

# Neutrino Physics

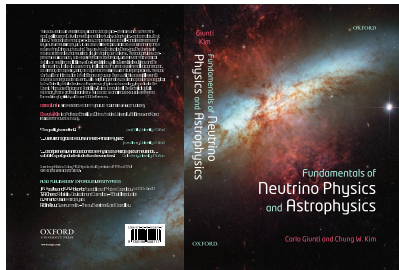
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Neutrino Unbound: <http://www.nu.to.infn.it>

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C. Giunti and C.W. Kim  
Fundamentals of Neutrino Physics  
and Astrophysics  
Oxford University Press  
15 March 2007 – 728 pages

C. Giunti – Neutrino Physics – May 2011 – 1

## Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Number of Flavor and Massive Neutrinos?
- Sterile Neutrinos

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## Part II: Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

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## Part III: Phenomenology

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

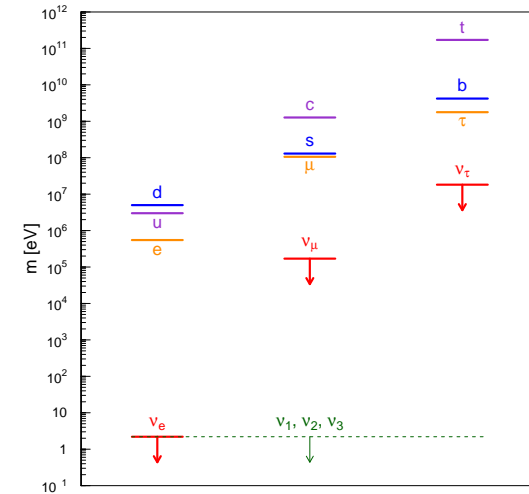
C. Giunti – Neutrino Physics – May 2011 – 4

## Part I

### Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
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- Number of Flavor and Massive Neutrinos?
- Sterile Neutrinos

### Fermion Mass Spectrum



### Dirac Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
  - Dirac Mass
  - Higgs Mechanism in SM
  - Dirac Lepton Masses
  - Three-Generations Dirac Neutrino Masses
  - Massive Chiral Lepton Fields
  - Massive Dirac Lepton Fields
  - Quantization
  - Mixing
  - Flavor Lepton Numbers
  - Total Lepton Number
  - Mixing Matrix
  - Standard Parameterization of Mixing Matrix
  - CP Violation
    - Example:  $\vartheta_{12} = 0$
    - Example:  $\vartheta_{13} = \pi/2$
    - Example:  $m_{\nu_2} = m_{\nu_3}$
- Jarlskog Rephasing Invariant

### Dirac Mass

- ▶ Dirac Equation:  $(i\partial - m)\nu(x) = 0$  ( $\partial \equiv \gamma^\mu \partial_\mu$ )
- ▶ Dirac Lagrangian:  $\mathcal{L}(x) = \bar{\nu}(x)(i\partial - m)\nu(x)$
- ▶ Chiral decomposition:  $\nu_L \equiv P_L \nu$ ,  $\nu_R \equiv P_R \nu$ ,  $\nu = \nu_L + \nu_R$ 

$$P_L \equiv \frac{1 - \gamma^5}{2}, \quad P_R \equiv \frac{1 + \gamma^5}{2}$$

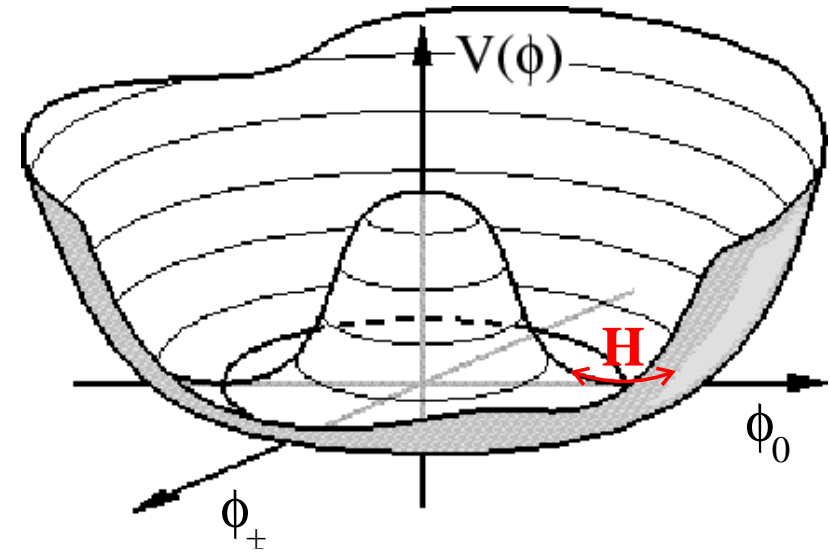
$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = P_R P_L = 0$$

$$\mathcal{L} = \bar{\nu}_L i \not{\partial} \nu_L + \bar{\nu}_R i \not{\partial} \nu_R - m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$
- ▶ In SM only  $\nu_L \Rightarrow$  no Dirac mass
- ▶ Oscillation experiments have shown that neutrinos are massive
- ▶ Simplest extension of the SM: add  $\nu_R$

## Higgs Mechanism in SM

- ▶ Higgs Doublet:  $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$   $|\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$
- ▶ Higgs Lagrangian:  $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$
- ▶ Higgs Potential:  $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$
- ▶  $\mu^2 < 0$  and  $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left( |\Phi|^2 - \frac{v^2}{2} \right)^2$ , with  $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$
- ▶ Vacuum:  $V_{\text{min}}$  for  $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- ▶ Spontaneous Symmetry Breaking:  $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$
- ▶ Unitary Gauge:  $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

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## Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \ell_R \quad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = -y^\ell \bar{\ell}_L \Phi \ell_R - y^\nu \bar{\ell}_L \tilde{\Phi} \nu_R + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{H,L} = & -\frac{y^\ell}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^\nu}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \text{H.c.} \end{aligned}$$

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$$\begin{aligned} \mathcal{L}_{H,L} = & -y^\ell \frac{v}{\sqrt{2}} \bar{\ell}_L \ell_R - y^\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R \\ & -\frac{y^\ell}{\sqrt{2}} \bar{\ell}_L \ell_R H - \frac{y^\nu}{\sqrt{2}} \bar{\nu}_L \nu_R H + \text{H.c.} \end{aligned}$$

$$m_\ell = y^\ell \frac{v}{\sqrt{2}} \quad m_\nu = y^\nu \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^\ell}{\sqrt{2}} = \frac{m_\ell}{v} \quad g_{\nu H} = \frac{y^\nu}{\sqrt{2}} = \frac{m_\nu}{v}$$

$$v = (\sqrt{2} G_F)^{1/2} = 246 \text{ GeV}$$

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## Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
$\nu'_{eR}$	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = - \sum_{\alpha, \beta = e, \mu, \tau} \left[ Y_{\alpha\beta}^{\ell} \overline{L'_{\alpha L}} \Phi \ell'_{\beta R} + Y_{\alpha\beta}^{\nu} \overline{L'_{\alpha L}} \tilde{\Phi} \nu'_{\beta R} \right] + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell'_L} Y^{\ell} \ell'_R + \overline{\nu'_L} Y^{\nu} \nu'_R \right] + \text{H.c.}$$

Diagonalization of  $Y^{\ell}$  and  $Y^{\nu}$  with unitary  $V_L^{\ell}, V_R^{\ell}, V_L^{\nu}, V_R^{\nu}$

$$\ell'_L = V_L^{\ell} \ell_L \quad \ell'_R = V_R^{\ell} \ell_R \quad \nu'_L = V_L^{\nu} \nu_L \quad \nu'_R = V_R^{\nu} \nu_R$$

Unitary transformations are allowed

because they leave invariant the kinetic terms in the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \overline{\ell'_L} i \not{\partial} \ell'_L + \overline{\ell'_R} i \not{\partial} \ell'_R + \overline{\nu'_L} i \not{\partial} \nu'_L + \overline{\nu'_R} i \not{\partial} \nu'_R \\ &= \overline{\ell}_L V_L^{\ell \dagger} i \not{\partial} V_L^{\ell} \ell_L + \dots \\ &= \overline{\ell}_L i \not{\partial} \ell_L + \overline{\ell}_R i \not{\partial} \ell_R + \overline{\nu}_L i \not{\partial} \nu_L + \overline{\nu}_R i \not{\partial} \nu_R \end{aligned}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \sum_{\alpha, \beta = e, \mu, \tau} \left[ Y_{\alpha\beta}^{\ell} \overline{\ell'_{\alpha L}} \ell'_{\beta R} + Y_{\alpha\beta}^{\nu} \overline{\nu'_{\alpha L}} \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell'_L} Y^{\ell} \ell'_R + \overline{\nu'_L} Y^{\nu} \nu'_R \right] + \text{H.c.}$$

$$\ell'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad \ell'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y^{\ell} \equiv \begin{pmatrix} Y_{ee}^{\ell} & Y_{e\mu}^{\ell} & Y_{e\tau}^{\ell} \\ Y_{\mu e}^{\ell} & Y_{\mu\mu}^{\ell} & Y_{\mu\tau}^{\ell} \\ Y_{\tau e}^{\ell} & Y_{\tau\mu}^{\ell} & Y_{\tau\tau}^{\ell} \end{pmatrix} \quad Y^{\nu} \equiv \begin{pmatrix} Y_{ee}^{\nu} & Y_{e\mu}^{\nu} & Y_{e\tau}^{\nu} \\ Y_{\mu e}^{\nu} & Y_{\mu\mu}^{\nu} & Y_{\mu\tau}^{\nu} \\ Y_{\tau e}^{\nu} & Y_{\tau\mu}^{\nu} & Y_{\tau\tau}^{\nu} \end{pmatrix}$$

$$M^{\ell} = \frac{v}{\sqrt{2}} Y^{\ell}$$

$$M^{\nu} = \frac{v}{\sqrt{2}} Y^{\nu}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell'_L} Y^{\ell} \ell'_R + \overline{\nu'_L} Y^{\nu} \nu'_R \right] + \text{H.c.}$$

$$\ell'_L = V_L^{\ell} \ell_L \quad \ell'_R = V_R^{\ell} \ell_R \quad \nu'_L = V_L^{\nu} \nu_L \quad \nu'_R = V_R^{\nu} \nu_R$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell}_L V_L^{\ell \dagger} Y^{\ell} V_R^{\ell} \ell_R + \overline{\nu}_L V_L^{\nu \dagger} Y^{\nu} V_R^{\nu} \nu_R \right] + \text{H.c.}$$

$$V_L^{\ell \dagger} Y^{\ell} V_R^{\ell} = Y^{\ell} \quad Y_{\alpha\beta}^{\ell} = y_{\alpha}^{\ell} \delta_{\alpha\beta} \quad (\alpha, \beta = e, \mu, \tau)$$

$$V_L^{\nu \dagger} Y^{\nu} V_R^{\nu} = Y^{\nu} \quad Y_{kj}^{\nu} = y_k^{\nu} \delta_{kj} \quad (k, j = 1, 2, 3)$$

Real and Positive  $y_{\alpha}^{\ell}, y_k^{\nu}$

$$V_L^{\dagger} Y^{\ell} V_R = Y^{\ell} \iff Y^{\ell} = V_L Y^{\ell} V_R^{\dagger}$$

## Massive Chiral Lepton Fields

$l_L = V_L^{\ell\dagger} l'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$l_R = V_R^{\ell\dagger} l'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger} \nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger} \nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{H,L} &= - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \bar{\ell}_L Y^\ell l_R + \bar{\mathbf{n}}_L Y^\nu \mathbf{n}_R \right] + \text{H.c.} \\ &= - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \sum_{\alpha=e,\mu,\tau} y_\alpha^\ell \bar{\ell}_{\alpha L} l_{\alpha R} + \sum_{k=1}^3 y_k^\nu \bar{\nu}_{kL} \nu_{kR} \right] + \text{H.c.} \end{aligned}$$

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## Quantization

$$\nu_k(x) = \int \frac{d^3p}{(2\pi)^3 2E_k} \sum_{h=\pm 1} \left[ a_k^{(h)}(p) u_k^{(h)}(p) e^{-ip \cdot x} + b_k^{(h)\dagger}(p) v_k^{(h)}(p) e^{ip \cdot x} \right]$$

$$p^0 = E_k = \sqrt{\vec{p}^2 + m_k^2} \quad \begin{aligned} (\not{p} - m_k) u_k^{(h)}(p) &= 0 \\ (\not{p} + m_k) v_k^{(h)}(p) &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} u_k^{(h)}(p) &= h u_k^{(h)}(p) \\ \frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} v_k^{(h)}(p) &= -h v_k^{(h)}(p) \end{aligned}$$

$$\begin{aligned} \{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} &= \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = (2\pi)^3 2E_k \delta^3(\vec{p} - \vec{p}') \delta_{hh'} \\ \{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} &= \{a_k^{(h)\dagger}(p), a_k^{(h')}(p')\} = 0 \\ \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} &= \{b_k^{(h)\dagger}(p), b_k^{(h')}(p')\} = 0 \\ \{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} &= \{a_k^{(h)\dagger}(p), b_k^{(h')}(p')\} = 0 \\ \{a_k^{(h)}(p), b_k^{(h')}(p')\} &= \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0 \end{aligned}$$

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## Massive Dirac Lepton Fields

$$\begin{aligned} l_\alpha &\equiv l_{\alpha L} + l_{\alpha R} \quad (\alpha = e, \mu, \tau) \\ \nu_k &= \nu_{kL} + \nu_{kR} \quad (k = 1, 2, 3) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{H,L} &= - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell v}{\sqrt{2}} \bar{\ell}_\alpha l_\alpha - \sum_{k=1}^3 \frac{y_k^\nu v}{\sqrt{2}} \bar{\nu}_k \nu_k \quad \text{Mass Terms} \\ &- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell}{\sqrt{2}} \bar{\ell}_\alpha l_\alpha H - \sum_{k=1}^3 \frac{y_k^\nu}{\sqrt{2}} \bar{\nu}_k \nu_k H \quad \text{Lepton-Higgs Couplings} \end{aligned}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^\ell v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \quad m_k = \frac{y_k^\nu v}{\sqrt{2}} \quad (k = 1, 2, 3)$$

Lepton-Higgs coupling  $\propto$  Lepton Mass

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## Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathcal{L}_1^{(CC)} = - \frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

$$\text{Weak Charged Current:} \quad j_W^\rho = j_{W,L}^\rho + j_{W,Q}^\rho$$

Leptonic Weak Charged Current

$$j_{W,L}^\rho = \sum_{\alpha=e,\mu,\tau} \bar{\nu}'_\alpha \gamma^\rho (1 - \gamma^5) \ell'_\alpha = 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}'_{\alpha L} \gamma^\rho \ell'_{\alpha L} = 2 \bar{\nu}'_L \gamma^\rho \ell'_L$$

$$\underline{\ell'_L = V_L^\ell \ell_L} \quad \underline{\nu'_L = V_L^\nu \mathbf{n}_L}$$

$$j_{W,L}^\rho = 2 \bar{\mathbf{n}}_L V_L^{\nu\dagger} \gamma^\rho V_L^\ell \ell_L = 2 \bar{\mathbf{n}}_L V_L^{\nu\dagger} V_L^\ell \gamma^\rho \ell_L = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger} V_L^\ell \quad \boxed{U = V_L^{\ell\dagger} V_L^\nu}$$

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## Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining  
**Flavor Lepton Numbers**  
as in the SM

	$L_e$	$L_\mu$	$L_\tau$		$L_e$	$L_\mu$	$L_\tau$
$(\nu_e, e^-)$	+1	0	0	$(\nu_e^c, e^+)$	-1	0	0
$(\nu_\mu, \mu^-)$	0	+1	0	$(\nu_\mu^c, \mu^+)$	0	-1	0
$(\nu_\tau, \tau^-)$	0	0	+1	$(\nu_\tau^c, \tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model: Lepton numbers are conserved

- ▶ Definition: Left-Handed Flavor Neutrino Fields

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- ▶ They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j_{W,L}^\rho = 2 \overline{\nu}_L \gamma^\rho \ell_L = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\rho \ell_{\alpha L}$$

- ▶ Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j_{W,L}^\rho = 2 (\overline{\nu}_{eL} \gamma^\rho e_L + \overline{\nu}_{\mu L} \gamma^\rho \mu_L + \overline{\nu}_{\tau L} \gamma^\rho \tau_L)$$

- ▶ In practice left-handed flavor neutrino fields are useful for calculations in the SM approximation of massless neutrinos (interactions).

- ▶ If neutrino masses must be taken into account, it is necessary to use

$$j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} U_{\alpha k}^* \overline{\nu}_{kL} \gamma^\rho \ell_{\alpha L}$$

$$\mathcal{L}_{\text{mass}}^D = - \begin{pmatrix} \overline{\nu}_{eL} & \overline{\nu}_{\mu L} & \overline{\nu}_{\tau L} \end{pmatrix} \begin{pmatrix} m_{ee}^D & m_{e\mu}^D & m_{e\tau}^D \\ m_{\mu e}^D & m_{\mu\mu}^D & m_{\mu\tau}^D \\ m_{\tau e}^D & m_{\tau\mu}^D & m_{\tau\tau}^D \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

$L_e, L_\mu, L_\tau$  are not conserved

$L$  is conserved:  $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

- ▶ Leptonic Weak Charged Current is invariant under the global U(1) gauge transformations

$$\ell_{\alpha L} \rightarrow e^{i\varphi_\alpha} \ell_{\alpha L} \quad \nu_{\alpha L} \rightarrow e^{i\varphi_\alpha} \nu_{\alpha L} \quad (\alpha = e, \mu, \tau)$$

- ▶ If neutrinos are massless (SM), Noether's theorem implies that there is, for each flavor, a conserved current:

$$j_\alpha^\rho = \overline{\nu}_{\alpha L} \gamma^\rho \nu_{\alpha L} + \overline{\ell}_\alpha \gamma^\rho \ell_\alpha \quad \partial_\rho j_\alpha^\rho = 0$$

and a conserved charge:

$$L_\alpha = \int d^3x j_\alpha^0(x) \quad \partial_0 L_\alpha = 0$$

$$\begin{aligned} :L_\alpha: &= \int \frac{d^3p}{(2\pi)^3 2E} \left[ a_{\nu_\alpha}^{(-)\dagger}(p) a_{\nu_\alpha}^{(-)}(p) - b_{\nu_\alpha}^{(+)\dagger}(p) b_{\nu_\alpha}^{(+)}(p) \right] \\ &+ \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{\ell_\alpha}^{(h)\dagger}(p) a_{\ell_\alpha}^{(h)}(p) - b_{\ell_\alpha}^{(h)\dagger}(p) b_{\ell_\alpha}^{(h)}(p) \right] \end{aligned}$$

## Total Lepton Number

- ▶ Lepton-Higgs Yukawa Lagrangian:

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \sum_{\alpha=e,\mu,\tau} y_{\alpha}^{\ell} \bar{l}_{\alpha L} l_{\alpha R} + \sum_{k=1}^3 y_k^{\nu} \bar{\nu}_{kL} \nu_{kR} \right] + \text{H.c.}$$

- ▶ Mixing:  $\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \iff \nu_{kL} = \sum_{\alpha=e,\mu,\tau} U_{\alpha k}^* \nu_{\alpha L}$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \sum_{\alpha=e,\mu,\tau} \left[ y_{\alpha}^{\ell} \bar{l}_{\alpha L} l_{\alpha R} + \bar{\nu}_{\alpha L} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \right] + \text{H.c.}$$

- ▶ Invariant for

$$l_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha L}, \quad \nu_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} \nu_{\alpha L}$$

$$l_{\alpha R} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha R}, \quad \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \rightarrow e^{i\varphi_{\alpha}} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$$

- ▶ But kinetic part of neutrino Lagrangian is not invariant

$$\mathcal{L}_{\text{kinetic}}^{(\nu)} = \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} i \not{\partial} \nu_{\alpha L} + \sum_{k=1}^3 \bar{\nu}_{kR} i \not{\partial} \nu_{kR}$$

because  $\sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$  is not a unitary combination of the  $\nu_{kR}$ 's

- ▶ Dirac neutrino masses violate conservation of Flavor Lepton Numbers
- ▶ Total Lepton Number is conserved, because Lagrangian is invariant under the global U(1) gauge transformations

$$\nu_{kL} \rightarrow e^{i\varphi} \nu_{kL}, \quad \nu_{kR} \rightarrow e^{i\varphi} \nu_{kR} \quad (k = 1, 2, 3)$$

$$l_{\alpha L} \rightarrow e^{i\varphi} l_{\alpha L}, \quad l_{\alpha R} \rightarrow e^{i\varphi} l_{\alpha R} \quad (\alpha = e, \mu, \tau)$$

- ▶ From Noether's theorem:

$$j^{\rho} = \sum_{k=1}^3 \bar{\nu}_k \gamma^{\rho} \nu_k + \sum_{\alpha=e,\mu,\tau} \bar{l}_{\alpha} \gamma^{\rho} l_{\alpha} \quad \partial_{\rho} j^{\rho} = 0$$

$$\text{Conserved charge: } L_{\alpha} = \int d^3x j_{\alpha}^0(x) \quad \partial_0 L_{\alpha} = 0$$

$$\begin{aligned} :L: &= \sum_{k=1}^3 \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{\nu_k}^{(h)\dagger}(p) a_{\nu_k}^{(h)}(p) - b_{\nu_k}^{(h)\dagger}(p) b_{\nu_k}^{(h)}(p) \right] \\ &+ \sum_{\alpha=e,\mu,\tau} \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{\ell_{\alpha}}^{(h)\dagger}(p) a_{\ell_{\alpha}}^{(h)}(p) - b_{\ell_{\alpha}}^{(h)\dagger}(p) b_{\ell_{\alpha}}^{(h)}(p) \right] \end{aligned}$$

## Mixing Matrix

- ▶ Leptonic Weak Charged Current:  $j_{W,L}^{\rho} = 2 \bar{\mathbf{n}}_L U^{\dagger} \gamma^{\rho} \ell_L$

$$U = V_L^{\ell\dagger} V_L^{\nu} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- ▶ Unitary  $N \times N$  matrix depends on  $N^2$  independent real parameters

$$N = 3 \implies \begin{aligned} \frac{N(N-1)}{2} &= 3 && \text{Mixing Angles} \\ \frac{N(N+1)}{2} &= 6 && \text{Phases} \end{aligned}$$

- ▶ Not all phases are physical observables
- ▶ Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- ▶ Weak Charged Current:  $j_{W,L}^{\rho} = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{kL} U_{\alpha k}^* \gamma^{\rho} l_{\alpha L}$

- ▶ Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)

$$\nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \quad l_{\alpha} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha} \quad (\alpha = e, \mu, \tau)$$

- ▶ Performing this transformation, the Charged Current becomes

$$j_{W,L}^{\rho} = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{kL} e^{-i\varphi_k} U_{\alpha k}^* e^{i\varphi_{\alpha}} \gamma^{\rho} l_{\alpha L}$$

$$j_{W,L}^{\rho} = 2 \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_1 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{kL} \underbrace{e^{-i(\varphi_k - \varphi_1)}}_2 U_{\alpha k}^* \underbrace{e^{i(\varphi_{\alpha} - \varphi_e)}}_2 \gamma^{\rho} l_{\alpha L}$$

- ▶ There are 5 arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix

- ▶ 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant  $\iff$  conservation of Total Lepton Number.

## Standard Parameterization of Mixing Matrix

- ▶ The mixing matrix contains 1 Physical Phase.
- ▶ It is convenient to express the  $3 \times 3$  unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13} \leq 2\pi$$

3 Mixing Angles  $\vartheta_{12}$ ,  $\vartheta_{23}$ ,  $\vartheta_{13}$  and 1 Phase  $\delta_{13}$

C. Giunti – Neutrino Physics – May 2011 – 29

C. Giunti – Neutrino Physics – May 2011 – 30

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} e^{i\delta_{23}} \\ 0 & -s_{23} e^{-i\delta_{13}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12} e^{-i\delta'_{12}} & 0 \\ -s'_{12} e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

C. Giunti – Neutrino Physics – May 2011 – 31

## CP Violation

- ▶  $U \neq U^* \implies$  CP Violation
- ▶ General conditions for CP violation (14 conditions):
  1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or  $\pi/2$  (6 conditions)
  3. The physical phase is different from 0 or  $\pi$  (2 conditions)
- ▶ These 14 conditions are combined into the single condition  $\det C \neq 0$

$$C = -i [M^{\nu} M^{\nu\dagger}, M^{\ell} M^{\ell\dagger}]$$

$$\det C = -2J (m_{\nu 2}^2 - m_{\nu 1}^2) (m_{\nu 3}^2 - m_{\nu 1}^2) (m_{\nu 3}^2 - m_{\nu 2}^2) (m_{\tau}^2 - m_{\mu}^2) (m_{\tau}^2 - m_{e}^2) (m_{\tau}^2 - m_{\mu}^2)$$

- ▶ Jarlskog rephasing invariant:  $J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13}$  (stand. par.)

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

C. Giunti – Neutrino Physics – May 2011 – 32



### Example: $\vartheta_{12} = 0$

$$U = R_{23}R_{13}W_{12}$$

$$W_{12} = \begin{pmatrix} \cos \vartheta_{12} & \sin \vartheta_{12} e^{-i\delta_{12}} & 0 \\ -\sin \vartheta_{12} e^{-i\delta_{12}} & \cos \vartheta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vartheta_{12} = 0 \quad \Rightarrow \quad W_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{1}$$

$$\text{real mixing matrix} \quad U = R_{23}R_{13}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix}$$

$$\lambda_{\mu 1} - \lambda_{\mu 2} = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi \quad \lambda_{\tau 1} - \lambda_{\mu 1} = \lambda_{\tau 2} - \lambda_{\mu 2} \pm \pi$$

$$\nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \quad \ell_\alpha \rightarrow e^{i\varphi_\alpha} \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$U \rightarrow \begin{pmatrix} e^{-i\varphi_e} & 0 & 0 \\ 0 & e^{-i\varphi_\mu} & 0 \\ 0 & 0 & e^{-i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{i(-\delta_{13} - \varphi_e + \varphi_3)} \\ |U_{\mu 1}| e^{i(\lambda_{\mu 1} - \varphi_\mu + \varphi_1)} & |U_{\mu 2}| e^{i(\lambda_{\mu 2} - \varphi_\mu + \varphi_2)} & 0 \\ |U_{\tau 1}| e^{i(\lambda_{\tau 1} - \varphi_\tau + \varphi_1)} & |U_{\tau 2}| e^{i(\lambda_{\tau 2} - \varphi_\tau + \varphi_2)} & 0 \end{pmatrix}$$

$$\varphi_1 = 0 \quad \varphi_\mu = \lambda_{\mu 1} \quad \varphi_\tau = \lambda_{\tau 1} \quad \varphi_2 = \varphi_\mu - \lambda_{\mu 2} = \lambda_{\mu 1} - \lambda_{\mu 2}$$

$$\varphi_2 = \varphi_\tau - \lambda_{\tau 2} \pm \pi = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi = \lambda_{\mu 1} - \lambda_{\mu 2} \quad \text{OK!}$$

$$U = \begin{pmatrix} 0 & 0 & \pm 1 \\ |U_{\mu 1}| & |U_{\mu 2}| & 0 \\ |U_{\tau 1}| & |U_{\tau 2}| & 0 \end{pmatrix}$$

### Example: $\vartheta_{13} = \pi/2$

$$U = R_{23}W_{13}R_{12}$$

$$W_{13} = \begin{pmatrix} \cos \vartheta_{13} & 0 & \sin \vartheta_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin \vartheta_{13} e^{i\delta_{13}} & 0 & \cos \vartheta_{13} \end{pmatrix}$$

$$\vartheta_{13} = \pi/2 \quad \Rightarrow \quad W_{13} = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -e^{i\delta_{13}} & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & 0 \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta_{13}} & 0 \end{pmatrix}$$

### Example: $m_{\nu 2} = m_{\nu 3}$

$$j_{W,L}^p = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$U = R_{12}R_{13}W_{23} \quad \Rightarrow \quad j_{W,L}^p = 2 \bar{\mathbf{n}}_L W_{23}^\dagger R_{13}^\dagger R_{12}^\dagger \gamma^\rho \ell_L$$

$$W_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \vartheta_{23} & \sin \vartheta_{23} e^{-i\delta_{23}} \\ 0 & -\sin \vartheta_{23} e^{-i\delta_{23}} & \cos \vartheta_{23} \end{pmatrix}$$

$$W_{23} \mathbf{n}_L = \mathbf{n}'_L \quad R_{12}R_{13} = U' \quad \Rightarrow \quad j_{W,L}^p = 2 \bar{\mathbf{n}}'_L U'^\dagger \gamma^\rho \ell_L$$

$\nu_2$  and  $\nu_3$  are indistinguishable

$$\text{drop the prime} \quad \Rightarrow \quad j_{W,L}^p = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$\text{real mixing matrix} \quad U = R_{12}R_{13}$$

## Jarlskog Rephasing Invariant

- Simplest rephasing invariants:  $|U_{\alpha k}| = U_{\alpha k} U_{\alpha k}^*$ ,  $U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$

$$\Im[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}] = \pm J$$

$$J = \Im[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}] = \Im \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- In standard parameterization:

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13}$$

$$= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos \vartheta_{13} \sin 2\vartheta_{13} \sin \delta_{13}$$

- Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way
- All measurable CP-violation effects depend on  $J$ .

## Maximal CP Violation

- Maximal CP violation is defined as the case in which  $|J|$  has its maximum possible value

$$|J|_{\max} = \frac{1}{6\sqrt{3}}$$

- In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4, \quad s_{13} = 1/\sqrt{3}, \quad \sin \delta_{13} = \pm 1$$

- This case is called **Trimaximal Mixing**. All the absolute values of the elements of the mixing matrix are equal to  $1/\sqrt{3}$ :

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp \frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

## GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- The unitarity of  $V_L^\ell$ ,  $V_R^\ell$  and  $V_L^\nu$  implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$j_{Z,L}^\rho = 2g_L^\nu \bar{\nu}_L^\rho \gamma^\rho \nu_L + 2g_L^\ell \bar{\ell}_L^\rho \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R^\rho \gamma^\rho \ell_R$$

$$= 2g_L^\nu \bar{\mathbf{n}}_L V_L^{\nu\dagger} \gamma^\rho V_L^\ell \mathbf{n}_L + 2g_L^\ell \bar{\ell}_L V_L^{\ell\dagger} \gamma^\rho V_L^\ell \ell_L + 2g_R^\ell \bar{\ell}_R V_R^{\ell\dagger} \gamma^\rho V_R^\ell \ell_R$$

$$= 2g_L^\nu \bar{\mathbf{n}}_L \gamma^\rho \mathbf{n}_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R$$

- The unitarity of  $U$  implies the same expression for the neutral weak current in terms of the flavor neutrino fields  $\nu_L = U \mathbf{n}_L$ :

$$j_{Z,L}^\rho = 2g_L^\nu \bar{\nu}_L U \gamma^\rho U^\dagger \nu_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R$$

$$= 2g_L^\nu \bar{\nu}_L \gamma^\rho \nu_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R$$

## Lepton Numbers Violating Processes

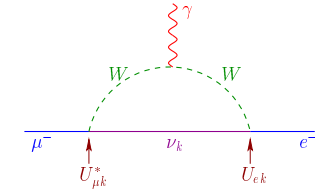
Dirac mass term allows  $L_e, L_\mu, L_\tau$  violating processes

Example:  $\mu^\pm \rightarrow e^\pm + \gamma$ ,  $\mu^\pm \rightarrow e^\pm + e^+ + e^-$

$$\mu^- \rightarrow e^- + \gamma$$

$\sum_k U_{\mu k}^* U_{ek} = 0 \Rightarrow$  only part of  $\nu_k$  propagator  $\propto m_k$  contributes

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \left| \underbrace{\sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2}}_{\text{BR}} \right|^2$$



Suppression factor:  $\frac{m_k}{m_W} \lesssim 10^{-11}$  for  $m_k \lesssim 1\text{eV}$

$$(\text{BR})_{\text{the}} \lesssim 10^{-47}$$

$$(\text{BR})_{\text{exp}} \lesssim 10^{-11}$$

## Majorana Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
  - Two-Component Theory of a Massless Neutrino
  - Majorana Equation
  - Majorana Lagrangian
  - Majorana Antineutrino?
  - Lepton Number
  - CP Symmetry
  - No Majorana Neutrino Mass in the SM
  - Effective Majorana Mass
  - Mixing of Three Majorana Neutrinos
  - Mixing Matrix

### Dirac-Majorana Mass Term

Number of Flavor C. Giunti – Neutrino Physics – May 2011 – 41

- $\psi_L$  and  $\psi_R$  have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \quad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ( $\psi_L \xrightarrow{P} \psi_R$ )
- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields  $\Rightarrow$  Two-component Theory of a Massless Neutrino (1957)
- $V - A$  Charged-Current Weak Interactions  $\Rightarrow \nu_L$
- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of  $\nu_R$

C. Giunti – Neutrino Physics – May 2011 – 43

## Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation:  $(i\gamma^\mu \partial_\mu - m)\psi = 0$
- Chiral decomposition of a Fermion Field:  $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^\mu \partial_\mu \psi_L = m\psi_R$$

$$i\gamma^\mu \partial_\mu \psi_R = m\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^\mu \partial_\mu \psi_L = 0$$

$$i\gamma^\mu \partial_\mu \psi_R = 0$$

- A massless fermion can be described by a single chiral field  $\psi_L$  or  $\psi_R$  (Weyl Spinor).

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## Majorana Equation

- Can a two-component spinor describe a massive fermion? Yes! (E. Majorana, 1937)
- Trick:  $\psi_R$  and  $\psi_L$  are not independent:  $\psi_R = C\overline{\psi}_L^T$
- $C\overline{\psi}_L^T$  is right-handed:  $P_R C\overline{\psi}_L^T = C\overline{\psi}_L^T$  ( $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ )
- Majorana Equation:  $i\gamma^\mu \partial_\mu \psi_L = mC\overline{\psi}_L^T$
- Majorana Field:  $\psi = \psi_L + \psi_R = \psi_L + C\overline{\psi}_L^T$
- Majorana Condition:  $\psi = C\overline{\psi}^T = \psi^C$

- Only two independent components:  $\psi = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

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## Majorana Lagrangian

- ▶  $\psi = \psi^C$  implies the equality of particle and antiparticle
- ▶ Only neutral fermions can be Majorana particles
- ▶ For a Majorana field, the electromagnetic current vanishes identically:

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}^C\gamma^\mu\psi^C = -\psi^T C^\dagger\gamma^\mu C\bar{\psi}^T = \bar{\psi}C\gamma^\mu{}^T C^\dagger\psi = -\bar{\psi}\gamma^\mu\psi = 0$$

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- ▶ Majorana Field:  $\nu = \nu_L + \nu_L^C$
- ▶ Majorana Condition:  $\nu^C = \nu$
- ▶ Majorana Lagrangian:  $\mathcal{L}^M = \frac{1}{2}\bar{\nu}(i\partial - m)\nu$
- ▶ The factor 1/2 distinguishes the Majorana Lagrangian from the Dirac Lagrangian
- ▶ Quantized Dirac Neutrino Field:
 
$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a^{(h)}(p) u^{(h)}(p) e^{-ip\cdot x} + b^{(h)\dagger}(p) v^{(h)}(p) e^{ip\cdot x} \right]$$
- ▶ Quantized Majorana Neutrino Field [ $b^{(h)}(p) = a^{(h)}(p)$ ]
 
$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a^{(h)}(p) u^{(h)}(p) e^{-ip\cdot x} + a^{(h)\dagger}(p) v^{(h)}(p) e^{ip\cdot x} \right]$$
- ▶ A Majorana field has half the degrees of freedom of a Dirac field

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Dirac Lagrangian

$$\begin{aligned} \mathcal{L}^D &= \bar{\nu}(i\partial - m)\nu \\ &= \bar{\nu}_L i\partial\nu_L + \bar{\nu}_R i\partial\nu_R - m(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R) \end{aligned}$$

$$\nu_R \rightarrow \nu_L^C = C\bar{\nu}_L^T$$

$$\frac{1}{2}\mathcal{L}^D \rightarrow \bar{\nu}_L i\partial\nu_L - \frac{m}{2}(-\nu_L^T C^\dagger\nu_L + \bar{\nu}_L C\bar{\nu}_L^T)$$

Majorana Lagrangian

$$\begin{aligned} \mathcal{L}^M &= \bar{\nu}_L i\partial\nu_L - \frac{m}{2}(-\nu_L^T C^\dagger\nu_L + \bar{\nu}_L C\bar{\nu}_L^T) \\ &= \bar{\nu}_L i\partial\nu_L - \frac{m}{2}(\bar{\nu}_L^C\nu_L + \bar{\nu}_L\nu_L^C) \end{aligned}$$

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## Majorana Antineutrino?

- ▶ A Majorana neutrino is the same as a Majorana antineutrino
- ▶ Neutrino interactions are described by the CC and NC Lagrangians

$$\mathcal{L}_{1,L}^{CC} = -\frac{g}{\sqrt{2}}(\bar{\nu}_L\gamma^\mu\ell_L W_\mu + \bar{\ell}_L\gamma^\mu\nu_L W_\mu^\dagger)$$

$$\mathcal{L}_{1,\nu}^{NC} = -\frac{g}{2\cos\vartheta_W}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu$$

- ▶ In practice, since detectable neutrinos are always ultrarelativistic, the neutrino mass can be neglected in interactions

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## Lepton Number

$$L = +1 \quad \leftarrow \quad \boxed{\nu = \nu^C} \quad \rightarrow \quad L = -1$$

$$\nu_L \implies L = +1 \quad \quad \nu_L^C \implies L = -1$$

$$\mathcal{L}^M = \bar{\nu}_L i \not{\partial} \nu_L - \frac{m}{2} (\bar{\nu}_L^C \nu_L + \bar{\nu}_L \nu_L^C)$$

Total Lepton Number is not conserved:  $\Delta L = \pm 2$

Best process to find violation of Total Lepton Number:

Neutrinoless Double- $\beta$  Decay

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + 2e^- + \cancel{2\nu_e} \quad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z - 2) + 2e^+ + \cancel{2\nu_e} \quad (\beta\beta_{0\nu}^+)$$

► In interaction amplitudes we neglect corrections of order  $m/E$

► Dirac:  $\left\{ \begin{array}{l} \nu_L \left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{array} \right. \\ \bar{\nu}_L \left\{ \begin{array}{l} \text{destroys right-handed antineutrinos} \\ \text{creates left-handed neutrinos} \end{array} \right. \end{array} \right.$

► Majorana:  $\left\{ \begin{array}{l} \nu_L \left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{array} \right. \\ \bar{\nu}_L \left\{ \begin{array}{l} \text{destroys right-handed neutrinos} \\ \text{creates left-handed neutrinos} \end{array} \right. \end{array} \right.$

► Common definitions:

Majorana neutrino with negative helicity  $\equiv$  neutrino

Majorana neutrino with positive helicity  $\equiv$  antineutrino

## CP Symmetry

► Under a CP transformation

$$U_{CP} \nu_L(x) U_{CP}^{-1} = \xi_\nu^{CP} \gamma^0 \nu_L^C(x_P)$$

$$U_{CP} \nu_L^C(x) U_{CP}^{-1} = -\xi_\nu^{CP*} \gamma^0 \nu_L(x_P)$$

$$U_{CP} \bar{\nu}_L(x) U_{CP}^{-1} = \xi_\nu^{CP*} \bar{\nu}_L^C(x_P) \gamma^0$$

$$U_{CP} \bar{\nu}_L^C(x) U_{CP}^{-1} = -\xi_\nu^{CP} \bar{\nu}_L(x_P) \gamma^0$$

with  $|\xi_\nu^{CP}|^2 = 1$ ,  $x^\mu = (x^0, \vec{x})$ , and  $x_P^\mu = (x^0, -\vec{x})$

► The theory is CP-symmetric if there are values of the phase  $\xi_\nu^{CP}$  such that the Lagrangian transforms as

$$U_{CP} \mathcal{L}(x) U_{CP}^{-1} = \mathcal{L}(x_P)$$

in order to keep invariant the action  $I = \int d^4x \mathcal{L}(x)$

► The Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^M(x) = -\frac{1}{2} m [\bar{\nu}_L^C(x) \nu_L(x) + \bar{\nu}_L(x) \nu_L^C(x)]$$

transforms as

$$U_{CP} \mathcal{L}_{\text{mass}}^M(x) U_{CP}^{-1} = -\frac{1}{2} m [ -(\xi_\nu^{CP})^2 \bar{\nu}_L(x_P) \nu_L^C(x_P) - (\xi_\nu^{CP*})^2 \bar{\nu}_L^C(x_P) \nu_L(x_P) ]$$

►  $U_{CP} \mathcal{L}_{\text{mass}}^M(x) U_{CP}^{-1} = \mathcal{L}_{\text{mass}}^M(x_P)$  for  $\xi_\nu^{CP} = \pm i$

► The one-generation Majorana theory is CP-symmetric

► The Majorana case is different from the Dirac case, in which the CP phase  $\xi_\nu^{CP}$  is arbitrary

## No Majorana Neutrino Mass in the SM

- ▶ Majorana Mass Term  $\propto [\nu_L^T C^\dagger \nu_L - \bar{\nu}_L C \bar{\nu}_L^T]$  involves only the neutrino left-handed chiral field  $\nu_L$ , which is present in the SM (one for each lepton generation)
- ▶ Eigenvalues of the weak isospin  $I$ , of its third component  $I_3$ , of the hypercharge  $Y$  and of the charge  $Q$  of the lepton and Higgs multiplets:

	$I$	$I_3$	$Y$	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	1/2	1/2 -1/2	-1	0 -1
lepton singlet $\ell_R$	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	1/2	1/2 -1/2	+1	1 0

- ▶  $\nu_L^T C^\dagger \nu_L$  has  $I_3 = 1$  and  $Y = -2 \implies$  needed Higgs triplet with  $Y = 2$

## Effective Majorana Mass

- ▶ Dimensional analysis: Fermion Field  $\sim [E]^{3/2}$  Boson Field  $\sim [E]$
- ▶ Dimensionless action:  $I = \int d^4x \mathcal{L}(x) \implies \mathcal{L}(x) \sim [E]^4$
- ▶ Kinetic terms:  $\bar{\psi} i \not{\partial} \psi \sim [E]^4$ ,  $(\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- ▶ Mass terms:  $m \bar{\psi} \psi \sim [E]^4$ ,  $m^2 \phi^\dagger \phi \sim [E]^4$
- ▶ CC weak interaction:  $g \bar{\nu}_L \gamma^\rho \ell_L W_\rho \sim [E]^4$
- ▶ Yukawa couplings:  $y \bar{L}_L \Phi \ell_R \sim [E]^4$
- ▶ Product of fields  $\mathcal{O}_d$  with energy dimension  $d \equiv \text{dim-}d$  operator
- ▶  $\mathcal{L}(\mathcal{O}_d) = C(\mathcal{O}_d) \mathcal{O}_d \implies C(\mathcal{O}_d) \sim [E]^{4-d}$
- ▶  $\mathcal{O}_{d>4}$  are **not renormalizable**

- ▶ SM Lagrangian includes all  $\mathcal{O}_{d \leq 4}$  invariant under  $SU(2)_L \times U(1)_Y$
- ▶ SM cannot be considered as the final theory of everything
- ▶ SM is an effective low-energy theory
- ▶ It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- ▶ It is plausible that at low-energy there are effective non-renormalizable  $\mathcal{O}_{d>4}$  [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- ▶ All  $\mathcal{O}_d$  must respect  $SU(2)_L \times U(1)_Y$ , because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- ▶  $\mathcal{O}_{d>4}$  is suppressed by a coefficient  $\mathcal{M}^{4-d}$ , where  $\mathcal{M}$  is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}} \mathcal{O}_5 + \frac{g_6}{\mathcal{M}^2} \mathcal{O}_6 + \dots$$

- ▶ Analogy with  $\mathcal{L}_{\text{eff}}^{(\text{CC})} \propto G_F (\bar{\nu}_e \gamma^\rho e_L) (\bar{e}_L \gamma_\rho \nu_e) + \dots$   
 $\mathcal{O}_6 \rightarrow (\bar{\nu}_e \gamma^\rho e_L) (\bar{e}_L \gamma_\rho \nu_e) + \dots \quad \frac{g_6}{\mathcal{M}^2} \rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$
- ▶  $\mathcal{M}^{4-d}$  is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM
- ▶ The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality
- ▶  $\mathcal{O}_5 \implies$  Majorana neutrino masses (Lepton number violation)
- ▶  $\mathcal{O}_6 \implies$  Baryon number violation (proton decay)

- ▶ Only one dim-5 operator:

$$\begin{aligned} \mathcal{O}_5 &= (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.} \\ &= \frac{1}{2} (L_L^T C^\dagger \sigma_2 \bar{\sigma} L_L) \cdot (\Phi^T \sigma_2 \bar{\sigma} \Phi) + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_5 = \frac{g_5}{2\mathcal{M}} (L_L^T C^\dagger \sigma_2 \bar{\sigma} L_L) \cdot (\Phi^T \sigma_2 \bar{\sigma} \Phi) + \text{H.c.}$$

- ▶ Electroweak Symmetry Breaking:  $\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}}$   $\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\mathcal{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \frac{g_5 v^2}{\mathcal{M}} \nu_L^T C^\dagger \nu_L + \text{H.c.} \implies \boxed{m = \frac{g_5 v^2}{\mathcal{M}}}$$

## Mixing of Three Majorana Neutrinos

$$\begin{aligned} \nu_L' &\equiv \begin{pmatrix} \nu_{eL}' \\ \nu_{\mu L}' \\ \nu_{\tau L}' \end{pmatrix} \\ \mathcal{L}_{\text{mass}}^{\text{M}} &= \frac{1}{2} \nu_L'^T C^\dagger M^L \nu_L' + \text{H.c.} \\ &= \frac{1}{2} \sum_{\alpha, \beta=e, \mu, \tau} \nu_{\alpha L}'^T C^\dagger M_{\alpha\beta}^L \nu_{\beta L}' + \text{H.c.} \end{aligned}$$

- ▶ In general, the matrix  $M^L$  is a complex symmetric matrix

$$\begin{aligned} \sum_{\alpha, \beta} \nu_{\alpha L}'^T C^\dagger M_{\alpha\beta}^L \nu_{\beta L}' &= - \sum_{\alpha, \beta} \nu_{\beta L}'^T M_{\alpha\beta}^L (C^\dagger)^T \nu_{\alpha L}' \\ &= \sum_{\alpha, \beta} \nu_{\beta L}'^T C^\dagger M_{\alpha\beta}^L \nu_{\alpha L}' = \sum_{\alpha, \beta} \nu_{\alpha L}'^T C^\dagger M_{\beta\alpha}^L \nu_{\beta L}' \end{aligned}$$

$$M_{\alpha\beta}^L = M_{\beta\alpha}^L \iff M^L = M^{L T}$$

- ▶ The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM

$$m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_{\text{D}}^2}{\mathcal{M}} \quad \text{natural explanation of smallness of neutrino masses (special case: See-Saw Mechanism)}$$

- ▶ Example:  $m_{\text{D}} \sim v \sim 10^2 \text{ GeV}$  and  $\mathcal{M} \sim 10^{15} \text{ GeV} \implies m \sim 10^{-2} \text{ eV}$

$$\mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \nu_L'^T C^\dagger M^L \nu_L' + \text{H.c.}$$

$$\nu_L' = V_L^\nu \mathbf{n}_L \implies \mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \nu_L'^T (V_L^\nu)^T C^\dagger M^L V_L^\nu \nu_L' + \text{H.c.}$$

$$(V_L^\nu)^T M^L V_L^\nu = M, \quad M_{kj} = m_k \delta_{kj} \quad (k, j = 1, 2, 3)$$

- ▶ Left-handed chiral fields with definite mass:  $\mathbf{n}_L = V_L^{\nu\dagger} \nu_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{M}} &= \frac{1}{2} (\mathbf{n}_L^T C^\dagger M \mathbf{n}_L - \bar{\mathbf{n}}_L M C \mathbf{n}_L^T) \\ &= \frac{1}{2} \sum_{k=1}^3 m_k (\nu_{kL}^T C^\dagger \nu_{kL} - \bar{\nu}_{kL} C \nu_{kL}^T) \end{aligned}$$

- ▶ Majorana fields of massive neutrinos:  $\nu_k = \nu_{kL} + \nu_{kL}^{\text{C}}$   $\nu_k^{\text{C}} = \nu_k$

$$\mathbf{n} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \implies \mathcal{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^3 \bar{\nu}_k (i\partial - m_k) \nu_k = \frac{1}{2} \bar{\mathbf{n}} (i\partial - M) \mathbf{n}$$

## Mixing Matrix

- ▶ Leptonic Weak Charged Current:

$$j_{W,L}^\rho = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L \quad \text{with} \quad U = V_L^{\ell\dagger} V_L^\nu$$

- ▶ Definition of the left-handed flavor neutrino fields:

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} \nu_L' = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- ▶ Leptonic Weak Charged Current has the SM form

$$j_{W,L}^\rho = 2 \bar{\nu}_L \gamma^\rho \ell_L = 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^\rho \ell_{\alpha L}$$

- ▶ Important difference with respect to Dirac case:  
Two additional CP-violating phases: Majorana phases

- ▶ Majorana Mass Term  $\mathcal{L}_{\text{mass}}^M = \frac{1}{2} \sum_{k=1}^3 m_k \nu_{kL}^T C^\dagger \nu_{kL} + \text{H.c.}$  is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \rightarrow e^{i\varphi_k} \nu_{kL} \quad (k = 1, 2, 3)$$

- ▶ Left-handed massive neutrino fields cannot be rephased in order to eliminate two Majorana phases factorized on the right of mixing matrix:

$$U = U^D D^M \quad D^M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- ▶  $U^D$  is analogous to a Dirac mixing matrix, with one Dirac phase
- ▶ Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- ▶ Jarlskog rephasing invariant:  $J = c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13} \sin \delta_{13}$

## Dirac-Majorana Mass Term

- ▶  $D^M = \text{diag}(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3})$ , but only two Majorana phases are physical

- ▶ All measurable quantities depend only on the differences of the Majorana phases

$$\ell_\alpha \rightarrow e^{i\varphi} \ell_\alpha \implies e^{i\lambda_k} \rightarrow e^{i(\lambda_k - \varphi)}$$

$e^{i(\lambda_k - \lambda_j)}$  remains constant

- ▶ Our convention:  $\lambda_1 = 0 \implies D^M = \text{diag}(1, e^{i\lambda_2}, e^{i\lambda_3})$
- ▶ CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:

$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
  - One Generation
  - Real Mass Matrix
  - Maximal Mixing
  - Dirac Limit
  - Pseudo-Dirac Neutrinos
  - See-Saw Mechanism
  - Majorana Neutrino Mass?
  - Fundamental Fields in QFT
  - Right-Handed Neutrino Mass Term
  - Singlet Majoron Model
  - Three-Generation Mixing



## One Generation

If  $\nu_R$  exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}} \bar{\nu}_R \nu_L + \text{H.c.} \quad \text{Dirac Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{L}} = \frac{1}{2} m_L \nu_L^T C^\dagger \nu_L + \text{H.c.} \quad \text{Majorana Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{R}} = \frac{1}{2} m_R \nu_R^T C^\dagger \nu_R + \text{H.c.} \quad \text{New Majorana Mass Term!}$$

## Real Mass Matrix

▶ CP is conserved if the mass matrix is real:  $M = M^*$

▶  $M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$  we consider real and positive  $m_R$  and  $m_{\text{D}}$  and real  $m_L$

▶ A real symmetric mass matrix can be diagonalized with  $U = \mathcal{O} \rho$

$$\mathcal{O} = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \quad \rho_k^2 = \pm 1$$

$$\begin{aligned} \text{▶ } \mathcal{O}^T M \mathcal{O} &= \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix} & \tan 2\vartheta &= \frac{2m_{\text{D}}}{m_R - m_L} \\ m'_{2,1} &= \frac{1}{2} \left[ m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_{\text{D}}^2} \right] \end{aligned}$$

▶  $m'_1$  is negative if  $m_L m_R < m_{\text{D}}^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m'_1 & 0 \\ 0 & \rho_2^2 m'_2 \end{pmatrix} \Rightarrow \boxed{m_k = \rho_k^2 m'_k}$$

▶ Column matrix of left-handed chiral fields:  $N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ C \bar{\nu}_R^T \end{pmatrix}$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} N_L^T C^\dagger M N_L + \text{H.c.} \quad M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

▶ The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

▶ Diagonalization:  $n_L = U^\dagger N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{Real } m_k \geq 0$$

$$\begin{aligned} \text{▶ } \mathcal{L}_{\text{mass}}^{\text{D+M}} &= \frac{1}{2} \sum_{k=1,2} m_k \nu_{kL}^T C^\dagger \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k \bar{\nu}_k \nu_k \\ &\quad \nu_k = \nu_{kL} + \nu_{kL}^c \end{aligned}$$

▶ Massive neutrinos are Majorana!  $\nu_k = \nu_k^c$

▶  $m'_2$  is always positive:

$$m_2 = m'_2 = \frac{1}{2} \left[ m_L + m_R + \sqrt{(m_L - m_R)^2 + 4m_{\text{D}}^2} \right]$$

▶ If  $m_L m_R \geq m_{\text{D}}^2$ , then  $m'_1 \geq 0$  and  $\rho_1^2 = 1$

$$m_1 = \frac{1}{2} \left[ m_L + m_R - \sqrt{(m_L - m_R)^2 + 4m_{\text{D}}^2} \right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \Rightarrow U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

▶ If  $m_L m_R < m_{\text{D}}^2$ , then  $m'_1 < 0$  and  $\rho_1^2 = -1$

$$m_1 = \frac{1}{2} \left[ \sqrt{(m_L - m_R)^2 + 4m_{\text{D}}^2} - (m_L + m_R) \right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \Rightarrow U = \begin{pmatrix} i \cos \vartheta & \sin \vartheta \\ -i \sin \vartheta & \cos \vartheta \end{pmatrix}$$

- ▶ If  $\Delta m^2$  is small, there are oscillations between active  $\nu_a$  generated by  $\nu_L$  and sterile  $\nu_s$  generated by  $\nu_R^C$ :

$$P_{\nu_a \rightarrow \nu_s}(L, E) = \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R) \sqrt{(m_L - m_R)^2 + 4m_D^2}$$

- ▶ It can be shown that the CP parity of  $\nu_k$  is  $\xi_k^{\text{CP}} = i\rho_k^2$ :

$$U_{\text{CP}} \nu_k(x) U_{\text{CP}}^{-1} = i\rho_k^2 \gamma^0 \nu_k(x_P)$$

- ▶ Special cases:

- ▶  $m_L = m_R \implies$  Maximal Mixing
- ▶  $m_L = m_R = 0 \implies$  Dirac Limit
- ▶  $|m_L|, m_R \ll m_D \implies$  Pseudo-Dirac Neutrinos
- ▶  $m_L = 0, m_D \ll m_R \implies$  See-Saw Mechanism

## Maximal Mixing

$$m_L = m_R$$

$$\vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_D$$

$$\left\{ \begin{array}{ll} \rho_1^2 = +1, & m_1 = m_L - m_D \quad \text{if } m_L \geq m_D \\ \rho_1^2 = -1, & m_1 = m_D - m_L \quad \text{if } m_L < m_D \end{array} \right.$$

$$m_2 = m_L + m_D$$

$$m_L < m_D$$

$$\left\{ \begin{array}{l} \nu_{1L} = \frac{-i}{\sqrt{2}} (\nu_L - \nu_R^C) \\ \nu_{2L} = \frac{1}{\sqrt{2}} (\nu_L + \nu_R^C) \end{array} \right.$$

$$\left\{ \begin{array}{l} \nu_1 = \nu_{1L} + \nu_{1L}^C = \frac{-i}{\sqrt{2}} [(\nu_L + \nu_R) - (\nu_L^C + \nu_R^C)] \\ \nu_2 = \nu_{2L} + \nu_{2L}^C = \frac{1}{\sqrt{2}} [(\nu_L + \nu_R) + (\nu_L^C + \nu_R^C)] \end{array} \right.$$

## Dirac Limit

$$m_L = m_R = 0$$

- ▶  $m'_{2,1} = \pm m_D \implies \left\{ \begin{array}{ll} \rho_1^2 = -1, & m_1 = m_D \\ \rho_2^2 = +1, & m_2 = m_D \end{array} \right.$
- ▶ The two Majorana fields  $\nu_1$  and  $\nu_2$  can be combined to give one Dirac field:

$$\nu = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) = \nu_L + \nu_R$$

- ▶ A Dirac field  $\nu$  can always be split in two Majorana fields:

$$\begin{aligned} \nu &= \frac{1}{2} [(\nu - \nu^C) + (\nu + \nu^C)] \\ &= \frac{i}{\sqrt{2}} \left( -i \frac{\nu - \nu^C}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left( \frac{\nu + \nu^C}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) \end{aligned}$$

- ▶ A Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities

## Pseudo-Dirac Neutrinos

$$|m_L|, m_R \ll m_D$$

- ▶  $m'_{2,1} \simeq \frac{m_L + m_R}{2} \pm m_D$
- ▶  $m'_1 < 0 \implies \rho_1^2 = -1 \implies m_{2,1} \simeq m_D \pm \frac{m_L + m_R}{2}$
- ▶ The two massive Majorana neutrinos have opposite CP parities and are almost degenerate in mass
- ▶ The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_D (m_L + m_R)$$

- ▶ The oscillations occur with practically maximal mixing:

$$\tan 2\vartheta = \frac{2m_D}{m_R - m_L} \gg 1 \implies \vartheta \simeq \pi/4$$

## See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$m_L = 0 \quad m_D \ll m_R$$

- ▶  $\mathcal{L}_{\text{mass}}^L$  is forbidden by SM symmetries  $\Rightarrow m_L = 0$
- ▶  $m_D \lesssim v \sim 100 \text{ GeV}$  is generated by SM Higgs Mechanism (protected by SM symmetries)
- ▶  $m_R$  is not protected by SM symmetries  $\Rightarrow m_R \sim \mathcal{M}_{\text{GUT}} \gg v$

$$\left. \begin{array}{l} m'_1 \simeq -\frac{m_D^2}{m_R} \\ m'_2 \simeq m_R \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \rho_1^2 = -1, \quad m_1 \simeq \frac{m_D^2}{m_R} \\ \rho_2^2 = +1, \quad m_2 \simeq m_R \end{array} \right. \quad \begin{array}{c} \textcircled{V_1} \\ \textcircled{V_2} \end{array} \Delta$$

- ▶ Natural explanation of smallness of neutrino masses
- ▶ Mixing angle is very small:  $\tan 2\vartheta = 2 \frac{m_D}{m_R} \ll 1$
- ▶  $\nu_1$  is composed mainly of active  $\nu_L$ :  $\nu_{1L} \simeq -i \nu_L$
- ▶  $\nu_2$  is composed mainly of sterile  $\nu_R$ :  $\nu_{2L} \simeq \nu_R^C$

C. Giunti – Neutrino Physics – May 2011 – 73

$$\mathcal{L}_5 = \frac{g}{\mathcal{M}} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

$$\mathcal{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \frac{(y^\nu)^2}{m_R} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

$$g = -\frac{(y^\nu)^2}{2} \quad \mathcal{M} = m_R$$

- ▶ See-saw mechanism is a particular case of the effective Lagrangian approach.
- ▶ See-saw mechanism is obtained when dimension-five operator is generated only by the presence of  $\nu_R$  with  $m_R \sim \mathcal{M}$ .
- ▶ In general, other terms can contribute to  $\mathcal{L}_5$ .

C. Giunti – Neutrino Physics – May 2011 – 75

## Connection with Effective Lagrangian Approach

- ▶ Dirac–Majorana neutrino mass term with  $m_L = 0$ :

$$\mathcal{L}^{\text{D+M}} = -m_D (\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R) + \frac{1}{2} m_R (\nu_R^T C^\dagger \nu_R + \nu_R^\dagger C \nu_R^*)$$

- ▶ Above the electroweak symmetry-breaking scale:

$$\mathcal{L}^{\text{D+M}} = -y^\nu (\bar{\nu}_R \tilde{\Phi}^\dagger L_L + \bar{L}_L \tilde{\Phi} \nu_R) + \frac{1}{2} m_R (\nu_R^T C^\dagger \nu_R + \nu_R^\dagger C \nu_R^*)$$

- ▶ If  $m_R \gg v \Rightarrow \nu_R$  is static  $\Rightarrow$  kinetic term in equation of motion can be neglected:

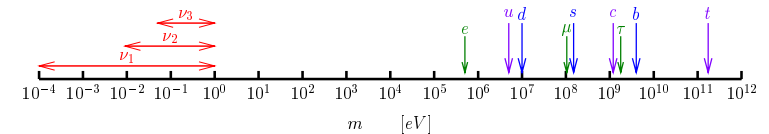
$$0 \simeq \frac{\partial \mathcal{L}^{\text{D+M}}}{\partial \nu_R} = m_R \nu_R^T C^\dagger - y^\nu \bar{L}_L \tilde{\Phi}$$

$$\nu_R \simeq -\frac{y^\nu}{m_R} \tilde{\Phi}^T C \bar{L}_L^T$$

$$\mathcal{L}^{\text{D+M}} \rightarrow \mathcal{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \frac{(y^\nu)^2}{m_R} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

C. Giunti – Neutrino Physics – May 2011 – 74

## Majorana Neutrino Mass?



known natural explanation of smallness of  $\nu$  masses

New High Energy Scale  $\mathcal{M} \Rightarrow \left\{ \begin{array}{l} \text{See-Saw Mechanism (if } \nu_R \text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{array} \right.$

both imply  $\left\{ \begin{array}{l} \text{Majorana } \nu \text{ masses} \Leftrightarrow |\Delta L| = 2 \Leftrightarrow \beta \beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \frac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}} \end{array} \right.$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

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## Fundamental Fields in QFT

- ▶ Each elementary particle is described by a field which is an irreducible representation of the Poincaré group (Lorentz group + space-time translations).
- ▶ In this way
  - ▶ Under Poincaré transformation an elementary particle remains itself.
  - ▶ Lagrangian is constructed with invariant products of elementary fields.
- ▶ Spinorial structure of a particle is determined by its representation under the restricted Lorentz group of proper and orthochronous Lorentz transformation (no space or time inversions).

C. Giunti – Neutrino Physics – May 2011 – 77

- ▶ Two-component left-handed Weyl (Majorana if massive) spinor:

$$\chi_L = \begin{pmatrix} \chi_{L1} \\ \chi_{L2} \end{pmatrix}$$

- ▶ Two-component right-handed Weyl (Majorana if massive) spinor:

$$\chi_R = \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \end{pmatrix}$$

- ▶ Four-component Dirac spinor:  $\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

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- ▶ Restricted Lorentz group is isomorphic to  $SU(2) \times SU(2)$ .

- ▶ Classification of fundamental representations:

$$(0, 0) \quad \text{scalar } \varphi$$

$$(1/2, 0) \quad \text{left-handed Weyl spinor } \chi_L \text{ (Majorana if massive)}$$

$$(0, 1/2) \quad \text{right-handed Weyl spinor } \chi_R \text{ (Majorana if massive)}$$

- ▶ All representations are constructed combining the two fundamental Weyl spinor representations.

$$(1/2, 1/2) \quad \text{four-vector } v^\mu \text{ (irreducible)}$$

$$(1/2, 0) + (0, 1/2) \quad \text{four-component Dirac spinor } \psi \text{ (reducible)}$$

- ▶ Two-component Weyl (Majorana if massive) spinor is more fundamental than four-component Dirac spinor.

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- ▶ Lorentz transformation:  $v^\mu \rightarrow v'^\mu = \Lambda^\mu{}_\nu v^\nu$

$$g_{\mu\nu} \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma = g_{\rho\sigma} \quad g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

- ▶ Restricted Lorentz transformation:  $\Lambda^\mu{}_\nu = [e^\omega]^\mu{}_\nu \quad \omega_{\mu\nu} = -\omega_{\nu\mu}$

$$\omega_{\mu\nu} = \begin{pmatrix} 0 & v_1 & v_2 & v_3 \\ -v_1 & 0 & \theta_3 & -\theta_2 \\ -v_2 & -\theta_3 & 0 & \theta_1 \\ -v_3 & \theta_2 & -\theta_1 & 0 \end{pmatrix}$$

- ▶ 6 parameters:

- ▶ 3 for rotations:  $\vec{\theta} = (\theta_1, \theta_2, \theta_3)$

- ▶ 3 for boosts:  $\vec{v} = (v_1, v_2, v_3)$

$$\begin{aligned} \chi_L &\rightarrow \chi'_L = \Lambda_L \chi_L & \Lambda_L &= e^{i(\vec{\theta} - i\vec{v}) \cdot \vec{\sigma} / 2} \\ \chi_R &\rightarrow \chi'_R = \Lambda_R \chi_R & \Lambda_R &= e^{i(\vec{\theta} + i\vec{v}) \cdot \vec{\sigma} / 2} \end{aligned}$$

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## Right-Handed Neutrino Mass Term

- Four-component form of two-component left-handed Weyl (Majorana if massive) spinor:

$$\psi_L = \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$$

- Majorana mass term:

$$\mathcal{L}_{\text{mass}}^L = \underbrace{\frac{1}{2} m_L \psi_L^T C^\dagger \psi_L}_{\text{four-component form}} + \text{H.c.} = -\frac{1}{2} m_L \underbrace{\chi_L^T i \sigma^2 \chi_L}_{\text{two-component form}} + \text{H.c.}$$

$$(1/2, 0) \times (1/2, 0) = \underbrace{(1, 0)}_{\text{symmetric}} + \underbrace{(0, 0)}_{\text{antisymmetric}} \quad \sigma^2 \text{ is antisymmetric!}$$

- Anticommutativity of spinors is necessary, otherwise

$$\chi_L^T i \sigma^2 \chi_L = (\chi_L^T i \sigma^2 \chi_L)^T = -\chi_L^T i \sigma^2 \chi_L = 0$$

Majorana mass term for  $\nu_R$  respects the  $SU(2)_L \times U(1)_Y$  Standard Model Symmetry!

$$\mathcal{L}_R^M = -\frac{1}{2} m (\overline{\nu_R^c} \nu_R + \overline{\nu_R} \nu_R^c)$$

Majorana mass term for  $\nu_R$  breaks Lepton number conservation!

Three possibilities:

- ▶ Lepton number can be explicitly broken
- ▶ Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- ▶ Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

## Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\begin{aligned} \mathcal{L}_\Phi &= -y_d (\overline{L}_L \Phi \nu_R + \overline{\nu_R} \Phi^\dagger L_L) \xrightarrow{\langle \Phi \rangle \neq 0} -m_D (\overline{\nu}_L \nu_R + \overline{\nu_R} \nu_L) \\ \mathcal{L}_\eta &= -y_s (\eta \overline{\nu_R^c} \nu_R + \eta^\dagger \overline{\nu_R} \nu_R^c) \xrightarrow{\langle \eta \rangle \neq 0} -\frac{1}{2} m_R (\overline{\nu_R^c} \nu_R + \overline{\nu_R} \nu_R^c) \end{aligned}$$

$$\eta = 2^{-1/2} (\langle \eta \rangle + \rho + i\chi) \quad \mathcal{L}_{\text{mass}} = -\frac{1}{2} (\overline{\nu_L^c} \overline{\nu_R}) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\frac{m_R}{\text{scale of } L \text{ violation}} \gg \frac{m_D}{\text{EW scale}} \implies \text{See-Saw: } m_1 \simeq \frac{m_D^2}{m_R}$$

$\rho =$  massive scalar,  $\chi =$  Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{iy_s}{\sqrt{2}} \chi \left[ \overline{\nu_2} \gamma^5 \nu_2 - \frac{m_D}{m_R} \left[ \overline{\nu_2} \gamma^5 \nu_1 + \overline{\nu_1} \gamma^5 \nu_2 \right] + \left( \frac{m_D}{m_R} \right)^2 \overline{\nu_1} \gamma^5 \nu_1 \right]$$

## Three-Generation Mixing

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{D+M}} &= \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}} \\ \mathcal{L}_{\text{mass}}^{\text{D}} &= -\sum_{s=1}^{N_s} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}} M_{s\alpha}^{\text{D}} \nu'_{\alpha L} + \text{H.c.} \\ \mathcal{L}_{\text{mass}}^{\text{L}} &= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}^T C^\dagger M_{\alpha\beta}^{\text{L}} \nu_{\beta L} + \text{H.c.} \\ \mathcal{L}_{\text{mass}}^{\text{R}} &= \frac{1}{2} \sum_{s,s'=1}^{N_s} \nu_{sR}^T C^\dagger M_{ss'}^{\text{R}} \nu_{s'R} + \text{H.c.} \end{aligned}$$

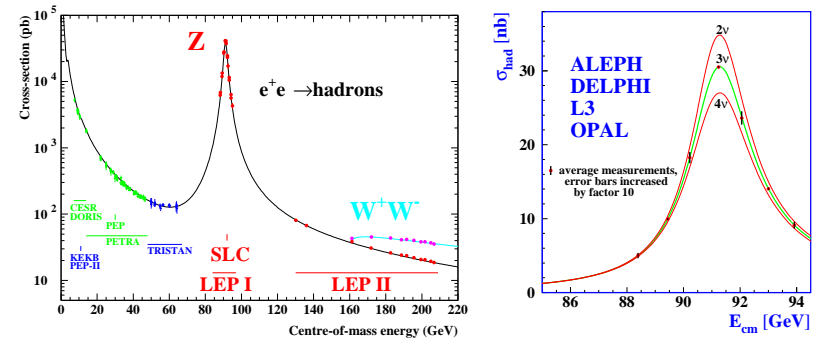
$$\mathbf{N}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'^C_R \equiv \begin{pmatrix} \nu'^C_{1R} \\ \vdots \\ \nu'^C_{N_s R} \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \mathbf{N}'_L{}^T C^\dagger M^{\text{D+M}} \mathbf{N}'_L + \text{H.c.} \quad M^{\text{D+M}} = \begin{pmatrix} M^{\text{L}} & M^{\text{D}T} \\ M^{\text{D}} & M^{\text{R}} \end{pmatrix}$$

- ▶ Diagonalization of the Dirac-Majorana Mass Term  $\implies$  massive Majorana neutrinos
- ▶ See-Saw Mechanism  $\implies$  right-handed neutrinos have large Majorana masses and are decoupled from the low-energy phenomenology.
- ▶ If all right-handed neutrinos have large Majorana masses, at low energy we have an effective mixing of three Majorana neutrinos.
- ▶ It is possible that not all right-handed neutrinos have large Majorana masses: some right-handed neutrinos may correspond to low-energy Majorana particles which belong to new physics beyond the Standard Model (as a light neutralino in supersymmetric models).
- ▶ Light anti- $\nu_R$  are called sterile neutrinos

$$\nu_R^c \rightarrow \nu_s \quad (\text{left-handed})$$

## Number of Flavor and Massive Neutrinos?



[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z \rightarrow \ell\bar{\ell}} + \sum_{q \neq t} \Gamma_{Z \rightarrow q\bar{q}} + \Gamma_{\text{inv}} \quad \Gamma_{\text{inv}} = N_\nu \Gamma_{Z \rightarrow \nu\bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$e^+ e^- \rightarrow Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \implies \nu_e \nu_\mu \nu_\tau$$

3 light active flavor neutrinos

$$\text{mixing} \implies \nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL} \quad \alpha = e, \mu, \tau \quad N \geq 3 \quad \text{no upper limit!}$$

Mass Basis:	$\nu_1$	$\nu_2$	$\nu_3$	$\nu_4$	$\nu_5$	$\dots$
Flavor Basis:	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$\nu_{s1}$	$\nu_{s2}$	$\dots$
	ACTIVE			STERILE		

## Sterile Neutrinos

- ▶ Sterile means no standard model interactions
- ▶ Obviously no electromagnetic interactions as normal active neutrinos
- ▶ Thus sterile means no standard weak interactions
- ▶ But sterile neutrinos are not absolutely sterile:
  - ▶ Gravitational Interactions
  - ▶ New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos
- ▶ Active neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ) can oscillate into sterile neutrinos ( $\nu_s$ )
- ▶ Observables:
  - ▶ Disappearance of active neutrinos
  - ▶ Indirect evidence through combined fit of data
- ▶ Powerful window on new physics beyond the Standard Model