

Phenomenology of Neutrino Mixing and Oscillations

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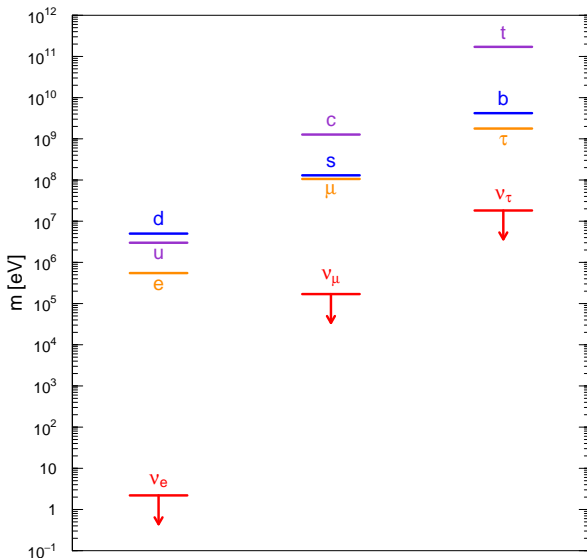
Outline

- Brief Introduction to Neutrino Masses and Mixing
- Three-Neutrino Mixing and Oscillations
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies
- Conclusions

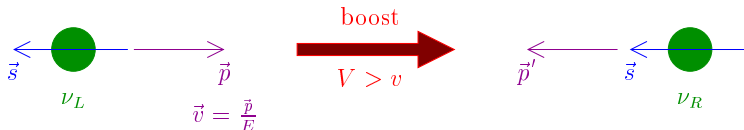
Brief Introduction to Neutrino Masses and Mixing

- Brief Introduction to Neutrino Masses and Mixing
 - Fermion Mass Spectrum
 - Standard Model: Massless Neutrinos
 - Extension of the SM: Massive Neutrinos
 - Lepton Numbers
 - Neutrino Mixing
 - Neutrino Oscillations
 - Two-Neutrino Mixing and Oscillations
- Three-Neutrino Mixing and Oscillations
- Absolute Scale of Neutrino Masses
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Fermion Mass Spectrum



Standard Model: Massless Neutrinos



Standard Model: $\nu_L, \nu_R^c \implies$ no Dirac mass term

$$\mathcal{L}^D \sim m^D \nu_L \nu_R \quad (\text{no } \nu_R, \nu_L^c)$$

Majorana Neutrino: $\nu^c = \nu$

$\nu_R^c = \nu_R \implies$ Majorana mass term

$$\mathcal{L}^M \sim m^M \nu_L \nu_R^c$$

Standard Model: Majorana mass term **not** allowed by $SU(2)_L \times U(1)_Y$
(no Higgs triplet)

Extension of the SM: Massive Neutrinos

Standard Model can be extended with ν_R ($e_L, e_R; u_L, u_R; d_L, d_R; \dots$)

$\nu_L + \nu_R \Rightarrow$ Dirac neutrino mass term $\mathcal{L}^D \sim m^D \nu_L \nu_R \Rightarrow m^D \lesssim 100 \text{ GeV}$

surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}_R^M \sim m_R^M \nu_L^c \nu_R$

total neutrino mass term $\mathcal{L}^{D+M} \sim \begin{pmatrix} \nu_L & \nu_L^c \end{pmatrix} \begin{pmatrix} 0 & m^D \\ m^D & m_R^M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}$

m_R^M can be arbitrarily large (not protected by SM symmetries)

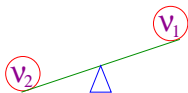
$m_R^M \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^M \gg m^D$

diagonalization of $\begin{pmatrix} 0 & m^D \\ m^D & m_R^M \end{pmatrix} \Rightarrow m_1 \simeq \frac{(m^D)^2}{m_R^M}, \quad m_2 \simeq m_R^M$

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

3-GEN \Rightarrow effective low-energy 3- ν mixing



see-saw mechanism

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Lepton Numbers

Standard Model:

Lepton numbers are conserved

	L_e	L_μ	L_τ		L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0	(ν_e^c, e^+)	-1	0	0
(ν_μ, μ^-)	0	+1	0	(ν_μ^c, μ^+)	0	-1	0
(ν_τ, τ^-)	0	0	+1	(ν_τ^c, τ^+)	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Dirac mass term $m^D \nu_L \nu_R \rightarrow (\nu_{eL} \quad \nu_{\mu L} \quad \nu_{\tau L}) \begin{pmatrix} m_{ee}^D & m_{e\mu}^D & m_{e\tau}^D \\ m_{\mu e}^D & m_{\mu\mu}^D & m_{\mu\tau}^D \\ m_{\tau e}^D & m_{\tau\mu}^D & m_{\tau\tau}^D \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^M \nu_L \nu_R^c \rightarrow (\nu_{eL} \quad \nu_{\mu L} \quad \nu_{\tau L}) \begin{pmatrix} m_{ee}^M & m_{e\mu}^M & m_{e\tau}^M \\ m_{\mu e}^M & m_{\mu\mu}^M & m_{\mu\tau}^M \\ m_{\tau e}^M & m_{\tau\mu}^M & m_{\tau\tau}^M \end{pmatrix} \begin{pmatrix} \nu_{eR}^c \\ \nu_{\mu R}^c \\ \nu_{\tau R}^c \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu_\alpha^c) = -L(\nu_\beta) \Rightarrow |\Delta L| = 2$

Neutrino Mixing

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

diagonalization of mass matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \sum_{k=1}^3 m_k \nu_k \nu_k$$

Neutrino Oscillations

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho (\nu_e L \gamma^\rho e_L + \nu_\mu L \gamma^\rho \mu_L + \nu_\tau L \gamma^\rho \tau_L)$$

Fields $\nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Rightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$ States

initial flavor: $\alpha = e \text{ or } \mu \text{ or } \tau$

$$|\nu_k(t, x)\rangle = e^{-iE_k t + ip_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, x)} |\nu_\beta\rangle$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t, x) = |\langle \nu_\beta | \nu_\alpha(t, x) \rangle|^2 = |\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, x)|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\implies t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

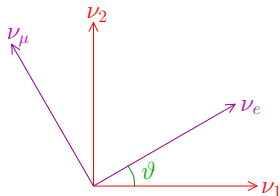
$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* e^{-im_k^2 L/2E} U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L/E) &= \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re} \left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left(\frac{\Delta m_{kj}^2 L}{4E} \right) \\ &\quad + 2 \sum_{k>j} \text{Im} \left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin \left(\frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^2 U_{\alpha k} |\nu_k\rangle \quad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\begin{aligned} |\nu_e\rangle &= \cos\vartheta |\nu_1\rangle + \sin\vartheta |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta |\nu_1\rangle + \cos\vartheta |\nu_2\rangle \end{aligned}$$

$$\Delta m^2 \equiv \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

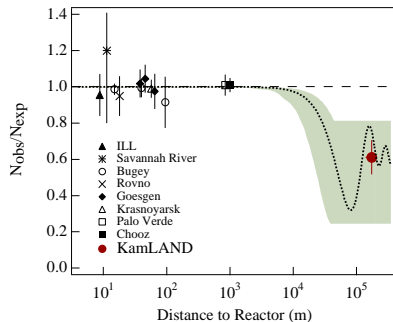
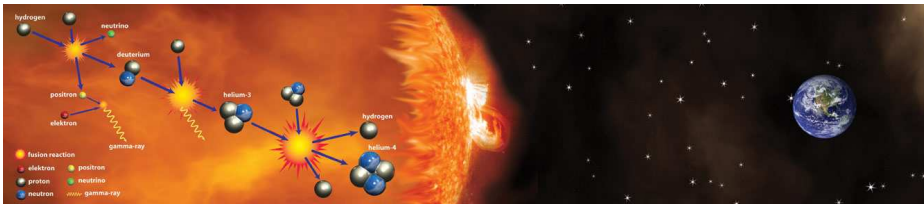
Transition Probability: $P_{\nu_e \rightarrow \nu_\mu} = P_{\nu_\mu \rightarrow \nu_e} = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$

Survival Probabilities: $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu} = 1 - P_{\nu_e \rightarrow \nu_\mu}$

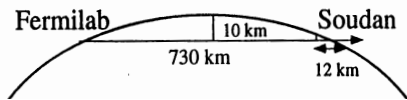
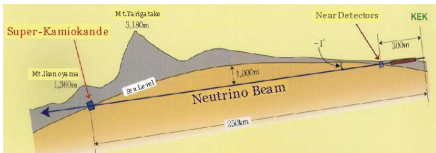
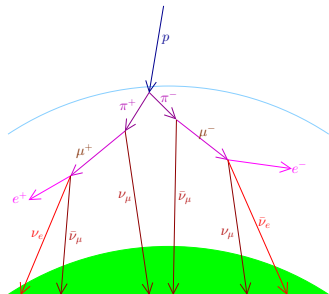
Three-Neutrino Mixing and Oscillations

- Brief Introduction to Neutrino Masses and Mixing
- Three-Neutrino Mixing and Oscillations
 - Solar Neutrinos and KamLAND
 - Atmospheric Neutrinos and LBL
 - Three-Neutrino Mixing
 - Allowed Three-Neutrino Schemes
 - Mixing Matrix
- Absolute Scale of Neutrino Masses
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Solar Neutrinos and KamLAND



Atmospheric Neutrinos and LBL



Solar
 $\nu_e \rightarrow \nu_\mu, \nu_\tau$

Reactor
 $\bar{\nu}_e$ disappearance

(Homestake
 Kamiokande
 GALLEX/GNO & SAGE
 Super-Kamiokande
 SNO
 BOREXino
 (KamLAND))

$\rightarrow \left\{ \begin{array}{l} \Delta m_{\text{SOL}}^2 \simeq (7.6 \pm 0.2) \times 10^{-5} \text{ eV}^2 \\ \tan^2 \vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06 \end{array} \right.$

Atmospheric
 $\nu_\mu \rightarrow \nu_\tau$

Accelerator
 ν_μ disappearance

(Kamiokande
 IMB
 Super-Kamiokande
 MACRO
 Soudan-2
 (K2K & MINOS))

$\rightarrow \left\{ \begin{array}{l} \Delta m_{\text{ATM}}^2 \simeq (2.4 \pm 0.1) \times 10^{-3} \text{ eV}^2 \\ \sin^2 \vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07 \end{array} \right.$

Two scales of Δm^2 : $\Delta m_{\text{ATM}}^2 \simeq 30 \Delta m_{\text{SOL}}^2$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \quad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

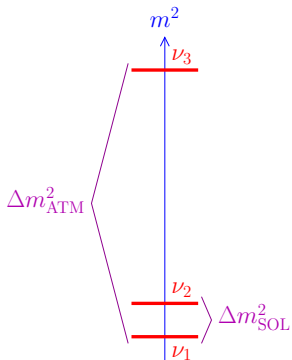
three massive fields: ν_1, ν_2, ν_3

$$\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = m_2^2 - m_1^2 + m_3^2 - m_2^2 + m_1^2 - m_3^2 = 0$$

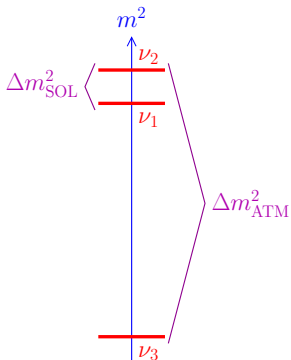
$$\Delta m_{\text{SOL}}^2 = \Delta m_{21}^2 \simeq (7.6 \pm 0.2) \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{\text{ATM}}^2 \simeq |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \simeq (2.4 \pm 0.1) \times 10^{-3} \text{ eV}^2$$

Allowed Three-Neutrino Schemes



"normal"



"inverted"

different signs of $\Delta m_{31}^2 \simeq \Delta m_{32}^2$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2|$$

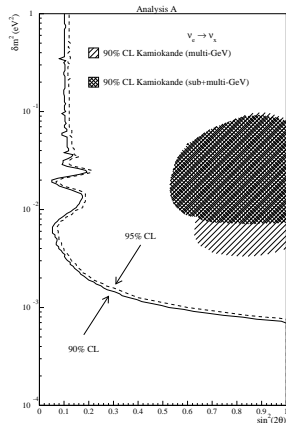
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

SOL →
↑
 ATM

$$\text{CHOOZ: } \begin{cases} \Delta m_{\text{CHOOZ}}^2 = \Delta m_{31}^2 = \Delta m_{\text{ATM}}^2 \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$$

$$|U_{e3}|^2 \lesssim 5 \times 10^{-2}$$

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

$$|U_{e1}|^2 \simeq \cos^2 \vartheta_{\text{SOL}} \quad |U_{e2}|^2 \simeq \sin^2 \vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2 \vartheta_{\text{ATM}} \quad |U_{\tau 3}|^2 \simeq \cos^2 \vartheta_{\text{ATM}}$$

Bilarge Mixing

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\sin^2 \vartheta_{12} = 0.304_{-0.016}^{+0.022}$$

$$\sin^2 \vartheta_{23} = 0.50_{-0.06}^{+0.07}$$

$$\sin^2 \vartheta_{13} < 0.035 \quad (90\% \text{ C.L.})$$

[Schwetz, Tortola, Valle, New J. Phys. 10 (2008) 113011]

Hint of $\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

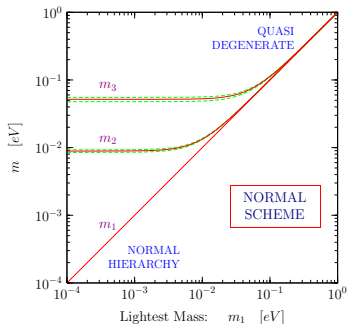
$$\sin^2 \vartheta_{13} = 0.016 \pm 0.010 \quad [\text{Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801}]$$

Absolute Scale of Neutrino Masses

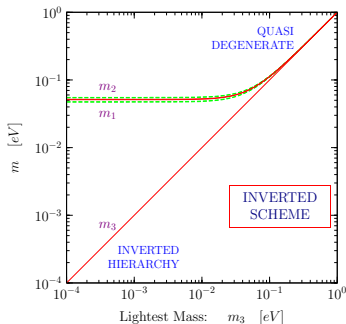
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 - Mass Hierarchy or Degeneracy?
 - Tritium Beta-Decay
 - Neutrinoless Double-Beta Decay
 - Cosmological Bound on Neutrino Masses
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- Conclusions

Mass Hierarchy or Degeneracy?

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$

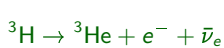
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$

$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2} \text{ eV}$

Tritium Beta-Decay

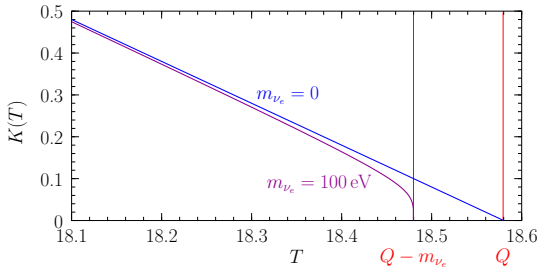


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) p E (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^3\text{H}} - M_{{}^3\text{He}} - m_e = 18.58 \text{ keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) p E}} = \left[(Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2} \right]^{1/2}$$



$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

Mainz & Troitsk

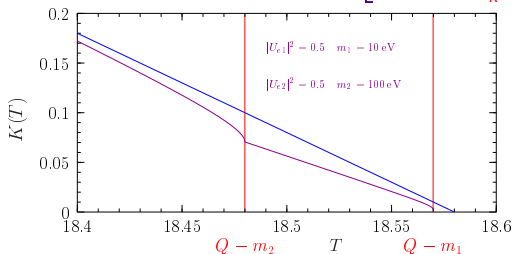
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2012)

[arXiv:0810.3281]

sensitivity: $m_{\nu_e} \simeq 0.2 \text{ eV} (3\sigma)$

$$\text{Neutrino Mixing} \implies K(T) = \left[(Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}$$



analysis of data is
different from the
no-mixing case:

$2N - 1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

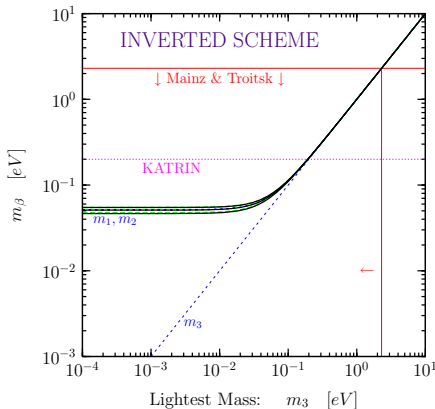
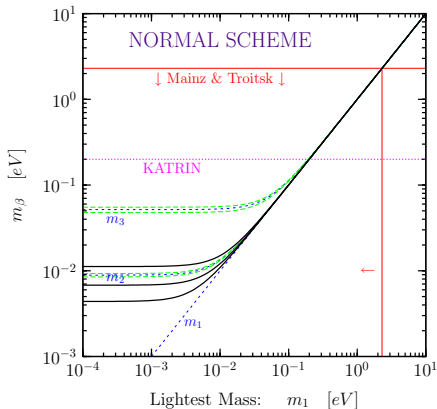
if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass:

$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$\begin{aligned} K^2 &= (Q - T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q - T)^2}} \simeq (Q - T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q - T)^2} \right] \\ &= (Q - T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T)^2} \right] \simeq (Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \end{aligned}$$

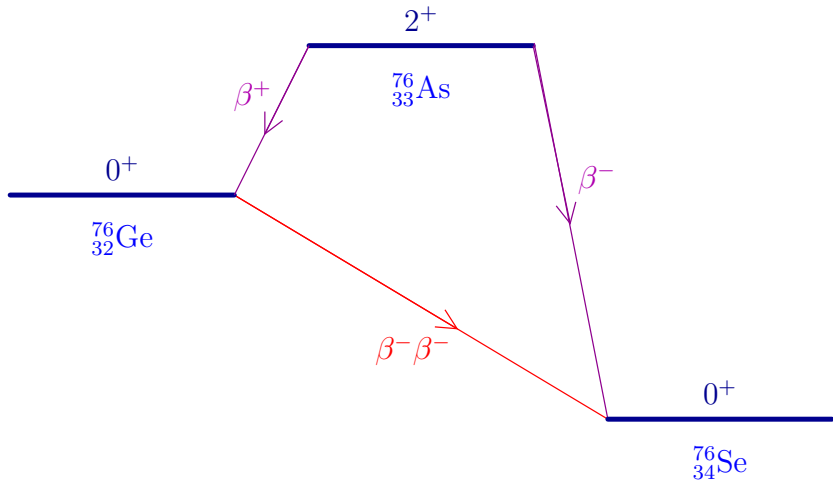
$$m_\beta^2 = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2} \text{ eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass:

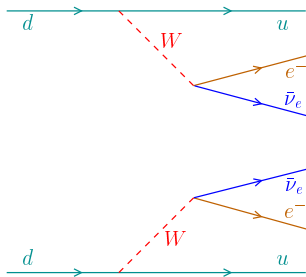
$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

Two-Neutrino Double- β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



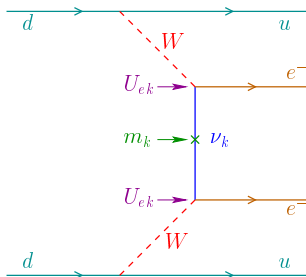
Neutrinoless Double- β Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 |m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

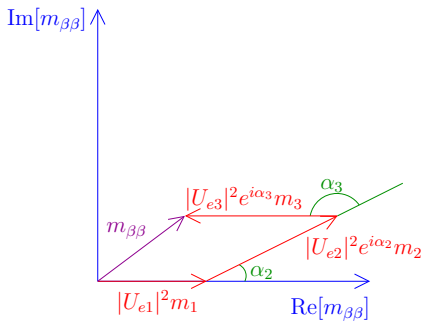
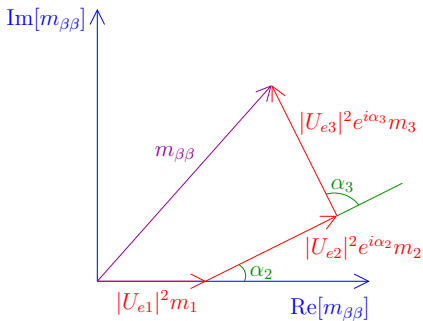


Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k \quad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

$$\alpha_2 = 2\lambda_2 \quad \alpha_3 = 2(\lambda_3 - \delta_{13})$$



Experimental Bounds

CUORICINO (^{130}Te) [PRC 78 (2008) 035502]

$$T_{1/2}^{0\nu} > 3 \times 10^{24} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.19 - 0.68 \text{ eV}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$T_{1/2}^{0\nu} > 1.9 \times 10^{25} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0 \text{ eV}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$T_{1/2}^{0\nu} > 1.57 \times 10^{25} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.33 - 1.35 \text{ eV}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$T_{1/2}^{0\nu} > 4.6 \times 10^{23} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.7 - 2.8 \text{ eV}$$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

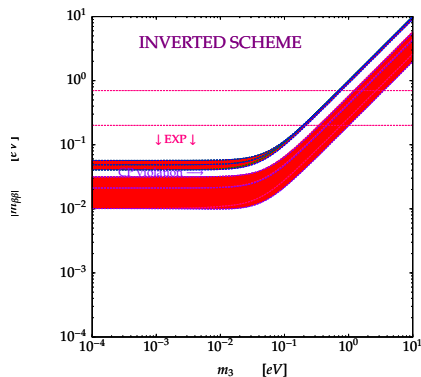
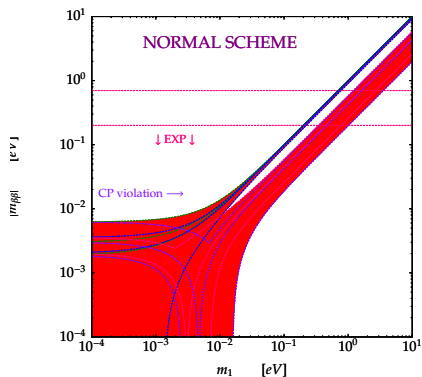
$$|m_{\beta\beta}| \sim \text{few } 10^{-1} \text{ eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few } 10^{-2} \text{ eV}$$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3$$

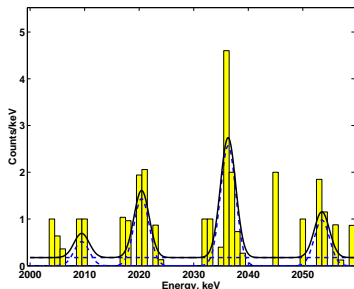


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2} \text{ eV} \Rightarrow$ NORMAL HIERARCHY

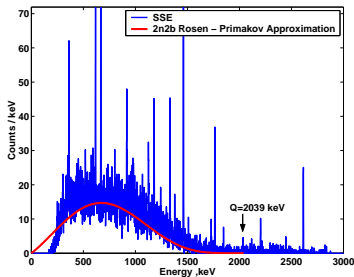
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$$T_{1/2}^{0\nu\text{bf}} = 1.19 \times 10^{25} \text{ y} \quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25} \text{ y} (3\sigma) \quad 4.2\sigma \text{ evidence}$$



pulse-shape selected spectrum



3.8 σ evidence

[PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Rightarrow \quad 0.22 \text{ eV} \lesssim |m_{\beta\beta}| \lesssim 1.6 \text{ eV}$$

if confirmed, very exciting (Majorana ν and large mass scale)

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in primeval plasma through weak interaction reactions
 $\nu\bar{\nu} \leftrightarrow e^+e^-$ $\bar{\nu}e \leftrightarrow \bar{\nu}e$ $\bar{\nu}N \leftrightarrow \bar{\nu}N$ $\nu_e n \leftrightarrow pe^-$ $\bar{\nu}_e p \leftrightarrow ne^+$ $n \leftrightarrow pe^- \bar{\nu}_e$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \implies T_{\text{dec}} \sim 1 \text{ MeV}$$

neutrino decoupling

$$\text{Relic Neutrinos: } T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945 \text{ K} \implies k T_\nu \simeq 1.676 \times 10^{-4} \text{ eV}$$

($T_\gamma = 2.725 \pm 0.001 \text{ K}$)

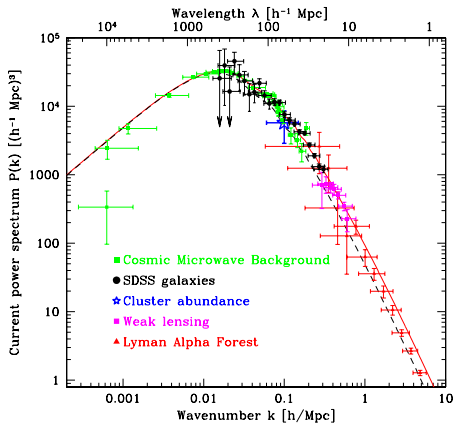
$$\text{number density: } n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \implies n_{\nu_k, \bar{\nu}_k} \simeq 0.1827 T_\nu^3 \simeq 112 \text{ cm}^{-3}$$

$$\text{density contribution: } \Omega_k = \frac{n_{\nu_k, \bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2} \frac{m_k}{94.14 \text{ eV}} \implies \Omega_\nu h^2 = \frac{\sum_k m_k}{94.14 \text{ eV}}$$

($\rho_c = \frac{3H^2}{8\pi G_N}$) [Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \implies \quad \sum_k m_k \lesssim 14 \text{ eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat Λ CDM model

$$(\Omega_M^0 = 0.28, h = 0.72, \Omega_B^0/\Omega_M^0 = 0.16)$$

Dashed Curve: $\sum_{k=1}^3 m_k = 1 \text{ eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1) \delta(\vec{x}_2) \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} P(\vec{k})$$

small scale suppression

$$\begin{aligned} \frac{\Delta P(k)}{P(k)} &\approx -8 \frac{\Omega_\nu}{\Omega_m} \\ &\approx -0.8 \left(\frac{\sum_k m_k}{1 \text{ eV}} \right) \left(\frac{0.1}{\Omega_m h^2} \right) \end{aligned}$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1 \text{ eV}}} \sqrt{\Omega_m} h \text{ Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia \implies Flat Λ CDM

$$T_0 = 13.7 \pm 0.2 \text{ Gyr} \quad h = 0.71_{-0.03}^{+0.04}$$
$$\Omega_0 = 1.02 \pm 0.02 \quad \Omega_b = 0.044 \pm 0.004 \quad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad (95\% \text{ conf.}) \implies \sum_{k=1}^3 m_k < 0.71 \text{ eV}$$

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12 \text{ Gyr} \quad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad (95\% \text{ C.L.})$$

$$\Omega_b = 0.0456 \pm 0.0015 \quad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^3 m_k < 0.67 \text{ eV} \quad (95\% \text{ C.L.}) \quad N_{\text{eff}} = 4.4 \pm 1.5$$

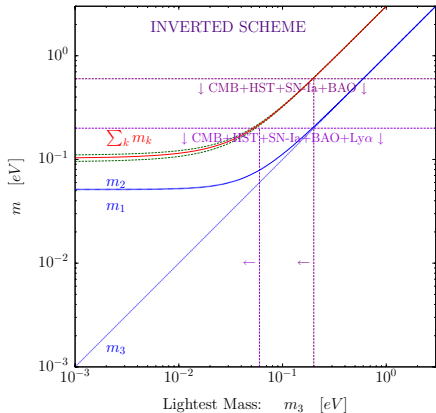
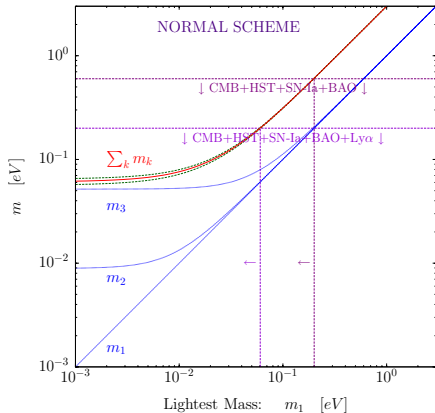
Flat Λ CDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + $\text{Ly}\alpha$	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ .

$$\sum_{k=1}^3 m_k \lesssim 0.6 \text{ eV} \quad (\sim 2\sigma) \quad \text{CMB + HST + SN-Ia + BAO}$$

$$\sum_{k=1}^3 m_k \lesssim 0.2 \text{ eV} \quad (\sim 2\sigma) \quad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$



FUTURE: IF $\sum_{k=1}^3 m_k \lesssim 9 \times 10^{-2} \text{ eV} \Rightarrow$ NORMAL HIERARCHY

Experimental Neutrino Anomalies

- Brief Introduction to Neutrino Masses and Mixing
- Three-Neutrino Mixing and Oscillations
- Absolute Scale of Neutrino Masses
- **Experimental Neutrino Anomalies**
 - LSND
 - MiniBooNE
 - Gallium Radioactive Source Experiments
- Conclusions

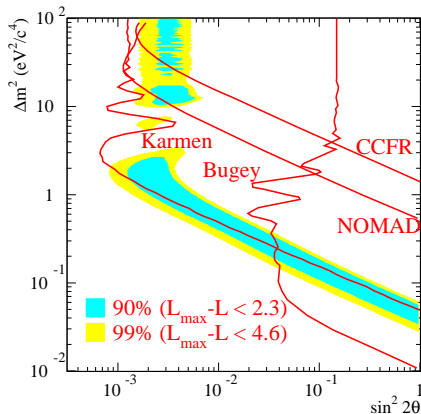
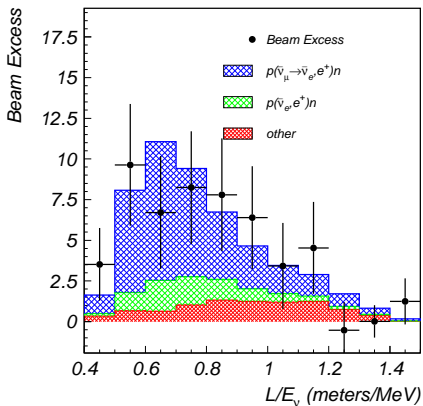
LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

$$L \simeq 30 \text{ m}$$

$$20 \text{ MeV} \leq E \leq 200 \text{ MeV}$$



$$\Delta m_{\text{LSND}}^2 \gtrsim 0.2 \text{ eV}^2 \quad (\gg \Delta m_{\text{ATM}}^2 \gg \Delta m_{\text{SOL}}^2)$$

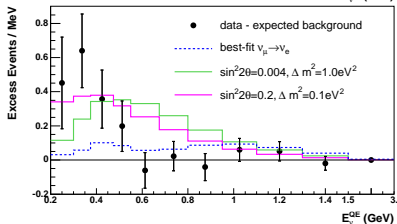
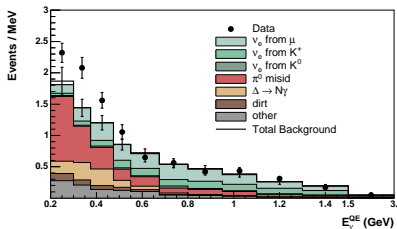
MiniBooNE

[PRL 98 (2007) 231801]

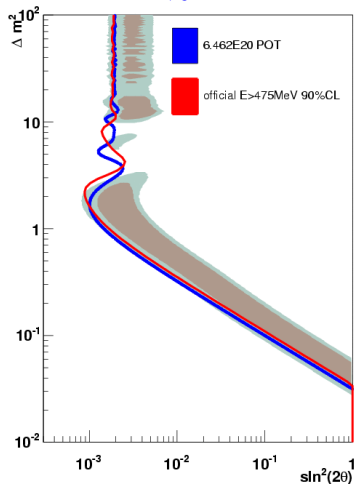
$$\nu_\mu \rightarrow \nu_e$$

$$L \simeq 541 \text{ m}$$

$$475 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$$



[arXiv:0812.2243]



[arXiv:0901.1648]

Low-Energy Anomaly!

Gallium Radioactive Source Experiments

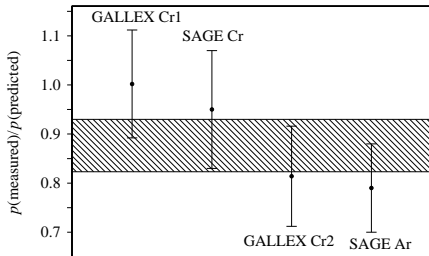
tests of solar neutrino detectors

GALLEX [PLB 342 (1995) 440; PLB 420 (1998) 114]

SAGE [PRL 77 (1996) 4708; PRC 59 (1999) 2246; PRC 73 (2006) 045805; arXiv:0901.2200]

Sources: $e^- + {}^{51}\text{Cr} \rightarrow {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \rightarrow {}^{37}\text{Cl} + \nu_e$

Detector: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$



[SAGE, PRC 73 (2006) 045805]

$$R_{\text{Ga}} = 0.87 \pm 0.05$$

[SAGE, arXiv:0901.2200]

Conclusions

$\nu_e \rightarrow \nu_\mu, \nu_\tau$ with $\Delta m_{\text{SOL}}^2 \simeq 8.3 \times 10^{-5} \text{ eV}^2$ (SOL, KamLAND)

$\nu_\mu \rightarrow \nu_\tau$ with $\Delta m_{\text{ATM}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$ (ATM, K2K, MINOS)



Bilarge 3ν -Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\implies m_\nu \lesssim 1 \text{ eV}$

FUTURE

Theory: Why lepton mixing \neq quark mixing?

(Due to Majorana nature of ν 's?)

Why only $|U_{e3}|^2 \ll 1$?

Explain experimental neutrino anomalies (sterile ν 's?).

Exp.: Measure $|U_{e3}| > 0 \implies$ CP viol., matter effects, mass hierarchy.

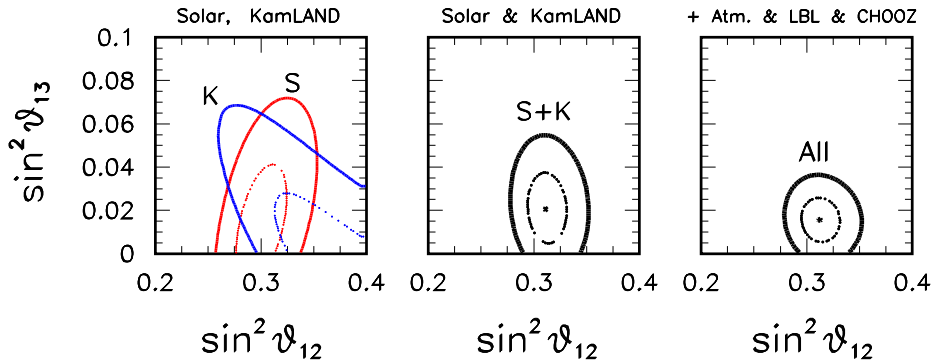
Check experimental neutrino anomalies.

Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale.

Improve β & $\beta\beta_{0\nu}$ Decay and Cosmology measurements.

Hint of $\vartheta_{13} > 0$

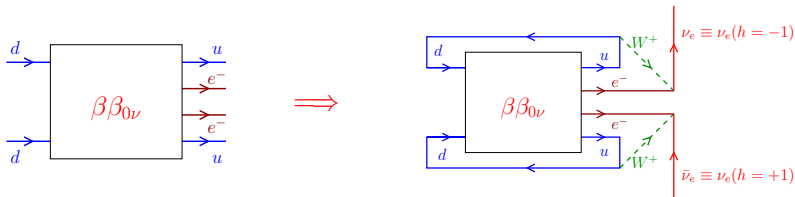
[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]



$$\sin^2 \vartheta_{13} = 0.016 \pm 0.010 \quad [\text{Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801}]$$

$$P_{\nu_e \rightarrow \nu_e}^{(-)} \simeq \begin{cases} (1 - \sin^2 \vartheta_{13})^2 (1 - 0.5 \sin^2 \vartheta_{12}) & \text{SOL low-energy \& KamLAND} \\ (1 - \sin^2 \vartheta_{13})^2 \sin^2 \vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

$\beta\beta_{0\nu}$ Decay \Leftrightarrow Majorana Neutrino Mass

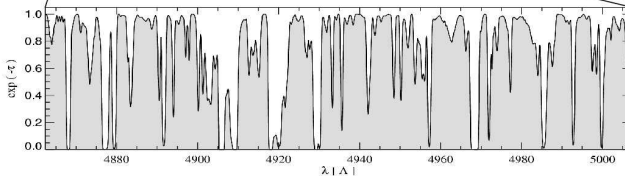
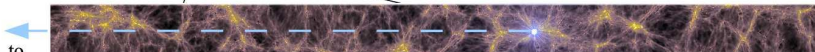
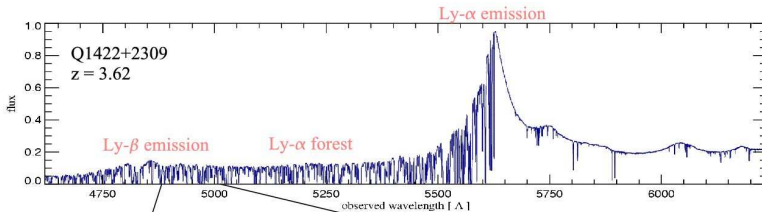


[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\mathcal{L}_{eL}^M = -\frac{1}{2} m_{ee} \left(\overline{\nu_{eL}^c} \nu_{eL} + \overline{\nu_{eL}} \nu_{eL}^c \right)$$

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α , β , γ wavelengths: $\lambda_{\alpha}^0 = 1215.67 \text{ \AA}$, $\lambda_{\beta}^0 = 1025.72 \text{ \AA}$, $\lambda_{\gamma}^0 = 972.54 \text{ \AA}$

Lyman- α forest: The region in which only Ly α photons can be absorbed: $[(1+z_q)\lambda_{\beta}^0, (1+z_q)\lambda_{\alpha}^0]$