Neutrino Physics

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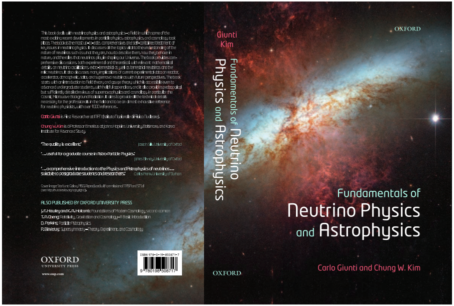

This book deals with neutrino physics and astrophysics—a field in which some of the most exciting recent developments in particle physics, astrophysics, and cosmology took place. The book is the most up-to-date, comprehensive, and self-contained treatment of key issues in neutrino physics. It discusses all the topics vital to the understanding of the nature of neutrinos such as what they are, how to describe them, how they behave in nature, and the roles that neutrinos play in shaping our Universe. The book provides comprehensive discussions, both experimental and theoretical, with relevant mathematical details, on neutrino oscillations, extra-terrestrial as well as terrestrial neutrinos and the relic neutrinos. It also discusses many implications of current experimental data on reactor, accelerator, atmospheric, solar, and supernova neutrinos with future perspectives. The book starts with an introduction to field theory and gauge theory which is accessible even to advanced undergraduate students, with helpful appendices, and it also provides pedagogical but sufficiently detailed reviews of supernova physics and cosmology, in particular the Cosmic Microwave Background Radiation. It aims to provide all the technical details necessary for the professionals in the field and to be an almost exhaustive reference for neutrino physicists, with over 1000 references.

Carlo Giunti is First Researcher at INFN (Istituto Nazionale di Fisica Nucleare).

Chung W. Kim is a Professor Emeritus at Johns Hopkins University, Baltimore, and Korea Institute for Advanced Study.

'The quality is excellent.' Joseph Silk, University of Oxford

'...useful for a graduate course in Astro-Particle Physics.' James Binney, University of Oxford

'...a comprehensive introduction to the Physics and Astrophysics of neutrinos ... suitable to postgraduate students and researchers.' Carlos Frenk, University of Durham

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Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Number of Flavor and Massive Neutrinos?
- Sterile Neutrinos

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- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

Part III

Phenomenology

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- Conclusions

Solar Neutrinos and KamLAND

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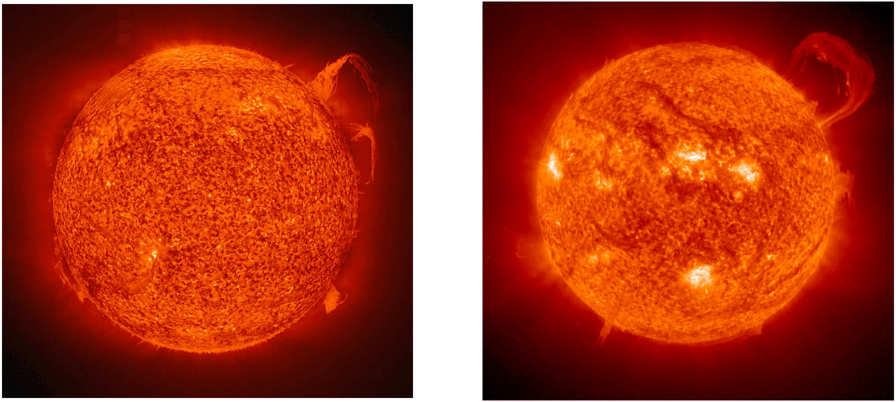
The Sun



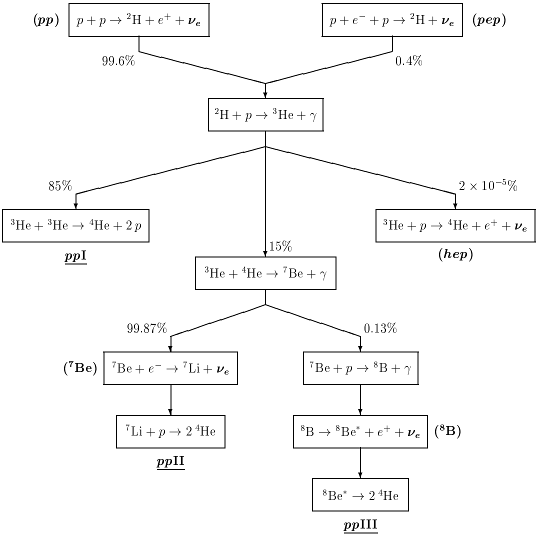
Extreme ultraviolet Imaging Telescope (EIT) 304 Å images of the Sun

emission in this spectral line (He II) shows the upper chromosphere

at a temperature of about 60,000 K

[The Solar and Heliospheric Observatory (SOHO), http://sohowww.nascom.nasa.gov/]

Standard Solar Model (SSM)



(pp) $p + p \to {}^2\text{H} + e^+ + \nu_e$

$p + e^- + p \to {}^2\text{H} + \nu_e$ **(pep)**

99.6%

0.4%

${}^2\text{H} + p \to {}^3\text{He} + \gamma$

85%

15%

$2 \times 10^{-5}\%$

${}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + 2\,p$

ppI

${}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma$

${}^3\text{He} + p \to {}^4\text{He} + e^+ + \nu_e$

(hep)

99.87%

0.13%

(^{7}Be) ${}^7\text{Be} + e^- \to {}^7\text{Li} + \nu_e$

${}^7\text{Be} + p \to {}^8\text{B} + \gamma$

${}^7\text{Li} + p \to 2\,{}^4\text{He}$

ppII

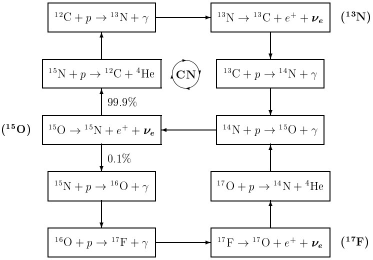
${}^8\text{B} \to {}^8\text{Be}^* + e^+ + \nu_e$ **(^{8}B)**

${}^8\text{Be}^* \to 2\,{}^4\text{He}$

ppIII

pp chain and CNO cycle

$$4\,p + 2\,e^- \to {}^4\text{He} + 2\,\nu_e + 26.731\text{ MeV}$$



${}^{12}\text{C} + p \to {}^{13}\text{N} + \gamma$

${}^{13}\text{N} \to {}^{13}\text{C} + e^+ + \nu_e$ **(^{13}N)**

${}^{15}\text{N} + p \to {}^{12}\text{C} + {}^4\text{He}$

CN

${}^{13}\text{C} + p \to {}^{14}\text{N} + \gamma$

99.9%

(^{15}O) ${}^{15}\text{O} \to {}^{15}\text{N} + e^+ + \nu_e$

${}^{14}\text{N} + p \to {}^{15}\text{O} + \gamma$

0.1%

${}^{15}\text{N} + p \to {}^{16}\text{O} + \gamma$

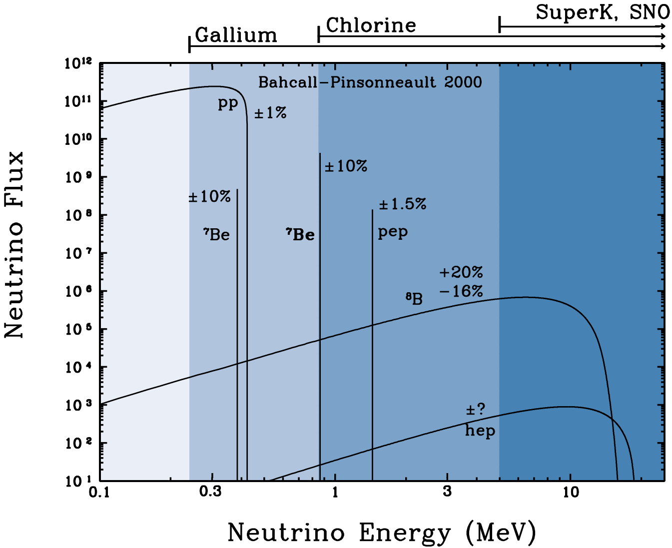
${}^{17}\text{O} + p \to {}^{14}\text{N} + {}^4\text{He}$

${}^{16}\text{O} + p \to {}^{17}\text{F} + \gamma$

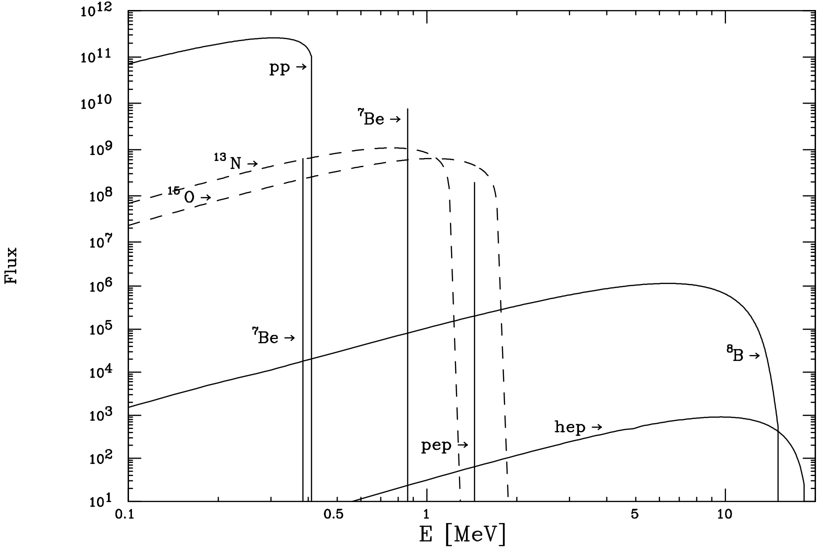
${}^{17}\text{F} \to {}^{17}\text{O} + e^+ + \nu_e$ **(^{17}F)**

Bahcall SSMs

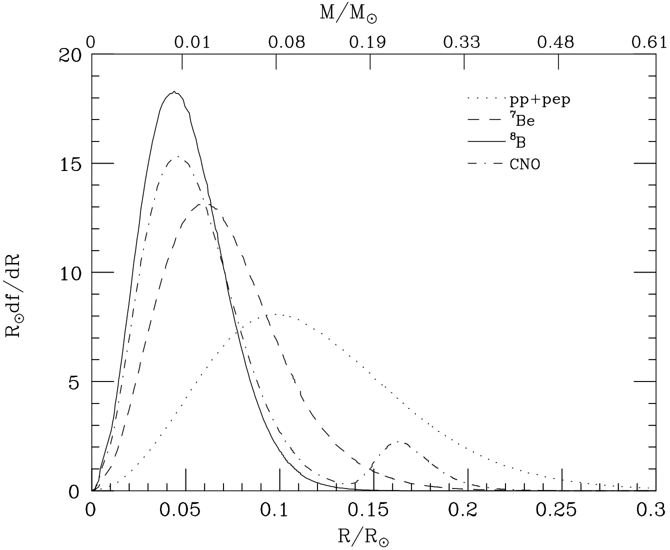
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



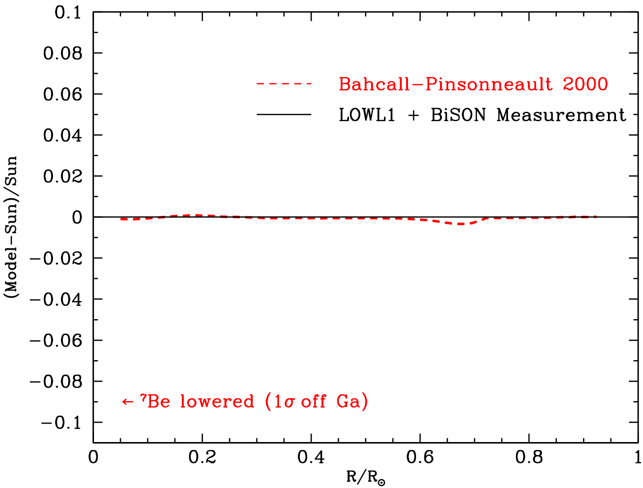
[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between $0.05\,R_\odot$ and $0.95\,R_\odot$ and is 0.08% for the deep interior region, $r < 0.25\,R_\odot$, in which neutrinos are produced

Homestake

$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$ [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e. $\Rightarrow \Phi_\mu \simeq 4\,\mathrm{m}^{-2}\,\mathrm{day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

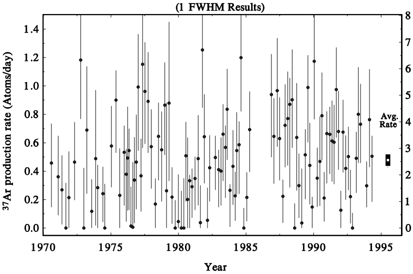
615 tons of tetrachloroethylene (C_2Cl_4), 2.16×10^{30} atoms of ^{37}Cl (133 tons)

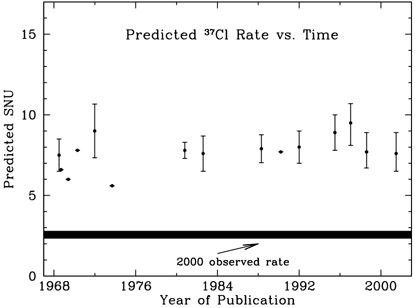
energy threshold: $E_{\mathrm{th}}^{\mathrm{Cl}} = 0.814\,\mathrm{MeV} \Longrightarrow {}^{8}\mathrm{B}\,,\ {}^{7}\mathrm{Be}\,,\ pep\,,\ hep\,,\ {}^{13}\mathrm{N}\,,\ {}^{15}\mathrm{O}\,,\ {}^{17}\mathrm{F}$

1970–1994, 108 extractions $\Longrightarrow$ $\dfrac{R_{\mathrm{Cl}}^{\mathrm{exp}}}{R_{\mathrm{Cl}}^{\mathrm{SSM}}} = 0.34 \pm 0.03$ [APJ 496 (1998) 505]

$R_{\mathrm{Cl}}^{\mathrm{exp}} = 2.56 \pm 0.23\,\mathrm{SNU}$ $R_{\mathrm{Cl}}^{\mathrm{SSM}} = 7.6^{+1.3}_{-1.1}\,\mathrm{SNU}$

$1\ \mathrm{SNU} = 10^{-36}$ events atom^{-1} s^{-1}





Gallium Experiments

SAGE, GALLEX, GNO

radiochemical experiments

$$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$$ [Kuzmin (1965)]

threshold: $E_{\text{th}}^{\text{Ga}} = 0.233\,\text{MeV} \Longrightarrow pp\,,\ {}^{7}\text{Be}\,,\ {}^{8}\text{B}\,,\ pep\,,\ hep\,,\ {}^{13}\text{N}\,,\ {}^{15}\text{O}\,,\ {}^{17}\text{F}$

SAGE+GALLEX+GNO $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{exp}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.56 \pm 0.03$

$R_{\text{Ga}}^{\text{exp}} = 72.4 \pm 4.7\,\text{SNU}$ $\qquad$ $R_{\text{Ga}}^{\text{SSM}} = 128^{+9}_{-7}\,\text{SNU}$

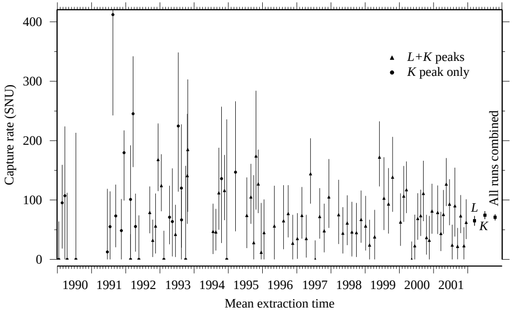
SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic ^{71}Ga, 2000 m deep, 4700 m.w.e. $\Rightarrow \Phi_\mu \simeq 2.6\,\text{m}^{-2}\,\text{day}^{-1}$

detector test: ^{51}Cr Source: $R = 0.95^{+0.11}_{-0.10}{}^{+0.06}_{-0.05}$ [PRC 59 (1999) 2246]

1990 – 2001 $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.54 \pm 0.05$ [astro-ph/0204245]



GALLEX: GALLium EXperiment

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

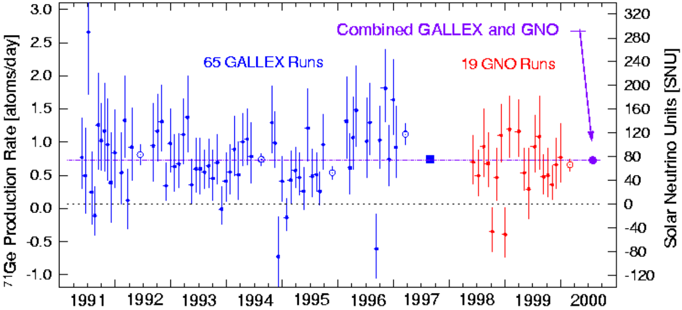
30.3 tons of gallium in 101 tons of gallium chloride ($GaCl_3$-HCl) solution

May 1991 – Jan 1997 $\Longrightarrow$ $\dfrac{R_{\mathrm{Ga}}^{\mathrm{GALLEX}}}{R_{\mathrm{Ga}}^{\mathrm{SSM}}} = 0.61 \pm 0.06$ [PLB 477 (1999) 127]

GNO: Gallium Neutrino Observatory

continuation of GALLEX: 30.3 tons of gallium

May 1998 – Jan 2000 $\Longrightarrow$ $\frac{R_{\text{Ga}}^{\text{GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.51 \pm 0.08$ [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

Kamiokande

water Cherenkov detector $\nu + e^- \to \nu + e^-$

Sensitive to ν_e, ν_μ, ν_τ, but $\sigma(\nu_e) \simeq 6\,\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}\,,\ hep$

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{SSM}}} = 0.55 \pm 0.08$$

[PRL 77 (1996) 1683]

Super-Kamiokande

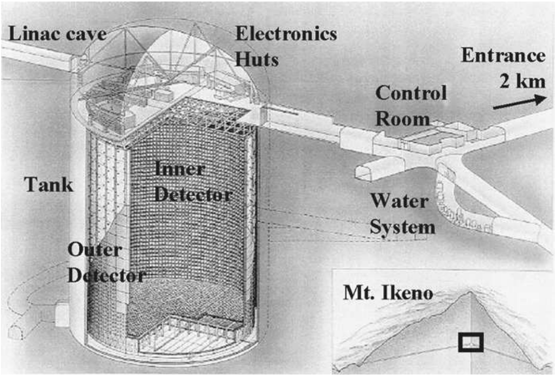
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}$, *hep*

1996 – 2001 (1496 days)

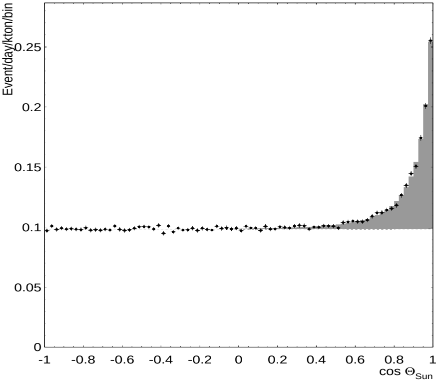
$$\frac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{SSM}}} = 0.465 \pm 0.015$$ [SK, PLB 539 (2002) 179]



the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

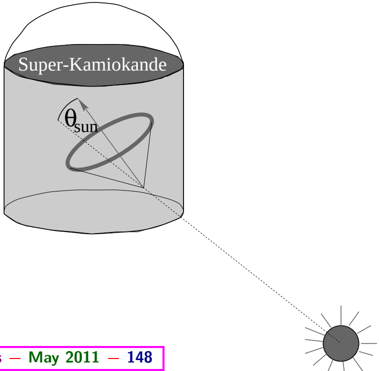
Super-Kamiokande $\cos\theta_{\mathrm{sun}}$ distribution



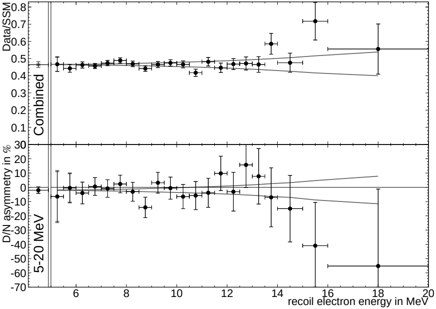
[Smy, hep-ex/0208004]

the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

the peak at $\cos\theta_{\mathrm{sun}} = 1$ is due to solar neutrinos

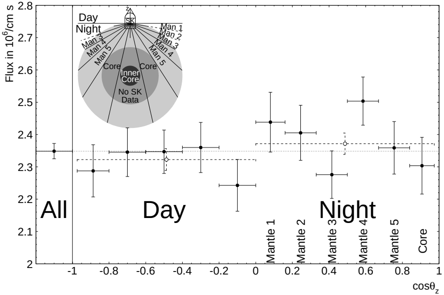


Super-Kamiokande energy spectrum normalized to BP2000 SSM



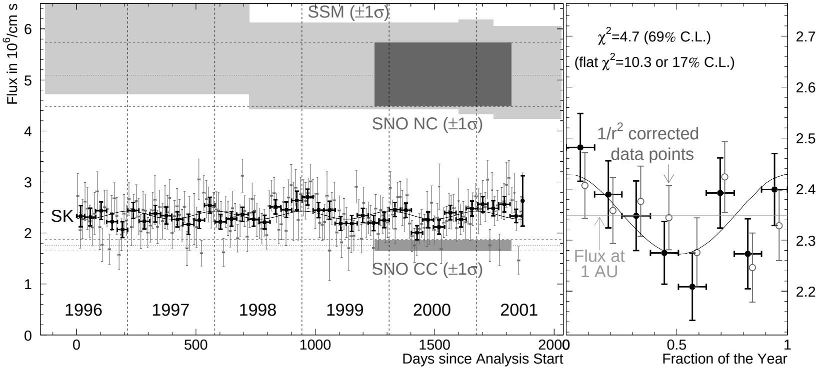
Day-Night asymmetry as a function of energy

solar zenith angle (θ_z) dependence of Super-Kamiokande data



[Smy, hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D_2O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

$$\begin{array}{ll} \text{CC:} & \nu_e + d \to p + p + e^- \\ \text{NC:} & \nu + d \to p + n + \nu \\ \text{ES:} & \nu + e^- \to \nu + e^- \end{array}$$

$$\left.\begin{array}{l} \text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2\,\text{MeV} \\ \text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2\,\text{MeV} \\ \text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0\,\text{MeV} \end{array}\right\} \Longrightarrow {}^8\text{B},\ hep$$

D_2O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.01 \pm 0.13$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.31 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.03 \pm 0.09$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

$$\Phi^{\text{SNO}}_{\nu_e} = 1.76 \pm 0.11 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1}$$

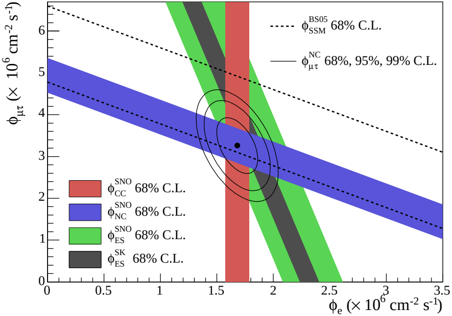
$$\Phi^{\text{SNO}}_{\nu_\mu,\nu_\tau} = 5.41 \pm 0.66 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1}$$

SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]



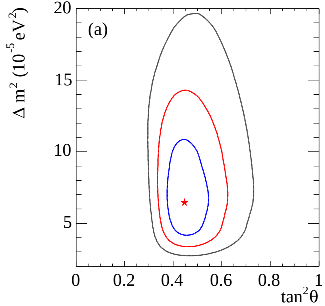
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5} \, \text{eV}^2$$

$$\tan^2 \vartheta \simeq 0.45$$



[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km

6.7% of flux from one reactor at 88 km
79% of flux from 26 reactors at 138–214 km
14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

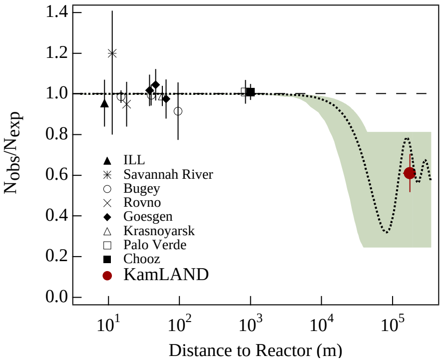
data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$
observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

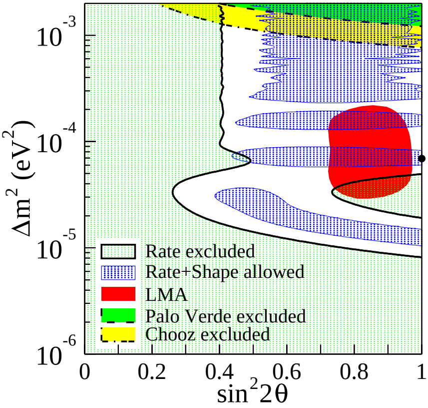
99.95% C.L. evidence
of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



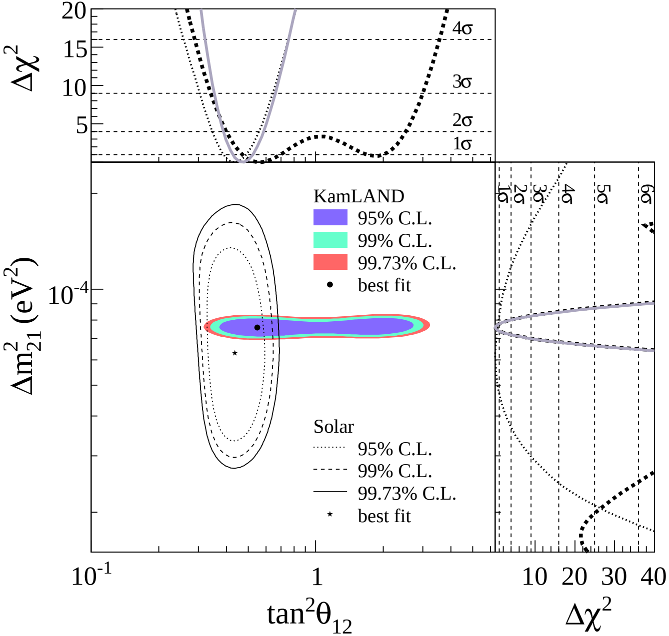
Shade: 95% C.L. LMA

$$\text{Curve:}\begin{cases}\Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2\\ \sin^2 2\vartheta = 0.83\end{cases}$$

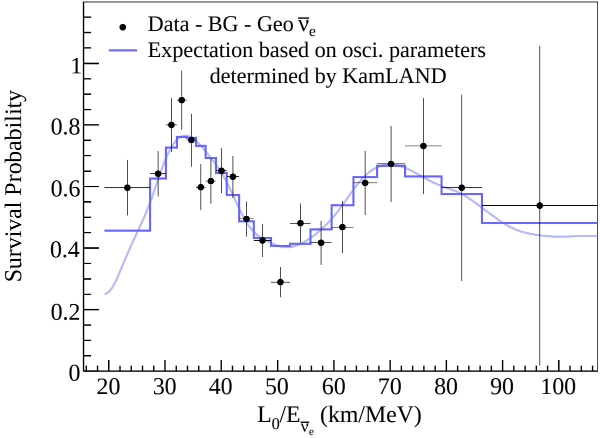


95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

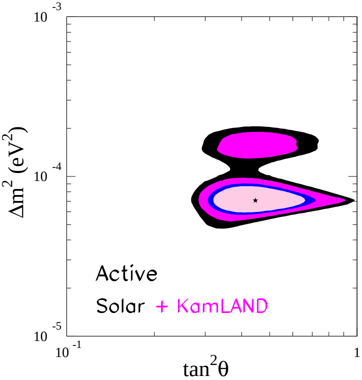


[KamLAND, PRL 100 (2008) 221803]



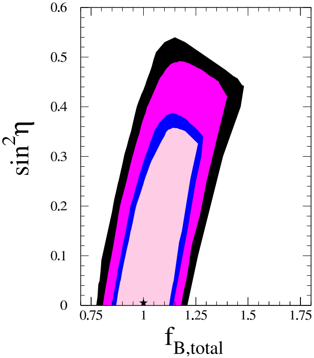
[KamLAND, PRL 100 (2008) 221803]

Sterile Neutrinos in Solar Neutrino Flux?



90%, 95%, 99%, 99.73% (3σ) C.L.

[Bahcall, Gonzalez-Garcia, Pena-Garay, JHEP 0302 (2003) 009]



$$\nu_e \to \cos\eta\,\nu_a + \sin\eta\,\nu_s$$

$$\sin^2\eta < 0.52\,(3\sigma)$$

$$f_{\mathrm{B,total}} = \frac{\Phi_{^8\mathrm{B}}}{\Phi_{^8\mathrm{B}}^{\mathrm{SSM}}} = 1.00 \pm 0.06$$

Determination of Solar Neutrino Fluxes

[Bahcall, Peña-Garay, hep-ph/0305159]

fit of solar and KamLAND neutrino data with fluxes as free parameters

+ luminosity constraint

$$\sum_r \alpha_r \Phi_r = K_\odot \quad (r = pp, pep, hep, {}^7\text{Be}, {}^8\text{B}, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F})$$

$$K_\odot \equiv \mathcal{L}_\odot / 4\pi (1\text{a.u.})^2 = 8.534 \times 10^{11}\,\text{MeV}\,\text{cm}^{-2}\,\text{s}^{-1}$$

solar constant

$$\Delta m^2 = 7.3^{+0.4}_{-0.6}\,\text{eV}^2 \qquad \tan^2\vartheta = 0.42^{+0.08}_{-0.06}\,\left({}^{+0.39}_{-0.19}\right)$$

$$\frac{\Phi_{^8\text{B}}}{\Phi^{\text{SSM}}_{^8\text{B}}} = 1.01^{+0.06}_{-0.06}\,\left({}^{+0.22}_{-0.17}\right)$$

moderate uncertainty

will improve with new SNO NC data (salt phase)

$$\frac{\Phi_{^7\text{Be}}}{\Phi^{\text{SSM}}_{^7\text{Be}}} = 0.97^{+0.28}_{-0.54}\,\left({}^{+0.85}_{-0.97}\right)$$

large uncertainty

needs ^{7}Be experiment (KamLAND, Borexino?)

$$\frac{\Phi_{pp}}{\Phi^{\text{SSM}}_{pp}} = 1.02^{+0.02}_{-0.02}\,\left({}^{+0.07}_{-0.07}\right)$$

small uncertainty

CNO luminosity: $\mathcal{L}_{\text{CNO}} / \mathcal{L}_\odot = 0.0^{+2.8}_{-0.0}\,\left({}^{+7.3}_{-0.0}\right)$

[Bahcall, Gonzalez-Garcia, Peña-Garay, PRL 90 (2003) 131301]

Details of Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{\mathrm{d}\ln A}{\mathrm{d}x}\right|_{\text{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2}EG_{\text{F}}N_e^{\text{c}}\exp\left(-\frac{x}{x_0}\right) \quad \Longrightarrow \quad \left|\frac{\mathrm{d}\ln A}{\mathrm{d}x}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3\times 10^{-15}\,\text{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad \Longrightarrow \quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad \gamma \simeq 2\times 10^4\left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad \Longrightarrow \quad P_{\text{c}} \ll 1 \quad \Longrightarrow \quad \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2}\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta^0_{\mathrm{M}} = \frac{\Delta m^2 \cos 2\vartheta - A^0_{\mathrm{CC}}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A^0_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, hep-ph/0305159]

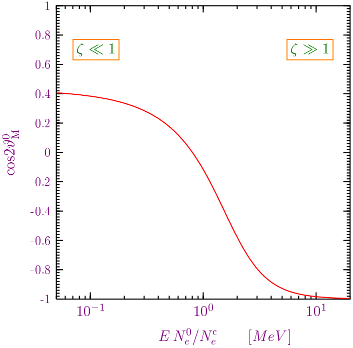
$$\zeta = \frac{A^0_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N^0_e}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right) \left(\frac{N^0_e}{N^c_e}\right)$$

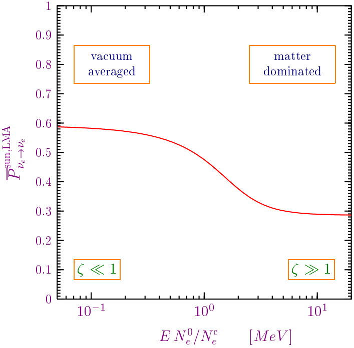
$$\zeta \ll 1 \implies \vartheta^0_{\mathrm{M}} \simeq \vartheta \implies \overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} \simeq 1 - \frac{1}{2}\sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \implies \vartheta^0_{\mathrm{M}} \simeq \pi/2 \implies \overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} \simeq \sin^2\vartheta$$

matter dominated survival probability

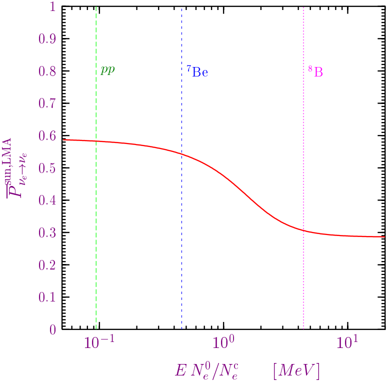




$\langle E\rangle_{pp} \simeq 0.27\,\text{MeV}\,,\ \langle r_0\rangle_{pp} \simeq 0.1\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c\rangle_{pp} \simeq 0.094\,\text{MeV}$

$E_{^7\text{Be}} \simeq 0.86\,\text{MeV}\,,\ \langle r_0\rangle_{^7\text{Be}} \simeq 0.06\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c\rangle_{^7\text{Be}} \simeq 0.46\,\text{MeV}$

$\langle E\rangle_{^8\text{B}} \simeq 6.7\,\text{MeV}\,,\ \langle r_0\rangle_{^8\text{B}} \simeq 0.04\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c\rangle_{^8\text{B}} \simeq 4.4\,\text{MeV}$



each neutrino experiment is mainly sensitive to one flux

each neutrino experiment is mainly sensitive to ϑ

accurate *pp* experiment can improve determination of ϑ

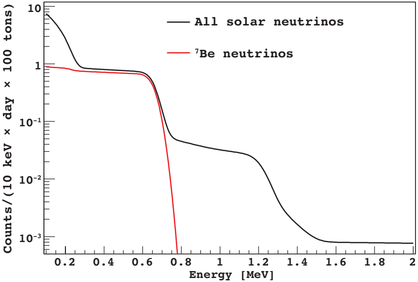
[Bahcall, Peña-Garay, hep-ph/0305159]

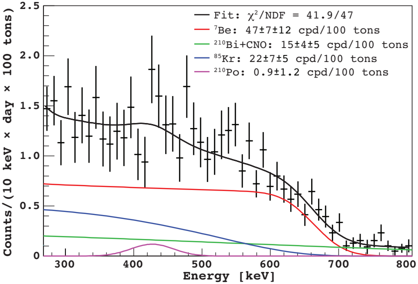
BOREXino

[BOREXino, arXiv:0708.2251]

Real-time measurement of ^{7}Be solar neutrinos ($0.862\,\text{MeV}$)

$$\nu + e \to \nu + e \qquad E = 0.862\,\text{MeV} \quad \Longrightarrow \quad \sigma_{\nu_e} \simeq 5.5\,\sigma_{\nu_\mu,\nu_\tau}$$





$$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad n_{\text{exp}} = 47 \pm 7 \pm 12\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$$

$$n_{\text{the}}^{\text{osc}} = 49 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad (n_{\text{the}}^{\text{no-osc}} - n_{\text{exp}})/\Delta n \simeq 1.9$$

Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
  - Atmospheric Neutrinos
  - Super-Kamiokande Up-Down Asymmetry
  - Fit of Super-Kamiokande Atmospheric Data
  - Kamiokande, Soudan-2, MACRO and MINOS
  - K2K
  - MINOS
  - Sterile Neutrinos in Atmospheric Neutrino Flux?
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
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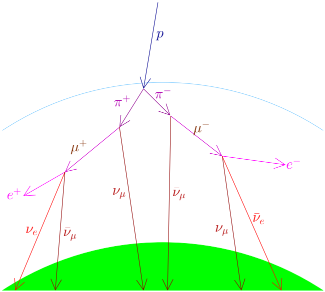
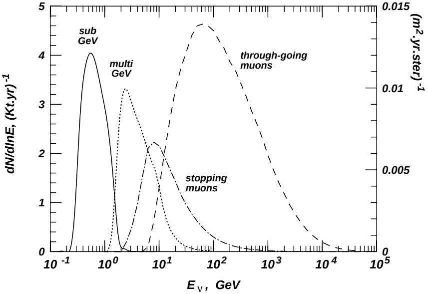
Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

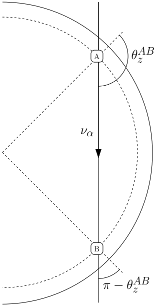
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB}) \qquad \phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z) = \phi^{(A)}_{\nu_\alpha}(\pi - \theta_z)$$

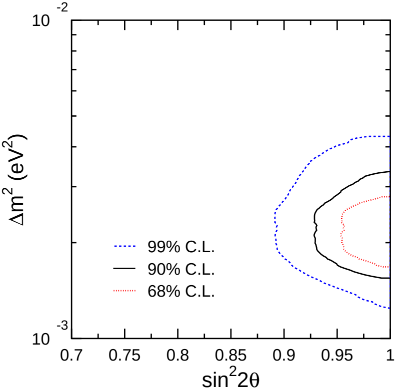
(December 1998)

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$\nu_\mu \to \nu_\tau$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20\text{events/yr}$
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\text{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$$\boxed{N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}}$$

$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

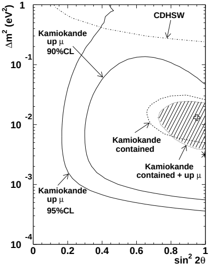
Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

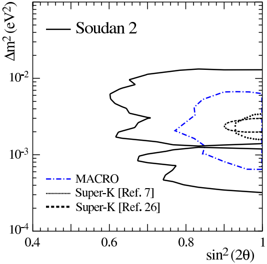
$L \simeq 732\,\text{km}$ $\langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

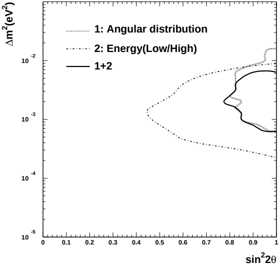
Kamiokande, Soudan-2, MACRO and MINOS



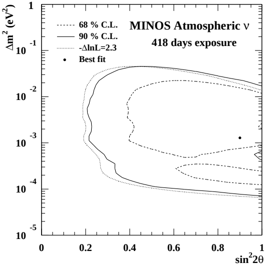
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



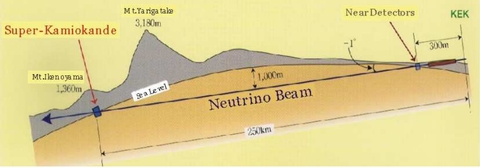
[MACRO, hep-ex/0304037]



[MINOS, hep-ex/0512036]

K2K

confirmation of atmospheric allowed region (June 2002)

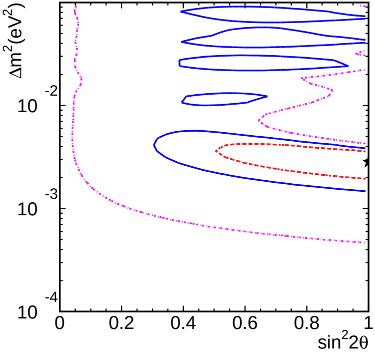


KEK to Kamioka

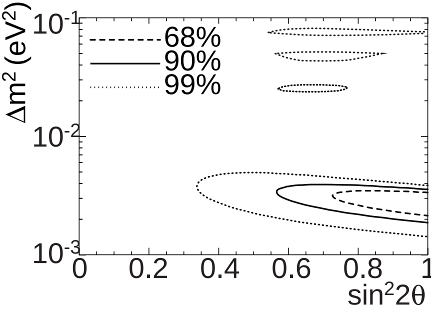
(Super-Kamiokande)

250 km

$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]



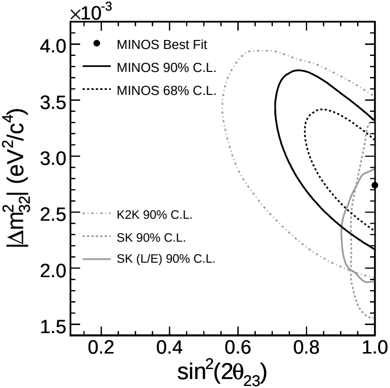
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

http://www-numi.fnal.gov/

Near Detector: 1 km

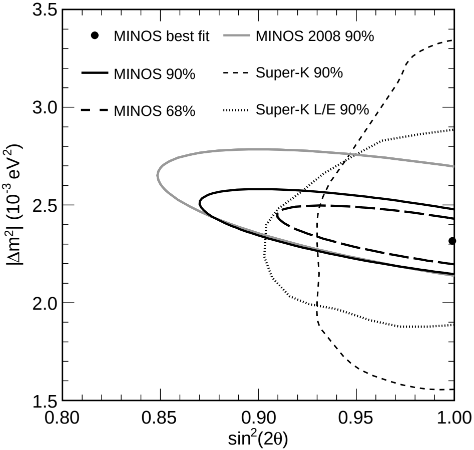


$$\nu_\mu \to \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \text{ @ } 68\%\,CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

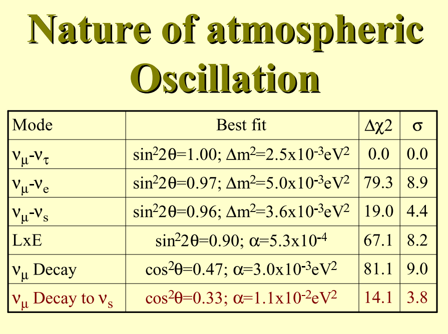


$$|\Delta m^2_{31}| = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\vartheta_{23} > 0.90 \quad (90\%\ \mathrm{C.L.})$$

[arXiv:1103.0340v1]

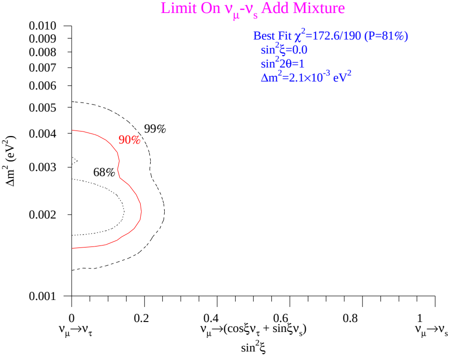
Sterile Neutrinos in Atmospheric Neutrino Flux?



Nature of atmospheric Oscillation

Mode	Best fit	$\Delta\chi 2$	σ
ν_μ-ν_τ	$\sin^2 2\theta=1.00$; $\Delta m^2=2.5\text{x}10^{-3}\text{eV}^2$	0.0	0.0
ν_μ-ν_e	$\sin^2 2\theta=0.97$; $\Delta m^2=5.0\text{x}10^{-3}\text{eV}^2$	79.3	8.9
ν_μ-ν_s	$\sin^2 2\theta=0.96$; $\Delta m^2=3.6\text{x}10^{-3}\text{eV}^2$	19.0	4.4
LxE	$\sin^2 2\theta=0.90$; $\alpha=5.3\text{x}10^{-4}$	67.1	8.2
ν_μ Decay	$\cos^2\theta=0.47$; $\alpha=3.0\text{x}10^{-3}\text{eV}^2$	81.1	9.0
ν_μ Decay to ν_s	$\cos^2\theta=0.33$; $\alpha=1.1\text{x}10^{-2}\text{eV}^2$	14.1	3.8

[Smy (SK), Moriond 2002]



[Nakaya (SK), hep-ex/0209036]

Phenomenology of Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
  - Experimental Evidences of Neutrino Oscillations
  - Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Homestake
Kamiokande
GALLEX/GNO & SAGE
Super-Kamiokande
SNO
BOREXino

Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$$\to \begin{cases} \Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2 \\ \tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Kamiokande
IMB
Super-Kamiokande
MACRO
Soudan-2

Accelerator
ν_μ disappearance

(K2K & MINOS)

$$\to \begin{cases} \Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07 \end{cases}$$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

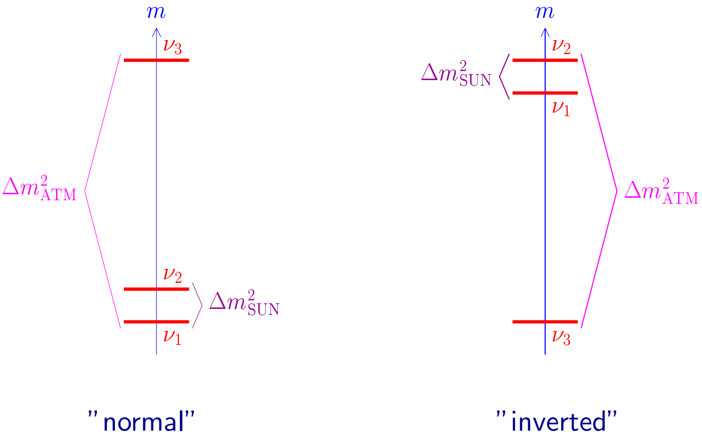
three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

SOL →

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

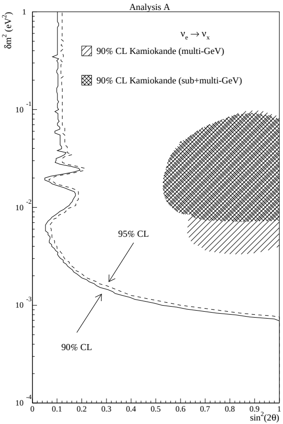
↑ ATM & LBL

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$

[Bilenky, Giunti, PLB 444 (1998) 379]



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau 3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

Effective ATM and LBL Oscillation Probability in Vacuum

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\beta} &= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i E_k t} \right|^2 * \left| e^{i E_1 t} \right|^2 \\
&= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i (E_k - E_1) t} \right|^2 \to \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2
\end{aligned}
$$

$$
E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \Delta m_{31}^2 \to \Delta m^2
$$

$$
P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right|^2
$$

$$
U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}
$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left| \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3} \left[1 - \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right] \right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E} \right) \\
&\qquad - 2\delta_{\alpha\beta} |U_{\alpha 3}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \sin^2 \frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E} \right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2 \right) \sin^2\left(\frac{\Delta m^2 L}{4E} \right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \left(\begin{matrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \\ U_{\tau 1} & U_{\tau 2} \end{matrix} \quad \underset{\substack{\uparrow \\ \text{LBL}}}{\boxed{\begin{matrix} U_{e3} \\ U_{\mu 3} \\ U_{\tau 3} \end{matrix}}}\right)$$

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \simeq 4|U_{e3}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu3}|^2 \left(1 - |U_{\mu3}|^2\right)$$

$$|U_{\mu3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - \sin^2 2\vartheta_{\mu\mu}}\right)$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2 |U_{\mu3}|^2$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\text{eV}^2 \qquad |\Delta m^2_{31}| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2\vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

$$\sin^2\vartheta_{13} < 0.035 \quad (90\%\ \text{C.L.})$$

[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]

Current Research

measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

Bilarge Mixing

$$\boxed{|U_{e3}|^2 \ll 1}$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Longrightarrow \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \qquad = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \Longrightarrow \vartheta_A \simeq \frac{\pi}{4} \Longrightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

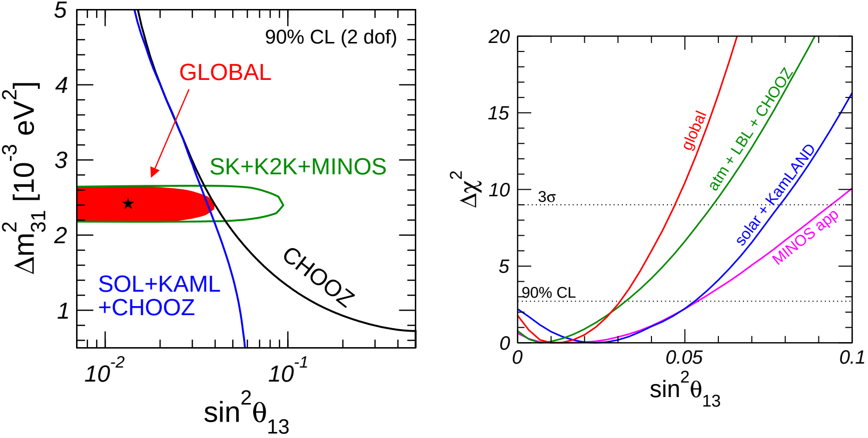
$$\text{Solar } \nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{CC}^{SNO}}{\Phi_{\nu_e}^{SSM}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6\,\text{MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \Longrightarrow U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

[Harrison, Perkins, Scott, hep-ph/0202074]

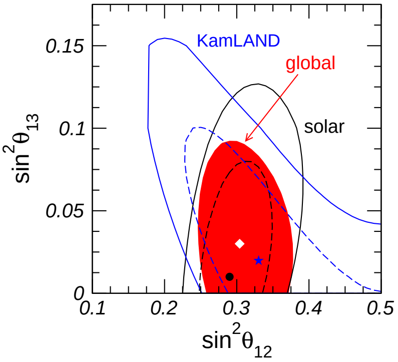


[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

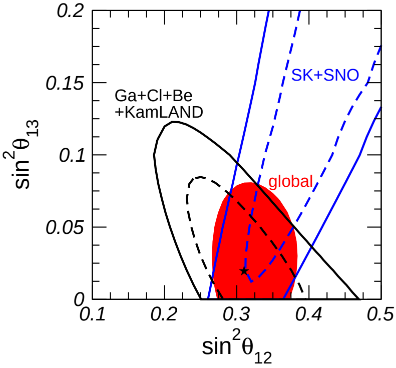
Hint of $\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]



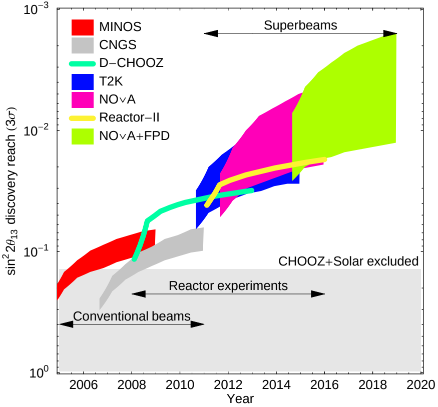
[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]



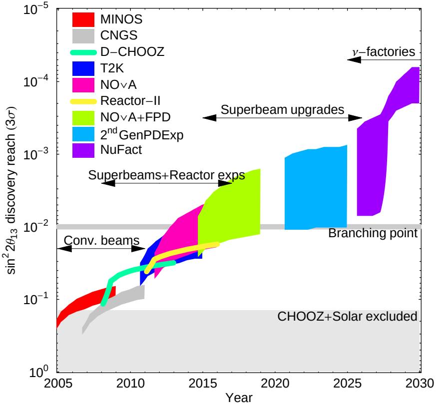
[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

$$P_{\overset{(-)}{\nu_e}\to\overset{(-)}{\nu_e}} \simeq \begin{cases} \left(1-\sin^2\vartheta_{13}\right)^2 \left(1-0.5\sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1-\sin^2\vartheta_{13}\right)^2 \sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

The Hunt for ϑ_{13}



3σ sensitivities. Bands reflect dependence of sensitivity on the CP violating phase δ_{13}.



"Branching point" refers to the decision between an upgraded superbeam and/or detector and a neutrino factory program. Neutrino factory is assumed to switch polarity after 2.5 years.

[Physics at a Fermilab Proton Driver, Albrow et al, hep-ex/0509019]

Effective LBL Oscillation Probabilities

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad \alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \qquad A = \frac{2EV}{\Delta m^2_{31} L} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \alpha \ll 1$$

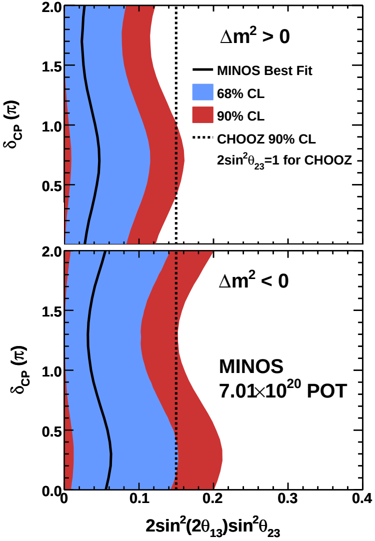
$$P^{\text{LBL}}_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2 \Delta - \alpha^2 \Delta^2 \sin^2 2\vartheta_{12}$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu \to \nu_e} \simeq\ & \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & + \alpha \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \delta_{13}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & + \alpha^2 \sin^2 2\vartheta_{12} \cos^2 \vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

[Mezzetto, Schwetz, arXiv:1003.5800]

MINOS

[arXiv:1006.0996v1]



54 ν_e events in FD

background: $49.1 \pm 7.0 \pm 2.7$

0.7σ excess

if $\delta_{13} = 0$

$$2\sin^2 2\vartheta_{13} \sin^2\vartheta_{23} < \begin{cases} 0.12 & \text{(NH)} \\ 0.20 & \text{(IH)} \end{cases}$$

(90% C.L.)

CP Violation

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to small ϑ_{13}
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

Mass Hierarchy

- $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $\cos 2\vartheta_{13} = \frac{2EV}{\Delta m^2_{13}}$

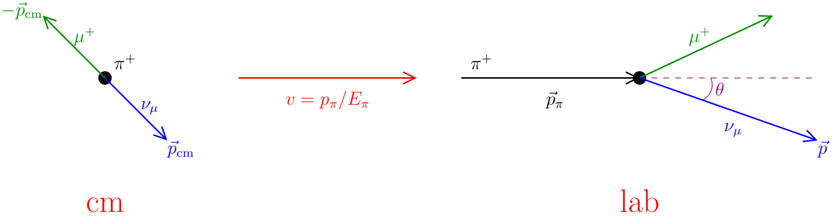
  Requires $\Delta m^2_{13} > 0$

- $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $\cos 2\vartheta_{13} = -\frac{2EV}{\Delta m^2_{13}}$

  Requires $\Delta m^2_{13} < 0$

Off-Axis Experiments

high-intensity WB beam
detector shifted by a small angle from axis of beam
almost monochromatic neutrino energy



$$E_{\rm cm} = p_{\rm cm} = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 29.79\,\text{MeV}$$

$$\gamma = \left(1 - v^2\right)^{-1/2} = E_\pi / m_\pi \gg 1$$

$$\begin{cases} E = \gamma\,(E_{\rm cm} + v\,p^z_{\rm cm}) \\ p^z = \gamma\,(v\,E_{\rm cm} + p^z_{\rm cm}) \end{cases}$$

$$p^z = p\,\cos\theta \qquad \Longrightarrow \qquad E = \frac{E_{\rm cm}}{\gamma\,(1 - v\,\cos\theta)}$$

$$\cos\theta \simeq 1 - \theta^2/2 \qquad \text{and} \qquad v \simeq 1$$

$$E = \frac{E_{\text{cm}}}{\gamma\,(1 - v\cos\theta)} \simeq \frac{\gamma\,(1+v)}{1 + \gamma^2\theta^2 v\,(1+v)/2}\,E_{\text{cm}} \simeq \frac{2\gamma}{1+\gamma^2\theta^2}\,E_{\text{cm}}$$

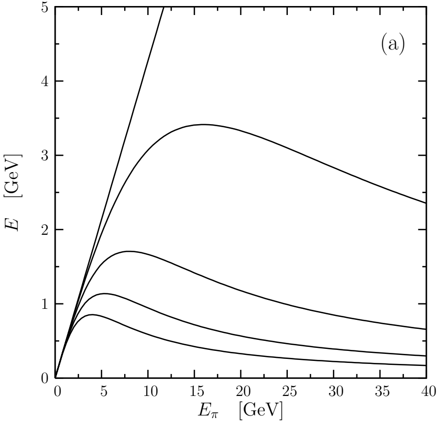
$$E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi}{1+\gamma^2\,\theta^2} = \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi\,m_\pi^2}{m_\pi^2 + E_\pi^2\,\theta^2}$$

- $\theta = 0 \quad \Longrightarrow \quad E \propto E_\pi$ WB beam
- $E_\pi\theta \gg m_\pi \quad \Longrightarrow \quad E \propto \dfrac{m_\pi^2}{E_\pi\,\theta^2}$ high-energy π^+ give low-energy ν_μ

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{1-\gamma^2\,\theta^2}{\left(1+\gamma^2\,\theta^2\right)^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq 0 \quad \text{for} \quad \theta = \gamma^{-1} = \frac{m_\pi}{E_\pi} \quad \Longrightarrow \quad E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{m_\pi}{2\theta} \simeq \frac{29.79\,\text{MeV}}{\theta}$$

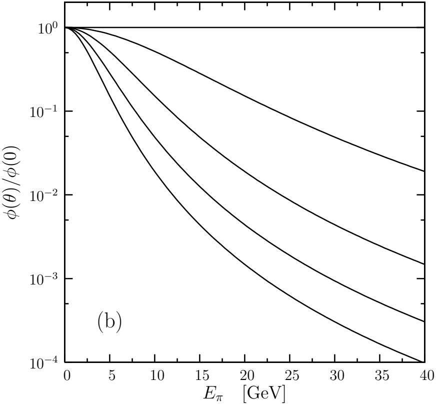
off-axis angle $\theta \simeq m_\pi / \langle E_\pi \rangle \quad \Longrightarrow \quad E \simeq \dfrac{29.79\,\text{MeV}}{\theta}$



$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

- E can be tuned on oscillation peak $E_{\text{peak}} = \Delta m^2 L / 2\pi$
- small $E \Longrightarrow$ short $L_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2} \Longrightarrow$ sensitivity to small values of Δm^2

$$\frac{\phi(\theta)}{\phi(0)} = \frac{1}{4}\left(\frac{2}{1+\gamma^2\,\theta^2}\right)^2$$



$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

flux suppression requires superbeam

Absolute Scale of Neutrino Masses

- Solar Neutrinos and KamLAND

- Atmospheric and LBL Oscillation Experiments

- Phenomenology of Three-Neutrino Mixing

- Absolute Scale of Neutrino Masses
  - Mass Hierarchy or Degeneracy?
  - Tritium Beta-Decay
  - Neutrinoless Double-Beta Decay
  - Cosmological Bound on Neutrino Masses

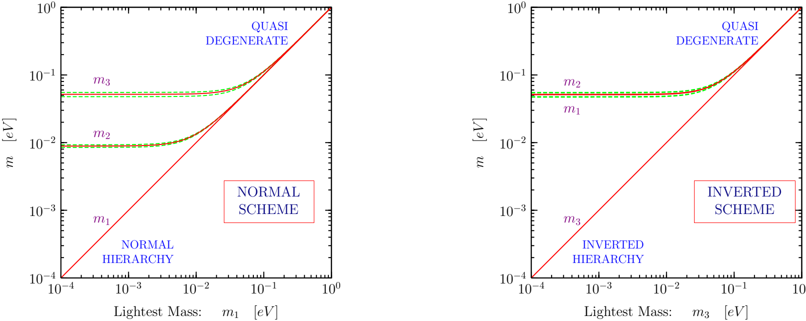
- Anomalies Beyond Three-Neutrino Mixing

- Conclusions

Mass Hierarchy or Degeneracy?

normal scheme

inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\mathrm{ATM}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

Tritium Beta-Decay

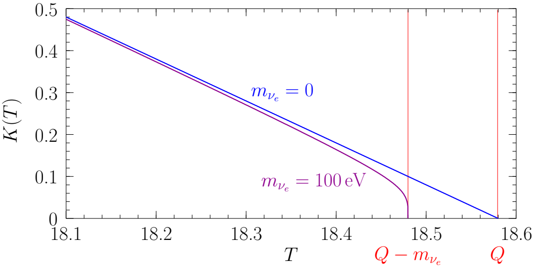
$^{3}\mathrm{H} \to {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_{e}$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$$

Mainz & Troitsk

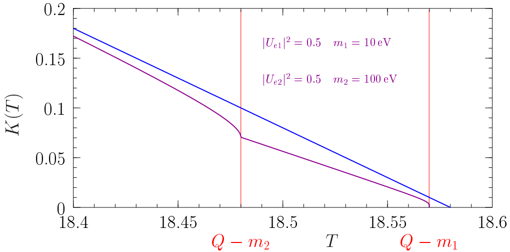
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2012)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters $\left(\sum_k |U_{ek}|^2 = 1 \right)$

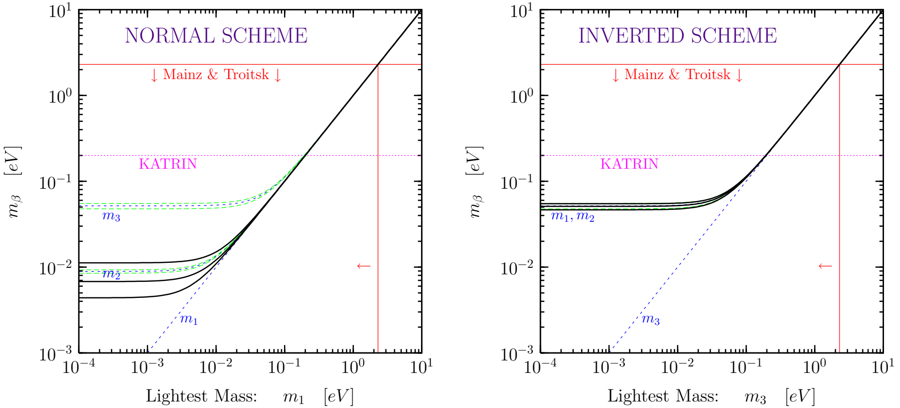
if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

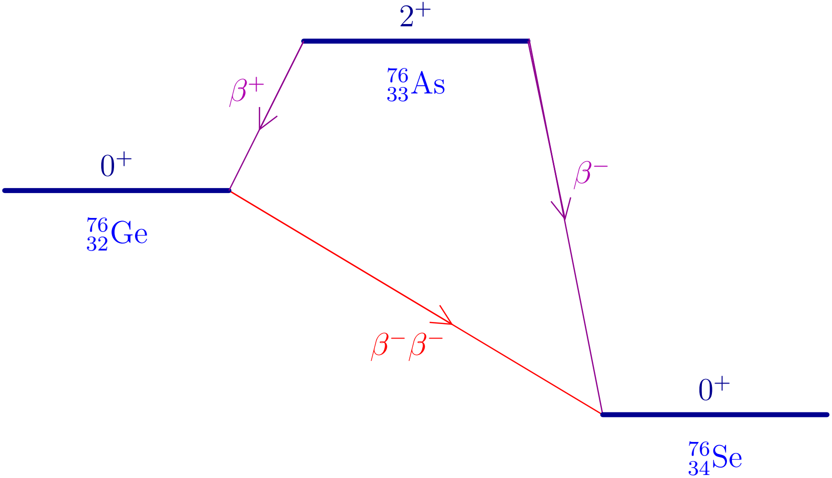
$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



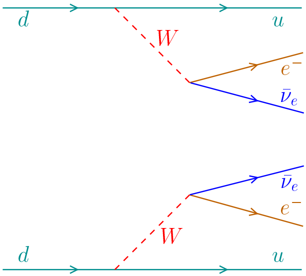
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu} \, |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

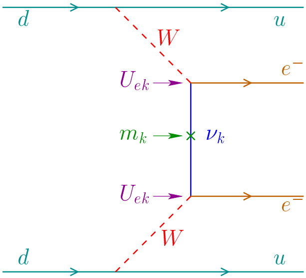


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, |m_{\beta\beta}|^2$$

effective
Majorana
mass $\quad m_{\beta\beta} = \sum_k U^2_{ek} \, m_k$

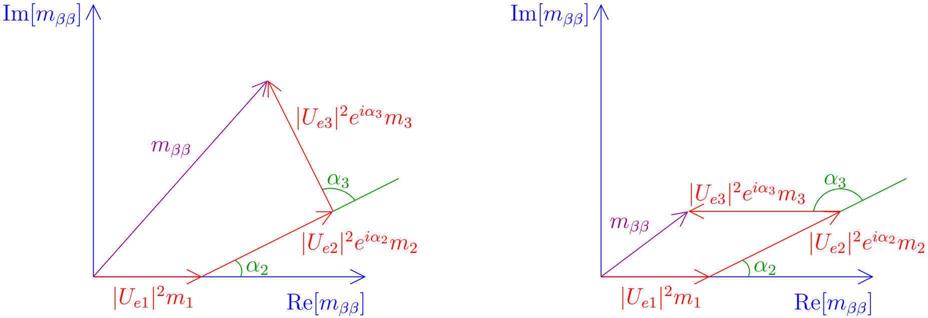


Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} ⇒ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$



Experimental Bounds

CUORICINO (^{130}Te) [arXiv:1012.3266]

$$\boxed{T_{1/2}^{0\nu} > 2.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T_{1/2}^{0\nu} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T_{1/2}^{0\nu} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T_{1/2}^{0\nu} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

$$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$$

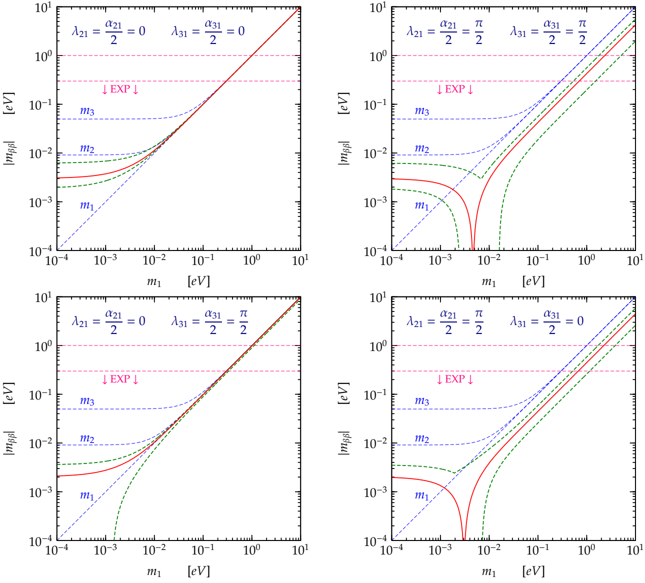
Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$

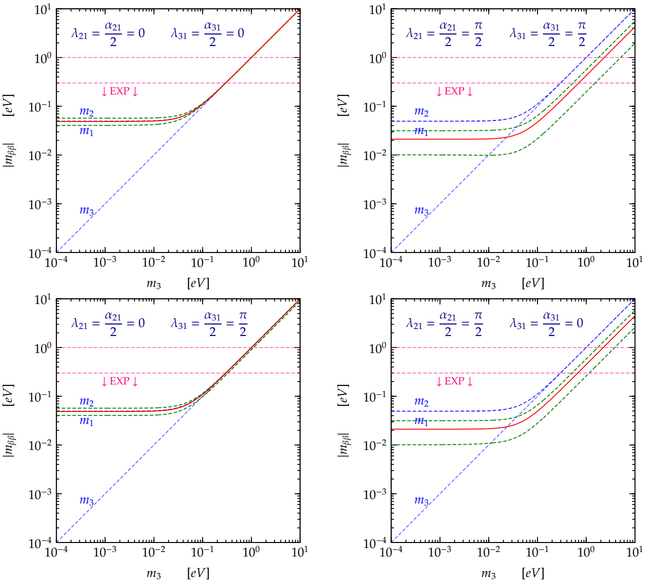
CP conservation

$$\alpha_{21} = 0 \, , \, \pi \qquad \alpha_{31} = 0 \, , \, \pi$$

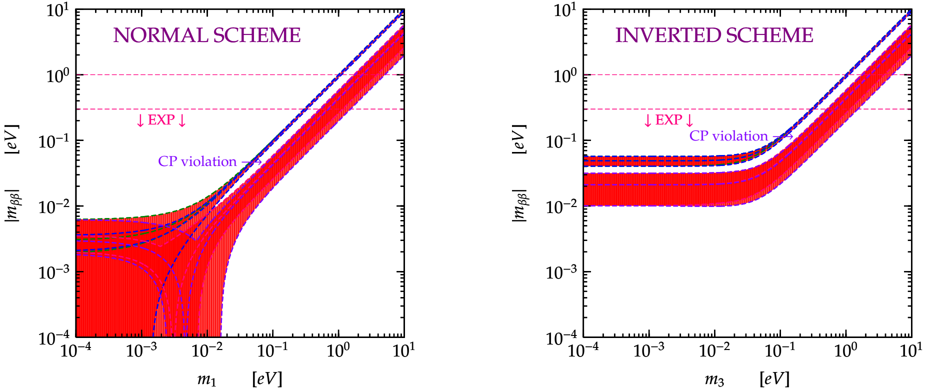
CP Conservation: Normal Scheme



CP Conservation: Inverted Scheme



$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$

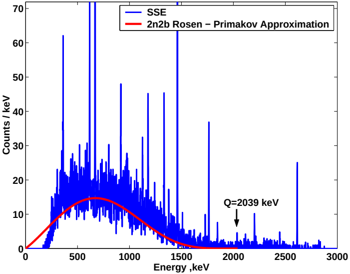


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

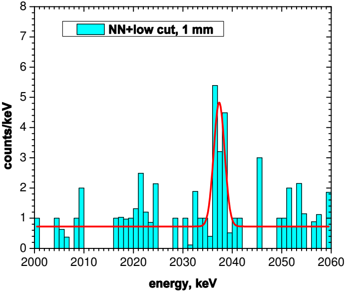
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

$T_{1/2}^{0\nu} = (2.23^{+0.44}_{-0.31}) \times 10^{25}$ y 6.5σ evidence   [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



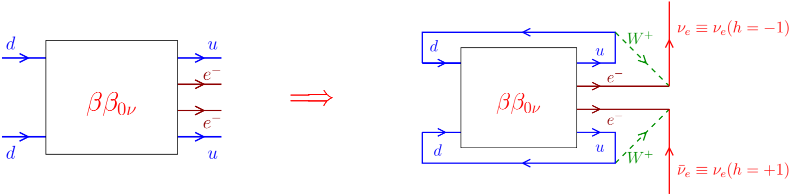
[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$ [MPLA 21 (2006) 1547]

if confirmed, very exciting (Majorana ν and large mass scale)

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

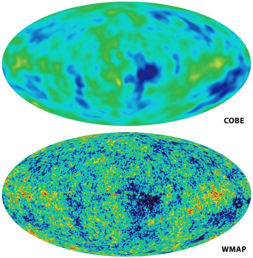


[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

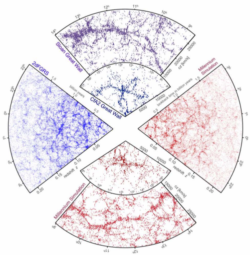
Majorana Mass Term

$$\boxed{\mathcal{L}_{eL}^{\mathrm{M}} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu_{eL}^{c}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu_{eL}^{c} \right)}$$

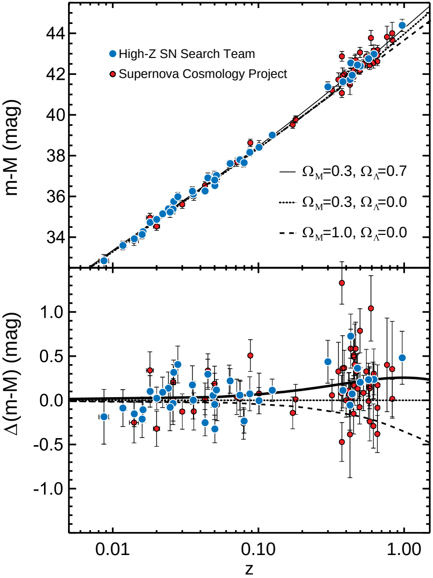
Cosmological Bound on Neutrino Masses



[WMAP, `http://map.gsfc.nasa.gov`]

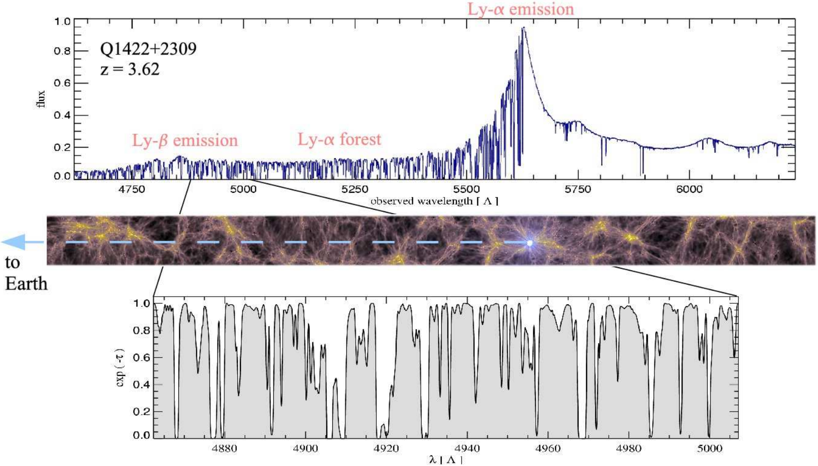


[Springel, Frenk, White, Nature 440 (2006) 1137]



[`http://cfa-www.harvard.edu/supernova/`]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_{\alpha}^{0} = 1215.67$ Å, $\lambda_{\beta}^{0} = 1025.72$ Å, $\lambda_{\gamma}^{0} = 972.54$ Å

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_{\beta}^{0}, (1+z_q)\lambda_{\alpha}^{0}]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

$$\text{Relic Neutrinos:} \quad T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$$

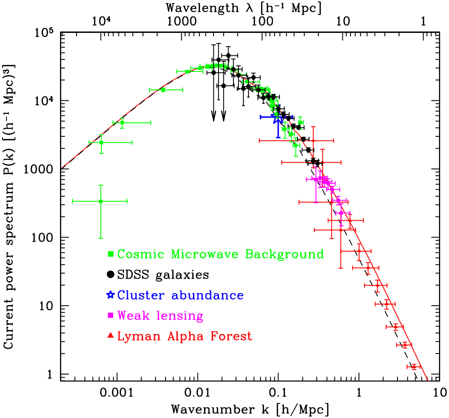
$$\text{number density:} \quad n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$$

$$\underset{\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)}{\text{density contribution:}} \quad \Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\,m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\,h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\mathrm{M}}^0 = 0.28\,,\ h = 0.72\,,\ \Omega_{\mathrm{B}}^0/\Omega_{\mathrm{M}}^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\mathrm{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right) \left(\frac{0.1}{\Omega_m\, h^2}\right)$$

for

$$k \gtrsim k_{\mathrm{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\sqrt{\Omega_m}\, h\,\mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

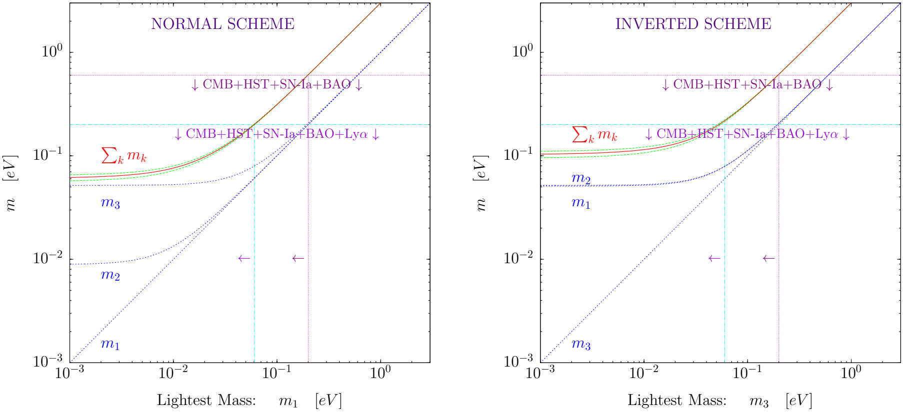
Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ.

$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO}$$

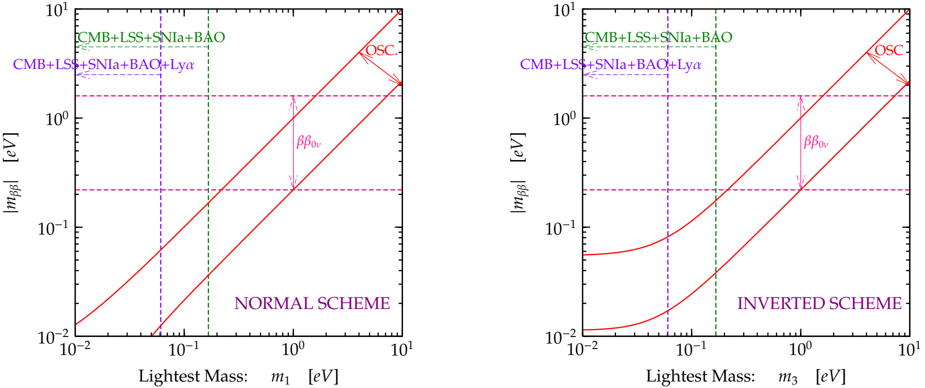
$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$



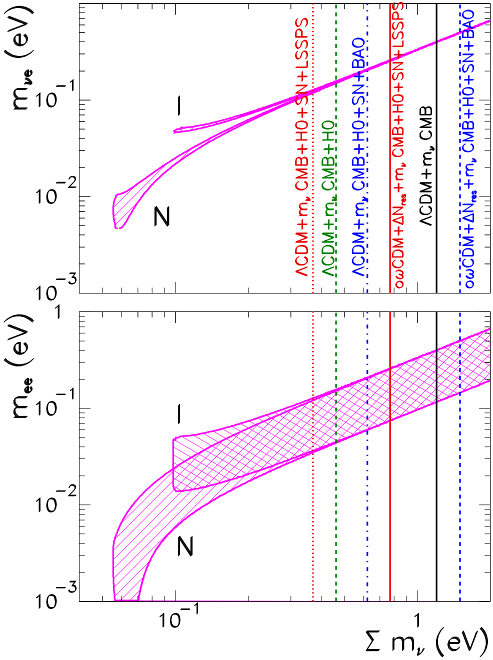
FUTURE: IF $\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ ($\sim 3\sigma$ range)

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]



tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal



95% allowed regions (2 dof)

95% upper bounds on $\sum m_\nu$

[Gonzalez-Garcia, Maltoni, Salvado,

JHEP08 (2010) 117, arXiv:1006.3795v2]

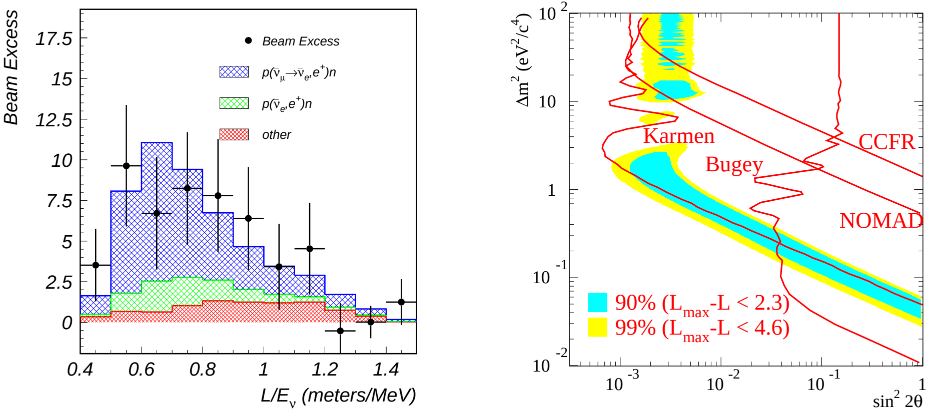
Anomalies Beyond Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- **Anomalies Beyond Three-Neutrino Mixing**
- Conclusions

LSND

[LSND, PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 30\,\text{m}$ $\quad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$

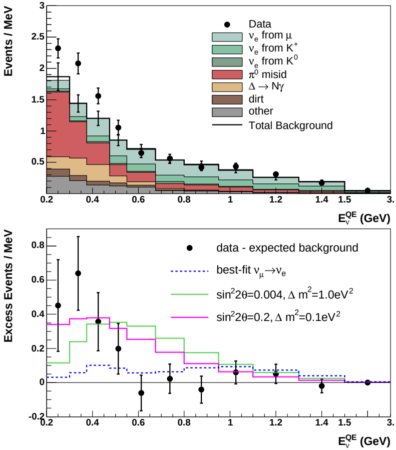


$$\Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}})$$

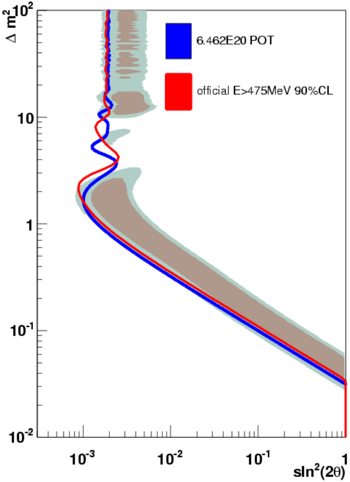
MiniBooNE Neutrinos

[PRL 98 (2007) 231801; PRL 102 (2009) 101802]

$\nu_\mu \to \nu_e$ $L \simeq 541\,\text{m}$ $475\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



[MiniBooNE, PRL 102 (2009) 101802, arXiv:0812.2243]



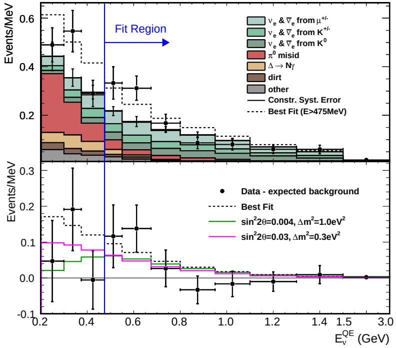
[Djurcic, arXiv:0901.1648]

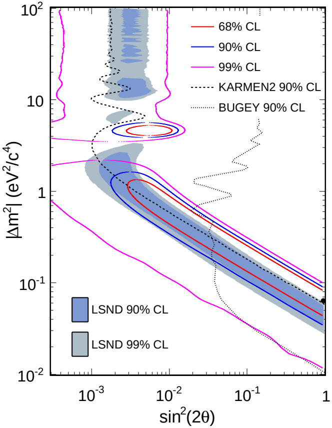
Low-Energy Anomaly!

MiniBooNE Antineutrinos

[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_\mu \to \bar{\nu}_e \qquad L \simeq 541\,\text{m} \qquad 475\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



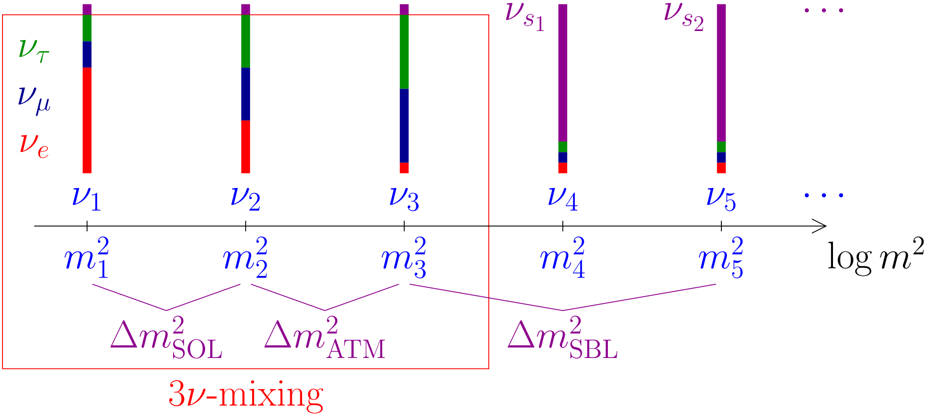


[MiniBooNE, PRL 105 (2010) 181801, arXiv:1007.1150]

Agreement with LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal!

Similar L/E but different L and $E \Longrightarrow$ Oscillations!

Beyond Three-Neutrino Mixing



Standard Model

- Neutrinos are the only massless fermions
- Neutrinos are the only fermions with only left-handed component ν_L

Extension of the SM: Massive Neutrinos

- Simplest extension: introduce right-handed component ν_R
- Dirac mass $m_{\text{D}} \overline{\nu_R} \nu_L$ $+$ Majorana mass $m_{\text{M}} \overline{\nu_R^c} \nu_R$
- $\nu_{eL}\,,\ \nu_{\mu L}\,,\ \nu_{\tau L}\ +\ \nu_{eR}\,,\ \nu_{\mu R}\,,\ \nu_{\tau R} \quad \Longrightarrow$ 6 massive Majorana neutrinos

Sterile Neutrinos

- Light anti-ν_R are called sterile neutrinos

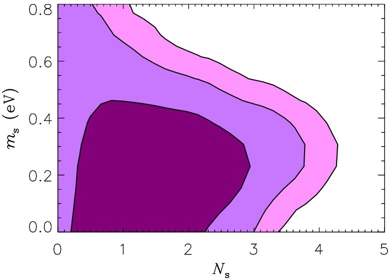
$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos
  - Indirect evidence through combined fit of data (current indication)
- Powerful window on new physics beyond the Standard Model

Cosmology

- N_s = number of thermalized sterile neutrinos (not necessarily integer)

- CMB and LSS in ΛCDM:



[Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301, arXiv:1006.5276]

$$N_s = 1.61 \pm 0.92 \qquad m_{\nu_s} < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

[Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, arXiv:1102.4774]

- BBN:

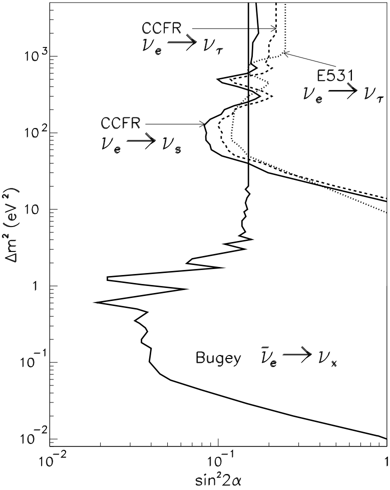
$$N_s = 0.22 \pm 0.59$$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]

$$N_s = 0.64^{+0.40}_{-0.35}$$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]

Direct Searches of Active-Sterile Transitions

CCFR

[PRD 59 (1999) 031101, arXiv:hep-ex/9809023]



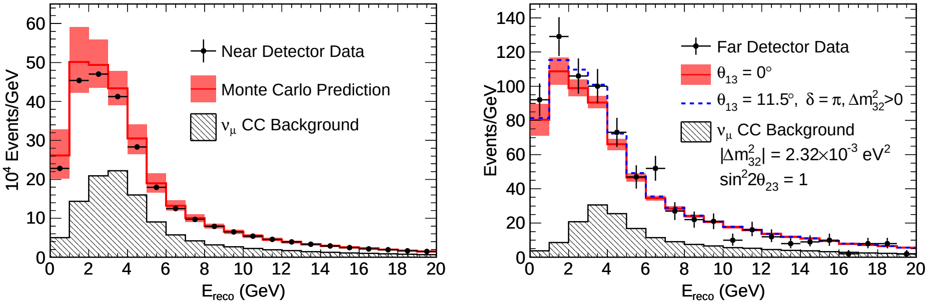
NC interactions $E \sim 100\,\text{GeV}$ $L \simeq 1.4\,\text{km}$

MINOS

NC sample: 89% efficiency and 61% purity

97% of ν_e-induced CC events misidentified as NC

[arXiv:1104.3922]



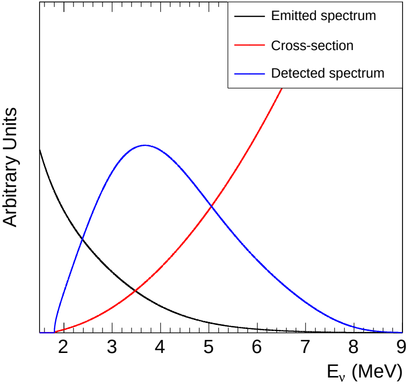
$L_{ND} = 1.04$ km $\qquad$ $L_{FD} = 735$ km

Model	ϑ_{13}	χ^2/D.O.F.	ϑ_{23}	ϑ_{24}	ϑ_{34}	f_s
$m_4 = m_1$	0	130.4/123	45.0^{+7}_{-7}	-	$0.0^{+17}_{-0.0}$	0.22
	11.5	128.5/123	45.6^{+7}_{-7}	-	$0.0^{+25}_{-0.0}$	0.40
$m_4 \gg m_3$	0	130.4/122	45.0^{+7}_{-7}	$0.0^{+5}_{-0.0}$	$0.0^{+17}_{-0.0}$	0.22
	11.5	128.5/122	45.6^{+7}_{-7}	$0.0^{+5}_{-0.0}$	$0.0^{+25}_{-0.0}$	0.40

90% C.L.

New Calculation of Reactor $\bar{\nu}_e$ Flux

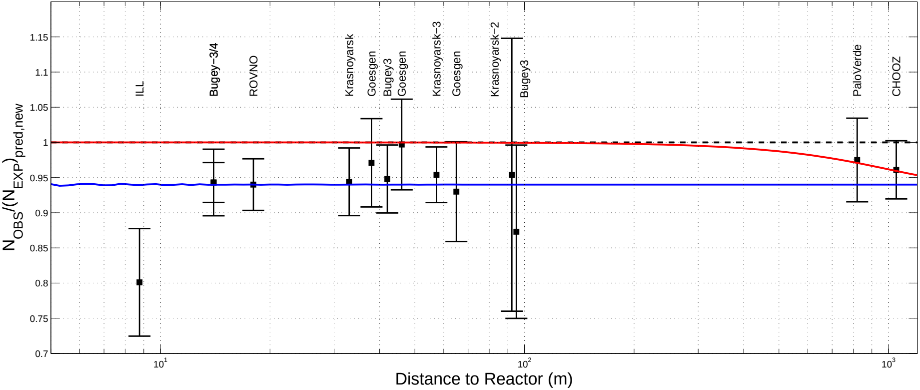
- Th. A. Mueller, D. Lhuillier, M. Fallot, A. Letourneau, S. Cormon, M. Fechner, L. Giot, T. Lasserre, J. Martino, G. Mention, A. Porta, F. Yermia, Improved Predictions of Reactor Antineutrino Spectra, arXiv:1101.2663
- detected flux normalization is increased by about 3%



- G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D. Lhuillier, M. Cribier, A. Letourneau, The Reactor Antineutrino Anomaly, arXiv:1101.2755

- ratio of observed and predicted event rates: 0.937 ± 0.027

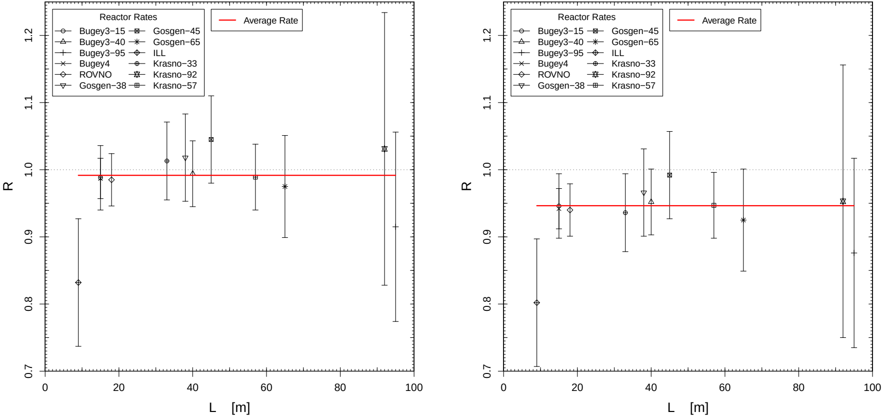
- deviation from unity at 98.4% C.L.: reactor antineutrino anomaly



Reactor Antineutrino Anomaly

Standard Reactor $\bar{\nu}_e$ Fluxes

New Reactor $\bar{\nu}_e$ Fluxes



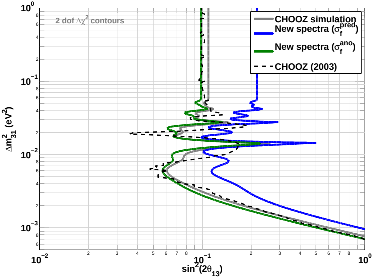
$\overline{R} = 0.992 \pm 0.024$

$\overline{R} = 0.946 \pm 0.024$

- New reactor neutrino flux has several implications: fit of solar and KamLAND data, determination of ϑ_{13}, short-baseline $\bar{\nu}_e$ disappearance, ...

Chooz limit on ϑ_{13}

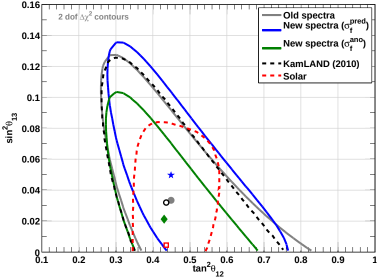
[Chooz, EPJC 27 (2003) 331]



[Mention et al., arXiv:1101.2755]

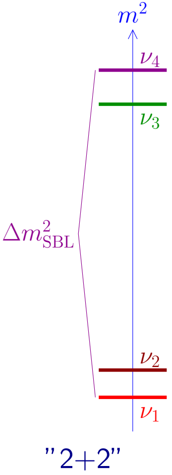
Hint of $\vartheta_{13} > 0$

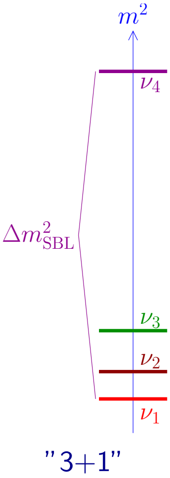
[Fogli et al., PRL 101 (2008) 141801]



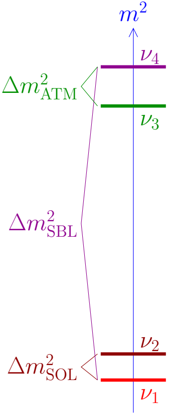
[Mention et al., arXiv:1101.2755]

Four-Neutrino Schemes: 2+2 and 3+1

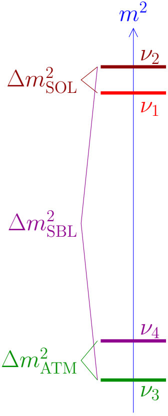




2+2 Four-Neutrino Schemes

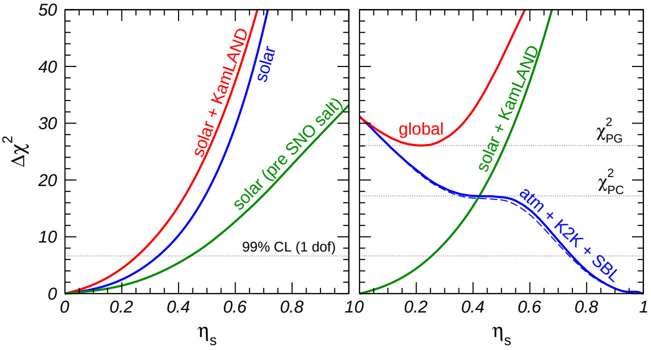


"normal"



"inverted"

2+2 Schemes are strongly disfavored by solar and atmospheric data



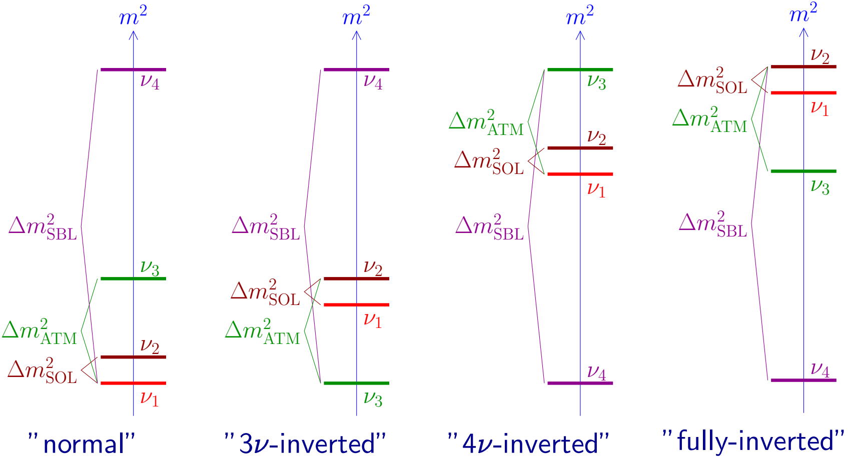
matter effects + SNO NC matter effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2$$
$$1 - \eta_s = |U_{s3}|^2 + |U_{s4}|^2$$

$$\text{99\% CL:} \quad \begin{cases} \eta_s < 0.25 & \text{(solar + KamLAND)} \\ \eta_s > 0.75 & \text{(atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

3+1 Four-Neutrino Schemes



Perturbation of 3-ν Mixing

$|U_{e4}|^2 \ll 1 \qquad |U_{\mu 4}|^2 \ll 1 \qquad |U_{\tau 4}|^2 \ll 1 \qquad |U_{s4}|^2 \simeq 1$

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2$$

$$= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \to \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \qquad \Delta m_{41}^2 \to \Delta m^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left| \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4} \left[1 - \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right] \right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E} \right) \\
&\qquad - 2\delta_{\alpha\beta} |U_{\alpha 4}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2 \right) \left(1 - \cos\frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2 \right) \sin^2 \frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E} \right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2 \right) \sin^2\left(\frac{\Delta m^2 L}{4E} \right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

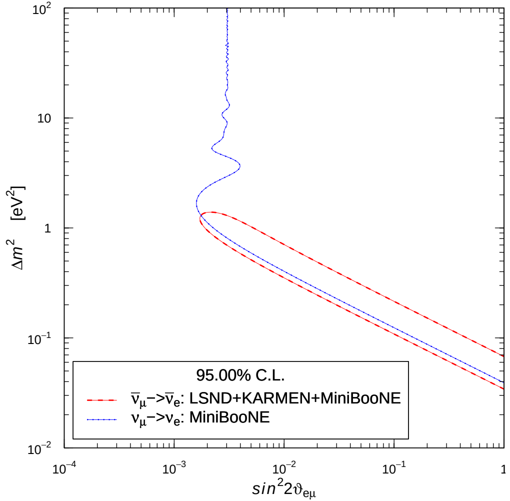
$$|U_{e4}|^2 \ll 1\,, \quad |U_{\mu 4}|^2 \ll 1\,, \quad |U_{\tau 4}|^2 \ll 1\,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{cccc} U_{e1} & U_{e2} & U_{e3} & \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \\ U_{s1} & U_{s2} & U_{s3} & \end{array} \underset{\substack{\uparrow \\ \text{SBL}}}{\boxed{\begin{array}{c} U_{e4} \\ U_{\mu 4} \\ U_{\tau 4} \\ U_{s4} \end{array}}}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$



3+1 Schemes

GoF = 32%

PGoF = 0.89%

- Tension between LSND + KARMEN + MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ and MiniBooNE $\nu_\mu \to \nu_e$ $\Longrightarrow$ CP Violation?
- 3+2 $\Longrightarrow$ CP Violation OK [Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004, hep-ph/0305255; Maltoni, Schwetz, PRD 76, 093005 (2007), arXiv:0705.0107; Karagiorgi et al, PRD 80 (2009) 073001, arXiv:0906.1997]
- 3+1+NSI $\Longrightarrow$ CP Violation OK [Akhmedov, Schwetz, JHEP 10 (2010) 115, arXiv:1007.4171]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$N_{\text{D}} =$ Number of Data $\qquad N_{\text{P}} =$ Number of Fitted Parameters

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF} \qquad \text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\,\mathrm{d}z \qquad p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\,\mathrm{d}z$

SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

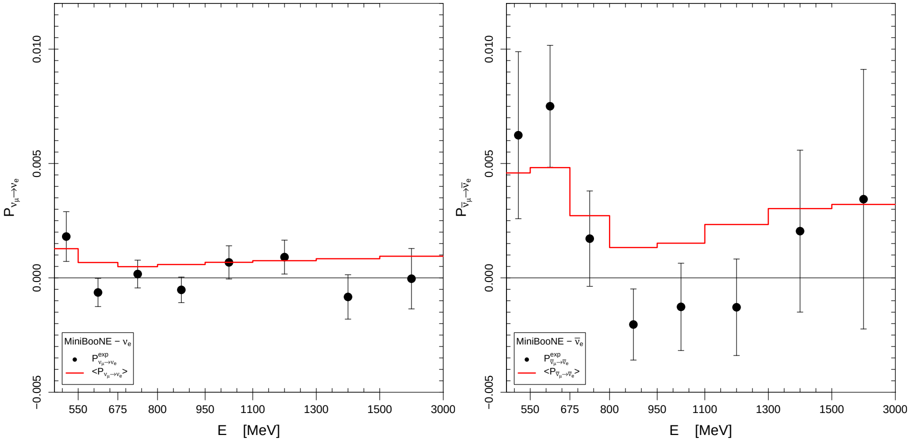
$$\begin{aligned} P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\phi_{51} \\ & + 4|U_{e4} U_{\mu 4} U_{e5} U_{\mu 5}| \cos\eta \left[\sin^2\phi_{41} + \sin^2\phi_{51} - \sin^2\phi_{54}\right] \\ & \overset{(+)}{-} 2|U_{e4} U_{\mu 4} U_{e5} U_{\mu 5}| \sin\eta \left[\sin(2\phi_{41}) - \sin(2\phi_{51}) + \sin(2\phi_{54})\right] \end{aligned}$$

$$\begin{aligned} P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = {} & 1 - 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2) \sin^2\phi_{41} + 4|U_{\alpha 5}|^2 (1 - |U_{\alpha 5}|^2) \sin^2\phi_{51} \\ & + 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \left[\sin^2\phi_{41} + \sin^2\phi_{51} - \sin^2\phi_{54}\right] \end{aligned}$$

MiniBooNE $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

		MB$\bar{\nu}$ + MBν
No Osc.	χ^2	35.5
	NDF	32
	GoF	0.31
Osc. (3+2)	χ^2_{min}	21.1
	NDF	25
	GoF	0.69
	Δm^2_{41}	0.087
	Δm^2_{51}	4.57
	$4\|U_{e4}\|^2\|U_{\mu 4}\|^2$	0.19
	$4\|U_{e5}\|^2\|U_{\mu 5}\|^2$	0.002
	η/π	1.40
	$4\|U_{\mu 4}\|^2(1-\|U_{\mu 4}\|^2)$	0.93
	$4\|U_{\mu 5}\|^2(1-\|U_{\mu 5}\|^2)$	0.0025
PGoF	$\Delta\chi^2_{\text{min}}$	5.72
	NDF	7
	PGoF	0.57

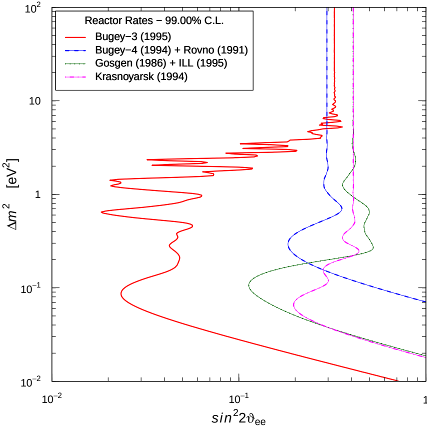
MiniBooNE $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$



$$
\begin{aligned}
P_{\overset{(-)}{\nu}_{\mu}\to\overset{(-)}{\nu}_{e}} = {} & 4|U_{e4}|^2|U_{\mu4}|^2\sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu5}|^2\sin^2\phi_{51} \\
& + 4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|\cos\eta\left[\sin^2\phi_{41} + \sin^2\phi_{51} - \sin^2\phi_{54}\right] \\
& \overset{(+)}{-}\ 2|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|\sin\eta\left[\sin(2\phi_{41}) - \sin(2\phi_{51}) + \sin(2\phi_{54})\right]
\end{aligned}
$$

Disappearance Constraints

$\bar{\nu}_e$ Disappearance

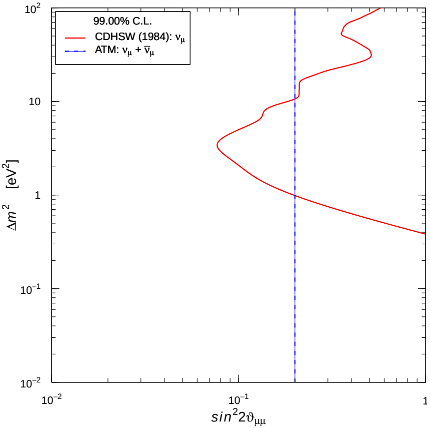


New Reactor $\bar{\nu}_e$ Fluxes

[Mueller et al., arXiv:1101.2663]

[Mention et al., arXiv:1101.2755]

ν_μ and $\bar{\nu}_\mu$ Disappearance



ATM constraint on $|U_{\mu 4}|^2$

[Maltoni, Schwetz, PRD 76 (2007) 093005, arXiv:0705.0107]

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

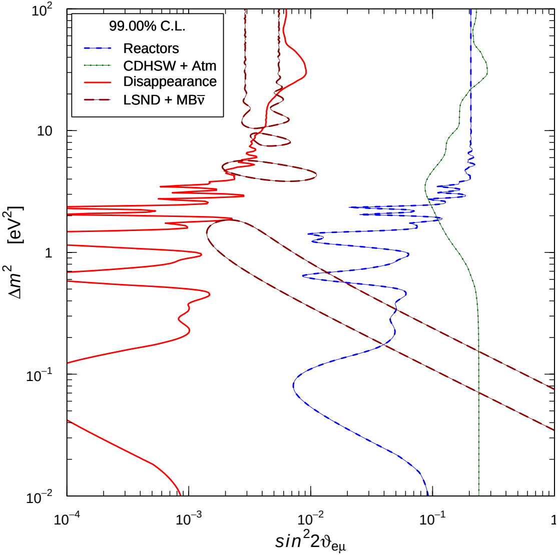
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

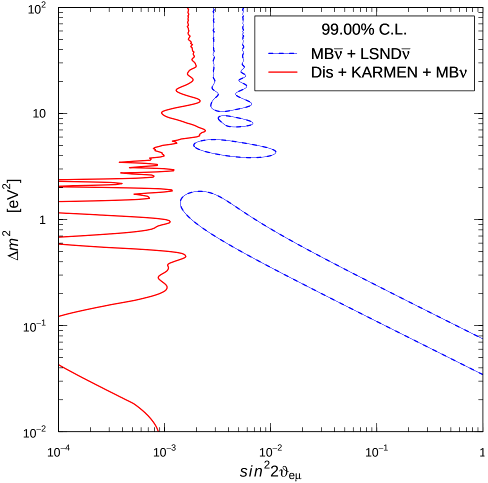
- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694, arXiv:hep-ph/9606411]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247, arXiv:hep-ph/9607372]

3+1 Schemes


99.00% C.L.
Reactors
CDHSW + Atm
Disappearance
LSND + MB$\bar{\nu}$
Δm^2 [eV^2]
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
$sin^2 2\vartheta_{e\mu}$



$\mathrm{PGoF} = 0.006\%$

- Strong tension between $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance and disappearance limits ($\bar{\nu}_e \to \bar{\nu}_e$ and mainly $\nu_\mu \to \nu_\mu$)
- Tension reduced in 3+2, 3+1+NSI
- CPT Violation?

[Barger, Marfatia, Whisnant, PLB 576 (2003) 303]
[Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

Global 3+2 Fit

[Kopp, Maltoni, Schwetz, arXiv:1103.4570]

Best fit: $$\begin{cases} \Delta m^2_{41} = 0.47\,\text{eV}^2 & \Delta m^2_{51} = 0.87\,\text{eV}^2 & \\ |U_{e4}| = 0.128 & |U_{e5}| = 0.138 & \\ |U_{\mu 4}| = 0.165 & |U_{\mu 5}| = 0.148 & \eta = 1.64\pi \end{cases}$$

$$\chi^2_{\text{min}}/\text{NDF} = 110.1/130$$

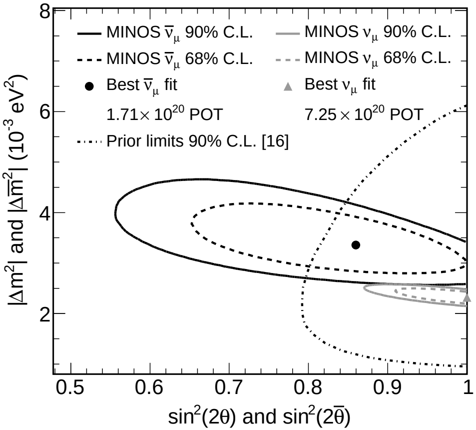
LSND + MiniBooNE($\bar{\nu}$) vs rest

$$\chi^2_{\text{PGof}}/\text{NDF}_{\text{PGof}} = 19.9/5 \qquad \text{PGof} = 0.13\%$$

MINOS Hint of CPT Violation?

LBL ν_μ and $\bar{\nu}_\mu$ disappearance $E \sim 3\,\text{GeV}$

Near Detector at 1.04 km Far Detector at 735 km



$$|\Delta m^2| = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta > 0.90 \quad (90\%\ \text{C.L.})$$

[arXiv:1103.0340v1]

$$|\Delta \overline{m}^2| = \left(3.36^{+0.46}_{-0.40}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\overline{\vartheta} = 0.86^{+0.11}_{-0.12}$$

[arXiv:1104.0344v2]

CDF Hint of CPT Violation?

Measurement of the mass difference between t and $\bar{t}$ quarks,

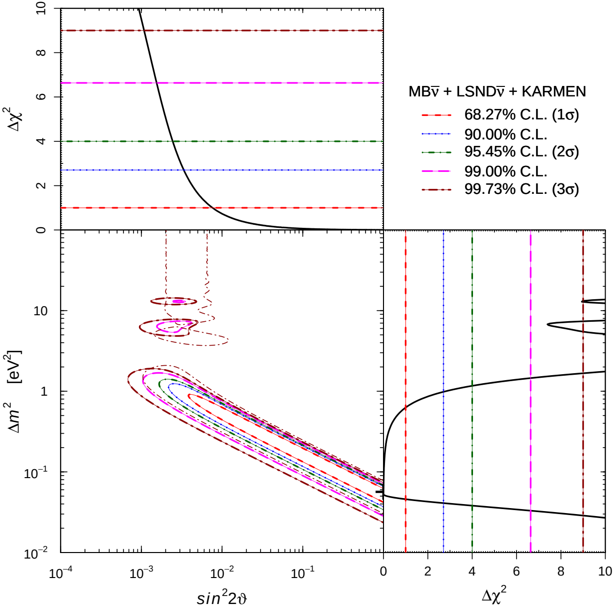
arXiv:1103.2782v1 [hep-ex]

$$m_t - m_{\bar{t}} = -3.3 \pm 1.4 \pm 1.0\,\mathrm{GeV}$$

"approximately two standard deviations away from the CPT hypothesis of zero mass difference"

Phenomenological Approach: Consider $\bar{\nu}$'s Only

$\bar{\nu}_\mu \to \bar{\nu}_e$



$\chi^2_{\text{min}} = 29.8$

$\text{NdF} = 26$

$\text{GoF} = 28\%$

$\sin^2 2\vartheta = 1.00$

$\Delta m^2 = 0.052\,\text{eV}^2$

Parameter Goodness-of-Fit

$\Delta\chi^2_{\text{min}} = 5.9$

$\text{NdF} = 4$

$\text{GoF} = 21\%$

[Giunti, Laveder, PRD 82 (2010) 093016, arXiv:1010.1395]

Conservation of Probability

$$\sum_{\alpha} P_{\bar{\nu}_{\alpha}\to\bar{\nu}_{e}} = 1$$

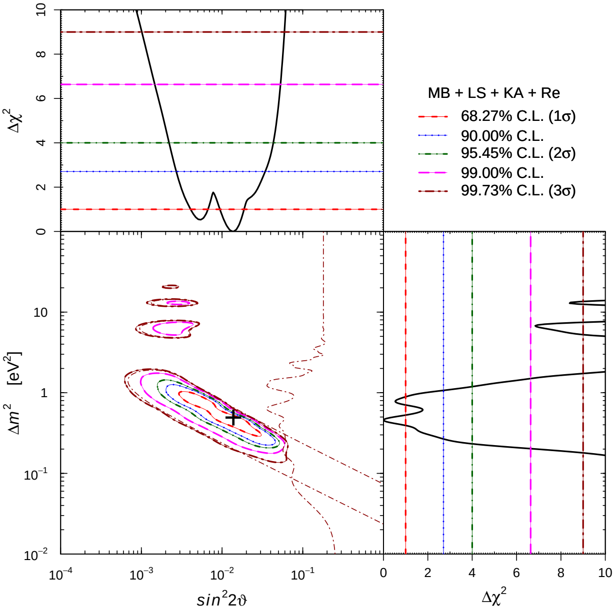
$$P_{\bar{\nu}_{e}\to\bar{\nu}_{e}} + P_{\bar{\nu}_{\mu}\to\bar{\nu}_{e}} + P_{\bar{\nu}_{\tau}\to\bar{\nu}_{e}} + P_{\bar{\nu}_{s}\to\bar{\nu}_{e}} = 1$$

$$P_{\bar{\nu}_{\mu}\to\bar{\nu}_{e}} = 1 - P_{\bar{\nu}_{e}\to\bar{\nu}_{e}} - P_{\bar{\nu}_{\tau}\to\bar{\nu}_{e}} - P_{\bar{\nu}_{s}\to\bar{\nu}_{e}}$$

$$\boxed{P_{\bar{\nu}_{\mu}\to\bar{\nu}_{e}} \leq 1 - P_{\bar{\nu}_{e}\to\bar{\nu}_{e}}}$$

Reactor $\bar{\nu}_e$ disappearance bound is unavoidable!

$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\bar{\nu}_e \to \bar{\nu}_e$



$\chi^2_{\text{min}} = 81.4$

$\text{NdF} = 82$

$\text{GoF} = 50\%$

$\sin^2 2\vartheta = 0.014$

$\Delta m^2 = 0.46\,\text{eV}^2$

Parameter Goodness-of-Fit

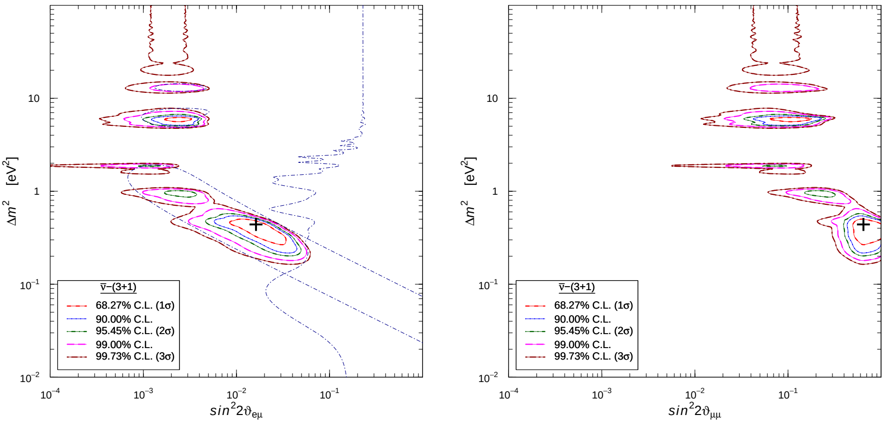
$\Delta\chi^2_{\text{min}} = 3.0$

$\text{NdF} = 2$

$\text{GoF} = 22\%$

[Giunti, Laveder, PRD 82 (2010) 093016, arXiv:1010.1395]

Antineutrino Oscillations in 3+1 Schemes



$\chi^2_{\text{min}} = 87.0$ NdF $= 83$ GoF $= 36\%$

$\Delta m^2 = 0.42\,\text{eV}^2$ $\sin^2 2\vartheta_{e\mu} = 0.016$ $\sin^2 2\vartheta_{ee} = 0.020$ $\sin^2 2\vartheta_{\mu\mu} = 0.65$

Prediction: large SBL $\bar{\nu}_\mu$ disappearance at $\Delta m^2 \gtrsim 0.1\,\text{eV}^2$

[Giunti, Laveder, PRD 83 (2011) 053006, arXiv:1012.0267]

Gallium Anomaly

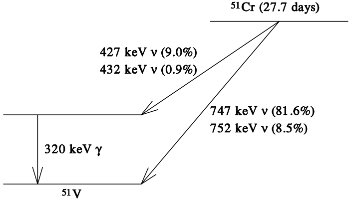
Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

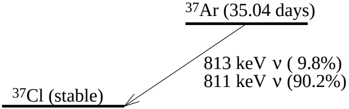
Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\qquad$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

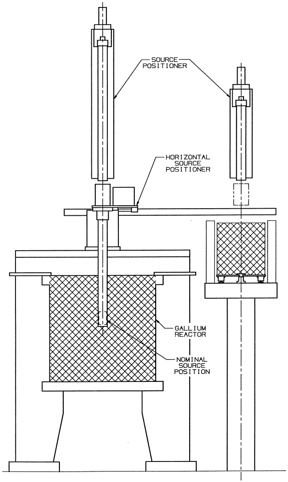
	^{51}Cr				^{37}Ar	
E [keV]	747	752	427	432	811	813
B.R.	0.8163	0.0849	0.0895	0.0093	0.902	0.098



[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]



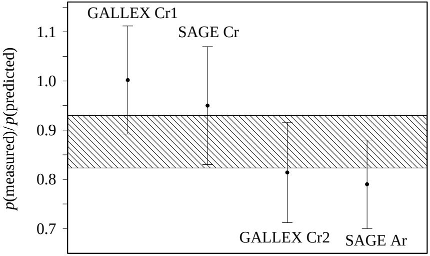
[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]



[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]

$$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$$

$$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$$



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

$$\begin{aligned}
R_{\text{B}}^{\text{Gallex-Cr1}} &= 0.953 \pm 0.11 \\
R_{\text{B}}^{\text{Gallex-Cr2}} &= 0.812^{+0.10}_{-0.11} \\
R_{\text{B}}^{\text{SAGE-Cr}} &= 0.95 \pm 0.12 \\
R_{\text{B}}^{\text{SAGE-Ar}} &= 0.791^{+0.084}_{-0.078}
\end{aligned}$$

$$\boxed{R_{\text{B}}^{\text{Ga}} = 0.86 \pm 0.05}$$

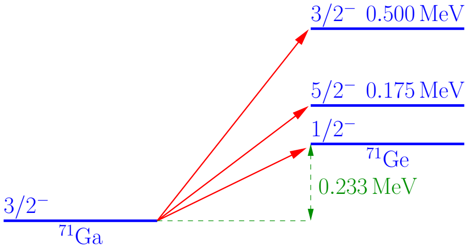
Bahcall cross section

Only exp uncertainties!

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391, hep-ph/9710491



- $\sigma_{\text{G.S.}}$ related to measured $\sigma(e^- + {}^{71}\mathrm{Ge} \to {}^{71}\mathrm{Ga} + \nu_e)$:

$$\sigma_{\text{G.S.}}({}^{51}\mathrm{Cr}) = 55.3 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\mathrm{Cr}) = \sigma_{\text{G.S.}}({}^{51}\mathrm{Cr}) \left(1 + 0.669\,\frac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\text{G.S.}}} + 0.220\,\frac{\mathrm{BGT}_{500\,\mathrm{keV}}}{\mathrm{BGT}_{\text{G.S.}}}\right)$

- Contribution of Excited States only 5%!

- Bahcall: [Bahcall, PRC 56 (1997) 3391, hep-ph/9710491]

  from $p + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + n$ measurements [Krofcheck et al., PRL 55 (1985) 1051]

  $$\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} < 0.056 \Rightarrow \frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = \frac{0.056}{2} \qquad \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0.146$$

  3σ lower limit: $\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0$

  3σ upper limit: $\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} < 0.056 \times 2 \qquad \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0.146 \times 2$

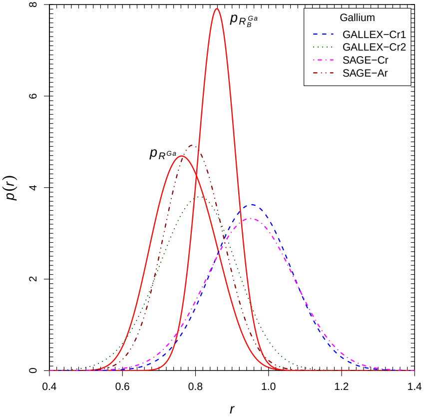
  $$\sigma({}^{51}\text{Cr}) = 58.1 \times 10^{-46}\,\text{cm}^2 \left(1^{+0.036}_{-0.028}\right)_{1\sigma} \Longrightarrow \boxed{R_{\text{B}}^{\text{Ga}} = 0.86 \pm 0.06}$$

- Haxton: [Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

  "a sophisticated shell model calculation is performed ... for the transition to the first excited state in ${}^{71}\text{Ge}$. The calculation predicts destructive interference between the (p, n) spin and spin-tensor matrix elements."

$$\sigma({}^{51}\text{Cr}) = 63.9 \times 10^{-46}\,\text{cm}^2 \,(1 \pm 0.106)_{1\sigma} \Longrightarrow \boxed{R_{\text{H}}^{\text{Ga}} = 0.78 \pm 0.13}$$

- $R_{\mathrm{H}}^{\mathrm{Ga}} = 0.78 \pm 0.13$ exp and the uncertainties added in quadrature
- $R^{\mathrm{Ga}} = R_{\mathrm{exp}}^{\mathrm{Ga}} / R_{\mathrm{the}}^{\mathrm{Ga}}$ probability distribution of ratio is not Gaussian



$$p_{R^{\mathrm{Ga}}}(r) = \int_{R_{\mathrm{gs}}^{\mathrm{Ga}}}^{\infty} p_{R_{\mathrm{exp}}^{\mathrm{Ga}}}(rs)\, p_{R_{\mathrm{the}}^{\mathrm{Ga}}}(s)\, s\, \mathrm{d}s$$

$$\boxed{R^{\mathrm{Ga}} = 0.76^{+0.09}_{-0.08}}$$

$$\begin{aligned} R^{\text{Gallex-Cr1}} &= 0.84^{+0.13}_{-0.12} \\ R^{\text{Gallex-Cr2}} &= 0.71^{+0.12}_{-0.11} \\ R^{\text{SAGE-Cr}} &= 0.84^{+0.14}_{-0.13} \\ R^{\text{SAGE-Ar}} &= 0.70^{+0.10}_{-0.09} \end{aligned}$$

[Giunti, Laveder, arXiv:1006.3244]

Gallium Radioactive Source Experiments
are
Short-BaseLine Neutrino Oscillation Experiments

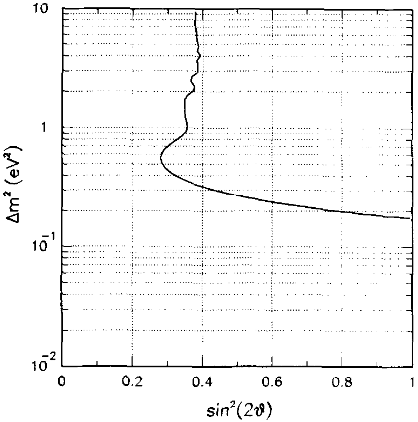


Fig. 1. Region of electron neutrino oscillation parameters ruled out at 90% C.L. by the GALLEX ^{51}Cr source experiment.

[Bahcall, Krastev, Lisi, PLB 348 (1995) 121]

$$P^{\mathrm{SBL}}_{\nu_e\to\nu_e}(L,E) = 1 - \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$R^k(\sin^2 2\vartheta, \Delta m^2) = \frac{\int_k \mathrm{d}V \, L^{-2} \sum_i b_i^k \, \sigma_i^k \, P^{\mathrm{SBL}}_{\nu_e\to\nu_e}(L,E_i)}{\sum_i b_i^k \, \sigma_i^k \int_k \mathrm{d}V \, L^{-2}}$$

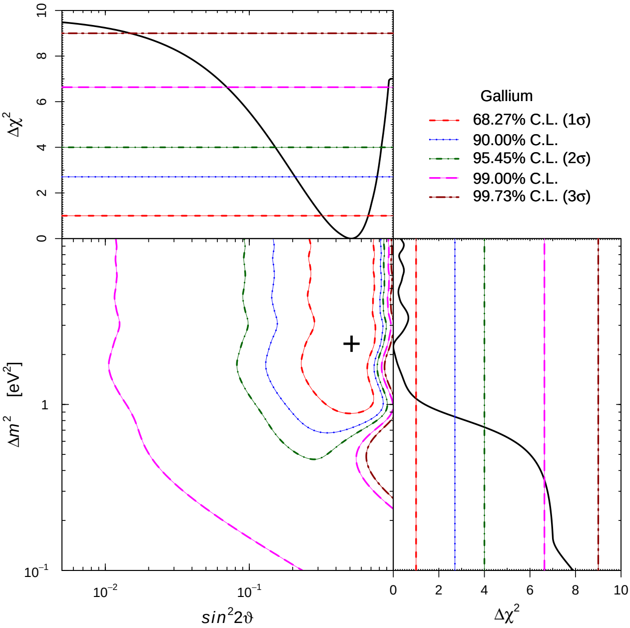
k = GALLEX-Cr1 , GALLEX-Cr2 , SAGE-Cr , SAGE-Ar

$R^k = R^k_{\mathrm{exp}}/R^k_{\mathrm{the}}$ fully correlated theoretical uncertainty!

$$p_{\vec{R}}(\vec{r}) = \int_{R^k_{\mathrm{gs}}}^{\infty} \left[\prod_k p_{R^k_{\mathrm{exp}}}(r^k s)\right] p_{R^k_{\mathrm{the}}}(s) \, s^4 \, \mathrm{d}s$$

$$\mathcal{L}(\sin^2 2\vartheta, \Delta m^2) = p_{\vec{R}}(\vec{R}(\sin^2 2\vartheta, \Delta m^2))$$

$$\chi^2(\sin^2 2\vartheta, \Delta m^2) = -2 \ln \mathcal{L}(\sin^2 2\vartheta, \Delta m^2) + \text{constant}$$



$\Delta\chi^2_{\text{No Osc.}} = 9.7$

No Osc. disfavored at 99.23 % C.L (2.7σ)

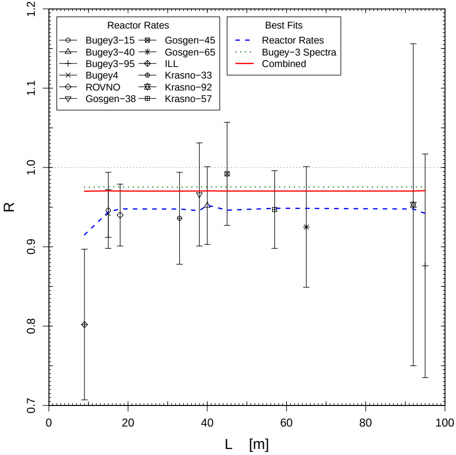
Osc.

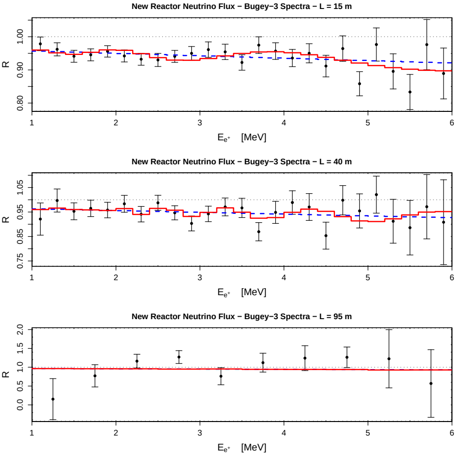
$\sin^2 2\vartheta_{\text{bf}} = 0.51$

$\Delta m^2_{\text{bf}} = 2.24\,\text{eV}^2$

[Giunti, Laveder, arXiv:1006.3244]

Reactor Antineutrino Anomaly

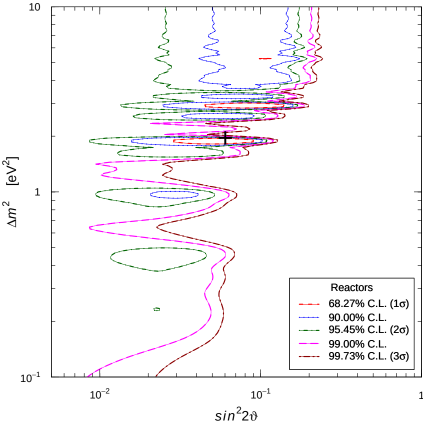


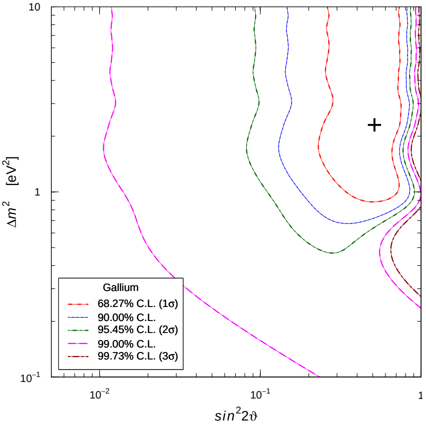


- $\overline{R}_{\text{rates}} = 0.946 \pm 0.024$
- Improved hint of oscillations given by Bugey energy spectrum with old reactor fluxes [Acero, Giunti, Laveder, PRD 78 (2008) 073009, arXiv:0711.4222]
- $\sin^2 2\vartheta_{\text{bf}} = 0.059 \qquad \Delta m^2_{\text{bf}} = 1.89\,\text{eV}^2$

Reactor Antineutrino Anomaly

Gallium Neutrino Anomaly





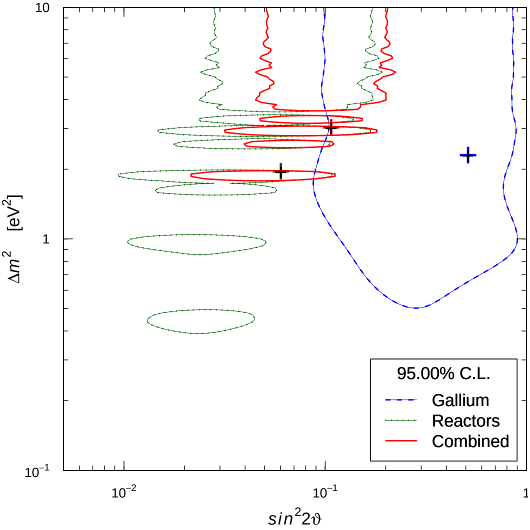
$$\sin^2 2\vartheta_{\text{bf}} = 0.059$$

$$\Delta m^2_{\text{bf}} = 1.89\,\text{eV}^2$$

$$\sin^2 2\vartheta_{\text{bf}} = 0.51$$

$$\Delta m^2_{\text{bf}} = 2.24\,\text{eV}^2$$

Gallium Anomaly + Reactor Anomaly



$\chi^2_{\text{min}} = 59.6$

NdF $= 71$

GoF $= 83\%$

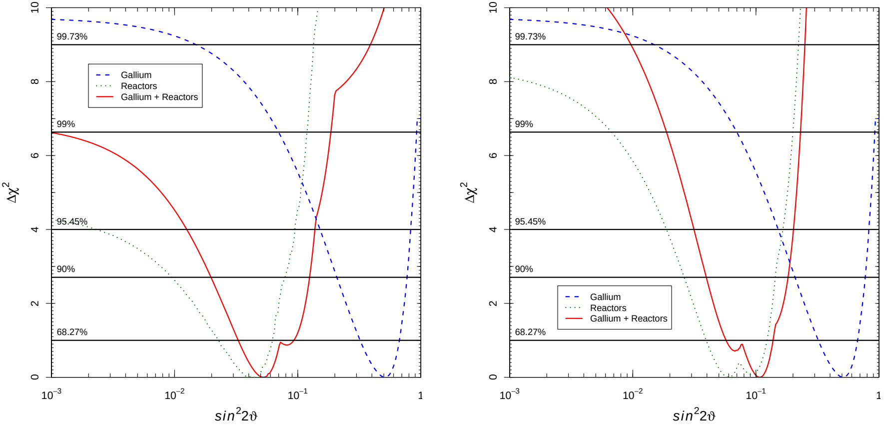
$\sin^2 2\vartheta = 0.11$

$\Delta m^2 = 2.95\,\text{eV}^2$

PGoF $= 4.6\%$

Old Reactor $\bar{\nu}_e$ Fluxes

New Reactor $\bar{\nu}_e$ Fluxes

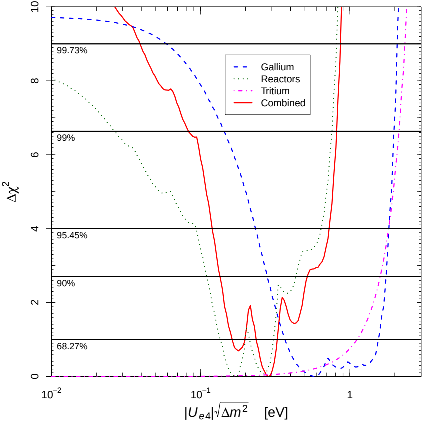


PGoF = 2.3%

PGoF = 4.6%

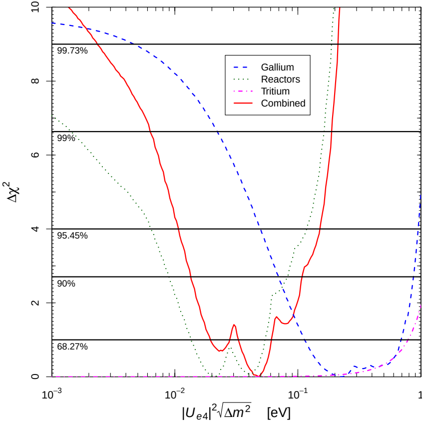
Implications of Gallium and Reactor Anomalies

β Decay



$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

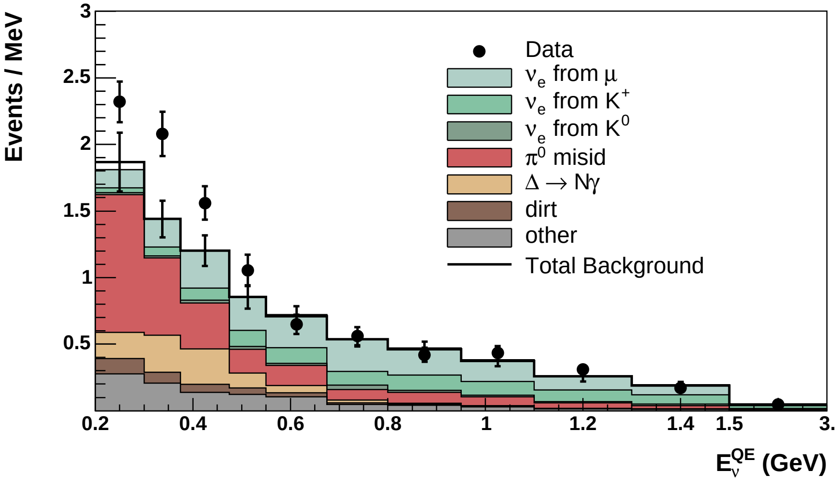
$(\beta\beta)_{0\nu}$ Decay



$$m_{\beta\beta} = \left| \sum_k U_{ek}^2 \, m_k \right|$$

[Giunti, Laveder, In Preparation]

MiniBooNE Low-Energy Anomaly



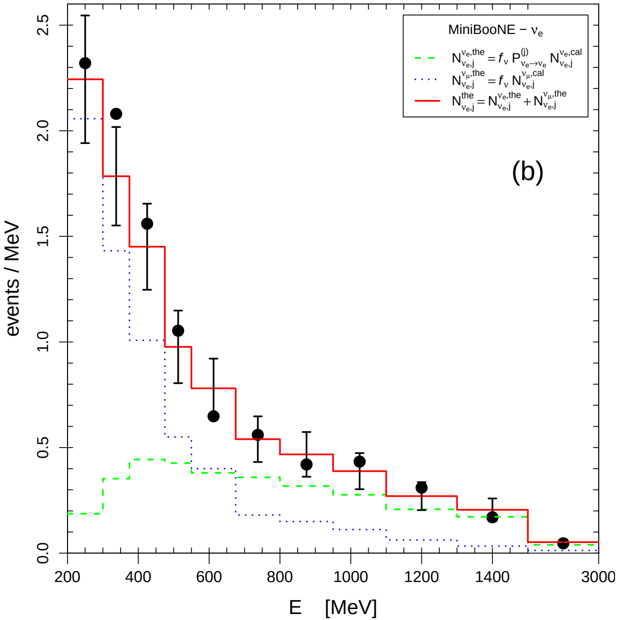
[PRL 102 (2009) 101802, arXiv:0812.2243]

Our Hypothesis: $N^{\text{the}}_{\nu,j} = f_\nu \left(P_{\nu_e \to \nu_e} N^{\text{cal}}_{\nu_e,j} + N^{\text{cal}}_{\nu_\mu,j} \right)$

[Giunti, Laveder, PRD 77 (2008) 093002, arXiv:0707.4593; PRD 80 (2009) 013005, arXiv:0902.1992]

$$\boxed{N^{\text{the}}_{\nu,j} = f_{\nu} \left(P_{\nu_e \to \nu_e} N^{\text{cal}}_{\nu_e,j} + N^{\text{cal}}_{\nu_\mu,j} \right)}$$

- Estimated 15% uncertainty of the calculated neutrino flux [MiniBooNE, PRD 79 (2009) 072002, arXiv:0806.1449] is consistent with measured ratio 1.21 ± 0.24 of detected and predicted charged-current quasi-elastic ν_μ events [MiniBooNE, PRL 100 (2008) 032301, arXiv:0706.0926]
- We fit MiniBooNE ν_e and ν_μ data using the info at `http://www-boone.fnal.gov/for_physicists/data_release/lowe/`



No Osc. & $f_\nu = 1$

$\chi^2_{\text{min}} = 14.3 + 5.4$

$\text{NdF} = 3 + 16$

$\text{GoF} = 41\%$

Our Hypothesis

$\chi^2_{\text{min}} = 2.0 + 7.6$

$\text{NdF} = 16$

$\text{GoF} = 89\%$

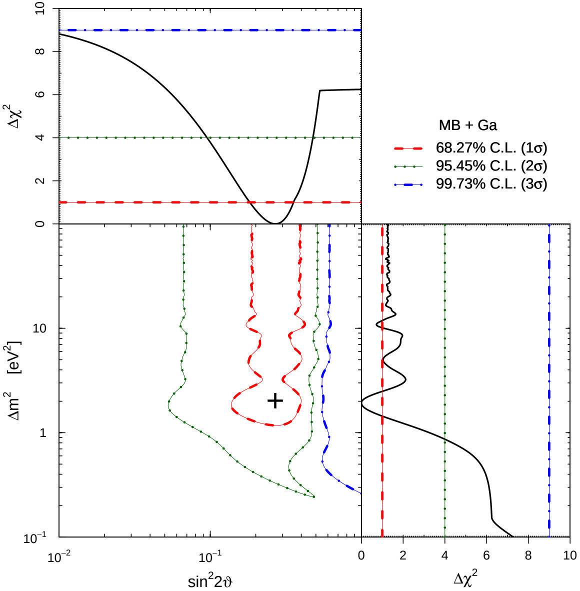
$f_\nu = 1.26$

$\sin^2 2\vartheta = 0.32$

$\Delta m^2 = 1.84 \, \text{eV}^2$

[Giunti, Laveder, PRD 82 (2010) 053005, arXiv:1005.4599]

MiniBooNE + Gallium



$\chi^2_{\text{min}} = 2.3 + 9.2$

$\text{NdF} = 20$

$\text{GoF} = 93\%$

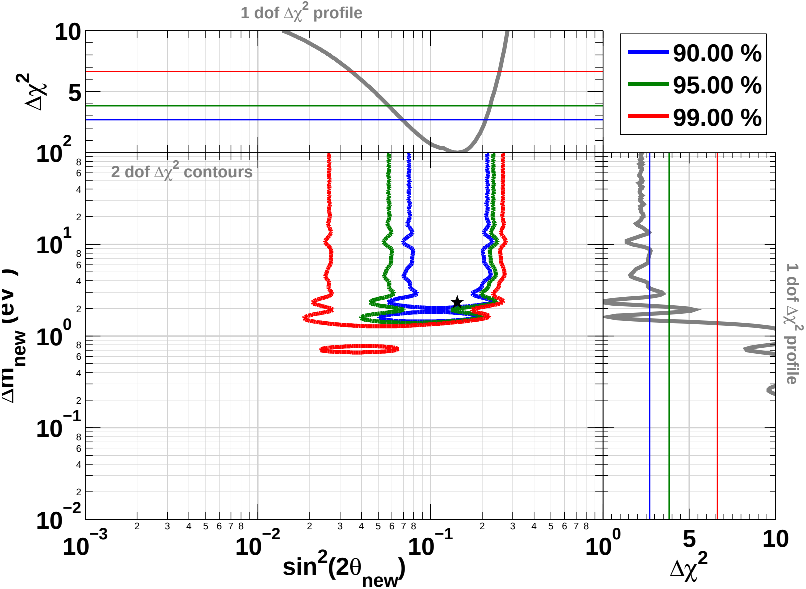
$\sin^2 2\vartheta = 0.27$

$\Delta m^2 = 1.92\,\text{eV}^2$

$\text{PGoF} = 93\%$

[Giunti, Laveder, PRD 82 (2010) 053005, arXiv:1005.4599]

MiniBooNE + Gallium + Reactors



[Mention et al., arXiv:1101.2755]

Future

- MiniBooNE is continuing to take antineutrino data.

$$\bar{\nu}_\mu \to \bar{\nu}_e + \bar{\nu}_\mu \to \bar{\nu}_\mu$$

- ICARUS@CERN-PS: $L \sim 1\text{km}$ $E \sim 1\text{GeV}$ [C. Rubbia et al, CERN-SPSC-2011-012]

$$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e + \overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu + \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$$

- MicroBooNE will test the MiniBooNE low-energy anomaly by measuring $\pi^0 \to 2\gamma$ background.

- ν_e disappearance: new SAGE Gallium source experiments with 2 spherical shells [Gavrin et al, arXiv:1006.2103]

- CPT test: ν_e and $\bar{\nu}_e$ disappearance

- Beta-Beam experiments: [Antusch, Fernandez-Martinez, PLB 665 (2008) 190, arXiv:0804.2820]

$$N(A,Z) \to N(A,Z+1) + e^- + \bar{\nu}_e \qquad (\beta^-)$$

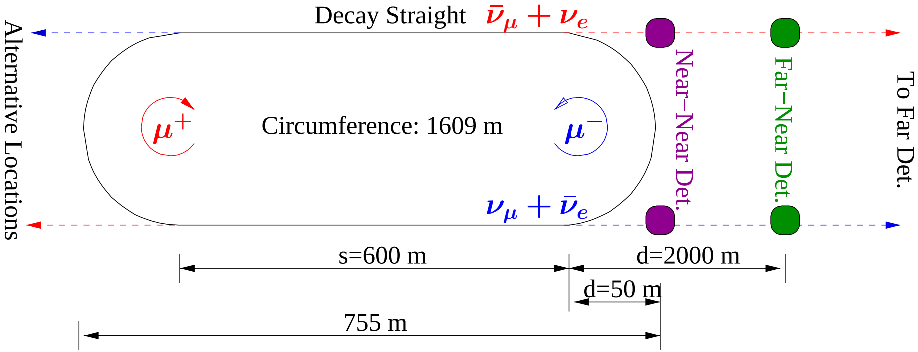
$$N(A,Z) \to N(A,Z-1) + e^+ + \nu_e \qquad (\beta^+)$$

- Neutrino Factory experiments: [Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

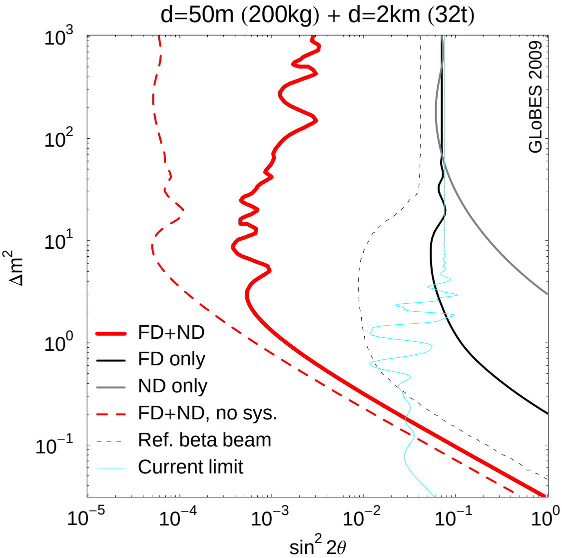
Neutrino Factory



[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

Near Detectors:	Scintillator or Iron Calorimeter with perfect flavor identification
Systematic Uncertainties:	Cross Section, Detector Normalization, Energy Resolution and Calibration, Backgrounds

ν_e Disappearance



[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

- New ν_e and $\bar{\nu}_e$ radioactive source experiments with low-threshold neutrino elastic scattering detectors.

- Borexino: $\overset{(-)}{\nu}_e + e^- \to \overset{(-)}{\nu}_e + e^-$ [Pallavicini, talk at BEYOND3NU]

  [Ianni, Montanino, Scioscia, EPJC 8 (1999) 609, arXiv:hep-ex/9901012]

- LENS (Low Energy Neutrino Spectroscopy): [Agarwalla, Raghavan, arXiv:1011.4509]

$$\nu_e + {}^{115}\text{In} \to {}^{115}\text{Sn} + e^- + 2\gamma \qquad E_{\text{th}} = 0.1\,\text{MeV}$$

$$\bar{\nu}_e + p \to n + e^+ \qquad E_{\text{th}} = 1.8\,\text{MeV}$$

- Spherical Gaseous TPC: [Vergados, Giomataris, Novikov, arXiv:1103.5307]
  - Targets: ^{131}Xe, ^{40}Ar, ^{20}Ne, ^{4}He.
  - Sources: ^{37}Ar, ^{51}Cr, ^{65}Zn, ^{32}P.

Conclusions

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

Conclusions - Three-Neutrino Mixing

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (SOL, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (ATM, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why only $|U_{e3}|^2 \ll 1$?

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy.

Find absolute mass scale.

Conclusions - Anomalies - 1

- Suggestive LSND and MiniBooNE agreement on SBL $\bar{\nu}_\mu \to \bar{\nu}_e$
- Hint in favor of sterile neutrinos is compatible with cosmological data, but mass is limited
- Two experimental tensions:
  - LSND and MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ vs MiniBooNE $\nu_\mu \to \nu_e$ (CP violation?)
  - LSND and MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ vs $\bar{\nu}_e$ and ν_μ disappearance limits
- CPT-invariant 3+1 Four-Neutrino Mixing is strongly disfavored (no CP violation and tension between appearance and disappearance)
- 3+2 can explain CP violation and reduce tension between appearance and disappearance with New Reactor $\bar{\nu}_e$ Fluxes
- 3+1+NSI has CP violation and reduced appearance-disappearance tension
- CPT-violating 3+1 Mixing $\Longrightarrow$ testable large SBL $\bar{\nu}_\mu$ disappearance

Conclusions - Anomalies - 2

- Interesting possible agreement of
  - Gallium Anomaly (SBL ν_e disappearance)
  - Reactor Anomaly (SBL $\bar{\nu}_e$ disappearance)
- Testable Predictions:
  - $m_\beta \sim 0.12 - 0.71\,\text{eV} \quad (2\sigma)$
  - $m_{\beta\beta} \sim 0.011 - 0.15\,\text{eV} \quad (2\sigma)$
- Exciting experimental results in favor of sterile neutrinos.
- More work to do because interpretation is not clear:
  - Explanation of all data needs at least two new physical effects.
  - Without CPT violation tensions do not disappear completely.
  - Possible that some experiments are giving misleading information.
- New short-baseline neutrino oscillation experiments are needed!