonian

$$\begin{aligned} \mathscr{H}_W(x) &= \frac{G_F}{\sqrt{2}} \cos\theta_C \bar{\nu}_e(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) \\ &= \frac{G_F}{\sqrt{2}} \cos\theta_C \sum_k U_{ek}^{*} \bar{\nu}_k(x) \gamma_\rho (1-\gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) \end{aligned}$$

$$\langle \nu_k, \mathbb{F} | \mathcal{H}_W(\tau) | \mathbb{I} \rangle = U_{ek}^{*} \, e^{i \Delta E_k t} \, T_k \qquad \text{with} \qquad \Delta E_k = E_k + E_{\mathbb{F}} - E_{\mathbb{I}}$$

$$\int_0^t d\tau \, e^{i \Delta E_k t} = e^{i \Delta E_k t/2} \, \frac{\sin(\Delta E_k t/2)}{\Delta E_k/2} \xrightarrow[\Delta E_k t \gg 1]{} 2\pi \, \delta(\Delta E_k) \, e^{i \Delta E_k t/2}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) = 4\pi^2 \left| \sum_k U_{ek}^* \, e^{i\Delta E_k t} \, \delta(\Delta E_k) \ T_k \right|^2$$

$T_k \simeq T_j$ $\delta(\Delta E_k)$ satisfied by wave packet

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) \propto \left| \sum_k U_{ek}^* \, e^{i\Delta E_k t} \right|^2$$

Two-Neutrino Mixing

$$P_{\mathbb{I}\to\mathbb{F}+\nu}(t) \propto \left| \cos\vartheta \, e^{i\Delta E_1 t} + \sin\vartheta \, e^{i\Delta E_2 t} \right|^2 = 1 + \sin 2\vartheta \, \cos\left(\frac{\Delta E t}{2}\right)$$

$$= 1 + \sin 2\vartheta \, \cos\left(\frac{\Delta m^2 t}{4M}\right)$$

$$\Delta E = \Delta E_2 - \Delta E_1 = E_2 - E_1 = \frac{\Delta m^2}{2M}$$

- Standard QFT: $P_{\mathbb{I}\to\mathbb{F}+\nu} = |\langle \nu, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 = \left| \sum_k \langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle \right|^2$
- S-matrix operator at first order in perturbation theory:

$$\mathsf{S} = 1 - i \int d^4x \, \mathscr{H}_W(x)$$

- Effective four-fermion interaction Hamiltonian:

$$\mathscr{H}_W(x) = \frac{G_F}{\sqrt{2}} \cos\theta_C \bar{\nu}_e(x) \gamma_\rho (1 - \gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x)$$

$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \sum_k U_{ek}^* \bar{\nu}_k(x) \gamma_\rho (1 - \gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x)$$

- $\langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle = U_{ek}^* \mathcal{M}_k$ with

$$\mathcal{M}_k = -i \frac{G_F}{\sqrt{2}} \cos\theta_C \int d^4x \langle \nu_k, \mathbb{F} | \bar{\nu}_k(x) \gamma_\rho (1 - \gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) | \mathbb{I} \rangle$$

- $P_{\mathbb{I}\to\mathbb{F}+\nu} = \left| \sum_k U_{ek}^* \mathcal{M}_k \right|^2$ different from standard $P = \sum_k |U_{ek}|^2 \, |\mathcal{M}_k|^2$

- Check: in the limit of massless neutrinos decay probability should reduce to the Standard Model decay probability

$$P_{\text{SM}} = |\mathcal{M}_{\text{SM}}|^2$$

with

$$\mathcal{M}_{\text{SM}} = -i \frac{G_F}{\sqrt{2}} \cos\theta_C \int d^4x \langle \nu_e, \mathbb{F} | \bar{\nu}_e(x) \gamma_\rho (1 - \gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) | \mathbb{I} \rangle$$

where ν_e is the Standard Model massless electron neutrino

$$\mathcal{M}_k = -i \frac{G_F}{\sqrt{2}} \cos\theta_C \int d^4x \langle \nu_k, \mathbb{F} | \bar{\nu}_k(x) \gamma_\rho (1 - \gamma^5) e(x) \bar{n}(x) \gamma^\rho (1 - g_A \gamma^5) p(x) | \mathbb{I} \rangle$$

$$\mathcal{M}_k \xrightarrow[m_k \to 0]{} \mathcal{M}_{\text{SM}}$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu} = \left| \sum_k U_{ek}^* \mathcal{M}_k \right|^2 \xrightarrow[m_k \to 0]{} |\mathcal{M}_{\text{SM}}|^2 \left| \sum_k U_{ek}^* \right|^2 \neq P_{\text{SM}}$$

WRONG!

- Correct normalized final neutrino state ($\langle \nu_e | \nu_e \rangle = 1$):

$$|\nu_e\rangle = \left(\sum_j |\langle \nu_j, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 \right)^{-1/2} \sum_k |\nu_k\rangle \, \langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle$$

$$= \left(\sum_j |U_{ej}|^2 |\mathcal{M}_j|^2 \right)^{-1/2} \sum_k U_{ek}^* \mathcal{M}_k \, |\nu_k\rangle$$

- Standard decay probability:

$$P_{\mathbb{I}\to\mathbb{F}+\nu_e} = |\langle \nu_e, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 = \sum_k |\langle \nu_k, \mathbb{F} | \mathsf{S} | \mathbb{I} \rangle|^2 = \sum_k |U_{ek}|^2 \, |\mathcal{M}_k|^2$$

$$P_{\mathbb{I}\to\mathbb{F}+\nu_e} \xrightarrow[m_k \to 0]{} P_{\text{SM}}$$

- In experiments which are not sensitive to the differences of the neutrino masses, as neutrino oscillation experiments,

$$\mathcal{M}_k \simeq \overline{\mathcal{M}} \quad \Longrightarrow \quad |\nu_e\rangle = \sum_k U_{ek}^* \, |\nu_k\rangle$$

Time-Dependent Perturbation Theory?

not appropriate because electron capture and decay are interrupted by

Schottky Mass Spectrometry

with ESR revolution frequency ~ 2 MHz, i.e. every

$\sim 5 \times 10^{-7}\,\mathrm{s}$

much smaller than ion lifetime

$T_{1/2}({}^{140}_{59}\mathrm{Pr}) \simeq 3.39\,\mathrm{m}$ $\qquad$ $T_{1/2}({}^{142}_{61}\mathrm{Pm}) \simeq 40.5\,\mathrm{s}$

and period of anomalous oscillations $T \simeq 7\,\mathrm{s}$

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interaction time: $\quad t_W \sim \dfrac{\hbar}{m_W} \simeq \dfrac{6.6 \times 10^{-22}\,\mathrm{MeV\,s}}{8.0 \times 10^{4}\,\mathrm{MeV}} \sim 10^{-26}\,\mathrm{s}$

$t \gg t_W$ in Time-Dependent Perturbation Theory

$\Downarrow$

Quantum Field Theory result

Lambiase, Papini, Scarpetta – nucl-th/0811.2302

Spin-rotation coupling in non-exponential decay of hydrogen-like heavy ions

14 November 2008

We discuss a model in which a recently reported modulation in the decay of the hydrogen-like ions ${}^{140}\mathrm{Pr}^{58+}$ and ${}^{142}\mathrm{Pm}^{60+}$ arises from the coupling of rotation to the spin of electron and nuclei (Thomas precession).

! electron and nucleus spins cannot precess independently !

! they are tied by spin-spin interaction, which generates hyperfine splitting !

! it is much stronger than precession force !

! precession is strongly suppressed by hyperfine splitting !