Neutrino Physics

Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

mailto://giunti@to.infn.it

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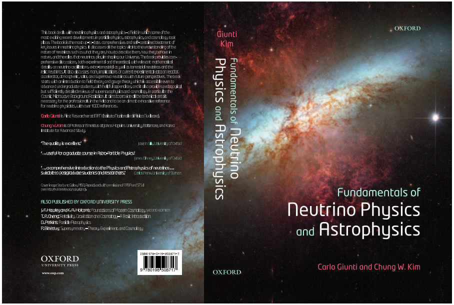


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics
and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Number of Flavor and Massive Neutrinos?
- Sterile Neutrinos

Part II: Neutrino Oscillations in Vacuum and in Matter

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

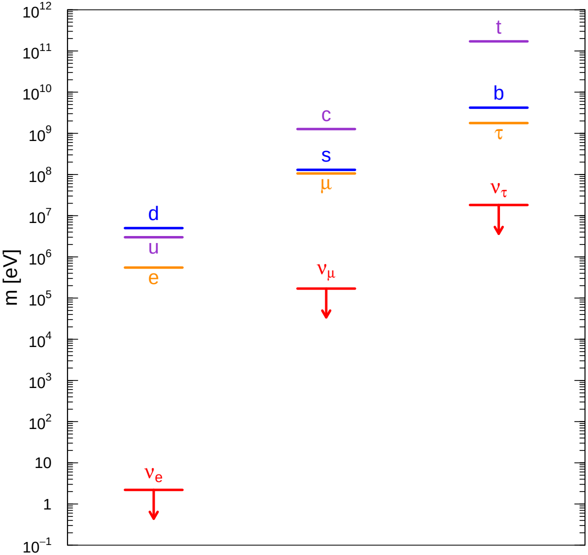
Part III: Experimental Results and Theoretical Implications

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies
- Conclusions

Part I

Theory of Neutrino Masses and Mixing

Fermion Mass Spectrum



Dirac Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
  - Dirac Mass
  - Higgs Mechanism in SM
  - Dirac Lepton Masses
  - Three-Generations Dirac Neutrino Masses
  - Massive Chiral Lepton Fields
  - Massive Dirac Lepton Fields
  - Quantization
  - Mixing
  - Flavor Lepton Numbers
  - Total Lepton Number
  - Mixing Matrix
  - Standard Parameterization of Mixing Matrix
  - CP Violation
  - Example: $\vartheta_{12} = 0$
  - Example: $\vartheta_{13} = \pi/2$
  - Example: $m_{\nu_2} = m_{\nu_3}$

Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$

- Dirac Lagrangian: $\mathscr{L}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$

- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

$$P_L \equiv \frac{1-\gamma^5}{2}\,, \quad P_R \equiv \frac{1+\gamma^5}{2}\,, \quad P_L^2 = P_R^2 = 1\,, \quad P_L P_R = P_R P_L = 0$$

$$\mathscr{L} = \overline{\nu_L}\, i\partial\!\!\!/ \nu_L + \overline{\nu_R}\, i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

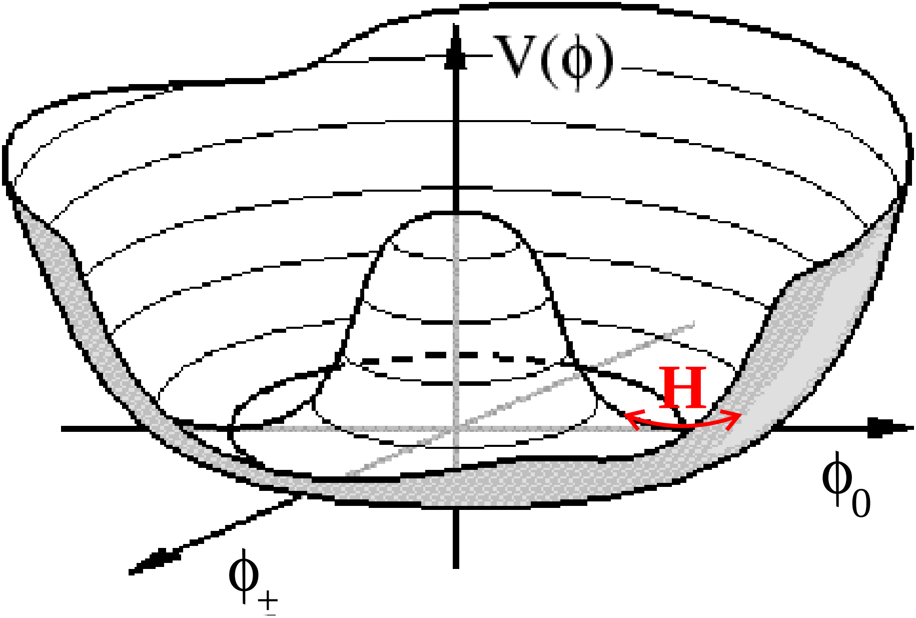
- In SM only $\nu_L \Longrightarrow$ no Dirac mass

- Oscillation experiments have shown that neutrinos are massive

- Simplest extension of the SM: add ν_R

Higgs Mechanism in SM

- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $\qquad |\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$
- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$
- Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$
- $\mu^2 < 0$ and $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$, with $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$
- Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- Spontaneous Symmetry Breaking: $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_Q$
- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$


V(ϕ)
H
ϕ0
ϕ±

Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^\ell \, \overline{L_L} \, \Phi \, \ell_R - y^\nu \, \overline{L_L} \, \widetilde{\Phi} \, \nu_R + \mathrm{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & - \frac{y^\ell}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & - \frac{y^\nu}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \mathrm{H.c.} \end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\, y^{\ell}\,\frac{v}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R - y^{\nu}\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R$$

$$-\,\frac{y^{\ell}}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R\,H - \frac{y^{\nu}}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R\,H + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell}\,\frac{v}{\sqrt{2}} \qquad\qquad m_{\nu} = y^{\nu}\,\frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_{\ell}}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_{\nu}}{v}$$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta}\, \overline{\ell'_{\alpha L}}\, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta}\, \overline{\nu'_{\alpha L}}\, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R \right] + \mathrm{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y'^{\ell} \equiv \begin{pmatrix} Y'^{\ell}_{ee} & Y'^{\ell}_{e\mu} & Y'^{\ell}_{e\tau} \\ Y'^{\ell}_{\mu e} & Y'^{\ell}_{\mu\mu} & Y'^{\ell}_{\mu\tau} \\ Y'^{\ell}_{\tau e} & Y'^{\ell}_{\tau\mu} & Y'^{\ell}_{\tau\tau} \end{pmatrix} \qquad Y'^{\nu} \equiv \begin{pmatrix} Y'^{\nu}_{ee} & Y'^{\nu}_{e\mu} & Y'^{\nu}_{e\tau} \\ Y'^{\nu}_{\mu e} & Y'^{\nu}_{\mu\mu} & Y'^{\nu}_{\mu\tau} \\ Y'^{\nu}_{\tau e} & Y'^{\nu}_{\tau\mu} & Y'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}}\, Y'^{\ell} \qquad M'^{\nu} = \frac{v}{\sqrt{2}}\, Y'^{\nu}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R\right] + \mathrm{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \mathbf{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \mathbf{n}_R$$

Kinetic terms are invariant under unitary transformations of the fields

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L} V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R + \overline{\boldsymbol{\nu}_L} V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \boldsymbol{\nu}_R\right] + \mathrm{H.c.}$$

$$V^{\ell\dagger}_L\ Y'^{\ell}\ V^{\ell}_R = Y^{\ell} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\, \delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L\ Y'^{\nu}\ V^{\nu}_R = Y^{\nu} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\, \delta_{kj} \qquad (k, j = 1, 2, 3)$$

Real and Positive y^{ℓ}_{α}, y^{ν}_{k}

$$V_L^\dagger \, Y' \, V_R = Y \qquad \Longleftrightarrow \qquad \begin{array}{ccccc} Y' & = & V_L & Y & V_R^\dagger \\ 2N^2 & & N^2 & N & N^2 \\ 18 & & 9 & 3 & 9 \end{array}$$

Massive Chiral Lepton Fields

$\boldsymbol{\ell}_L = V_L^{\ell\dagger}\,\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\boldsymbol{\ell}_R = V_R^{\ell\dagger}\,\boldsymbol{\ell}'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned}\mathscr{L}_{H,\mathrm{L}} &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L}\,Y^{\ell}\,\boldsymbol{\ell}_R + \overline{\mathbf{n}_L}\,Y^{\nu}\,\mathbf{n}_R\right] + \mathrm{H.c.} \\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y_\alpha^{\ell}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y_k^{\nu}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \mathrm{H.c.}\end{aligned}$$

Massive Dirac Lepton Fields

$$\ell_\alpha \equiv \ell_{\alpha L} + \ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \qquad (k = 1, 2, 3)$$

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell \, v}{\sqrt{2}} \overline{\ell_\alpha} \, \ell_\alpha - \sum_{k=1}^{3} \frac{y_k^\nu \, v}{\sqrt{2}} \overline{\nu_k} \, \nu_k \qquad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell}{\sqrt{2}} \overline{\ell_\alpha} \, \ell_\alpha \, H - \sum_{k=1}^{3} \frac{y_k^\nu}{\sqrt{2}} \overline{\nu_k} \, \nu_k \, H \qquad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^\ell \, v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \qquad\qquad m_k = \frac{y_k^\nu \, v}{\sqrt{2}} \qquad (k = 1, 2, 3)$$

Lepton-Higgs coupling $\propto$ Lepton Mass

Quantization

$$\nu_k(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2E_k} \sum_{h=\pm 1} \left[a_k^{(h)}(p)\, u_k^{(h)}(p)\, e^{-ip\cdot x} + b_k^{(h)\dagger}(p)\, v_k^{(h)}(p)\, e^{ip\cdot x} \right]$$

$$p^0 = E_k = \sqrt{\vec{p}^2 + m_k^2}$$

$$(\not{p} - m_k)\, u_k^{(h)}(p) = 0$$
$$(\not{p} + m_k)\, v_k^{(h)}(p) = 0$$

$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, u_k^{(h)}(p) = h\, u_k^{(h)}(p)$$
$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, v_k^{(h)}(p) = -h\, v_k^{(h)}(p)$$

$$\{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} = \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = (2\pi)^3\, 2E_k\, \delta^3(\vec{p} - \vec{p}')\, \delta_{hh'}$$
$$\{a_k^{(h)}(p), a_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), a_k^{(h')\dagger}(p')\} = 0$$
$$\{b_k^{(h)}(p), b_k^{(h')}(p')\} = \{b_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$
$$\{a_k^{(h)}(p), b_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$
$$\{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')}(p')\} = 0$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathsf{I}}^{(\mathsf{CC})} = -\frac{g}{2\sqrt{2}}\, j_W^\rho\, W_\rho + \mathsf{H.c.}$$

Weak Charged Current: $\quad j_W^\rho = j_{W,\mathsf{L}}^\rho + j_{W,\mathsf{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathsf{L}}^\rho = \sum_{\alpha=e,\mu,\tau} \overline{\nu'_\alpha}\,\gamma^\rho\left(1-\gamma^5\right)\,\ell'_\alpha = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu'_{\alpha L}}\,\gamma^\rho\,\ell'_{\alpha L} = 2\,\overline{\boldsymbol{\nu}'_L}\,\gamma^\rho\,\boldsymbol{\ell}'_L$$

$$\underline{\boldsymbol{\ell}'_L = V_L^\ell\,\boldsymbol{\ell}_L} \qquad \underline{\boldsymbol{\nu}'_L = V_L^\nu\,\mathbf{n}_L}$$

$$j_{W,\mathsf{L}}^\rho = 2\,\overline{\mathbf{n}_L}\,V_L^{\nu\dagger}\,\gamma^\rho\,V_L^\ell\,\boldsymbol{\ell}_L = 2\,\overline{\mathbf{n}_L}\,V_L^{\nu\dagger}\,V_L^\ell\,\gamma^\rho\,\boldsymbol{\ell}_L = 2\,\overline{\mathbf{n}_L}\,U^\dagger\,\gamma^\rho\,\boldsymbol{\ell}_L$$

Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger}\,V_L^\ell \qquad \boxed{U = V_L^{\ell\dagger}\,V_L^\nu}$$

- Definition: Left-Handed Flavor Neutrino Fields

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j^{\rho}_{W,\mathrm{L}} = 2\left(\overline{\nu_{eL}}\,\gamma^{\rho}\,e_L + \overline{\nu_{\mu L}}\,\gamma^{\rho}\,\mu_L + \overline{\nu_{\tau L}}\,\gamma^{\rho}\,\tau_L\right)$$

- In practice left-handed flavor neutrino fields are useful for calculations in the SM approximation of massless neutrinos (interactions).

- If neutrino masses must be taken into account, it is necessary to use

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{k=1}^{3}\sum_{\alpha=e,\mu,\tau}U^{*}_{\alpha k}\,\overline{\nu_{kL}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Standard Model: Lepton numbers are conserved

$$\mathscr{L}_{\text{mass}}^{\text{D}} = -\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\text{D}} & m_{e\mu}^{\text{D}} & m_{e\tau}^{\text{D}} \\ m_{\mu e}^{\text{D}} & m_{\mu\mu}^{\text{D}} & m_{\mu\tau}^{\text{D}} \\ m_{\tau e}^{\text{D}} & m_{\tau\mu}^{\text{D}} & m_{\tau\tau}^{\text{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

L is conserved: $\quad L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

- Leptonic Weak Charged Current is invariant under the global U(1) gauge transformations

$$\ell_{\alpha L} \to e^{i\varphi_\alpha}\,\ell_{\alpha L} \qquad \nu_{\alpha L} \to e^{i\varphi_\alpha}\,\nu_{\alpha L} \qquad (\alpha = e, \mu, \tau)$$

- If neutrinos are massless (SM), Noether's theorem implies that there is, for each flavor, a conserved current:

$$j_\alpha^\rho = \overline{\nu_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L} + \overline{\ell_\alpha}\,\gamma^\rho\,\ell_\alpha \qquad \partial_\rho j_\alpha^\rho = 0$$

and a conserved charge:

$$\mathrm{L}_\alpha = \int \mathrm{d}^3x\, j_\alpha^0(x) \qquad \partial_0 \mathrm{L}_\alpha = 0$$

$$:\mathrm{L}_\alpha: \; = \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E}\left[a_{\nu_\alpha}^{(-)\dagger}(p)\,a_{\nu_\alpha}^{(-)}(p) - b_{\nu_\alpha}^{(+)\dagger}(p)\,b_{\nu_\alpha}^{(+)}(p)\right]$$
$$+ \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E}\sum_{h=\pm1}\left[a_{\ell_\alpha}^{(h)\dagger}(p)\,a_{\ell_\alpha}^{(h)}(p) - b_{\ell_\alpha}^{(h)\dagger}(p)\,b_{\ell_\alpha}^{(h)}(p)\right]$$

- Lepton-Higgs Yukawa Lagrangian:

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y^{\nu}_{k}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \mathrm{H.c.}$$

- Mixing: $\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\,\nu_{kL} \quad \Longleftrightarrow \quad \nu_{kL} = \sum_{\alpha=e,\mu,\tau} U^{*}_{\alpha k}\,\nu_{\alpha L}$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\sum_{\alpha=e,\mu,\tau}\left[y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \overline{\nu_{\alpha L}}\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}\right] + \mathrm{H.c.}$$

- Invariant for

$$\ell_{\alpha L} \to e^{i\varphi_\alpha}\,\ell_{\alpha L}\,, \quad \nu_{\alpha L} \to e^{i\varphi_\alpha}\,\nu_{\alpha L}$$

$$\ell_{\alpha R} \to e^{i\varphi_\alpha}\,\ell_{\alpha R}\,, \quad \sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR} \to e^{i\varphi_\alpha}\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}$$

- But kinetic part of neutrino Lagrangian is not invariant

$$\mathscr{L}^{(\nu)}_{\text{kinetic}} = \sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,i\partial\!\!\!/\,\nu_{\alpha L} + \sum_{k=1}^{3}\overline{\nu_{kR}}\,i\partial\!\!\!/\,\nu_{kR}$$

because $\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}$ is not a unitary combination of the ν_{kR}'s

Total Lepton Number

- Dirac neutrino masses violate conservation of Flavor Lepton Numbers
- Total Lepton Number is conserved, because Lagrangian is invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi}\,\nu_{kL}\,, \qquad \nu_{kR} \to e^{i\varphi}\,\nu_{kR} \qquad (k=1,2,3)$$
$$\ell_{\alpha L} \to e^{i\varphi}\,\ell_{\alpha L}\,, \qquad \ell_{\alpha R} \to e^{i\varphi}\,\ell_{\alpha R} \qquad (\alpha=e,\mu,\tau)$$

- From Noether's theorem:

$$j^\rho = \sum_{k=1}^{3} \overline{\nu_k}\,\gamma^\rho\,\nu_k + \sum_{\alpha=e,\mu,\tau} \overline{\ell_\alpha}\,\gamma^\rho\,\ell_\alpha \qquad \partial_\rho j^\rho = 0$$

Conserved charge: $\mathrm{L}_\alpha = \int \mathrm{d}^3x\, j^0_\alpha(x) \qquad \partial_0 \mathrm{L}_\alpha = 0$

$$:\mathrm{L}: \; = \sum_{k=1}^{3} \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)\dagger}_{\nu_k}(p)\, a^{(h)}_{\nu_k}(p) - b^{(h)\dagger}_{\nu_k}(p)\, b^{(h)}_{\nu_k}(p) \right]$$
$$+ \sum_{\alpha=e,\mu,\tau} \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)\dagger}_{\ell_\alpha}(p)\, a^{(h)}_{\ell_\alpha}(p) - b^{(h)\dagger}_{\ell_\alpha}(p)\, b^{(h)}_{\ell_\alpha}(p) \right]$$

Mixing Matrix

- Leptonic Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\boldsymbol{\ell}_L$

- $U = V_L^{\ell\dagger}\,V_L^{\nu} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \qquad \Longrightarrow \qquad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \quad \text{Mixing Angles} \\[2ex] \dfrac{N(N+1)}{2} = 6 & \quad \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, U^{*}_{\alpha k}\, \gamma^{\rho}\, \ell_{\alpha L}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)
$$\nu_k \to e^{i\varphi_k}\,\nu_k \quad (k=1,2,3)\,, \qquad \ell_\alpha \to e^{i\varphi_\alpha}\,\ell_\alpha \quad (\alpha=e,\mu,\tau)$$
- Performing this transformation, the Charged Current becomes
$$j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, e^{-i\varphi_k}\, U^{*}_{\alpha k}\, e^{i\varphi_\alpha}\, \gamma^{\rho}\, \ell_{\alpha L}$$
$$j^{\rho}_{W,\mathrm{L}} = 2 \underbrace{e^{-i(\varphi_1-\varphi_e)}}_{1} \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, \underbrace{e^{-i(\varphi_k-\varphi_1)}}_{2}\, U^{*}_{\alpha k}\, \underbrace{e^{i(\varphi_\alpha-\varphi_e)}}_{2}\, \gamma^{\rho}\, \ell_{\alpha L}$$
- There are 5 arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant $\Longleftrightarrow$ conservation of Total Lepton Number.

- The mixing matrix contains 1 Physical Phase.

- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

  3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos\vartheta_{ab} \qquad s_{ab} \equiv \sin\vartheta_{ab} \qquad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \qquad 0 \leq \delta_{13} \leq 2\pi$$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} e^{i\delta_{23}} \\ 0 & -s_{23} e^{-i\delta_{13}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12} e^{-i\delta'_{12}} & 0 \\ -s'_{12} e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

CP Violation

- $U \neq U^* \quad \Longrightarrow \quad$ CP Violation
- General conditions for CP violation (14 conditions):
  1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
  3. The physical phase is different from 0 or π (2 conditions)
- These 14 conditions are combined into the single condition $\det C \neq 0$

$$C = -i \, [M'^{\nu} \, M'^{\nu\dagger}, \, M'^{\ell} \, M'^{\ell\dagger}]$$

$$\begin{aligned} \det C = -2 \, J \left(m_{\nu_2}^2 - m_{\nu_1}^2 \right) \left(m_{\nu_3}^2 - m_{\nu_1}^2 \right) \left(m_{\nu_3}^2 - m_{\nu_2}^2 \right) \\ \left(m_{\mu}^2 - m_{e}^2 \right) \left(m_{\tau}^2 - m_{e}^2 \right) \left(m_{\tau}^2 - m_{\mu}^2 \right) \end{aligned}$$

- Jarlskog rephasing invariant: $J = \mathfrak{Im}\left[U_{e2} \, U_{e3}^* \, U_{\mu 2}^* \, U_{\mu 3} \right]$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Example: $\vartheta_{12} = 0$

$$U = R_{23} R_{13} W_{12}$$

$$W_{12} = \begin{pmatrix} \cos\vartheta_{12} & \sin\vartheta_{12} e^{-i\delta_{12}} & 0 \\ -\sin\vartheta_{12} e^{-i\delta_{12}} & \cos\vartheta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vartheta_{12} = 0 \qquad \Longrightarrow \qquad W_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{1}$$

real mixing matrix $\qquad U = R_{23} R_{13}$

Example: $\vartheta_{13} = \pi/2$

$$U = R_{23} W_{13} R_{12}$$

$$W_{13} = \begin{pmatrix} \cos\vartheta_{13} & 0 & \sin\vartheta_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin\vartheta_{13} e^{i\delta_{13}} & 0 & \cos\vartheta_{13} \end{pmatrix}$$

$$\vartheta_{13} = \pi/2 \qquad \Longrightarrow \qquad W_{13} = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -e^{i\delta_{13}} & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & 0 \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta_{13}} & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix}$$

$$\lambda_{\mu 1} - \lambda_{\mu 2} = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi \qquad \lambda_{\tau 1} - \lambda_{\mu 1} = \lambda_{\tau 2} - \lambda_{\mu 2} \pm \pi$$

$$\nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1, 2, 3) \, , \qquad \ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$U \to \begin{pmatrix} e^{-i\varphi_e} & 0 & 0 \\ 0 & e^{-i\varphi_\mu} & 0 \\ 0 & 0 & e^{-i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{i(-\delta_{13} - \varphi_e + \varphi_3)} \\ |U_{\mu 1}| e^{i(\lambda_{\mu 1} - \varphi_\mu + \varphi_1)} & |U_{\mu 2}| e^{i(\lambda_{\mu 2} - \varphi_\mu + \varphi_2)} & 0 \\ |U_{\tau 1}| e^{i(\lambda_{\tau 1} - \varphi_\tau + \varphi_1)} & |U_{\tau 2}| e^{i(\lambda_{\tau 2} - \varphi_\tau + \varphi_2)} & 0 \end{pmatrix}$$

$$\varphi_1 = 0 \qquad \varphi_\mu = \lambda_{\mu 1} \qquad \varphi_\tau = \lambda_{\tau 1} \qquad \varphi_2 = \varphi_\mu - \lambda_{\mu 2} = \lambda_{\mu 1} - \lambda_{\mu 2}$$

$$\varphi_2 = \varphi_\tau - \lambda_{\tau 2} \pm \pi = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi = \lambda_{\mu 1} - \lambda_{\mu 2} \qquad \text{OK!}$$

$$U = \begin{pmatrix} 0 & 0 & \pm 1 \\ |U_{\mu 1}| & |U_{\mu 2}| & 0 \\ |U_{\tau 1}| & -|U_{\tau 2}| & 0 \end{pmatrix}$$

Example: $m_{\nu_2} = m_{\nu_3}$

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L$$

$$U = R_{12} R_{13} W_{23} \qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,W_{23}^{\dagger} R_{13}^{\dagger} R_{12}^{\dagger}\,\gamma^{\rho}\,\ell_L$$

$$W_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_{23} & \sin\vartheta_{23} e^{-i\delta_{23}} \\ 0 & -\sin\vartheta_{23} e^{-i\delta_{23}} & \cos\vartheta_{23} \end{pmatrix}$$

$$W_{23}\mathbf{n}_L = \mathbf{n}'_L \qquad R_{12} R_{13} = U' \qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}'_L}\,U'^{\dagger}\,\gamma^{\rho}\,\ell_L$$

ν_2 and ν_3 are indistinguishable

drop the prime $\qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L$

real mixing matrix $\qquad U = R_{12} R_{13}$

Jarlskog Rephasing Invariant

- Simplest rephasing invariants: $|U_{\alpha k}| = U_{\alpha k} U_{\alpha k}^*\,, \quad U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$

$$\Im\mathfrak{m}\Big[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}\Big] = \pm J$$

$$J = \Im\mathfrak{m}\Big[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\Big] = \Im\mathfrak{m} \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \, \sin 2\vartheta_{23} \, \cos\vartheta_{13} \, \sin 2\vartheta_{13} \, \sin\delta_{13} \end{aligned}$$

- Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way
- All measurable CP-violation effects depend on J.

Maximal CP Violation

- Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\max} = \frac{1}{6\sqrt{3}}$$

- In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4 \,, \quad s_{13} = 1/\sqrt{3} \,, \quad \sin\delta_{13} = \pm 1$$

- This case is called Trimaximal Mixing. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp\frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- The unitarity of V_L^ℓ, V_R^ℓ and V_L^ν implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$
\begin{aligned}
j^\rho_{Z,\mathrm{L}} =& 2\, g_L^\nu\, \overline{\nu'_L}\, \gamma^\rho\, \nu'_L + 2\, g_L^l\, \overline{\ell'_L}\, \gamma^\rho \ell'_L + 2\, g_R^l\, \overline{\ell'_R}\, \gamma^\rho \ell'_R \\
=& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, V_L^{\nu\dagger}\, \gamma^\rho\, V_L^\nu\, \mathbf{n}_L + 2\, g_L^l\, \overline{\ell_L}\, V_L^{\ell\dagger}\, \gamma^\rho\, V_L^\ell\, \ell_L + 2\, g_R^l\, \overline{\ell_R}\, V_R^{\ell\dagger}\, \gamma^\rho\, V_R^\ell\, \ell_R \\
=& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, \gamma^\rho\, \mathbf{n}_L + 2\, g_L^l\, \overline{\ell_L}\, \gamma^\rho \ell_L + 2\, g_R^l\, \overline{\ell_R}\, \gamma^\rho \ell_R
\end{aligned}
$$

- The unitarity of U implies the same expression for the neutral weak current in terms of the flavor neutrino fields $\nu_L = U\, \mathbf{n}_L$:

$$
\begin{aligned}
j^\rho_{Z,\mathrm{L}} &= 2\, g_L^\nu\, \overline{\nu_L}\, U\, \gamma^\rho\, U^\dagger\, \nu_L + 2\, g_L^l\, \overline{\ell_L}\, \gamma^\rho \ell_L + 2\, g_R^l\, \overline{\ell_R}\, \gamma^\rho \ell_R \\
&= 2\, g_L^\nu\, \overline{\nu_L}\, \gamma^\rho\, \nu_L + 2\, g_L^l\, \overline{\ell_L}\, \gamma^\rho \ell_L + 2\, g_R^l\, \overline{\ell_R}\, \gamma^\rho \ell_R
\end{aligned}
$$

Lepton Numbers Violating Processes

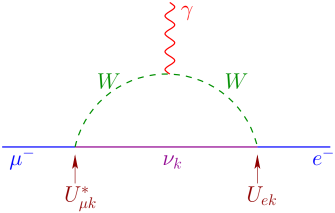
Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \to e^\pm + \gamma\,,\qquad \mu^\pm \to e^\pm + e^+ + e^-$

$$\boxed{\mu^- \to e^- + \gamma}$$

$\sum_k U_{\mu k}^* U_{ek} = 0 \Longrightarrow$ only part of ν_k propagator $\propto m_k$ contributes

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor: $\quad \frac{m_k}{m_W} \lesssim 10^{-11} \quad$ for $\quad m_k \lesssim 1\,\text{eV}$

$(\text{BR})_{\text{the}} \lesssim 10^{-47} \qquad (\text{BR})_{\text{exp}} \lesssim 10^{-11}$

Majorana Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
  - Two-Component Theory of a Massless Neutrino
  - Majorana Equation
  - Majorana Lagrangian
  - Majorana Antineutrino?
  - Lepton Number
  - CP Symmetry
  - No Majorana Neutrino Mass in the SM
  - Effective Majorana Mass
  - Mixing of Three Majorana Neutrinos
  - Mixing Matrix
- Dirac-Majorana Mass Term
- Number of Flavor and Massive Neutrinos?

Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^{\mu}\partial_{\mu}\psi_L = m\,\psi_R$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^{\mu}\partial_{\mu}\psi_L = 0$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = 0$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor).

- ψ_L and ψ_R have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \qquad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ($\psi_L \overset{P}{\rightleftharpoons} \psi_R$)

- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- $V - A$ Charged-Current Weak Interactions $\Longrightarrow$ ν_L

- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of ν_R

Majorana Equation

- Can a two-component spinor describe a massive fermion? Yes! (E. Majorana, 1937)
- Trick: ψ_R and ψ_L are not independent: $\boxed{\psi_R = \mathcal{C}\,\overline{\psi_L}^T}$
- $\mathcal{C}\,\overline{\psi_L}^T$ is right-handed: $P_R \mathcal{C}\,\overline{\psi_L}^T = \mathcal{C}\,\overline{\psi_L}^T$ $\quad (\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu)$
- Majorana Equation: $\boxed{i\gamma^\mu \partial_\mu \psi_L = m\,\mathcal{C}\,\overline{\psi_L}^T}$
- Majorana Field: $\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\,\overline{\psi_L}^T$
- Majorana Condition: $\boxed{\psi = \mathcal{C}\,\overline{\psi}^T = \psi^C}$
- Only two independent components: $\psi = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

- $\psi = \psi^C$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi^C}\gamma^\mu\psi^C = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\psi}^T = \overline{\psi}\mathcal{C}\gamma^{\mu\,T}\mathcal{C}^\dagger\psi = -\overline{\psi}\gamma^\mu\psi = 0$$

Majorana Lagrangian

Dirac Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{D}} &= \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu \\
&= \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R)
\end{aligned}
$$

$$
\nu_R \quad \to \quad \nu_L^C = \mathcal{C}\,\overline{\nu_L}^T
$$

$$
\frac{1}{2}\mathscr{L}^{\mathrm{D}} \quad \to \quad \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right)
$$

Majorana Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{M}} &= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right) \\
&= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right)
\end{aligned}
$$

- Majorana Field: $\nu = \nu_L + \nu_L^C$
- Majorana Condition: $\nu^C = \nu$
- Majorana Lagrangian: $\mathscr{L}^{\mathrm{M}} = \dfrac{1}{2}\,\overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu$
- The factor $1/2$ distinguishes the Majorana Lagrangian from the Dirac Lagrangian
- Quantized Dirac Neutrino Field:
$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2E} \sum_{h=\pm 1} \left[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\,\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x} \right]$$
- Quantized Majorana Neutrino Field $[b^{(h)}(p) = a^{(h)}(p)]$
$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2E} \sum_{h=\pm 1} \left[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + a^{(h)\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x} \right]$$
- A Majorana field has half the degrees of freedom of a Dirac field

Majorana Antineutrino?

- A Majorana neutrino is the same as a Majorana antineutrino
- Neutrino interactions are described by the CC and NC Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_L}\, \gamma^\mu\, \ell_L\, W_\mu + \overline{\ell_L}\, \gamma^\mu\, \nu_L\, W_\mu^\dagger \right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\, \overline{\nu_L}\, \gamma^\mu\, \nu_L\, Z_\mu$$

- In practice, since detectable neutrinos are always ultrarelativistic, the neutrino mass can be neglected in interactions

- In interaction amplitudes we neglect corrections of order m/E

- Dirac: $\begin{cases} \nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{cases} \\ \\ \overline{\nu_L} \begin{cases} \text{destroys right-handed antineutrinos} \\ \text{creates left-handed neutrinos} \end{cases} \end{cases}$

- Majorana: $\begin{cases} \nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{cases} \\ \\ \overline{\nu_L} \begin{cases} \text{destroys right-handed neutrinos} \\ \text{creates left-handed neutrinos} \end{cases} \end{cases}$

- Common definitions:

  Majorana neutrino with negative helicity $\equiv$ neutrino
  Majorana neutrino with positive helicity $\equiv$ antineutrino

Lepton Number

$$\cancel{L=+1} \quad \longleftarrow \quad \boxed{\nu = \nu^C} \quad \longrightarrow \quad \cancel{L=-1}$$

$$\nu_L \quad \Longrightarrow \quad L=+1 \qquad\qquad \nu_L^C \quad \Longrightarrow \quad L=-1$$

$$\mathscr{L}^{\mathrm{M}} = \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta_{0\nu}^+)$$

CP Symmetry

- Under a CP transformation

$$
\begin{aligned}
\mathsf{U}_{\mathrm{CP}} \nu_L(x) \mathsf{U}_{\mathrm{CP}}^{-1} &= \xi_\nu^{\mathrm{CP}} \, \gamma^0 \, \nu_L^C(x_{\mathrm{P}}) \\
\mathsf{U}_{\mathrm{CP}} \nu_L^C(x) \mathsf{U}_{\mathrm{CP}}^{-1} &= -\xi_\nu^{\mathrm{CP}*} \, \gamma^0 \, \nu_L(x_{\mathrm{P}}) \\
\mathsf{U}_{\mathrm{CP}} \overline{\nu_L}(x) \mathsf{U}_{\mathrm{CP}}^{-1} &= \xi_\nu^{\mathrm{CP}*} \, \overline{\nu_L^C}(x_{\mathrm{P}}) \, \gamma^0 \\
\mathsf{U}_{\mathrm{CP}} \overline{\nu_L^C}(x) \mathsf{U}_{\mathrm{CP}}^{-1} &= -\xi_\nu^{\mathrm{CP}} \, \overline{\nu_L}(x_{\mathrm{P}}) \, \gamma^0
\end{aligned}
$$

with $|\xi_\nu^{\mathrm{CP}}|^2 = 1$, $x^\mu = (x^0, \vec{x})$, and $x_{\mathrm{P}}^\mu = (x^0, -\vec{x})$

- The theory is CP-symmetric if there are values of the phase ξ_ν^{CP} such that the Lagrangian transforms as

$$\mathsf{U}_{\mathrm{CP}} \mathscr{L}(x) \mathsf{U}_{\mathrm{CP}}^{-1} = \mathscr{L}(x_{\mathrm{P}})$$

in order to keep invariant the action $I = \int \mathrm{d}^4x \, \mathscr{L}(x)$

- The Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{\text{M}}(x) = -\frac{1}{2}\, m \left[\overline{\nu_L^C}(x)\, \nu_L(x) + \overline{\nu_L}(x)\, \nu_L^C(x) \right]$$

transforms as

$$\begin{aligned} \text{U}_{\text{CP}} \mathscr{L}_{\text{mass}}^{\text{M}}(x) \text{U}_{\text{CP}}^{-1} = & -\frac{1}{2}\, m \Big[-(\xi_\nu^{\text{CP}})^2\, \overline{\nu_L}(x_{\text{P}})\, \nu_L^C(x_{\text{P}}) \\ & \qquad -(\xi_\nu^{\text{CP}*})^2\, \overline{\nu_L^C}(x_{\text{P}})\, \nu_L(x_{\text{P}}) \Big] \end{aligned}$$

- $\text{U}_{\text{CP}} \mathscr{L}_{\text{mass}}^{\text{M}}(x) \text{U}_{\text{CP}}^{-1} = \mathscr{L}_{\text{mass}}^{\text{M}}(x_{\text{P}})$ for $\boxed{\xi_\nu^{\text{CP}} = \pm i}$

- The one-generation Majorana theory is CP-symmetric

- The Majorana case is different from the Dirac case, in which the CP phase ξ_ν^{CP} is arbitrary

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T \, \mathcal{C}^\dagger \, \nu_L - \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM (one for each lepton generation)
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed Higgs triplet with $Y = 2$

Effective Majorana Mass

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int \mathrm{d}^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4, \quad (\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4, \quad m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $g \, \overline{\nu_L} \gamma^\rho \ell_L \, W_\rho \sim [E]^4$
- Yukawa couplings: $y \, \overline{L_L} \Phi \ell_R \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-d operator
- $\mathscr{L}_{(\mathscr{O}_d)} = C_{(\mathscr{O}_d)} \mathscr{O}_d \quad \Longrightarrow \quad C_{(\mathscr{O}_d)} \sim [E]^{4-d}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d\leq 4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{4-d}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

- Analogy with $\quad \mathscr{L}_{\mathrm{eff}}^{(\mathrm{CC})} \propto G_{\mathrm{F}}\,(\overline{\nu_{eL}}\gamma^{\rho} e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots$

$$\mathscr{O}_6 \to (\overline{\nu_{eL}}\gamma^{\rho} e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots \qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Only one dim-5 operator:

$$\mathscr{O}_5 = (L_L^T\,\sigma_2\,\Phi)\,\mathcal{C}^\dagger\,(\Phi^T\,\sigma_2\,L_L) + \text{H.c.}$$
$$= \frac{1}{2}\,(L_L^T\,\mathcal{C}^\dagger\,\sigma_2\,\vec{\tau}\,L_L)\cdot(\Phi^T\,\sigma_2\,\vec{\tau}\,\Phi) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}}\,(L_L^T\,\mathcal{C}^\dagger\,\sigma_2\,\vec{\tau}\,L_L)\cdot(\Phi^T\,\sigma_2\,\vec{\tau}\,\Phi) + \text{H.c.}$$

- Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = \dfrac{1}{2}\,\dfrac{g_5\,v^2}{\mathcal{M}}\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.} \quad \Longrightarrow \quad \boxed{m = \dfrac{g_5\,v^2}{\mathcal{M}}}$

- The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM

- $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_{\mathrm{D}}^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses (special case: See-Saw Mechanism)

- Example: $m_{\mathrm{D}} \sim v \sim 10^2\,\mathrm{GeV}$ and $\mathcal{M} \sim 10^{15}\,\mathrm{GeV} \implies m \sim 10^{-2}\,\mathrm{eV}$

Mixing of Three Majorana Neutrinos

- $\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_L \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}'_L + \text{H.c.}$$

$$= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} = - \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, M^L_{\alpha\beta} \, (\mathcal{C}^\dagger)^T \, \nu'_{\alpha L}$$

$$= \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\beta\alpha} \, \nu'_{\beta L}$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \quad \Longleftrightarrow \quad M^L = M^{L\,T}$$

- $\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^T_L \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}'_L + \text{H.c.}$
- $\boldsymbol{\nu}'_L = V^\nu_L \, \mathbf{n}_L \qquad \Longrightarrow \qquad \mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^T_L \, (V^\nu_L)^T \, \mathcal{C}^\dagger \, M^L \, V^\nu_L \, \boldsymbol{\nu}'_L + \text{H.c.}$
- $(V^\nu_L)^T \, M^L \, V^\nu_L = M \,, \qquad M_{kj} = m_k \, \delta_{kj} \qquad (k, j = 1, 2, 3)$
- Left-handed chiral fields with definite mass: $\mathbf{n}_L = V^{\nu\dagger}_L \, \boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\text{M}}_{\text{mass}} &= \frac{1}{2} \left(\mathbf{n}^T_L \, \mathcal{C}^\dagger \, M \, \mathbf{n}_L - \overline{\mathbf{n}_L} \, M \, \mathcal{C} \, \mathbf{n}^T_L \right) \\ &= \frac{1}{2} \sum_{k=1}^{3} m_k \left(\nu^T_{kL} \, \mathcal{C}^\dagger \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu^T_{kL} \right) \end{aligned}$$

- Majorana fields of massive neutrinos: $\nu_k = \nu_{kL} + \nu^C_{kL} \qquad \boxed{\nu^C_k = \nu_k}$
- $\mathbf{n} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Longrightarrow \mathscr{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} \overline{\nu_k} \, (i\partial\!\!\!/ - m_k) \, \nu_k = \frac{1}{2} \, \overline{\mathbf{n}} \, (i\partial\!\!\!/ - M) \, \mathbf{n}$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L \qquad \text{with} \qquad U = V^{\ell\dagger}_L\,V^{\nu}_L$$

- Definition of the left-handed flavor neutrino fields:

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V^{\ell\dagger}_L\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- Leptonic Weak Charged Current has the SM form

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} = \frac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + \mathrm{H.c.}$ is not invariant under the global $\mathrm{U}(1)$ gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \qquad (k = 1, 2, 3)$$

- Left-handed massive neutrino fields cannot be rephased in order to eliminate two Majorana phases factorized on the right of mixing matrix:

$$\boxed{U = U^{\mathrm{D}} \, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is analogous to a Dirac mixing matrix, with one Dirac phase
- Standard parameterization:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- Jarlskog rephasing invariant: $J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$

- $D^{\mathrm{M}} = \mathrm{diag}\left(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3}\right)$, but only two Majorana phases are physical

- All measurable quantities depend only on the differences of the Majorana phases
$$\ell_\alpha \to e^{i\varphi}\ell_\alpha \quad \Longrightarrow \quad e^{i\lambda_k} \to e^{i(\lambda_k - \varphi)}$$
$e^{i(\lambda_k - \lambda_j)}$ remains constant

- Our convention: $\lambda_1 = 0 \Longrightarrow D^{\mathrm{M}} = \mathrm{diag}\left(1, e^{i\lambda_2}, e^{i\lambda_3}\right)$

- CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:
$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

Dirac-Majorana Mass Term

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
  - One Generation
  - Real Mass Matrix
  - Maximal Mixing
  - Dirac Limit
  - Pseudo-Dirac Neutrinos
  - See-Saw Mechanism
  - Majorana Neutrino Mass?
  - Right-Handed Neutrino Mass Term
  - Singlet Majoron Model
  - Three-Generation Mixing
- Number of Flavor and Massive Neutrinos?

One Generation

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \mathscr{L}_{\text{mass}}^{\text{D}} + \mathscr{L}_{\text{mass}}^{L} + \mathscr{L}_{\text{mass}}^{R}$$

$$\mathscr{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}}\,\overline{\nu_R}\,\nu_L + \text{H.c.}$$ Dirac Mass Term

$$\mathscr{L}_{\text{mass}}^{L} = \frac{1}{2}\,m_L\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.}$$ Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{R} = \frac{1}{2}\,m_R\,\nu_R^T\,\mathcal{C}^\dagger\,\nu_R + \text{H.c.}$$ New Majorana Mass Term!

- Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\,\overline{\nu_R}^T \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \text{H.c.} \qquad M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

- The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- Diagonalization: $n_L = U^\dagger\, N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T\, M\, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \text{Real } m_k \geq 0$$

- $\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k\, \nu_{kL}^T\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_k}\, \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^C$$

- Massive neutrinos are Majorana! $\qquad \nu_k = \nu_k^C$

Real Mass Matrix

- CP is conserved if the mass matrix is real: $M = M^*$
- $M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$ we consider real and positive m_R and m_{D} and real m_L
- A real symmetric mass matrix can be diagonalized with $U = \mathcal{O}\,\rho$

$$\mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \qquad \rho_k^2 = \pm 1$$

- $\mathcal{O}^T\,M\,\mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix} \qquad \tan 2\vartheta = \dfrac{2m_{\text{D}}}{m_R - m_L}$

$$m'_{2,1} = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

- m'_1 is negative if $m_L m_R < m_{\text{D}}^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m'_1 & 0 \\ 0 & \rho_2^2 m'_2 \end{pmatrix} \quad \Longrightarrow \quad \boxed{m_k = \rho_k^2\,m'_k}$$

- m_2' is always positive:

$$m_2 = m_2' = \frac{1}{2}\left[m_L + m_R + \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

- If $m_L m_R \geq m_{\text{D}}^2$, then $m_1' \geq 0$ and $\rho_1^2 = 1$

$$m_1 = \frac{1}{2}\left[m_L + m_R - \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

- If $m_L m_R < m_{\text{D}}^2$, then $m_1' < 0$ and $\rho_1^2 = -1$

$$m_1 = \frac{1}{2}\left[\sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2} - (m_L + m_R)\right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix}$$

- If Δm^2 is small, there are oscillations between active ν_a generated by ν_L and sterile ν_s generated by ν_R^C:

$$P_{\nu_a \to \nu_s}(L, E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 \, L}{4 \, E}\right)$$

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R) \sqrt{(m_L - m_R)^2 + 4 \, m_{\mathrm{D}}^2}$$

- It can be shown that the CP parity of ν_k is $\xi_k^{\mathrm{CP}} = i \, \rho_k^2$:

$$\mathrm{U}_{\mathrm{CP}} \nu_k(x) \mathrm{U}_{\mathrm{CP}}^{-1} = i \, \rho_k^2 \, \gamma^0 \, \nu_k(x_{\mathrm{P}})$$

- Special cases:
  - $m_L = m_R \quad \Longrightarrow \quad$ Maximal Mixing
  - $m_L = m_R = 0 \quad \Longrightarrow \quad$ Dirac Limit
  - $|m_L|, m_R \ll m_{\mathrm{D}} \quad \Longrightarrow \quad$ Pseudo-Dirac Neutrinos
  - $m_L = 0 \quad m_{\mathrm{D}} \ll m_R \quad \Longrightarrow \quad$ See-Saw Mechanism

Maximal Mixing

$$\boxed{m_L = m_R}$$

$$\vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_\mathrm{D}$$

$$\begin{cases} \rho_1^2 = +1\,, \quad m_1 = m_L - m_\mathrm{D} \quad \text{if} \quad m_L \geq m_\mathrm{D} \\ \rho_1^2 = -1\,, \quad m_1 = m_\mathrm{D} - m_L \quad \text{if} \quad m_L < m_\mathrm{D} \end{cases}$$

$$m_2 = m_L + m_\mathrm{D}$$

$$\underline{m_L < m_\mathrm{D}}$$

$$\begin{cases} \nu_{1L} = \frac{-i}{\sqrt{2}} \left(\nu_L - \nu_R^C \right) \\ \nu_{2L} = \frac{1}{\sqrt{2}} \left(\nu_L + \nu_R^C \right) \end{cases}$$

$$\begin{cases} \nu_1 = \nu_{1L} + \nu_{1L}^C = \frac{-i}{\sqrt{2}} \left[(\nu_L + \nu_R) - \left(\nu_L^C + \nu_R^C \right) \right] \\ \nu_2 = \nu_{2L} + \nu_{2L}^C = \frac{1}{\sqrt{2}} \left[(\nu_L + \nu_R) + \left(\nu_L^C + \nu_R^C \right) \right] \end{cases}$$

Dirac Limit

$$\boxed{m_L = m_R = 0}$$

- $m'_{2,1} = \pm m_{\mathrm{D}} \quad \Longrightarrow \quad \begin{cases} \rho_1^2 = -1\,, & m_1 = m_{\mathrm{D}} \\ \rho_2^2 = +1\,, & m_2 = m_{\mathrm{D}} \end{cases}$
- The two Majorana fields ν_1 and ν_2 can be combined to give one Dirac field:

$$\nu = \frac{1}{\sqrt{2}}\,(i\nu_1 + \nu_2) = \nu_L + \nu_R$$

- A Dirac field ν can always be split in two Majorana fields:

$$\begin{aligned} \nu &= \frac{1}{2}\left[\left(\nu - \nu^C\right) + \left(\nu + \nu^C\right)\right] \\ &= \frac{i}{\sqrt{2}}\left(-i\,\frac{\nu - \nu^C}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\nu + \nu^C}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\,(i\nu_1 + \nu_2) \end{aligned}$$

- A Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities

Pseudo-Dirac Neutrinos

$$\boxed{|m_L|\,,\ m_R \ll m_{\mathrm{D}}}$$

- $m'_{2,1} \simeq \dfrac{m_L + m_R}{2} \pm m_{\mathrm{D}}$

- $m'_1 < 0 \quad \Longrightarrow \quad \rho_1^2 = -1 \quad \Longrightarrow \quad m_{2,1} \simeq m_{\mathrm{D}} \pm \dfrac{m_L + m_R}{2}$

- The two massive Majorana neutrinos have opposite CP parities and are almost degenerate in mass

- The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_{\mathrm{D}} \, (m_L + m_R)$$

- The oscillations occur with practically maximal mixing:

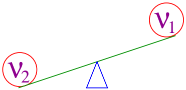
$$\tan 2\vartheta = \frac{2 m_{\mathrm{D}}}{m_R - m_L} \gg 1 \quad \Longrightarrow \quad \vartheta \simeq \pi/4$$

See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

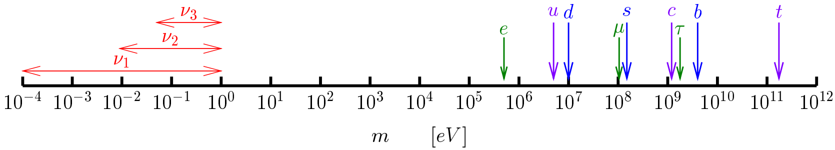
$$\boxed{m_L = 0 \qquad m_{\mathrm{D}} \ll m_R}$$

- $\mathscr{L}^L_{\text{mass}}$ is forbidden by SM symmetries $\Longrightarrow m_L = 0$
- $m_{\mathrm{D}} \lesssim v \sim 100\,\mathrm{GeV}$ is generated by SM Higgs Mechanism (protected by SM symmetries)
- m_R is not protected by SM symmetries $\Longrightarrow m_R \sim \mathcal{M}_{\mathrm{GUT}} \gg v$
- $$\left.\begin{array}{l} m'_1 \simeq -\dfrac{m_{\mathrm{D}}^2}{m_R} \\ m'_2 \simeq m_R \end{array}\right\} \quad \Longrightarrow \quad \left\{\begin{array}{ll} \rho_1^2 = -1\,, & m_1 \simeq \dfrac{m_{\mathrm{D}}^2}{m_R} \\ \rho_2^2 = +1\,, & m_2 \simeq m_R \end{array}\right.$$



- Natural explanation of smallness of neutrino masses
- Mixing angle is very small: $\tan 2\vartheta = 2\,\dfrac{m_{\mathrm{D}}}{m_R} \ll 1$
- ν_1 is composed mainly of active ν_L: $\nu_{1L} \simeq -i\,\nu_L$
- ν_2 is composed mainly of sterile ν_R: $\nu_{2L} \simeq \nu_R^C$

Majorana Neutrino Mass?



known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M}$ $\Rightarrow$ $\begin{cases} \text{See-Saw Mechanism (if } \nu_R\text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{cases}$

both imply $\begin{cases} \text{Majorana } \nu \text{ masses} \iff |\Delta L| = 2 \iff \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}} \end{cases}$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Right-Handed Neutrino Mass Term

Majorana mass term for ν_R respects the $\text{SU}(2)_L \times \text{U}(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^{\text{M}} = -\frac{1}{2}\, m \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

Majorana mass term for ν_R breaks Lepton number conservation!

Three possibilities:

- Lepton number can be explicitly broken
- Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left(\overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle\neq 0]{} -m_{\text{D}} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left(\eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle\neq 0]{} -\tfrac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left(\langle\eta\rangle + \rho + i\,\chi \right) \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2} \left(\overline{\nu_L^c}\ \overline{\nu_R} \right) \left(\begin{smallmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{smallmatrix} \right) \left(\begin{smallmatrix} \nu_L \\ \nu_R^c \end{smallmatrix} \right) + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_{\text{D}}} \Longrightarrow \text{See-Saw: } \boxed{m_1 \simeq \frac{m_{\text{D}}^2}{m_R}}$$

ρ = massive scalar, χ = Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\, \chi \left[\overline{\nu_2} \gamma^5 \nu_2 - \frac{m_{\text{D}}}{m_R} \left[\overline{\nu_2} \gamma^5 \nu_1 + \overline{\nu_1} \gamma^5 \nu_2 \right) + \left(\frac{m_{\text{D}}}{m_R} \right)^2 \overline{\nu_1} \gamma^5 \nu_1 \right]$$

Three-Generation Mixing

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{L}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$\mathscr{L}^{\text{D}}_{\text{mass}} = -\sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}}\, M^{\text{D}}_{s\alpha}\, \nu'_{\alpha L} + \text{H.c.}$$

$$\mathscr{L}^{L}_{\text{mass}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L}\, \mathcal{C}^{\dagger}\, M^{L}_{\alpha\beta}\, \nu'_{\beta L} + \text{H.c.}$$

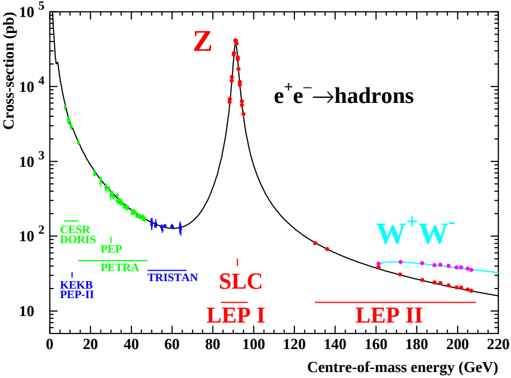
$$\mathscr{L}^{R}_{\text{mass}} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'^{T}_{sR}\, \mathcal{C}^{\dagger}\, M^{R}_{ss'}\, \nu'_{s'R} + \text{H.c.}$$

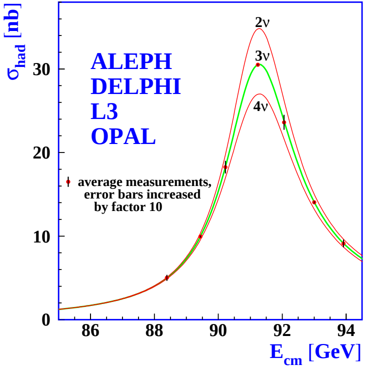
$$\mathbf{N}'_L \equiv \begin{pmatrix} \nu'_L \\ \nu'^{C}_R \end{pmatrix} \qquad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \nu'^{C}_R \equiv \begin{pmatrix} \nu'^{C}_{1R} \\ \vdots \\ \nu'^{C}_{N_S R} \end{pmatrix}$$

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \frac{1}{2} \mathbf{N}'^{T}_L\, \mathcal{C}^{\dagger}\, M^{\text{D+M}}\, \mathbf{N}'_L + \text{H.c.} \qquad M^{\text{D+M}} = \begin{pmatrix} M^{L} & M^{\text{D}\,T} \\ M^{\text{D}} & M^{R} \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term $\Longrightarrow$ massive Majorana neutrinos
- See-Saw Mechanism $\Longrightarrow$ sterile right-handed neutrinos have large masses and are decoupled from the low-energy phenomenology
- At low energy we have an effective mixing of three Majorana neutrinos

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

$e^+e^- \to Z \to \nu\bar{\nu} \quad \Rightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$ active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

STERILE NEUTRINOS

singlets of SM $\Longrightarrow$ no interactions!

active $\to$ sterile transitions are possible if $\nu_4, \ldots$ are light

$\Downarrow$

disappearance of active neutrinos

Sterile Neutrinos

- Sterile means No Standard Model Interactions
- Obviously no electromagnetic interactions as normal active neutrinos
- Thus Sterile means No Standard Weak Interactions
- But Sterile Neutrinos are not absolutely sterile:
  - Gravitational Interactions
  - New Non-Standard Interactions of the Physics Beyond the Standard Model which generates the masses of sterile neutrinos

Part II

Neutrino Oscillations in Vacuum and in Matter

Neutrino Oscillations in Vacuum

- Neutrino Oscillations in Vacuum
  - Ultrarelativistic Approximation
  - Easy Example of Neutrino Production
  - Neutrino Oscillations in Vacuum
  - Neutrinos and Antineutrinos
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered} \nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2} \end{gathered}}$$

$$\begin{array}{ll}
\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\
\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\
\bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\
\nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\
\nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV}
\end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO), $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Easy Example of Neutrino Production

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\boxed{\nu_\mu = \sum_k U_{\mu k} \nu_k}$$

two-body decay $\Longrightarrow$ fixed kinematics

$$\boxed{E_k^2 = p_k^2 + m_k^2}$$

π at rest:
$$\begin{cases} p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \\[2ex] E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \end{cases}$$

0^{th} order: $m_k = 0 \Rightarrow p_k = E_k = E = \dfrac{m_\pi}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1^{st} order: $\boxed{E_k \simeq E + \xi\,\dfrac{m_k^2}{2E}}$ $\boxed{p_k \simeq E - (1-\xi)\,\dfrac{m_k^2}{2E}}$

$$\xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{\mathrm{CC}} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \quad$ States

initial flavor: $\alpha \; = \; e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + ip_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U^*_{\alpha k} \, e^{-iE_k t + ip_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U^*_{\alpha k} U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha \to \nu_\beta}(t,x) = \left|\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k}\right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}^*\, e^{-im_k^2 L/2E}\, U_{\beta k}\right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

$$\begin{array}{lcl} \mathrm{C} & \Longrightarrow & \text{Particle} \leftrightarrows \text{Antiparticle} \\ \mathrm{P} & \Longrightarrow & \text{Left-Handed} \leftrightarrows \text{Right-Handed} \end{array}$$

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \xrightarrow{\mathrm{CP}} \nu_{\alpha L}^{\mathrm{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathrm{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $\quad U \leftrightarrows U^* \quad$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT, CP and T Symmetries

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
  - CPT Symmetry
  - CP Symmetry
  - T Symmetry
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\implies$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $\quad U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$ $\boxed{\text{CPT} \quad \Rightarrow \quad A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

violation of CP in neutrino oscillations is proportional to

$|U_{e3}| = \sin\vartheta_{13}$ and $\sin\delta_{13}$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\mathrm{T}} P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $A^{\mathrm{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\begin{aligned}\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\mathrm{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\ &= A^{\mathrm{T}}_{\alpha\beta} + A^{\mathrm{CP}}_{\beta\alpha} = A^{\mathrm{T}}_{\alpha\beta} - A^{\mathrm{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\mathrm{T}}_{\alpha\beta} = A^{\mathrm{CP}}_{\alpha\beta}}\end{aligned}$$

$$\boxed{A^{\mathrm{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

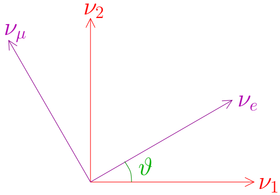
Jarlskog rephasing invariant: $\mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

Two-Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
  - Two-Neutrino Mixing and Oscillations
  - Types of Experiments
  - Average over Energy Resolution of the Detector
  - Anatomy of Exclusion Plots
- Neutrino Oscillations in Matter

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

two-neutrino mixing transition probability

$$\alpha \neq \beta \qquad\qquad \alpha, \beta = e, \mu, \tau$$

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\beta}(L, E) &= \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{m}]}{E[\text{MeV}]}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}\right)
\end{aligned}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \, \frac{E \, [\text{MeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{m} = 2.47 \, \frac{E \, [\text{GeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{km}$$

Types of Experiments

Two-Neutrino Mixing

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

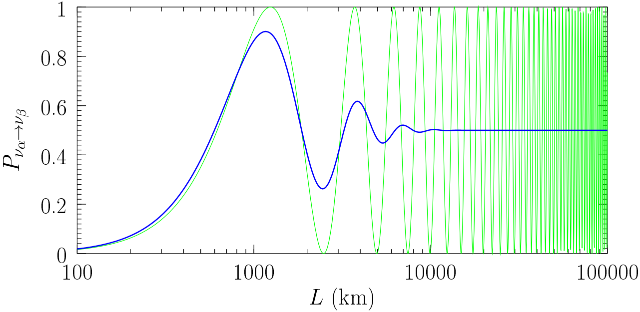
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

Average over Energy Resolution of the Detector

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\,\sin^2 2\vartheta\left[1-\cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

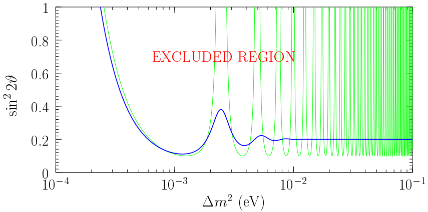


$\Delta m^2 = 10^{-3}\,\text{eV}$ $\quad$ $\sin^2 2\vartheta = 1$ $\quad$ $\langle E\rangle = 1\,\text{GeV}$ $\quad$ $\Delta E = 0.2\,\text{GeV}$

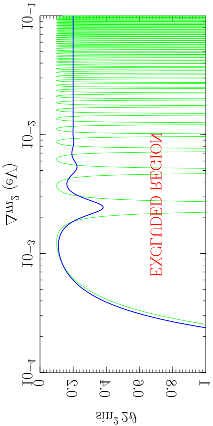
$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\,\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha\neq\beta)$$

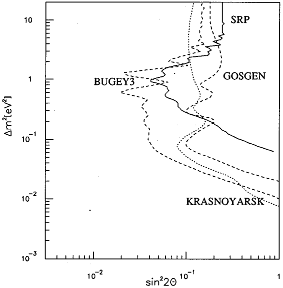
$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E)\, \mathrm{d}E \right] \qquad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \rangle \leq P^{\text{max}}_{\nu_\alpha \to \nu_\beta} \quad \Longrightarrow \quad \sin^2 2\vartheta \leq \frac{2\, P^{\text{max}}_{\nu_\alpha \to \nu_\beta}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E)\, \mathrm{d}E}$$

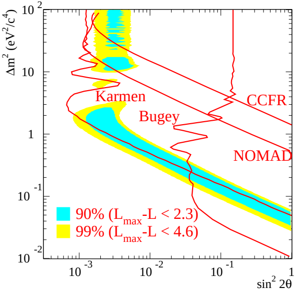


rotate
and
mirror

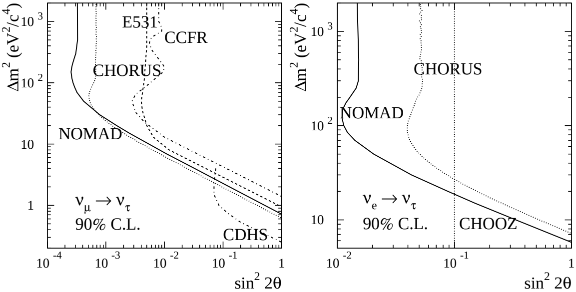




Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$

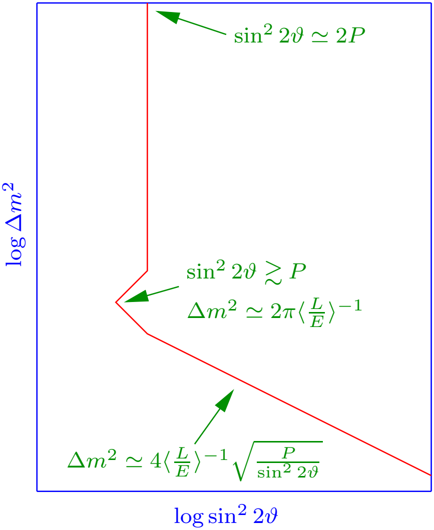


Accelerator SBL Experiments: $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{\tau}$ and $\overset{(-)}{\nu}_{e} \to \overset{(-)}{\nu}_{\tau}$

Anatomy of Exclusion Plots



- $\Delta m^2 \gg \langle L/E \rangle^{-1}$

  $$P \simeq \frac{1}{2}\sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P$$

- $\text{Min}\left\langle \cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle \geq -1$

  $$\sin^2 2\vartheta = \frac{2\,P}{1-\text{Min}\left\langle\cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle} \geq P$$

  $$\Delta m^2 \simeq 2\pi\langle L/E \rangle^{-1}$$

- $\Delta m^2 \ll 2\pi\langle L/E \rangle^{-1}$

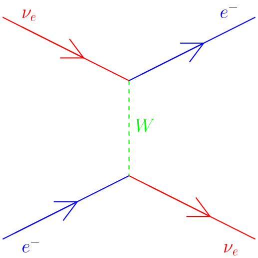
  $$\cos\left(\frac{\Delta m^2 L}{2E}\right) \simeq 1 - \frac{1}{2}\left(\frac{\Delta m^2 L}{2E}\right)^2$$

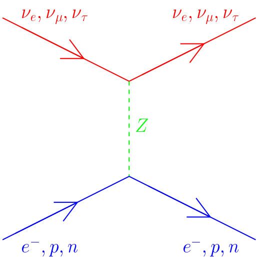
  $$\Delta m^2 \simeq 4\left\langle\frac{L}{E}\right\rangle^{-1}\sqrt{\frac{P}{\sin^2 2\vartheta}}$$

Neutrino Oscillations in Matter

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter
  - Effective Potentials in Matter
  - Matter Effects
  - Evolution of Neutrino Flavors in Matter
  - Two-Neutrino Mixing
  - Constant Matter Density
  - MSW Effect (Resonant Transitions in Matter)
  - Averaged Survival Probability
  - Crossing Probability
  - Solar Neutrinos
  - Electron Neutrino Regeneration in the Earth

Effective Potentials in Matter





$$\boxed{V_{\mathrm{CC}} = \sqrt{2} G_{\mathrm{F}} N_e} \qquad V_{\mathrm{NC}}^{(e^-)} = -V_{\mathrm{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\mathrm{NC}} = V_{\mathrm{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\mathrm{F}} N_n}$$

$$V_e = V_{\mathrm{CC}} + V_{\mathrm{NC}} \qquad V_\mu = V_\tau = V_{\mathrm{NC}}$$

only $V_{\mathrm{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\mathrm{CC}} = -V_{\mathrm{CC}}$ $\qquad \overline{V}_{\mathrm{NC}} = -V_{\mathrm{NC}}$

Matter Effects

a flavor neutrino ν_α with momentum p is described by

$$|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* \, |\nu_k(p)\rangle$$

$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

in matter $\quad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$

$V_\alpha =$ effective potential due to coherent interactions with the medium

forward elastic CC and NC scattering

Evolution of Neutrino Flavors in Matter

Schrödinger picture: $$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle \,, \qquad |\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes: $$\varphi_\beta(p,t) = \langle \nu_\beta(p) | \nu(p,t) \rangle \,, \qquad \varphi_\beta(p,0) = \delta_{\alpha\beta}$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu(p,t) \rangle = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle$$

$$\begin{aligned} \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle &= \sum_\rho \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\rho(p) \rangle \underbrace{\langle \nu_\rho(p) | \nu(p,t) \rangle}_{\varphi_\rho(p,t)} \\ &= \sum_\rho \sum_{k,j} U_{\beta k} \underbrace{\langle \nu_k(p) | \mathcal{H}_0 | \nu_j(p) \rangle}_{\delta_{kj} E_k} U^*_{\rho j} \, \varphi_\rho(p,t) \end{aligned}$$

$$\langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle = \sum_\rho \underbrace{\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\rho(p) \rangle}_{\delta_{\beta\rho} V_\beta} \varphi_\rho(p,t) = V_\beta \, \varphi_\beta(p,t)$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \, E_k \, U^*_{\rho k} + \delta_{\beta\rho} \, V_\beta \right) \varphi_\rho}$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $\quad E = p \quad t = x$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

$$i \frac{\text{d}}{\text{d}x} \varphi_\beta(p,x) = (p + V_{\text{NC}}) \varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x)\, e^{ipx + i \int_0^x V_{\text{NC}}(x')\,\text{d}x'}$$

$$\Downarrow$$

$$i \frac{\text{d}}{\text{d}x} \psi_\beta = e^{ipx + i \int_0^x V_{\text{NC}}(x')\,\text{d}x'} \left(-p - V_{\text{NC}} + i \frac{\text{d}}{\text{d}x} \right) \varphi_\beta$$

$$\boxed{i \frac{\text{d}}{\text{d}x} \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \psi_\rho}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2 E V_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$

$$
\begin{aligned}
U\,\mathbb{M}^2\,U^\dagger &= \begin{pmatrix} \cos^2\vartheta\, m_1^2 + \sin^2\vartheta\, m_2^2 & \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) \\ \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) & \sin^2\vartheta\, m_1^2 + \cos^2\vartheta\, m_2^2 \end{pmatrix} \\
&= \underbrace{\frac{1}{2}\,\Sigma m^2}_{\uparrow} + \frac{1}{2}\begin{pmatrix} -\Delta m^2\cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}
\end{aligned}
$$

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

$$\boxed{\begin{array}{l}P_{\nu_e\to\nu_\mu}(x) = \left|\psi_\mu(x)\right|^2\\ P_{\nu_e\to\nu_e}(x) = \left|\psi_e(x)\right|^2 = 1 - P_{\nu_e\to\nu_\mu}(x)\end{array}}$$

Constant Matter Density

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x}=0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\left[\underset{\substack{\uparrow\\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}}+\frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A^{\mathrm{R}}_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\mathrm{R}}_e = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1(0)\\ \psi_2(0)\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

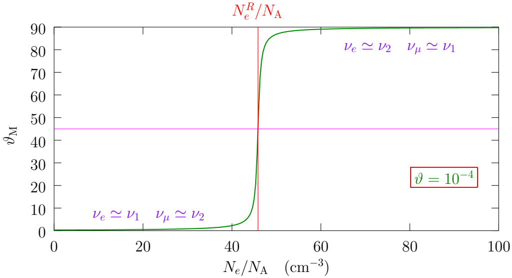
$$\psi_1(x) = \cos\vartheta_{\mathrm{M}}\exp\left(i\,\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

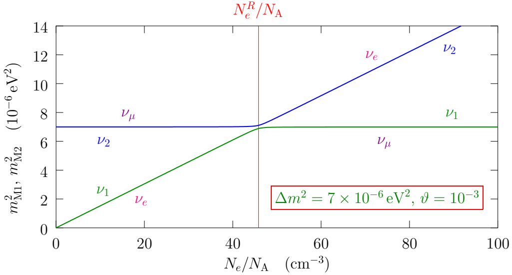
$$\psi_2(x) = \sin\vartheta_{\mathrm{M}}\exp\left(-i\,\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 = |-\sin\vartheta_{\mathrm{M}}\psi_1(x) + \cos\vartheta_{\mathrm{M}}\psi_2(x)|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)





$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \Bigg[\underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E} \begin{pmatrix} -\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}} \end{pmatrix} + \underset{\substack{\uparrow \\ \text{maximum near resonance}}}{\begin{pmatrix} 0 & -i\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} \\ i\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0 \end{pmatrix}} \Bigg] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} & -\sin\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} & \cos\vartheta^0_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} \end{pmatrix}$$

$$\begin{aligned} \psi_1(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{11} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{21} \right] \\ & \times \exp\left(i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \\ \psi_2(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{12} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{22} \right] \\ & \times \exp\left(-i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\, \mathrm{d}x' \right) \end{aligned}$$

Averaged Survival Probability

$$\psi_e(x) = \cos\vartheta_\text{M}^x \, \psi_1(x) + \sin\vartheta_\text{M}^x \, \psi_2(x)$$

neglect interference (averaged over energy spectrum)

$$\begin{aligned}\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = {} & \cos^2\vartheta_\text{M}^x \, \cos^2\vartheta_\text{M}^0 \, |\mathcal{A}_{11}^\text{R}|^2 + \cos^2\vartheta_\text{M}^x \, \sin^2\vartheta_\text{M}^0 \, |\mathcal{A}_{21}^\text{R}|^2 \\ & + \sin^2\vartheta_\text{M}^x \, \cos^2\vartheta_\text{M}^0 \, |\mathcal{A}_{12}^\text{R}|^2 + \sin^2\vartheta_\text{M}^x \, \sin^2\vartheta_\text{M}^0 \, |\mathcal{A}_{22}^\text{R}|^2\end{aligned}$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^\text{R}|^2 = |\mathcal{A}_{21}^\text{R}|^2 = P_\text{c} \qquad\qquad |\mathcal{A}_{11}^\text{R}|^2 = |\mathcal{A}_{22}^\text{R}|^2 = 1 - P_\text{c}$$

$P_\text{c} \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_\text{c}\right) \cos2\vartheta_\text{M}^0 \, \cos2\vartheta_\text{M}^x}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_{\rm c} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $$\gamma = \left. \frac{\Delta m^2_{\rm M}/2E}{2\left|{\rm d}\vartheta_{\rm M}/{\rm d}x\right|} \right|_{\rm R} = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{{\rm d}\ln A_{\rm CC}}{{\rm d}x}\right|_{\rm R}}$$

$A \propto x$ $\quad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $\quad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $\quad F = 1 - \tan^2\vartheta$

[Pizzochero, PRD 36 (1987) 2293]

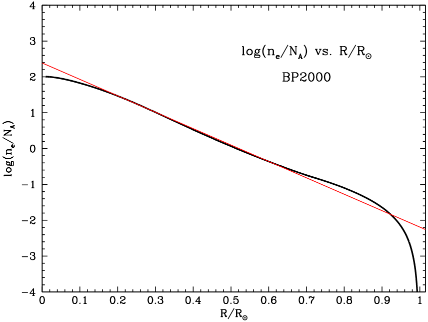
[Toshev, PLB 196 (1987) 170]

[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $N_e(x) \simeq N_e^{\rm c} \exp\left(-\frac{x}{x_0}\right)$ $\qquad N_e^{\rm c} = 245\, N_{\rm A}/\mathrm{cm}^3 \qquad x_0 = \frac{R_\odot}{10.54}$



$$\overline{P}^{\rm sun}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\rm c}\right) \cos 2\vartheta^0_{\rm M}\, \cos 2\vartheta$$

$$P_{\rm c} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{\mathrm{d}\ln A_{\rm CC}}{\mathrm{d}x}\right|_{\rm R}}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{\rm CC} = 2\sqrt{2} E G_{\rm F} N_e$$

practical prescription:

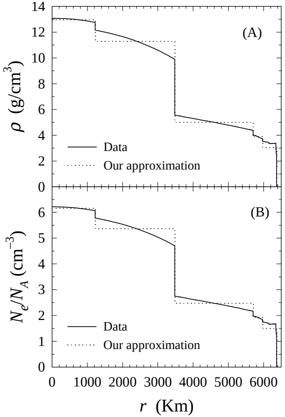
[Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } \left|\mathrm{d}\ln A_{\rm CC}/\mathrm{d}x\right|_{\rm R} & \text{for } x \leq 0.904 R_\odot \\ \left|\mathrm{d}\ln A_{\rm CC}/\mathrm{d}x\right|_{\rm R} \to \frac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e\to\nu_e} = \overline{P}^{\text{sun}}_{\nu_e\to\nu_e} + \frac{\left(1-2\overline{P}^{\text{sun}}_{\nu_e\to\nu_e}\right)\left(P^{\text{earth}}_{\nu_2\to\nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



[Giunti, Kim, Monteno, NP B 521 (1998) 3]

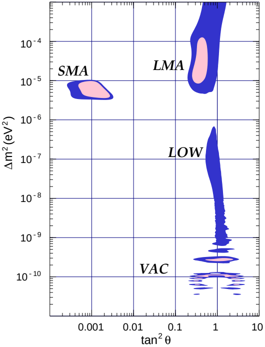
$P^{\text{earth}}_{\nu_2\to\nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

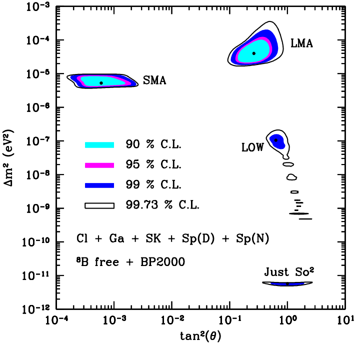
Wave functions of flavor neutrinos are joined at the boundaries of steps.

Phenomenology of Solar Neutrinos

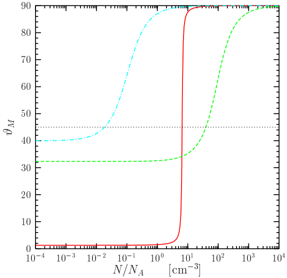
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\mathrm{eV}^2\,,$	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\mathrm{eV}^2\,,$	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\mathrm{eV}^2\,,$	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\mathrm{eV}^2\,,$	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\mathrm{eV}^2\,,$	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



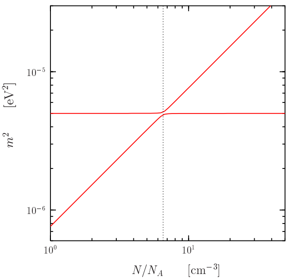
[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



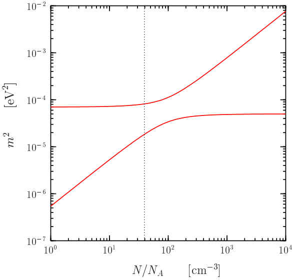
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\text{eV}^2$
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\text{eV}^2$
(typical LMA) $\tan^2\vartheta = 0.4$

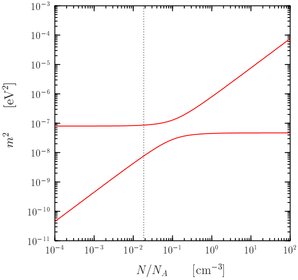
dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\text{eV}^2$
(typical LOW) $\tan^2\vartheta = 0.7$



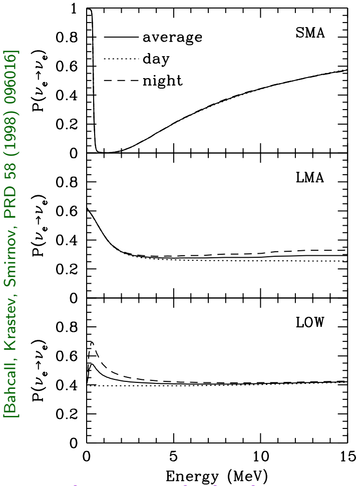
typical SMA



typical LMA



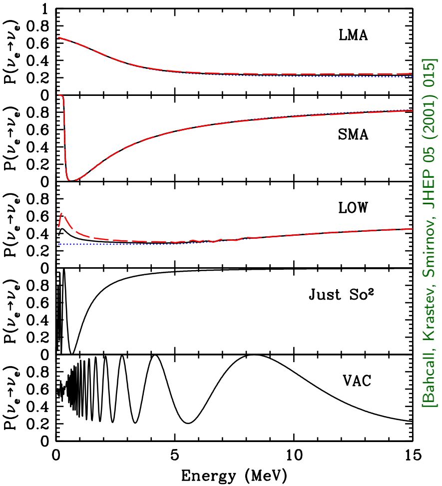
typical LOW



SMA: $\Delta m^2 = 5.0 \times 10^{-6}\,\text{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$

LMA: $\Delta m^2 = 1.6 \times 10^{-5}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.57$

LOW: $\Delta m^2 = 7.9 \times 10^{-8}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.95$



LMA: $\Delta m^2 = 4.2 \times 10^{-5}\,\text{eV}^2$ $\tan^2\vartheta = 0.26$

SMA: $\Delta m^2 = 5.2 \times 10^{-6}\,\text{eV}^2$ $\tan^2\vartheta = 5.5 \times 10^{-4}$

LOW: $\Delta m^2 = 7.6 \times 10^{-8}\,\text{eV}^2$ $\tan^2\vartheta = 0.72$

Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\,\text{eV}^2$ $\tan^2\vartheta = 1.0$

VAC: $\Delta m^2 = 1.4 \times 10^{-10}\,\text{eV}^2$ $\tan^2\vartheta = 0.38$

In Neutrino Oscillations Dirac = Majorana

Evolution of Amplitudes: $$i \frac{d\psi_\alpha}{dx} = \frac{1}{2E} \sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2 D \quad \Longrightarrow \quad DM^2 D^\dagger = M^2$$

$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$

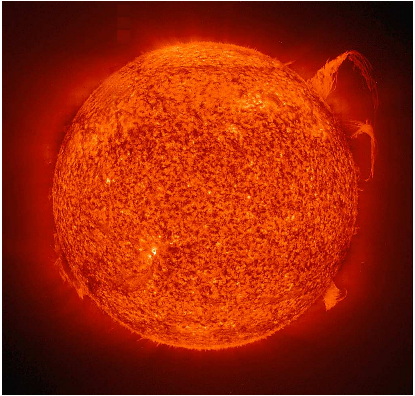
Part III

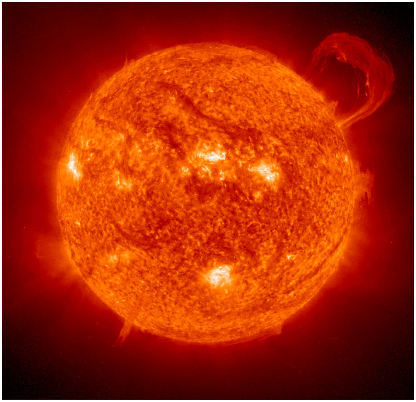
Experimental Results and Theoretical Implications

Solar Neutrinos and KamLAND

- Solar Neutrinos and KamLAND
  - The Sun
  - Standard Solar Model (SSM)
  - Homestake
  - Gallium Experiments
  - SAGE: Soviet-American Gallium Experiment
  - GALLEX: GALLium EXperiment
  - GNO: Gallium Neutrino Observatory
  - Kamiokande
  - Super-Kamiokande
  - SNO: Sudbury Neutrino Observatory
  - KamLAND
  - Sterile Neutrinos in Solar Neutrino Flux?
  - Determination of Solar Neutrino Fluxes
  - Details of Solar Neutrino Oscillations
  - BOREXino

The Sun





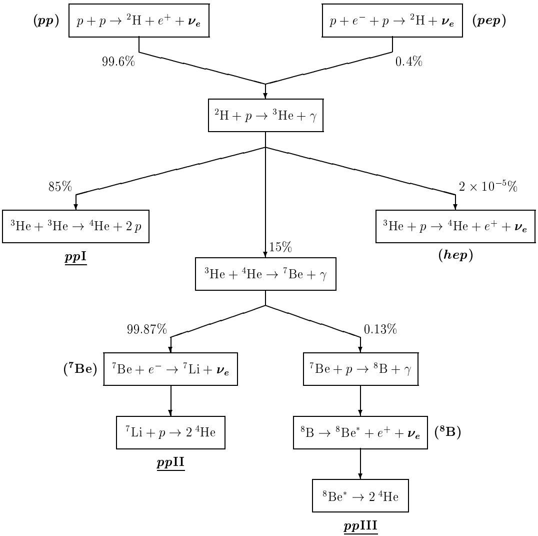
Extreme ultraviolet Imaging Telescope (EIT) 304 $\AA$ images of the Sun

emission in this spectral line (He II) shows the upper chromosphere

at a temperature of about 60,000 K

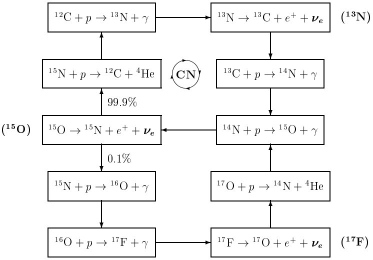
[The Solar and Heliospheric Observatory (SOHO), http://sohowww.nascom.nasa.gov/]

Standard Solar Model (SSM)



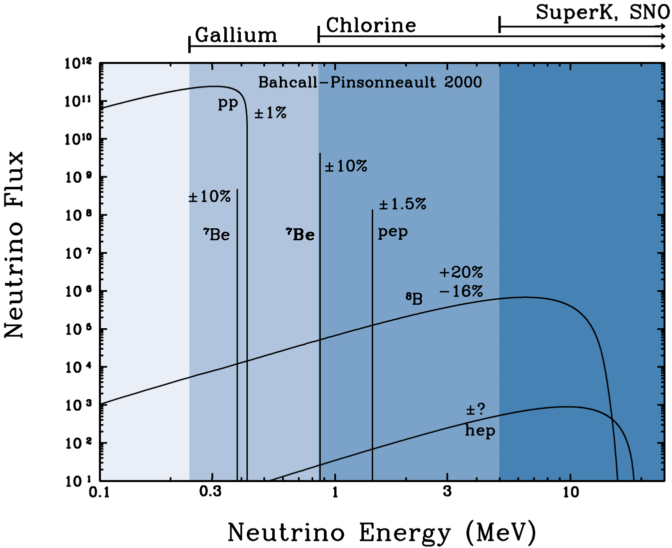
pp chain and CNO cycle

$$4\,p + 2\,e^- \to {}^4\mathrm{He} + 2\,\nu_e + 26.731\ \mathrm{MeV}$$

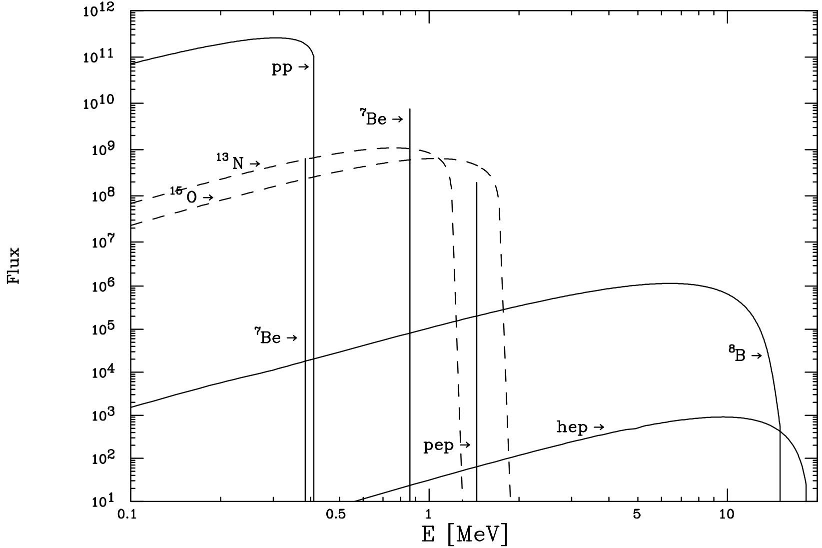


Bahcall SSMs

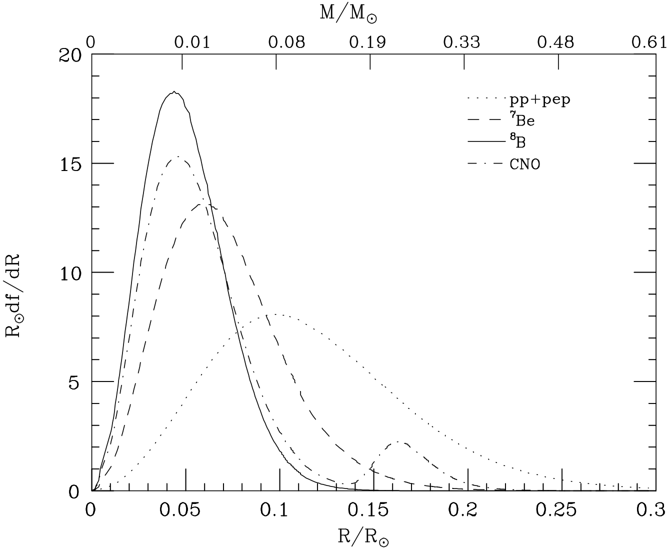
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



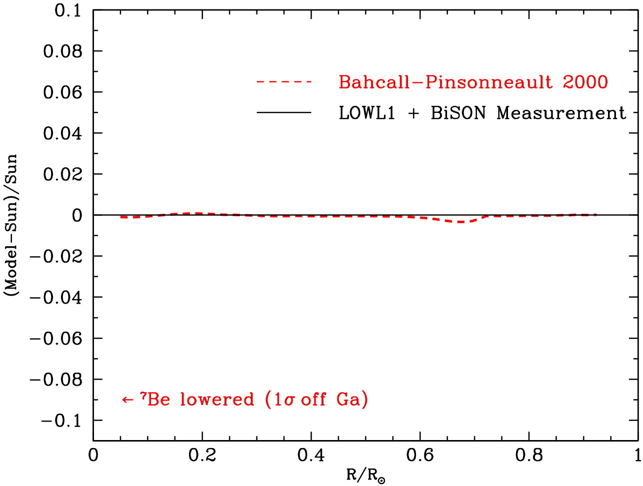
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between $0.05\,R_\odot$ and $0.95\,R_\odot$ and is 0.08% for the deep interior region, $r < 0.25\,R_\odot$, in which neutrinos are produced

Homestake

$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$ [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e. $\Rightarrow \Phi_\mu \simeq 4\,\text{m}^{-2}\,\text{day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

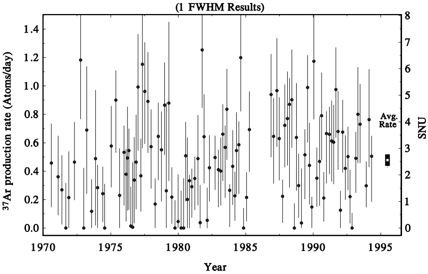
615 tons of tetrachloroethylene (C_2Cl_4), 2.16×10^{30} atoms of ^{37}Cl (133 tons)

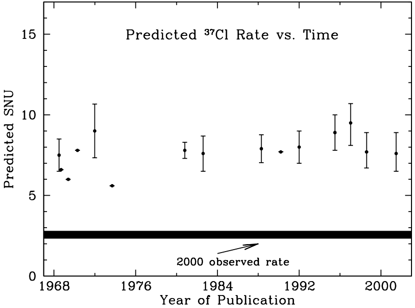
energy threshold: $E_{\text{th}}^{\text{Cl}} = 0.814\,\text{MeV} \Longrightarrow {}^{8}\text{B}\,,\ {}^{7}\text{Be}\,,\ pep\,,\ hep\,,\ {}^{13}\text{N}\,,\ {}^{15}\text{O}\,,\ {}^{17}\text{F}$

1970–1994, 108 extractions $\Longrightarrow$ $\dfrac{R_{\text{Cl}}^{\text{exp}}}{R_{\text{Cl}}^{\text{SSM}}} = 0.34 \pm 0.03$ [APJ 496 (1998) 505]

$R_{\text{Cl}}^{\text{exp}} = 2.56 \pm 0.23\,\text{SNU}$ $R_{\text{Cl}}^{\text{SSM}} = 7.6^{+1.3}_{-1.1}\,\text{SNU}$

1 SNU $= 10^{-36}$ events atom^{-1} s^{-1}





Gallium Experiments

SAGE, GALLEX, GNO

radiochemical experiments

$$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$$ [Kuzmin (1965)]

threshold: $E_{\text{th}}^{\text{Ga}} = 0.233\,\text{MeV} \Longrightarrow pp\,,\ {}^{7}\text{Be}\,,\ {}^{8}\text{B}\,,\ pep\,,\ hep\,,\ {}^{13}\text{N}\,,\ {}^{15}\text{O}\,,\ {}^{17}\text{F}$

SAGE+GALLEX+GNO $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{exp}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.56 \pm 0.03$

$R_{\text{Ga}}^{\text{exp}} = 72.4 \pm 4.7\,\text{SNU}$ $\qquad$ $R_{\text{Ga}}^{\text{SSM}} = 128^{+9}_{-7}\,\text{SNU}$

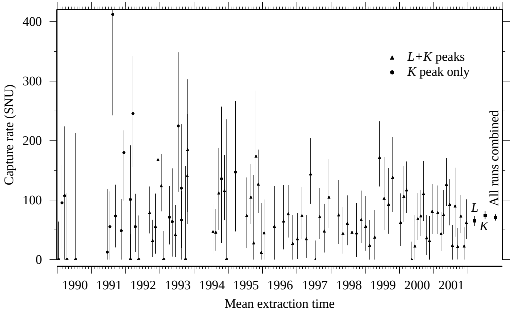
SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic ^{71}Ga, 2000 m deep, 4700 m.w.e. $\Rightarrow \Phi_{\mu} \simeq 2.6\,\text{m}^{-2}\,\text{day}^{-1}$

detector test: ^{51}Cr Source: $R = 0.95^{+0.11}_{-0.10}{}^{+0.06}_{-0.05}$ [PRC 59 (1999) 2246]

1990 – 2001 $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.54 \pm 0.05$ [astro-ph/0204245]



GALLEX: GALLium EXperiment

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

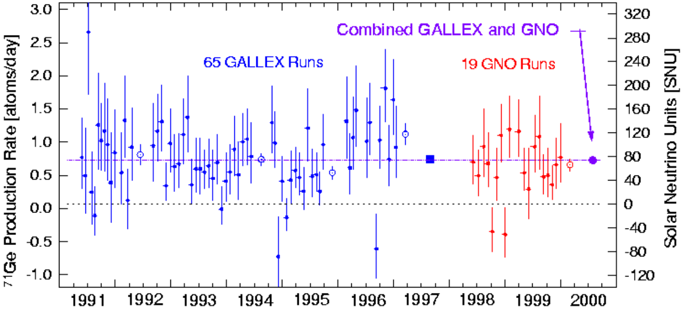
30.3 tons of gallium in 101 tons of gallium chloride ($GaCl_3$-HCl) solution

May 1991 – Jan 1997 $\Longrightarrow$ $$\frac{R_{\mathrm{Ga}}^{\mathrm{GALLEX}}}{R_{\mathrm{Ga}}^{\mathrm{SSM}}} = 0.61 \pm 0.06$$ [PLB 477 (1999) 127]

GNO: Gallium Neutrino Observatory

continuation of GALLEX: 30.3 tons of gallium

May 1998 – Jan 2000 $\Longrightarrow$ $$\frac{R_{\text{Ga}}^{\text{GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.51 \pm 0.08$$ [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

Kamiokande

water Cherenkov detector $\nu + e^- \to \nu + e^-$

Sensitive to ν_e, ν_μ, ν_τ, but $\sigma(\nu_e) \simeq 6\,\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75\,\text{MeV} \Longrightarrow {}^8\text{B}$, *hep*

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{SSM}}} = 0.55 \pm 0.08$$ [PRL 77 (1996) 1683]

Super-Kamiokande

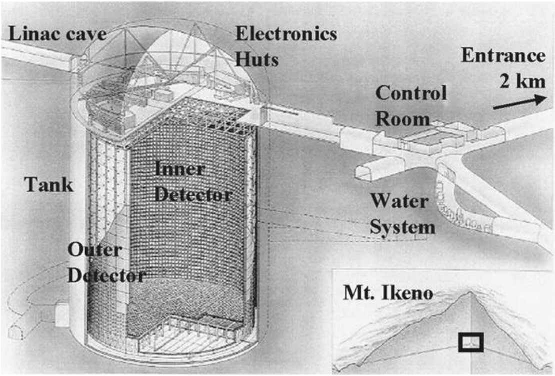
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}$, *hep*

1996 – 2001 (1496 days)

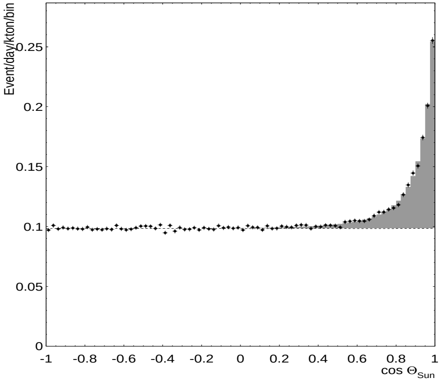
$$\frac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{SSM}}} = 0.465 \pm 0.015$$ [SK, PLB 539 (2002) 179]



the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

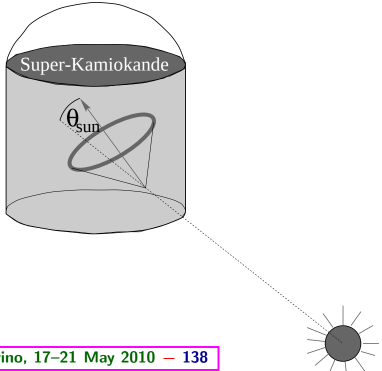
Super-Kamiokande $\cos\theta_{\rm sun}$ distribution



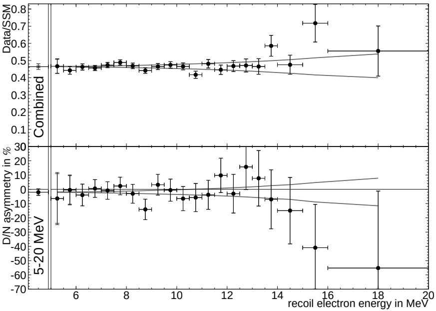
[Smy, hep-ex/0208004]

the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

the peak at $\cos\theta_{\rm sun} = 1$ is due to solar neutrinos

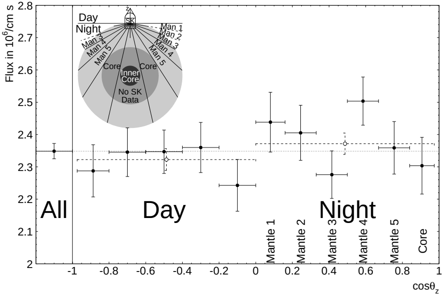


Super-Kamiokande energy spectrum normalized to BP2000 SSM



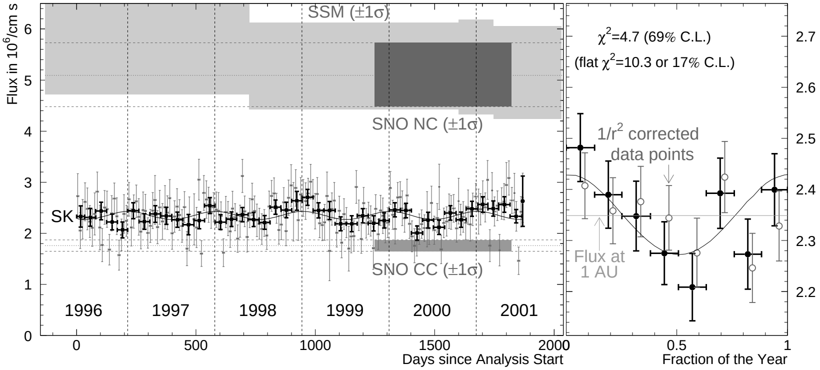
Day-Night asymmetry as a function of energy

solar zenith angle (θ_z) dependence of Super-Kamiokande data



[Smy, hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D_2O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

$$
\begin{array}{ll}
\text{CC:} & \nu_e + d \to p + p + e^- \\
\text{NC:} & \nu + d \to p + n + \nu \\
\text{ES:} & \nu + e^- \to \nu + e^-
\end{array}
$$

$$
\left.
\begin{array}{l}
\text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2\,\text{MeV} \\
\text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2\,\text{MeV} \\
\text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0\,\text{MeV}
\end{array}
\right\} \Longrightarrow {}^8\text{B},\ hep
$$

D_2O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.01 \pm 0.13$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.31 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.03 \pm 0.09$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

$$\Phi_{\nu_e}^{\text{SNO}} = 1.76 \pm 0.11 \times 10^6 \,\text{cm}^{-2}\,\text{s}^{-1}$$

$$\Phi_{\nu_\mu,\nu_\tau}^{\text{SNO}} = 5.41 \pm 0.66 \times 10^6 \,\text{cm}^{-2}\,\text{s}^{-1}$$

SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

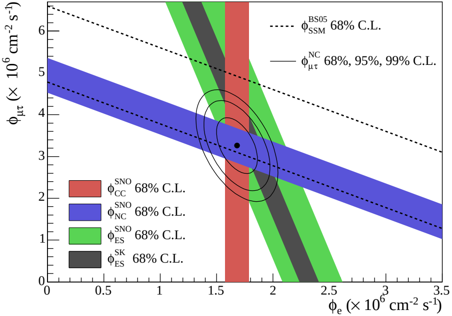
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

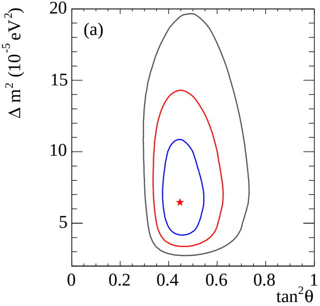
$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2$$

$$\tan^2\vartheta \simeq 0.45$$





[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km

- 6.7% of flux from one reactor at 88 km
- 79% of flux from 26 reactors at 138–214 km
- 14.3% of flux from other reactors at $>$295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$

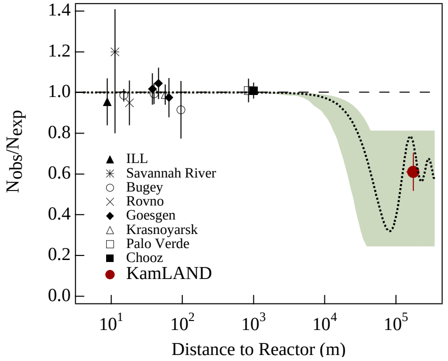
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$

observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

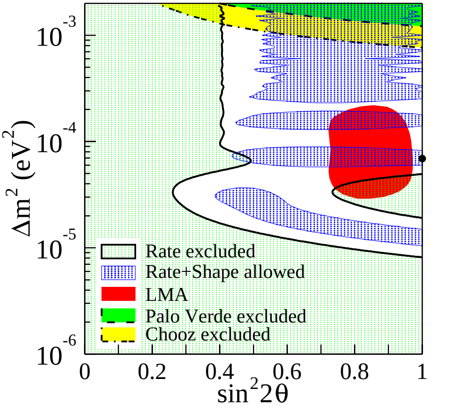
$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

99.95% C.L. evidence of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



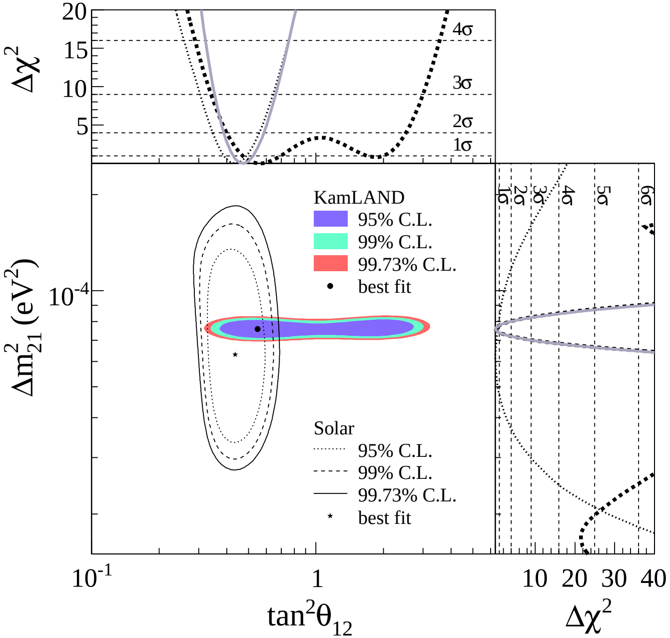
Shade: 95% C.L. LMA



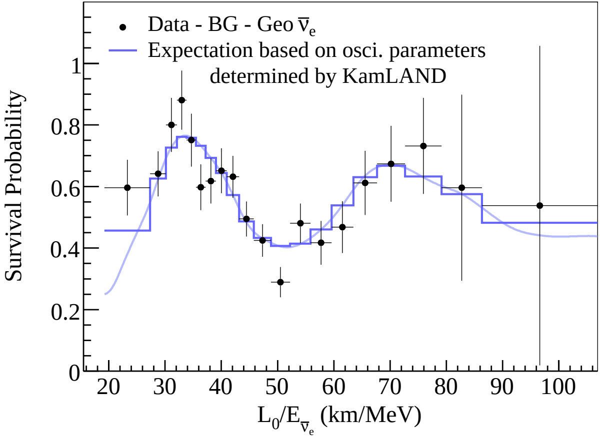
95% C.L.

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

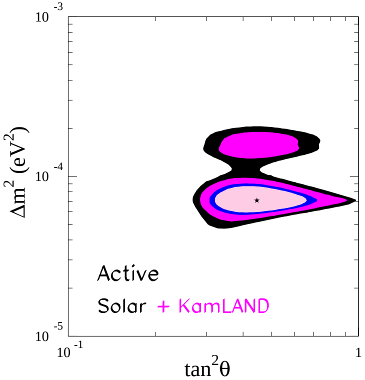


[KamLAND, PRL 100 (2008) 221803]



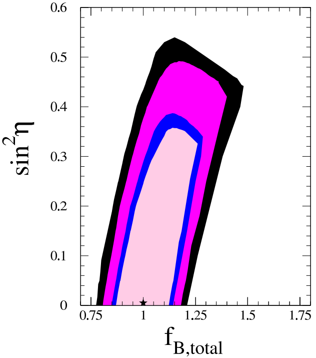
[KamLAND, PRL 100 (2008) 221803]

Sterile Neutrinos in Solar Neutrino Flux?



90%, 95%, 99%, 99.73% (3σ) C.L.

[Bahcall, Gonzalez-Garcia, Pena-Garay, JHEP 0302 (2003) 009]



$$\nu_e \to \cos\eta\,\nu_a + \sin\eta\,\nu_s$$

$$\sin^2\eta < 0.52\,(3\sigma)$$

$$f_{\text{B,total}} = \frac{\Phi_{^8\text{B}}}{\Phi_{^8\text{B}}^{\text{SSM}}} = 1.00 \pm 0.06$$

Determination of Solar Neutrino Fluxes

[Bahcall, Peña-Garay, hep-ph/0305159]

fit of solar and KamLAND neutrino data with fluxes as free parameters

+ luminosity constraint

$$\sum_r \alpha_r \, \Phi_r = K_\odot \quad (r = pp, pep, hep, {}^7\text{Be}, {}^8\text{B}, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F})$$

$$K_\odot \equiv \mathcal{L}_\odot / 4\pi (1\text{a.u.})^2 = 8.534 \times 10^{11} \, \text{MeV} \, \text{cm}^{-2} \, \text{s}^{-1}$$

solar constant

$$\Delta m^2 = 7.3^{+0.4}_{-0.6} \, \text{eV}^2 \qquad \tan^2\vartheta = 0.42^{+0.08}_{-0.06} \, \left({}^{+0.39}_{-0.19}\right)$$

$$\frac{\Phi_{^8\text{B}}}{\Phi_{^8\text{B}}^{\text{SSM}}} = 1.01^{+0.06}_{-0.06} \, \left({}^{+0.22}_{-0.17}\right)$$

moderate uncertainty

will improve with new SNO NC data (salt phase)

$$\frac{\Phi_{^7\text{Be}}}{\Phi_{^7\text{Be}}^{\text{SSM}}} = 0.97^{+0.28}_{-0.54} \, \left({}^{+0.85}_{-0.97}\right)$$

large uncertainty

needs ^{7}Be experiment (KamLAND, Borexino?)

$$\frac{\Phi_{pp}}{\Phi_{pp}^{\text{SSM}}} = 1.02^{+0.02}_{-0.02} \, \left({}^{+0.07}_{-0.07}\right)$$

small uncertainty

CNO luminosity: $\mathcal{L}_{\text{CNO}} / \mathcal{L}_\odot = 0.0^{+2.8}_{-0.0} \, \left({}^{+7.3}_{-0.0}\right)$

[Bahcall, Gonzalez-Garcia, Peña-Garay, PRL 90 (2003) 131301]

Details of Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2$ $\quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta\left|\frac{d\ln A}{dx}\right|_{\text{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2}EG_{\text{F}}N^{\text{c}}_e\exp\left(-\frac{x}{x_0}\right) \quad\Longrightarrow\quad \left|\frac{d\ln A}{dx}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3\times 10^{-15}\,\text{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad\Longrightarrow\quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad\qquad \gamma \simeq 2\times 10^4\left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad\Longrightarrow\quad P_{\text{c}} \ll 1 \quad\Longrightarrow\quad \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2}\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta_{\mathrm{M}}^{0} = \frac{\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, hep-ph/0305159]

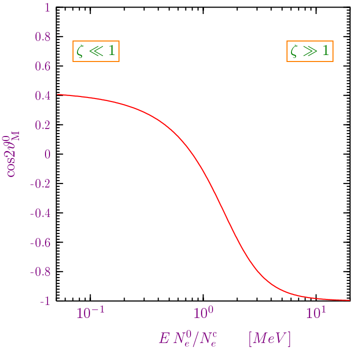
$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right) \left(\frac{N_e^0}{N_e^{\mathrm{c}}}\right)$$

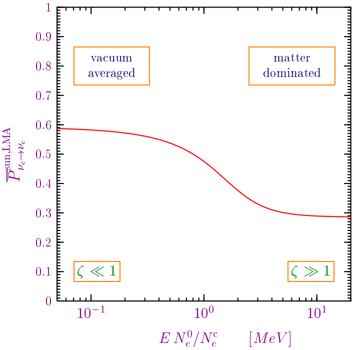
$$\zeta \ll 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \vartheta \implies \overline{P}_{\nu_e \to \nu_e}^{\mathrm{sun}} \simeq 1 - \tfrac{1}{2} \sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \pi/2 \implies \overline{P}_{\nu_e \to \nu_e}^{\mathrm{sun}} \simeq \sin^2 \vartheta$$

matter dominated survival probability

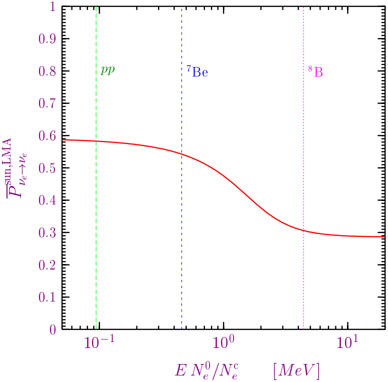




$\langle E \rangle_{pp} \simeq 0.27\,\text{MeV}\,,\ \langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot \quad \Longrightarrow \quad \langle E\, N_e^0 / N_e^c \rangle_{pp} \simeq 0.094\,\text{MeV}$

$E_{^7\text{Be}} \simeq 0.86\,\text{MeV}\,,\ \langle r_0 \rangle_{^7\text{Be}} \simeq 0.06\,R_\odot \quad \Longrightarrow \quad \langle E\, N_e^0 / N_e^c \rangle_{^7\text{Be}} \simeq 0.46\,\text{MeV}$

$\langle E \rangle_{^8\text{B}} \simeq 6.7\,\text{MeV}\,,\ \langle r_0 \rangle_{^8\text{B}} \simeq 0.04\,R_\odot \quad \Longrightarrow \quad \langle E\, N_e^0 / N_e^c \rangle_{^8\text{B}} \simeq 4.4\,\text{MeV}$



each neutrino experiment is mainly sensitive to one flux

each neutrino experiment is mainly sensitive to ϑ

accurate *pp* experiment can improve determination of ϑ

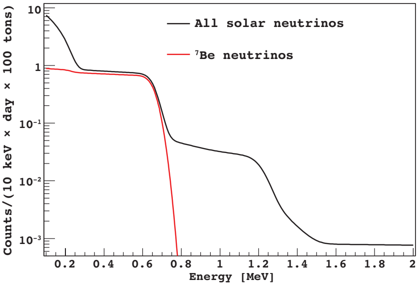
[Bahcall, Peña-Garay, hep-ph/0305159]

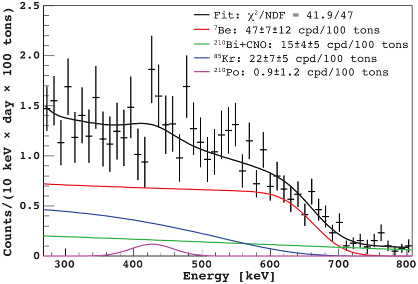
BOREXino

[BOREXino, arXiv:0708.2251]

Real-time measurement of ^{7}Be solar neutrinos (0.862 MeV)

$$\nu + e \to \nu + e \qquad E = 0.862\,\text{MeV} \quad \Longrightarrow \quad \sigma_{\nu_e} \simeq 5.5\,\sigma_{\nu_\mu,\nu_\tau}$$





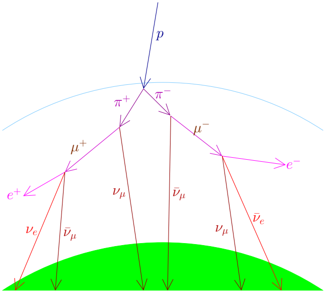
$$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad n_{\text{exp}} = 47 \pm 7 \pm 12\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$$

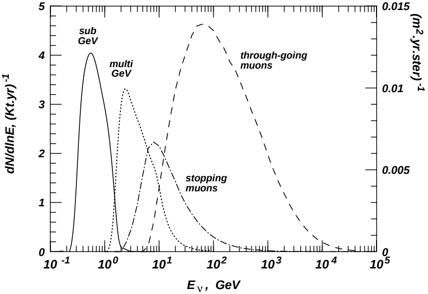
$$n_{\text{the}}^{\text{osc}} = 49 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad (n_{\text{the}}^{\text{no-osc}} - n_{\text{exp}})/\Delta n \simeq 1.9$$

Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
  - Atmospheric Neutrinos
  - Super-Kamiokande Up-Down Asymmetry
  - Fit of Super-Kamiokande Atmospheric Data
  - Kamiokande, Soudan-2, MACRO and MINOS
  - K2K
  - MINOS
  - Sterile Neutrinos in Atmospheric Neutrino Flux?
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies

Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \qquad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim$ 5%

uncertainty on fluxes: $\sim$ 30%

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

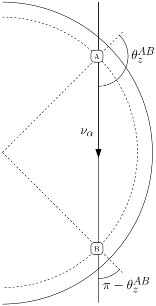
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB}) \qquad \phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z) = \phi^{(A)}_{\nu_\alpha}(\pi - \theta_z)$$

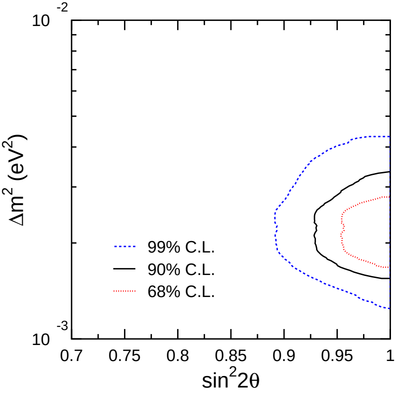
(December 1998)

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$\nu_\mu \to \nu_\tau$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20$events/yr
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\text{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$\boxed{N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}}$

$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

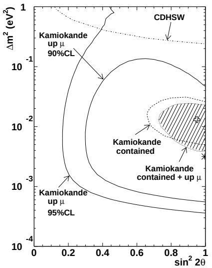
Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

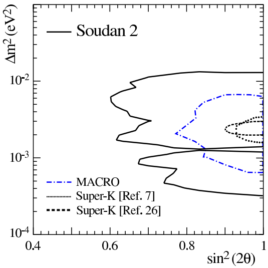
$L \simeq 732\,\text{km}$ $\langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

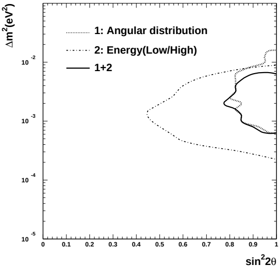
Kamiokande, Soudan-2, MACRO and MINOS



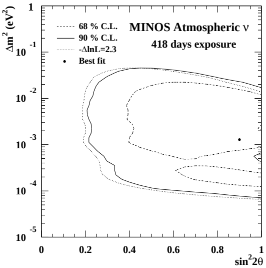
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



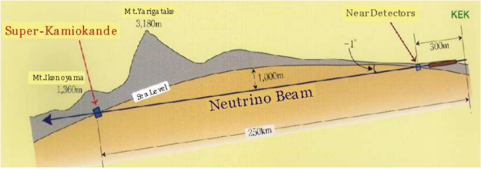
[MACRO, hep-ex/0304037]



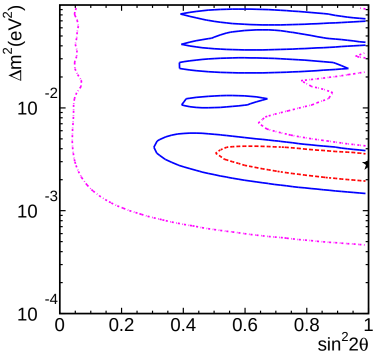
[MINOS, hep-ex/0512036]

K2K

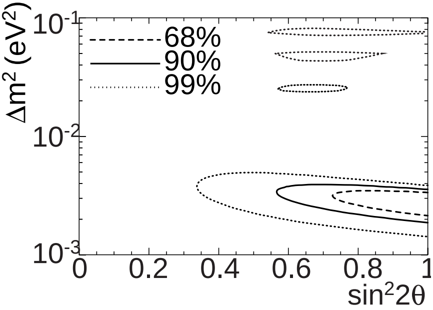
confirmation of atmospheric allowed region (June 2002)



KEK to Kamioka
(Super-Kamiokande)
250 km
$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]

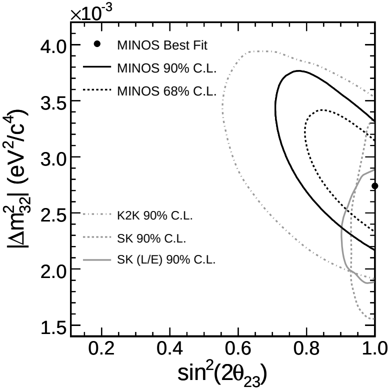


[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

http://www-numi.fnal.gov/



Near Detector: 1 km

$$\nu_\mu \to \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \ @\ 68\%\, CL$$

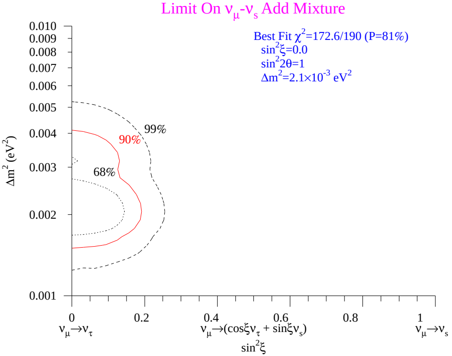
[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

Sterile Neutrinos in Atmospheric Neutrino Flux?

Nature of atmospheric Oscillation

Mode	Best fit	$\Delta\chi 2$	σ
ν_μ-ν_τ	$\sin^2 2\theta=1.00$; $\Delta m^2=2.5\text{x}10^{-3}\text{eV}^2$	0.0	0.0
ν_μ-ν_e	$\sin^2 2\theta=0.97$; $\Delta m^2=5.0\text{x}10^{-3}\text{eV}^2$	79.3	8.9
ν_μ-ν_s	$\sin^2 2\theta=0.96$; $\Delta m^2=3.6\text{x}10^{-3}\text{eV}^2$	19.0	4.4
LxE	$\sin^2 2\theta=0.90$; $\alpha=5.3\text{x}10^{-4}$	67.1	8.2
ν_μ Decay	$\cos^2\theta=0.47$; $\alpha=3.0\text{x}10^{-3}\text{eV}^2$	81.1	9.0
ν_μ Decay to ν_s	$\cos^2\theta=0.33$; $\alpha=1.1\text{x}10^{-2}\text{eV}^2$	14.1	3.8

[Smy (SK), Moriond 2002]



[Nakaya (SK), hep-ex/0209036]

FUTURE

MINOS: $\nu_\mu \to \nu_\mu$, $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_{e,\mu,\tau}$ (NC)

CNGS: ICARUS: $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$ OPERA: $\nu_\mu \to \nu_\tau$

Phenomenology of Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
  - Experimental Evidences of Neutrino Oscillations
  - Three-Neutrino Mixing
  - Allowed Three-Neutrino Schemes
  - Mixing Matrix
  - The Hunt for ϑ_{13}
  - Bilarge Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$ Reactor $\bar{\nu}_e$ disappearance	(Homestake, Kamiokande, GALLEX/GNO & SAGE, Super-Kamiokande, SNO, BOREXino) (KamLAND)	$\to$	$\Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$ $\tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06$
Atmospheric $\nu_\mu \to \nu_\tau$ Accelerator ν_μ disappearance	(Kamiokande, IMB, Super-Kamiokande, MACRO, Soudan-2) (K2K & MINOS)	$\to$	$\Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$ $\sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

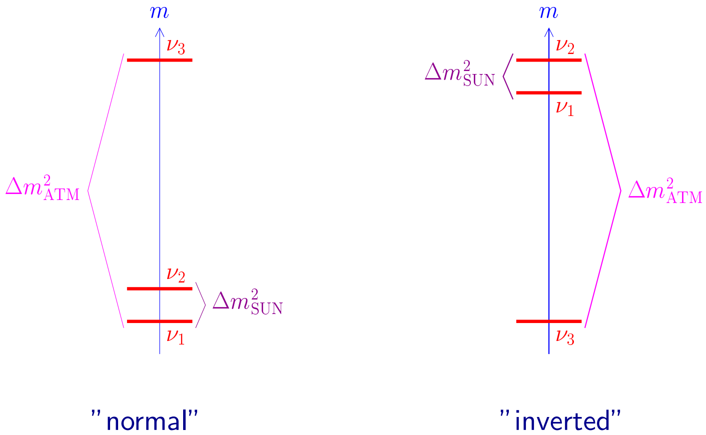
three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

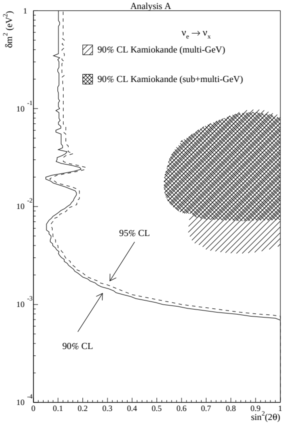
$$\begin{array}{r} \text{SOL} \rightarrow \\ \\ \\ \end{array} U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

$$\begin{array}{r} \uparrow \\ \text{ATM} \end{array}$$

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1-|U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

Effective ATM and LBL Oscillation Probability in Vacuum

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2$$

$$= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \to \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(\frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \Delta m_{31}^2 \to \Delta m^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(\frac{\Delta m^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}$$

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\beta} &= \left| \delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3} \left[1 - \exp\left(\frac{\Delta m^2 L}{2E} \right) \right] \right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E} \right) \\
&\qquad - 2\delta_{\alpha\beta} |U_{\alpha 3}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \sin^2 \frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$
\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)
$$

$$
\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2 \right) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)
$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \simeq 4|U_{e3}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu3}|^2 \left(1 - |U_{\mu3}|^2\right)$$

$$|U_{\mu3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - \sin^2 2\vartheta_{\mu\mu}}\right)$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2 |U_{\mu3}|^2$$

$$
U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}
$$

$$
= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}
$$

$$
\sin^2 \vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2 \vartheta_{23} = 0.50^{+0.07}_{-0.06}
$$

$$
\sin^2 \vartheta_{13} < 0.035 \quad (90\% \text{ C.L.})
$$

[Schwetz, Tortola, Valle, New J. Phys. 10 (2008) 113011]

Hint of $\vartheta_{13} > 0$

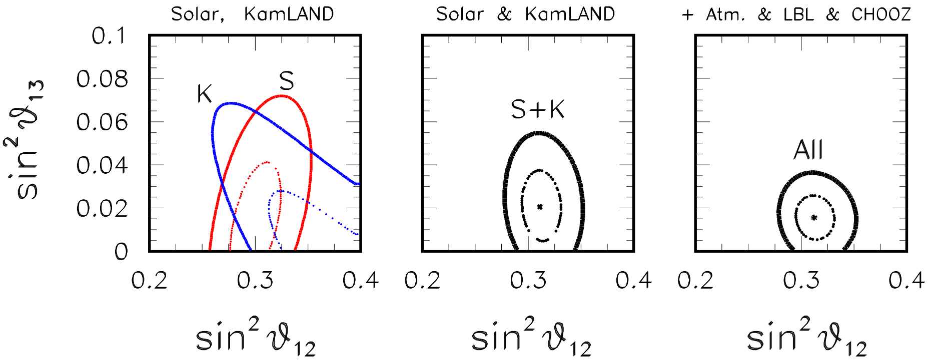
[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2 \vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]

future: measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

Hint of $\vartheta_{13} > 0$

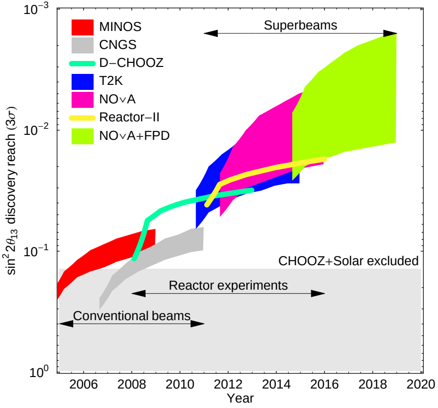
[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

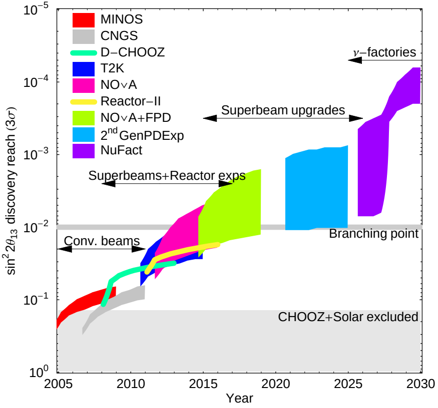


$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]

$$P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \simeq \begin{cases} \left(1-\sin^2\vartheta_{13}\right)^2 \left(1-0.5\sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1-\sin^2\vartheta_{13}\right)^2 \sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

The Hunt for ϑ_{13}





3σ sensitivities. Bands reflect dependence of sensitivity on the CP violating phase δ_{13}.

"Branching point" refers to the decision between an upgraded superbeam and/or detector and a neutrino factory program. Neutrino factory is assumed to switch polarity after 2.5 years.

[Physics at a Fermilab Proton Driver, Albrow et al, hep-ex/0509019]

Bilarge Mixing

$$\boxed{|U_{e3}|^2 \ll 1}$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Longrightarrow \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \qquad = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \Longrightarrow \vartheta_A \simeq \frac{\pi}{4} \Longrightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\text{Solar } \nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{CC}^{SNO}}{\Phi_{\nu_e}^{SSM}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6\,\text{MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \Longrightarrow U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

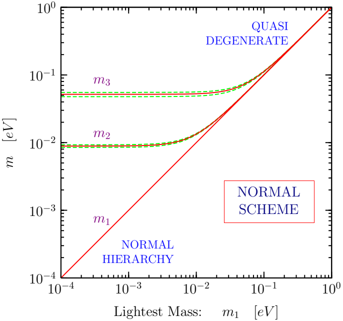
[Harrison, Perkins, Scott, hep-ph/0202074]

Absolute Scale of Neutrino Masses

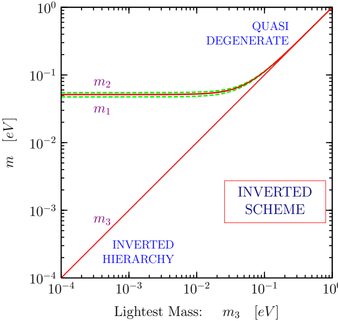
- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
  - Mass Hierarchy or Degeneracy?
  - Tritium Beta-Decay
  - Neutrinoless Double-Beta Decay
  - Bounds from Neutrino Oscillations
  - $\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass
  - Cosmological Bound on Neutrino Masses
- Experimental Neutrino Anomalies

Mass Hierarchy or Degeneracy?

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

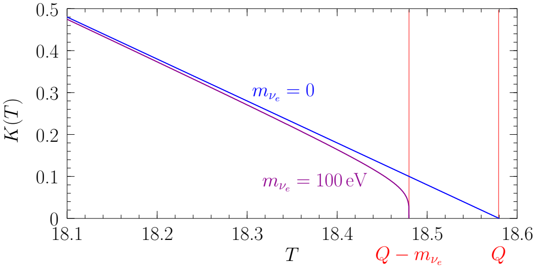
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$

Mainz & Troitsk

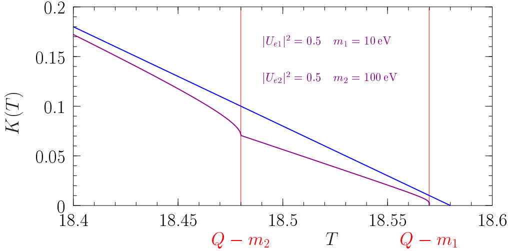
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

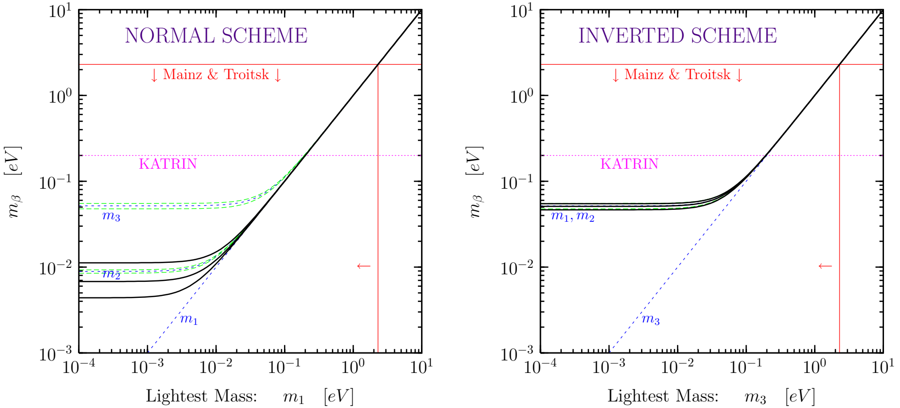
if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

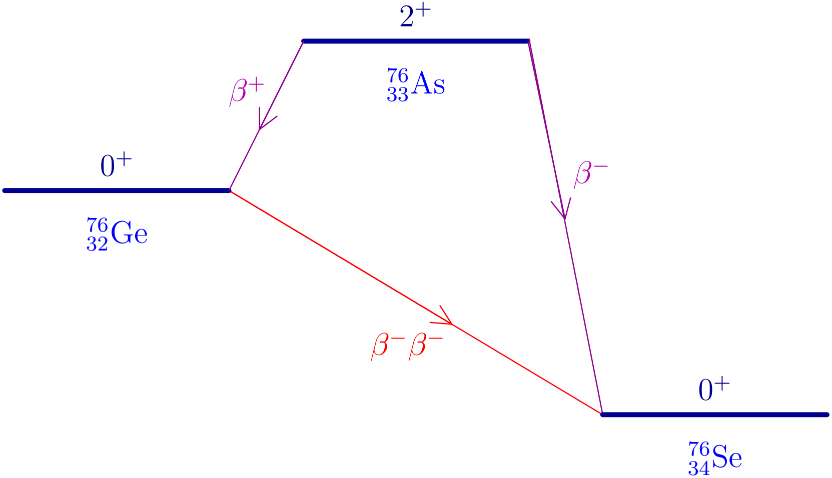
$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



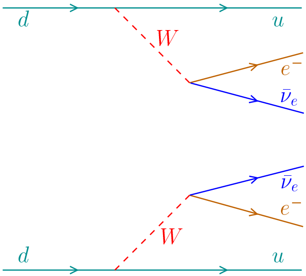
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_{k} U_{ek}^{2}\, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

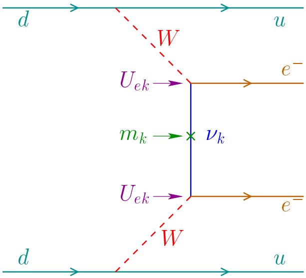


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$

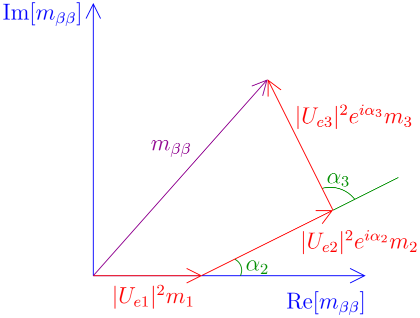


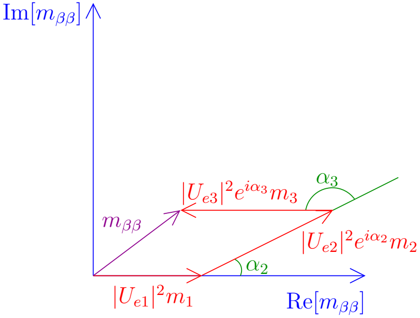
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$





Experimental Bounds

CUORICINO (^{130}Te) [PRC 78 (2008) 035502]

$$\boxed{T_{1/2}^{0\nu} > 3 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.19 - 0.68\,\text{eV}}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T_{1/2}^{0\nu} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T_{1/2}^{0\nu} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T_{1/2}^{0\nu} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

$$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$$

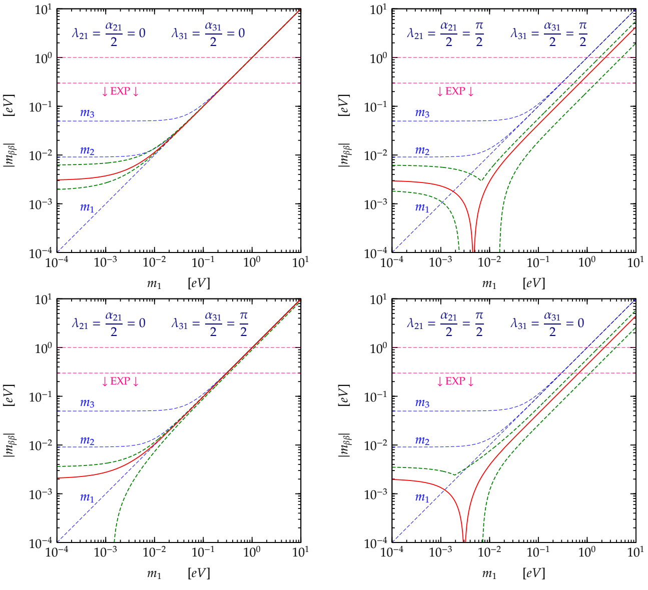
Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$

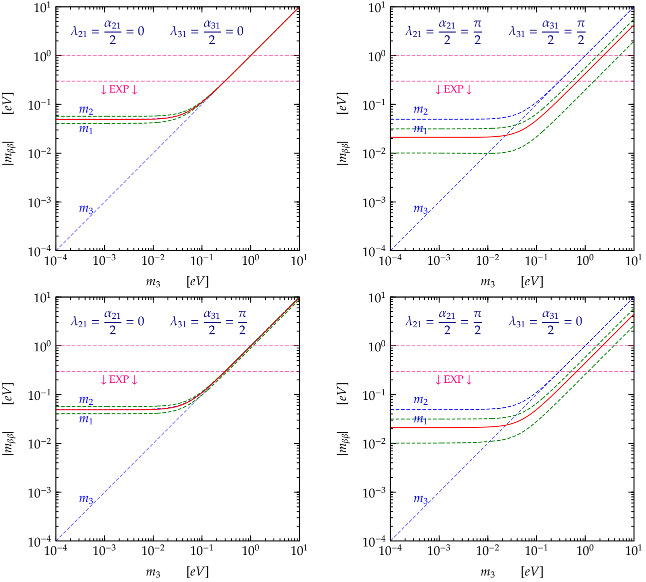
CP conservation

$$\alpha_{21} = 0 \, , \, \pi \qquad \alpha_{31} = 0 \, , \, \pi$$

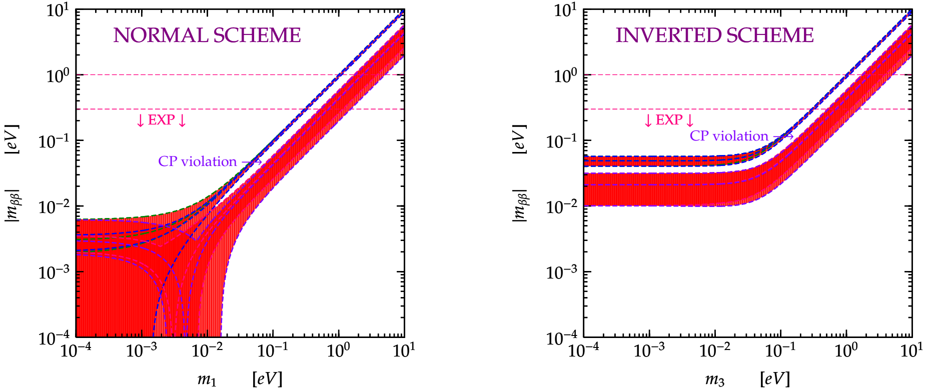
CP Conservation: Normal Scheme



CP Conservation: Inverted Scheme



$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3$$

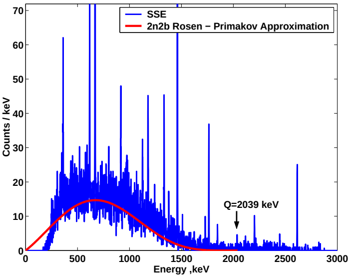


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

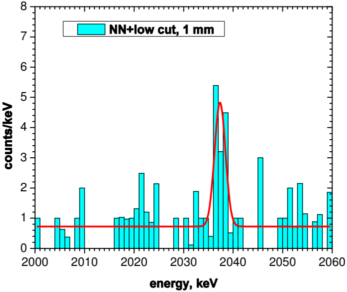
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

$$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\,\mathrm{y}$$ 6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



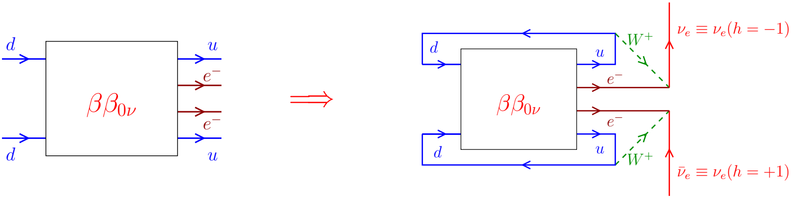
[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

$$|m_{\beta\beta}| = 0.32 \pm 0.03\,\mathrm{eV}$$ [MPLA 21 (2006) 1547]

if confirmed, very exciting (Majorana ν and large mass scale)

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

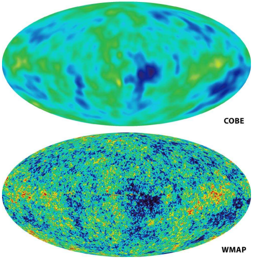


[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

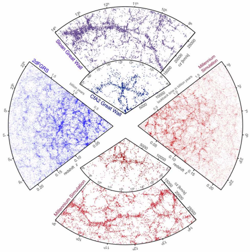
Majorana Mass Term

$$\boxed{\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)}$$

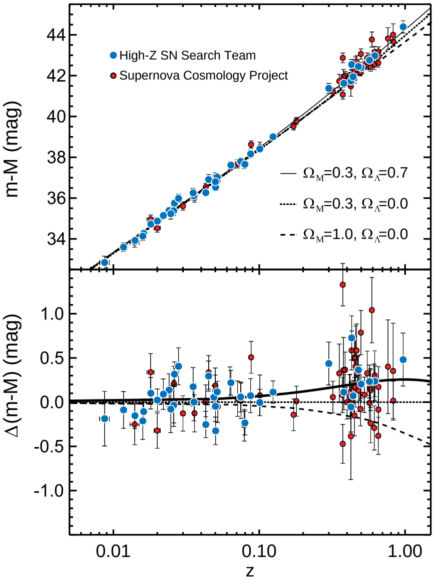
Cosmological Bound on Neutrino Masses



[WMAP, http://map.gsfc.nasa.gov]

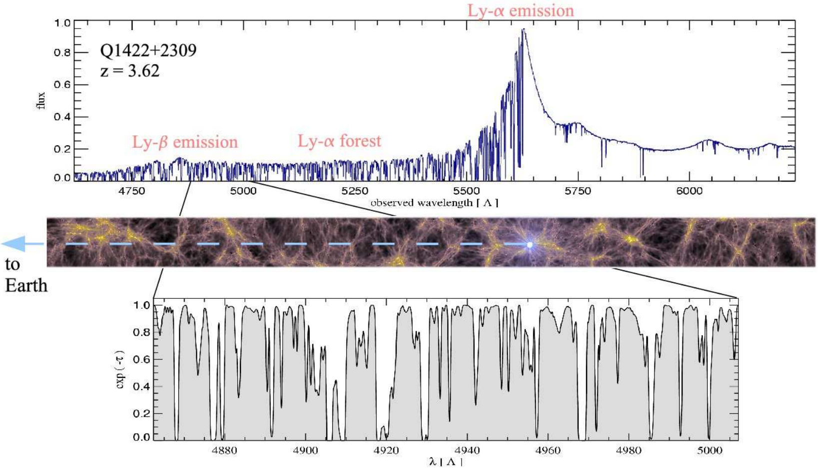


[Springel, Frenk, White, Nature 440 (2006) 1137]



[http://cfa-www.harvard.edu/supernova/]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67$ Å, $\lambda_\beta^0 = 1025.72$ Å, $\lambda_\gamma^0 = 972.54$ Å

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim \overbrace{T^2/M_P \sim \sqrt{G_N T^4}}^{\text{weak interactions freeze out}} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

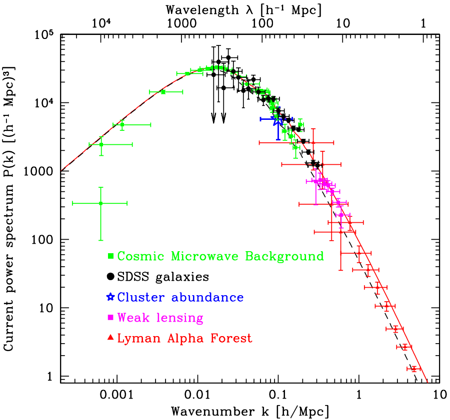
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\, T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$ $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\, h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \qquad \Longrightarrow \qquad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\mathrm{M}}^{0} = 0.28\,,\ h = 0.72\,,\ \Omega_{\mathrm{B}}^{0}/\Omega_{\mathrm{M}}^{0} = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\mathrm{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3 k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\mathrm{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\,\sqrt{\Omega_m}\, h\,\mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

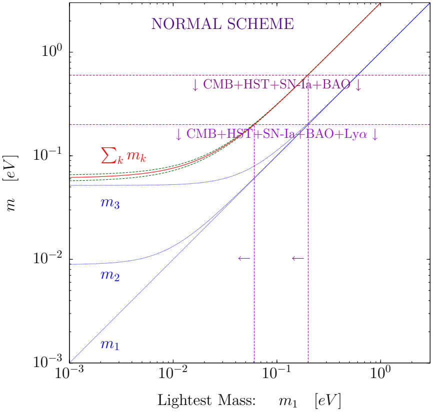
Flat ΛCDM

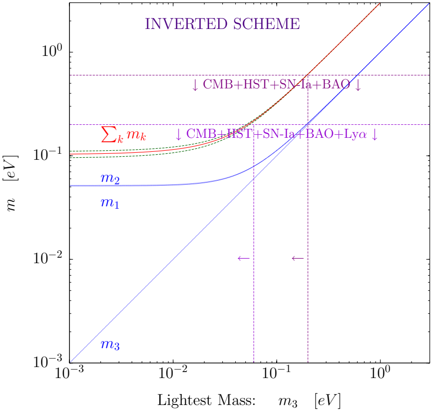
Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ.

$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO}$$

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$

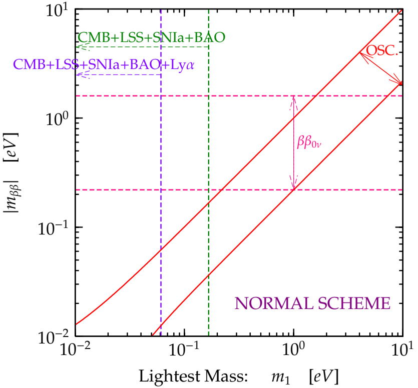


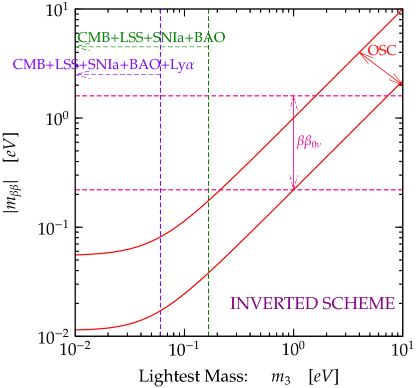


FUTURE: IF $\displaystyle\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ ($\sim 3\sigma$ range)

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]





tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

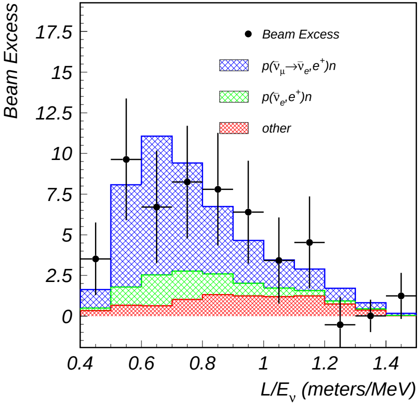
Experimental Neutrino Anomalies

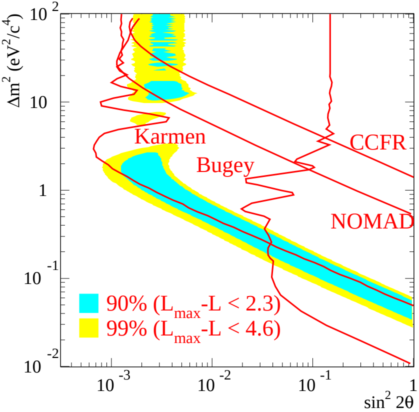
- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies
  - LSND
  - Four-Neutrino Schemes: 2+2 and 3+1
    - 2+2 Four-Neutrino Schemes
    - 3+1 Four-Neutrino Schemes
  - MiniBooNE
  - CCFR
  - MINOS
  - Gallium Anomaly

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

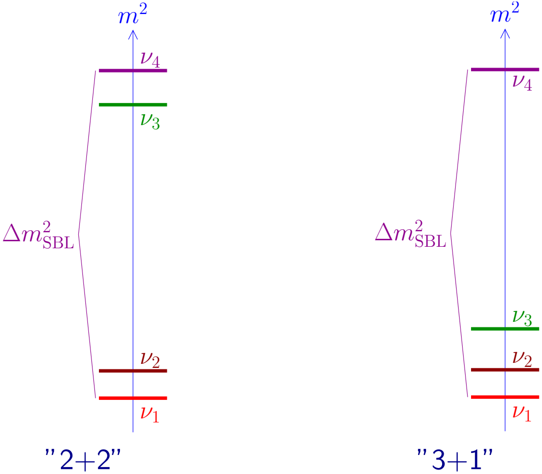
$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad L \simeq 30\,\mathrm{m}$ $\qquad 20\,\mathrm{MeV} \leq E \leq 200\,\mathrm{MeV}$



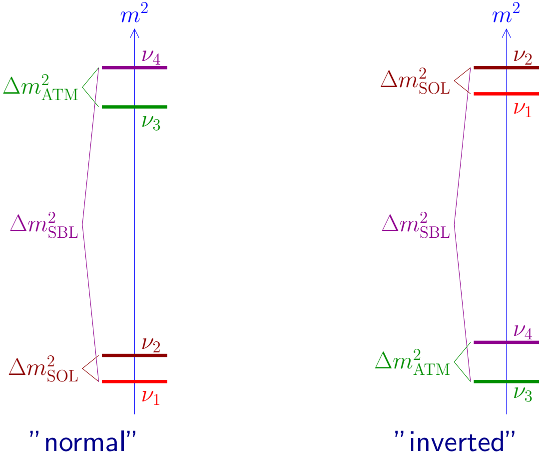


$$\Delta m^2_{\mathrm{LSND}} \gtrsim 0.2\,\mathrm{eV}^2 \qquad (\gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}})$$

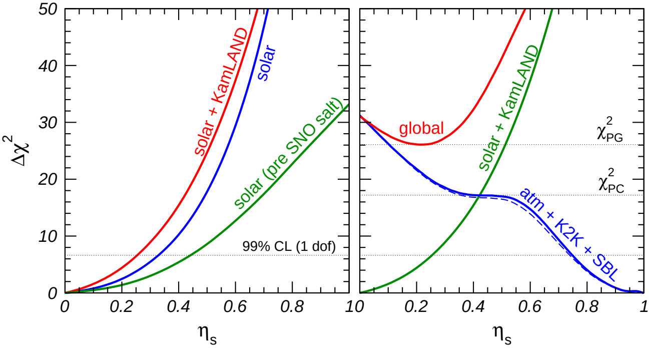
Four-Neutrino Schemes: 2+2 and 3+1



2+2 Four-Neutrino Schemes



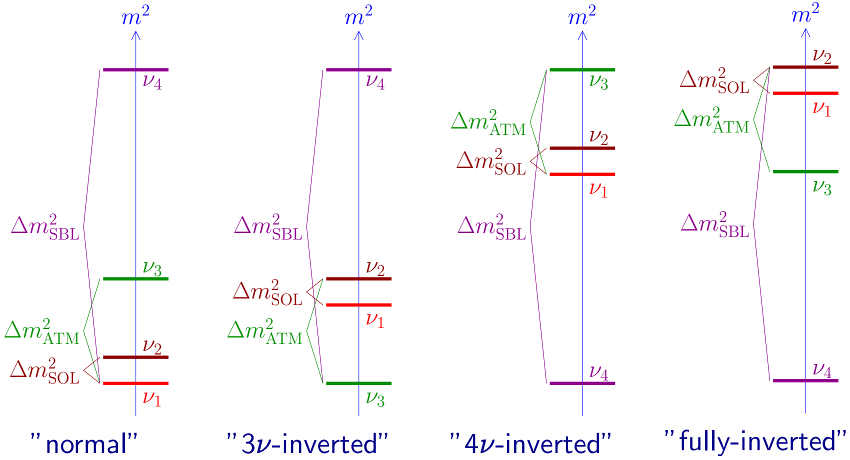
2+2 Schemes are strongly disfavored by solar and atmospheric data



[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 \qquad 99\%\ \text{CL:} \begin{cases} \eta_s < 0.25 & \text{(solar + KamLAND)} \\ \eta_s > 0.75 & \text{(atmospheric + K2K)} \end{cases}$$

3+1 Four-Neutrino Schemes



Perturbation of 3-ν Mixing

$$|U_{e4}|^2 \ll 1 \qquad |U_{\mu 4}|^2 \ll 1 \qquad |U_{\tau 4}|^2 \ll 1 \qquad |U_{s4}|^2 \simeq 1$$

Effective SBL Oscillation Probability in 3+1 Schemes

$$P_{\nu_\alpha\to\nu_\beta} = \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t}\right|^2 * \left|e^{iE_1 t}\right|^2$$

$$= \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k-E_1)t}\right|^2 \to \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(\frac{\Delta m_{k1}^2 L}{2E}\right)\right|^2$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \qquad \Delta m_{41}^2 \to \Delta m^2$$

$$P_{\nu_\alpha\to\nu_\beta} = \left|U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(\frac{\Delta m^2 L}{2E}\right)\right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$P_{\nu_\alpha\to\nu_\beta} = \left|\delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}\left[1 - \exp\left(\frac{\Delta m^2 L}{2E}\right)\right]\right|^2$$

$$= \delta_{\alpha\beta} + |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E}\right)$$

$$- 2\delta_{\alpha\beta}|U_{\alpha 4}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 2|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\left(1 - \cos\frac{\Delta m^2 L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\sin^2\frac{\Delta m^2 L}{2E}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 4|U_{\alpha 4}|^2\left(1 - |U_{\alpha 4}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

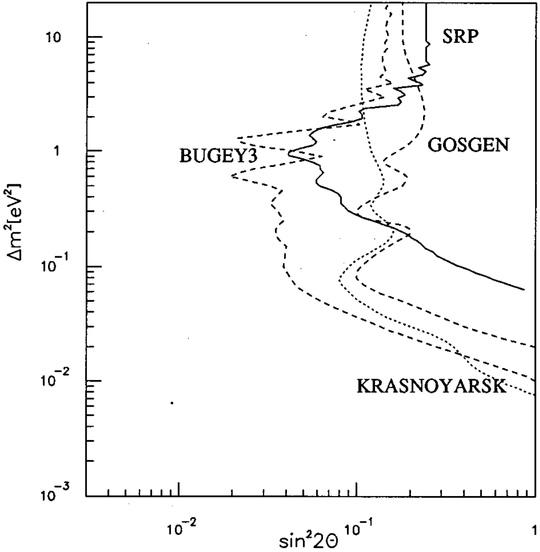
$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

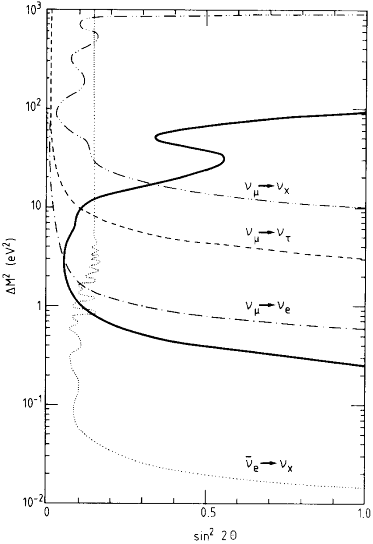
- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ imply strong limit on $\sin^2 2\vartheta_{\mu e}$

ν_e Disappearance

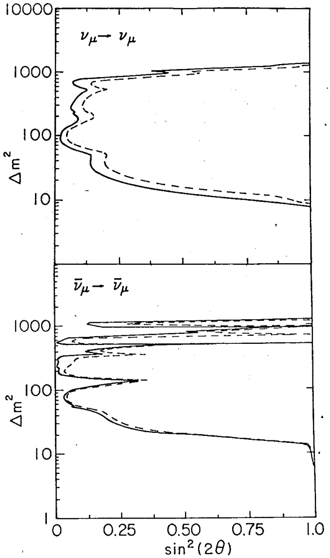


[Savannah River (SRP), PRD 53 (1996) 6054]

ν_μ Disappearance

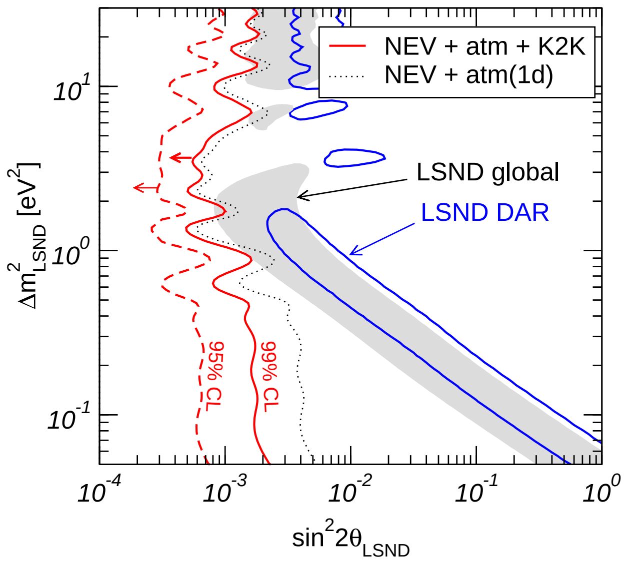


[CDHSW, PLB 134 (1984) 281]



[CCFR, Z. Phys. C 27 (1985) 53]

$\nu_\mu \to \nu_e$

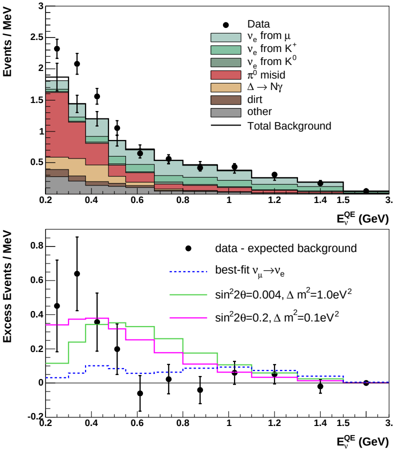


[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

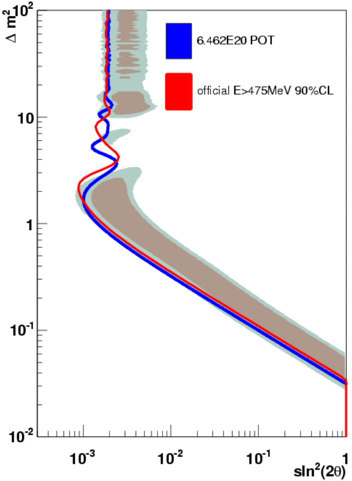
MiniBooNE

[PRL 98 (2007) 231801]

$\nu_\mu \to \nu_e \qquad L \simeq 541\,\text{m} \qquad 475\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



[PRL 102 (2009) 101802, arXiv:0812.2243]



[arXiv:0901.1648]

Low-Energy Anomaly!

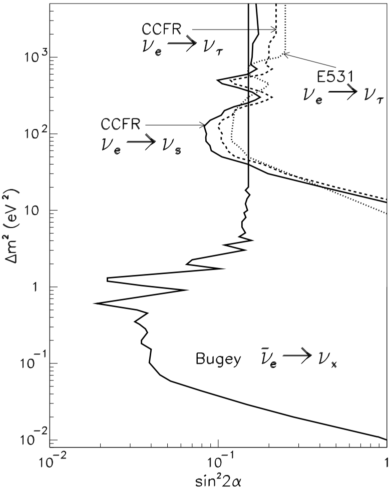
- The LSND signal is strongly disfavored:
  - Not seen by other $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ experiments
  - Disfavored by combined fit of data

- Possibility of a $\Delta m^2 \gtrsim 1\,\text{eV}^2$ relevant for SBL experiments independent of LSND signal remains interesting: chance to discover Sterile Neutrinos and open powerful window on New Physics

- There are also direct searches of active-sterile transitions:
  - Solar + KamLAND: mixing smaller than 0.25 at 99% CL (constrained by matter effects and by SNO NC measurement)
  - Atmospheric + K2K: mixing smaller than 0.25 at 99% CL (constrained by matter effects)
  - Bounds from observation of NC interactions in SBL (CCFR) and LBL (MINOS) experiments

CCFR

[PRD 59 (1999) 031101, arXiv:hep-ex/9809023]



$E \sim 100\,\mathrm{GeV}$ $\qquad$ $L \simeq 1.4\,\mathrm{km}$

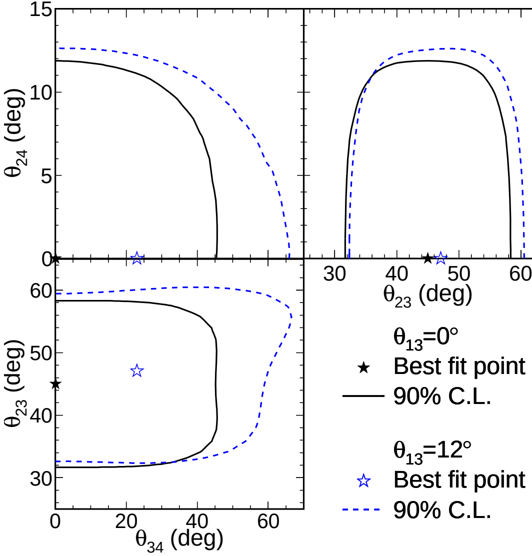
MINOS

[PRD 81 (2010) 052004, arXiv:1001.0336]

- LBL ν_μ disappearance and $\nu_\mu \to \nu_e$ experiment with $E \sim 3\,\text{GeV}$ and
  - Near Detector at 1.04 km
  - Far Detector at 734 km
- Events classified in two groups: CC and NC
- Information on $\nu_\mu \to \nu_s$ from difference between near and far NC energy spectrum
- Analysis complicated because there are five contributions to NC sample:
  1. Genuine NC interactions
  2. Misidentified ν_μ CC interactions
  3. ν_τ CC interactions
  4. Possible ν_e CC interactions originating from $\nu_\mu \to \nu_e$ oscillations
  5. CC interactions of ν_e beam component
- Assumed 4-ν Mixing with Mixing Matrix

$$U = R_{34}(\theta_{34}) R_{24}(\theta_{24}, \delta_2) R_{14}(\theta_{14}) R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta_1) R_{12}(\theta_{12}, \delta_3)$$

3+1 Schemes



[MINOS, PRD 81 (2010) 052004, arXiv:1001.0336]

Gallium Anomaly

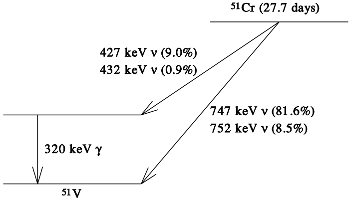
Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

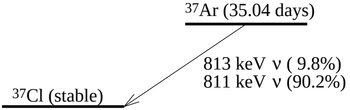
Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

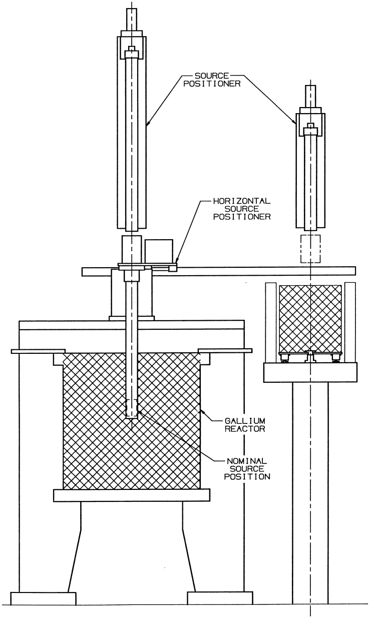
	^{51}Cr				^{37}Ar	
E [keV]	747	752	427	432	811	813
B.R.	0.8163	0.0849	0.0895	0.0093	0.902	0.098



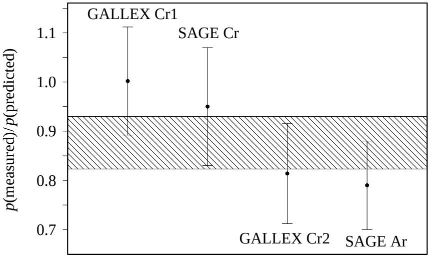
[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]



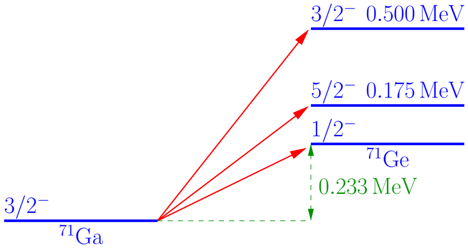
[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

$$\boxed{R_{\mathrm{Ga}} = 0.88 \pm 0.05}$$

- Deficit could be partly due to overestimate of
$$\sigma(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-)$$
- Calculation: Bahcall, PRC 56 (1997) 3391, hep-ph/9710491



- $\sigma_{\text{G.S.}}$ related to measured $\sigma(e^- + {}^{71}\text{Ge} \to {}^{71}\text{Ga} + \nu_e)$:
$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = 55.3 \times 10^{-46}\,\text{cm}^2\,(1 \pm 0.004)_{3\sigma}$$
- $\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr})\left(1 + 0.669\,\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} + 0.220\,\frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}}\right)$
- Contribution of Excited States only 5%!

- Bahcall: [Bahcall, PRC 56 (1997) 3391, hep-ph/9710491]

  from $p + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + n$ measurements [Krofcheck et al., PRL 55 (1985) 1051]

$$\frac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} < 0.056 \Rightarrow \frac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} = \frac{0.056}{2} \qquad \frac{\mathrm{BGT}_{500\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} = 0.146$$

3σ lower limit: $\dfrac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} = \dfrac{\mathrm{BGT}_{500\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} = 0$

3σ upper limit: $\dfrac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} < 0.056 \times 2 \qquad \dfrac{\mathrm{BGT}_{500\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} = 0.146 \times 2$

$$\sigma({}^{51}\mathrm{Cr}) = 58.1 \times 10^{-46}\,\mathrm{cm}^2 \left(1^{+0.036}_{-0.028}\right)_{1\sigma}$$

- Haxton: [Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

  "a sophisticated shell model calculation is performed ... for the transition to the first excited state in ${}^{71}\mathrm{Ge}$. The calculation predicts destructive interference between the (p, n) spin and spin-tensor matrix elements."

$$\sigma({}^{51}\mathrm{Cr}) = 63.9 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.106)_{1\sigma}$$

Gallium Radioactive Source Experiments
are
Short-BaseLine Neutrino Oscillation Experiments

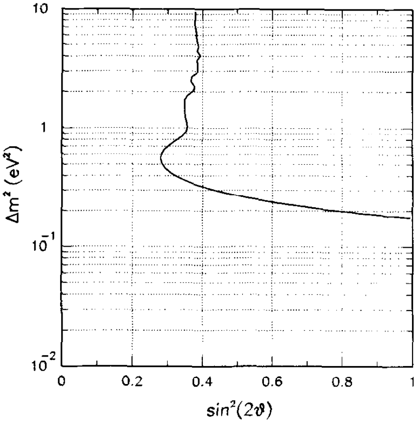


Fig. 1. Region of electron neutrino oscillation parameters ruled out at 90% C.L. by the GALLEX ^{51}Cr source experiment.

[Bahcall, Krastev, Lisi, PLB 348 (1995) 121]

	GALLEX		SAGE	
	Cr1	Cr2	Cr	Ar
R	0.953 ± 0.11	$0.812^{+0.10}_{-0.11}$	0.95 ± 0.12	$0.79 \pm {}^{+0.09}_{-0.10}$
$\langle L \rangle$	1.9 m		0.6 m	

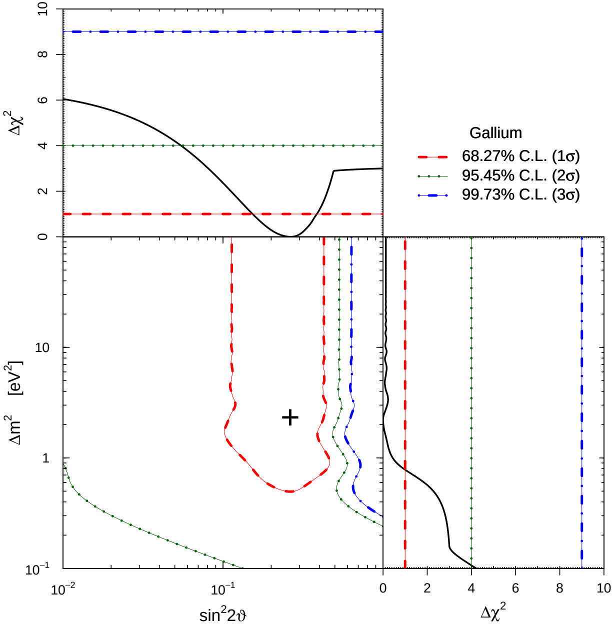
$$\boxed{R_{\text{Ga}} = 0.87 \pm 0.05}$$

$$P_{\nu_e \to \nu_e}(L, E) = 1 - \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$L_{\text{osc}} \lesssim 0.5 \quad \text{m} \quad \Longrightarrow \quad \Delta m^2 \gtrsim 1\,\text{eV}^2 \quad \Longrightarrow \quad \nu_e \to \nu_s$$

$$R = \frac{\int \text{d}V\, L^{-2} \sum_i (\text{B.R.})_i\, \sigma_i\, P_{\nu_e \to \nu_e}(L, E_i)}{\sum_i (\text{B.R.})_i\, \sigma_i \int \text{d}V\, L^{-2}}$$

[Acero, Giunti, Laveder, PRD 78 (2008) 073009, arXiv:0711.4222]



No Osc.

$\chi^2_{\text{min}} = 8.3$

$\text{NdF} = 2$

$\text{GoF} = 8.1\%$

Osc.

$\chi^2_{\text{min}} = 1.8$

$\text{NdF} = 2$

$\text{GoF} = 40\%$

$\sin^2 2\vartheta = 0.26$

$\Delta m^2 = 2.20\,\text{eV}^2$

[Acero, Giunti, Laveder, PRD 78 (2008) 073009, arXiv:0711.4222]

Future Promising Searches of SBL Oscillations

- SAGE is planning a new source experiment (ν_e disappearance)
- Beta-Beam experiments:

$$N(A,Z) \to N(A,Z+1) + e^- + \bar{\nu}_e \qquad (\beta^-)$$

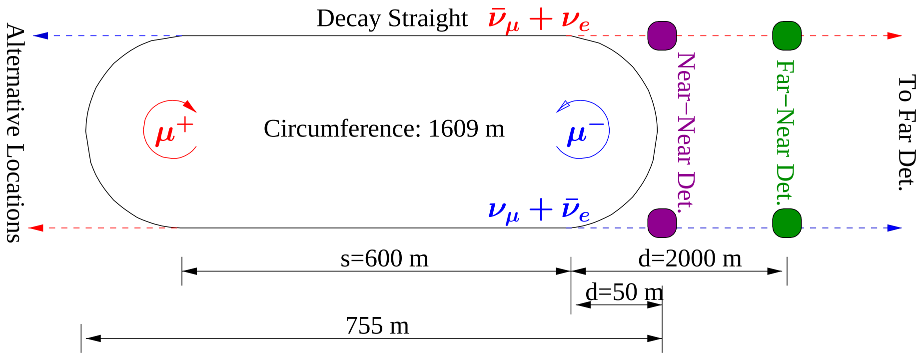
$$N(A,Z) \to N(A,Z-1) + e^+ + \nu_e \qquad (\beta^+)$$

- Neutrino Factory experiments:

$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

Neutrino Factory

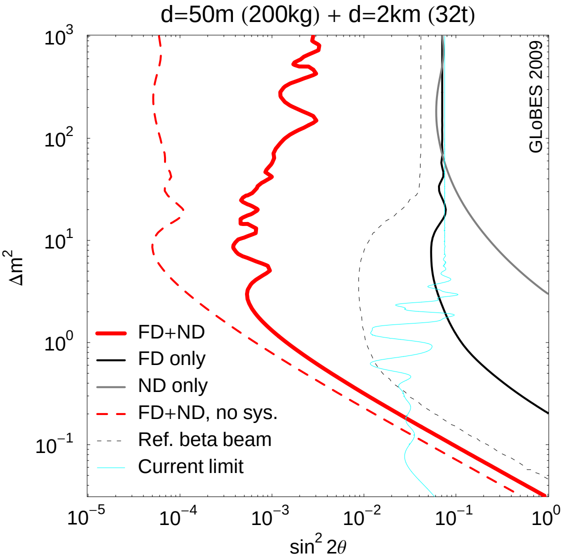


[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

Near Detectors: Scintillator or Iron Calorimeter with perfect flavor identification

Systematic Uncertainties: Cross Section, Detector Normalization, Energy Resolution and Calibration, Backgrounds

ν_e Disappearance



[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

Conclusions

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies
- Conclusions
  - Conclusions - Three-Neutrino Mixing
  - Conclusions - Anomalies

Conclusions - Three-Neutrino Mixing

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (SOL, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (ATM, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why only $|U_{e3}|^2 \ll 1$?

Explain experimental neutrino anomalies (sterile ν's?).

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy.

Check experimental neutrino anomalies.

Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale.

Improve β & $\beta\beta_{0\nu}$ Decay and Cosmology measurements.

Conclusions - Anomalies

- Existence of sterile neutrinos is possible
- Likely connected with neutrino mass generation
- Active-Sterile transitions have been searched in several experiments and discussed in global phenomenological analyses of data
- LSND indication of 4-Neutrino Mixing is disfavored
- Gallium Anomaly may be due to $\nu_e \to \nu_s$ oscillations with $\sin^2 2\vartheta \gtrsim 0.1$ and $\Delta m^2 \gtrsim 1\,\text{eV}^2$
- SBL oscillations can be explored with high precision in
  - Beta-Beam experiments (pure ν_e or $\bar{\nu}_e$ beam from nuclear decay)
  - Neutrino Factory experiments (ν_e and $\bar{\nu}_\mu$ from μ^+ decay, or $\bar{\nu}_e$ and ν_μ from μ^- decay)