Theory of Neutrino Oscillations and Phenomenology of Sterile Neutrinos

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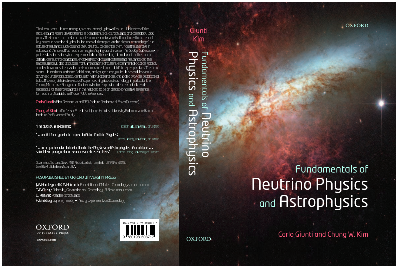
Neutrinos in Nuclear-, Particle- and Astrophysics



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics
and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part II

Phenomenology of Sterile Neutrinos

Three-Neutrino Mixing Paradigm

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Homestake
Kamiokande
GALLEX/GNO & SAGE
Super-Kamiokande
SNO
BOREXino

Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

→ $\Delta m^2_{\text{SOL}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$
$\sin^2\vartheta_{\text{SOL}} \simeq 0.32$

Atmospheric
$\nu_\mu \to \nu_\tau$

Kamiokande
IMB
Super-Kamiokande
MACRO
Soudan-2

Accelerator
ν_μ disappearance

(K2K & MINOS)

→ $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$
$\sin^2\vartheta_{\text{ATM}} \simeq 0.50$

Two scales of Δm^2 $\Longleftrightarrow$ Three-Neutrino Mixing

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2$$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

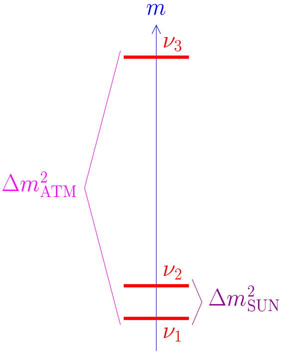
three massive fields: ν_1, ν_2, ν_3

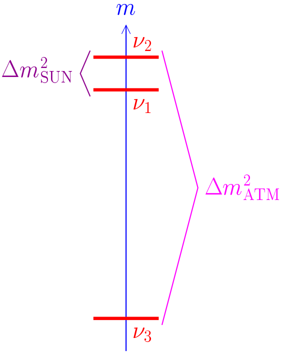
$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes





"normal"

"inverted"

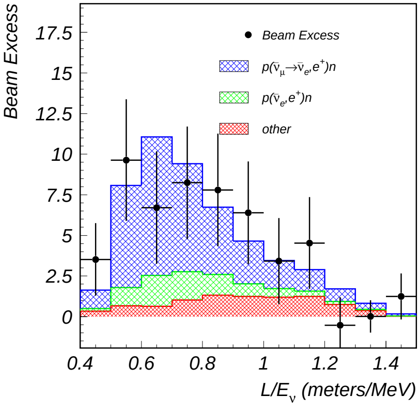
different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

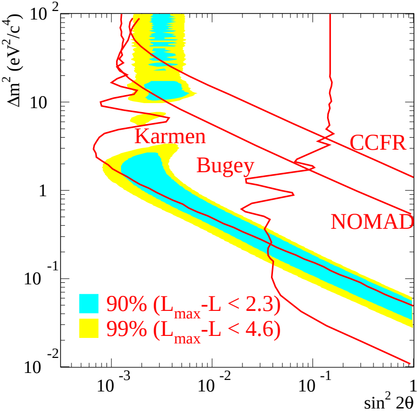
absolute scale is not determined by neutrino oscillation data

LSND

[LSND, PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad L \simeq 30\,\text{m}$ $\qquad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



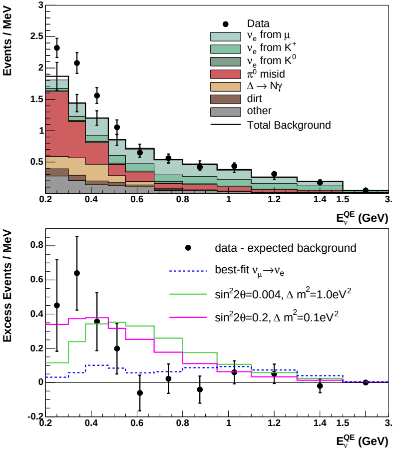


$$\Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}})$$

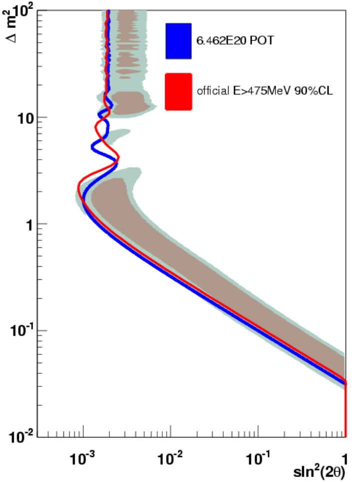
MiniBooNE Neutrinos

[PRL 98 (2007) 231801; PRL 102 (2009) 101802]

$\nu_\mu \to \nu_e$ $L \simeq 541\,\mathrm{m}$ $475\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$



[MiniBooNE, PRL 102 (2009) 101802, arXiv:0812.2243]



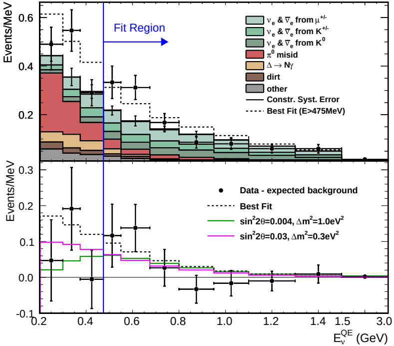
[Djurcic, arXiv:0901.1648]

Low-Energy Anomaly!

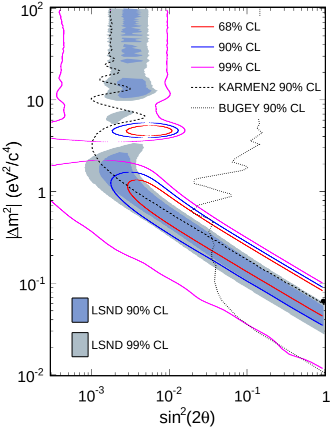
MiniBooNE Antineutrinos

[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 541\,\text{m} \quad 475\,\text{MeV} \le E \lesssim 3\,\text{GeV}$



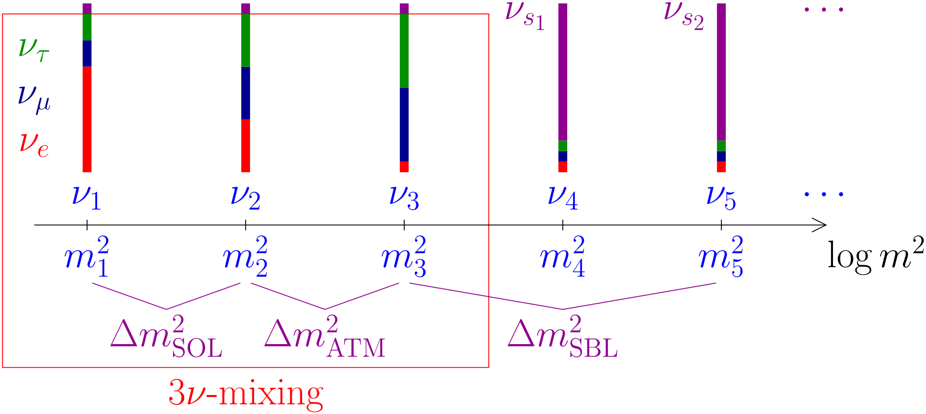
[MiniBooNE, PRL 105 (2010) 181801, arXiv:1007.1150]



Agreement with LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal!

Similar L/E but different L and E $\Longrightarrow$ Oscillations!

Beyond Three-Neutrino Mixing



Standard Model

- Neutrinos are the only massless fermions
- Neutrinos are the only fermions with only left-handed component ν_L

Extension of the SM: Massive Neutrinos

- Simplest extension: introduce right-handed component ν_R
- Dirac mass $m_{\text{D}}\overline{\nu_R}\nu_L$ + Majorana mass $m_{\text{M}}\overline{\nu_R^c}\nu_R$
- $\nu_{eL}\,,\ \nu_{\mu L}\,,\ \nu_{\tau L}\ +\ \nu_{eR}\,,\ \nu_{\mu R}\,,\ \nu_{\tau R}\quad\Longrightarrow$ 6 massive Majorana neutrinos

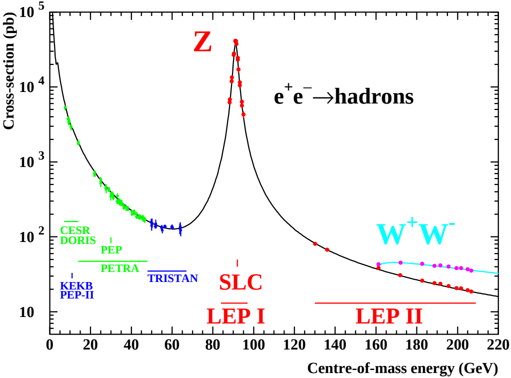
Sterile Neutrinos

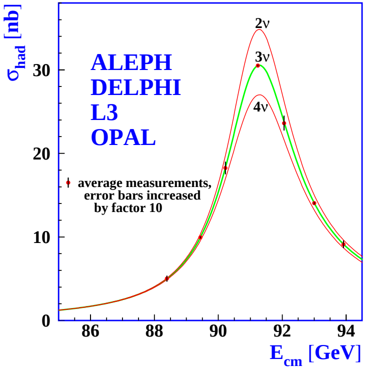
- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos
  - Indirect evidence through combined fit of data (current indication)
- Powerful window on new physics beyond the Standard Model

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$

no upper limit!

Mass Basis: ν_1 ν_2 ν_3 ν_4 ν_5 $\cdots$

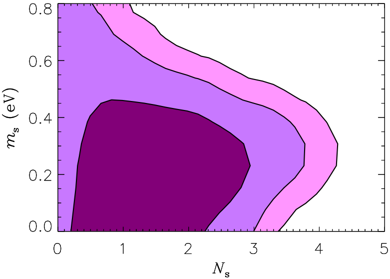
Flavor Basis: ν_e ν_μ ν_τ ν_{s_1} ν_{s_2} $\cdots$

ACTIVE STERILE

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Cosmology

- N_s = number of thermalized sterile neutrinos (not necessarily integer)
- CMB and LSS in ΛCDM:



[Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301, arXiv:1006.5276]

$$N_s = 1.61 \pm 0.92 \qquad m_{\nu_s} < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

[Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, arXiv:1102.4774]

- BBN:

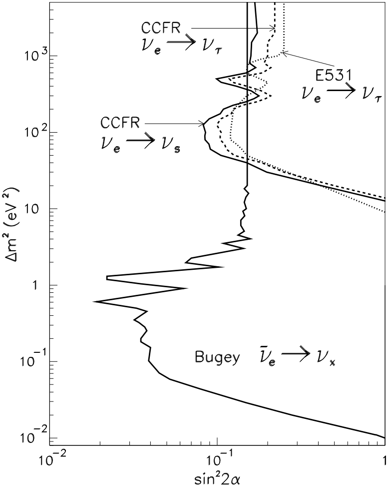
$$N_s = 0.22 \pm 0.59$$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]

$$N_s = 0.64^{+0.40}_{-0.35}$$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]

Direct Searches of Active-Sterile Transitions

CCFR

[PRD 59 (1999) 031101, arXiv:hep-ex/9809023]



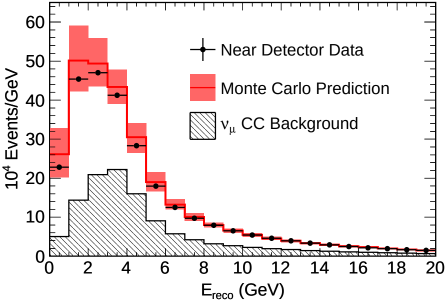
NC interactions $E \sim 100\,\text{GeV}$ $L \simeq 1.4\,\text{km}$

MINOS

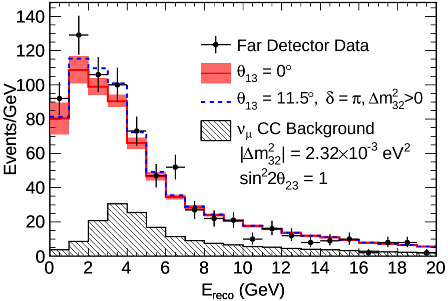
NC sample: 89% efficiency and 61% purity

97% of ν_e-induced CC events misidentified as NC

[arXiv:1104.3922]



$L_{ND} = 1.04\,\mathrm{km}$



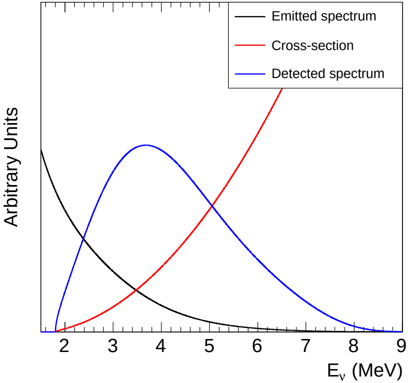
$L_{FD} = 735\,\mathrm{km}$

Model	ϑ_{13}	χ^2/D.O.F.	ϑ_{23}	ϑ_{24}	ϑ_{34}	f_s
$m_4 = m_1$	0	130.4/123	45.0^{+7}_{-7}	-	$0.0^{+17}_{-0.0}$	0.22
	11.5	128.5/123	45.6^{+7}_{-7}	-	$0.0^{+25}_{-0.0}$	0.40
$m_4 \gg m_3$	0	130.4/122	45.0^{+7}_{-7}	$0.0^{+5}_{-0.0}$	$0.0^{+17}_{-0.0}$	0.22
	11.5	128.5/122	45.6^{+7}_{-7}	$0.0^{+5}_{-0.0}$	$0.0^{+25}_{-0.0}$	0.40

90% C.L.

New Calculation of Reactor $\bar{\nu}_e$ Flux

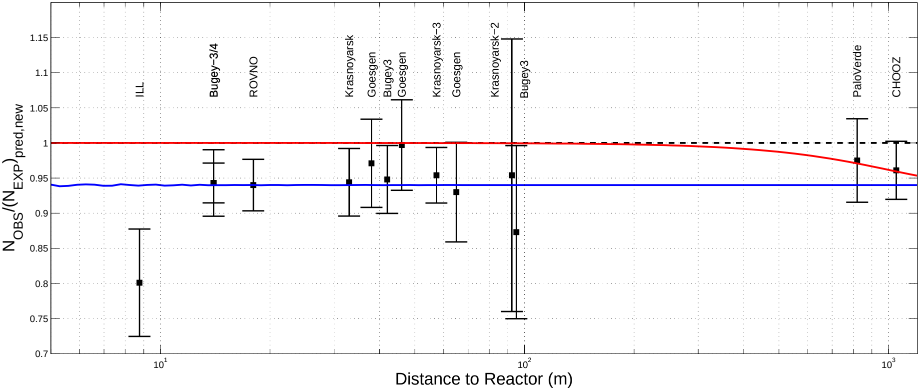
- Th. A. Mueller, D. Lhuillier, M. Fallot, A. Letourneau, S. Cormon, M. Fechner, L. Giot, T. Lasserre, J. Martino, G. Mention, A. Porta, F. Yermia, Improved Predictions of Reactor Antineutrino Spectra, arXiv:1101.2663
- detected flux normalization is increased by about 3%



- G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D. Lhuillier, M. Cribier, A. Letourneau, The Reactor Antineutrino Anomaly, arXiv:1101.2755

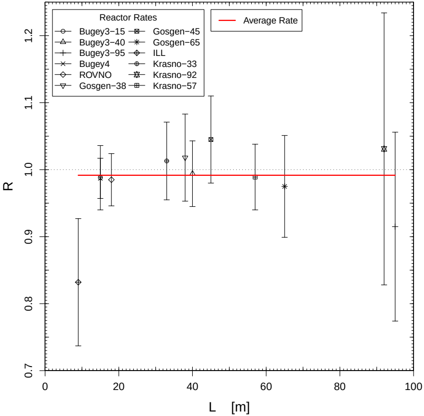
- ratio of observed and predicted event rates: 0.937 ± 0.027

- deviation from unity at 98.4% C.L.: reactor antineutrino anomaly



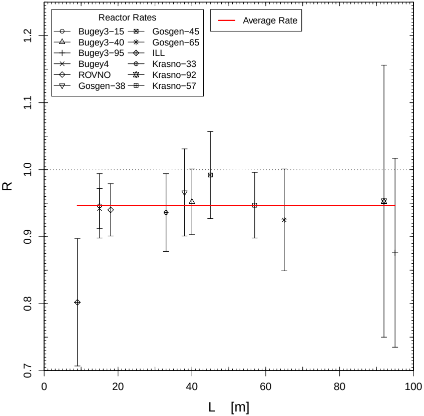
Reactor Antineutrino Anomaly

Standard Reactor $\bar{\nu}_e$ Fluxes



$$\overline{R} = 0.992 \pm 0.024$$

New Reactor $\bar{\nu}_e$ Fluxes

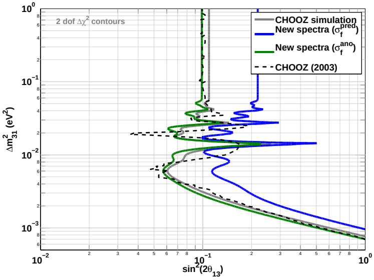


$$\overline{R} = 0.946 \pm 0.024$$

- New reactor neutrino flux has several implications: fit of solar and KamLAND data, determination of ϑ_{13}, short-baseline $\bar{\nu}_e$ disappearance, ...

Chooz limit on ϑ_{13}

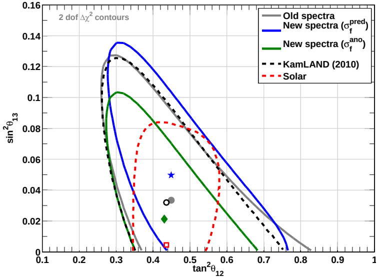
[Chooz, EPJC 27 (2003) 331]



[Mention et al., arXiv:1101.2755]

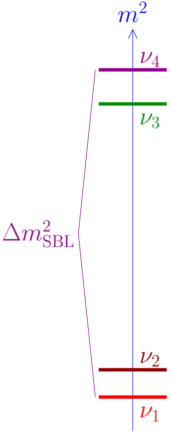
Hint of $\vartheta_{13} > 0$

[Fogli et al., PRL 101 (2008) 141801]

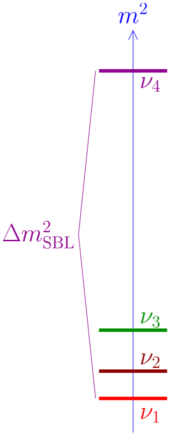


[Mention et al., arXiv:1101.2755]

Four-Neutrino Schemes: 2+2 and 3+1

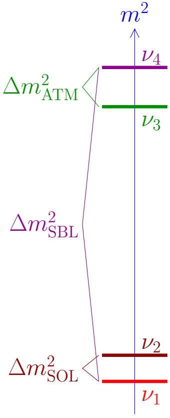


"2+2"

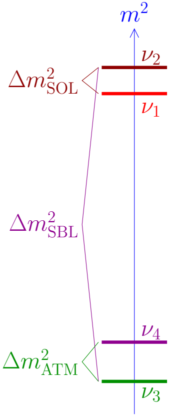


"3+1"

2+2 Four-Neutrino Schemes

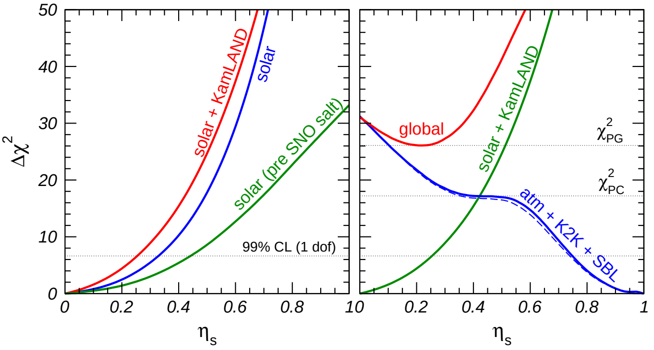


"normal"



"inverted"

2+2 Schemes are strongly disfavored by solar and atmospheric data



matter effects + SNO NC

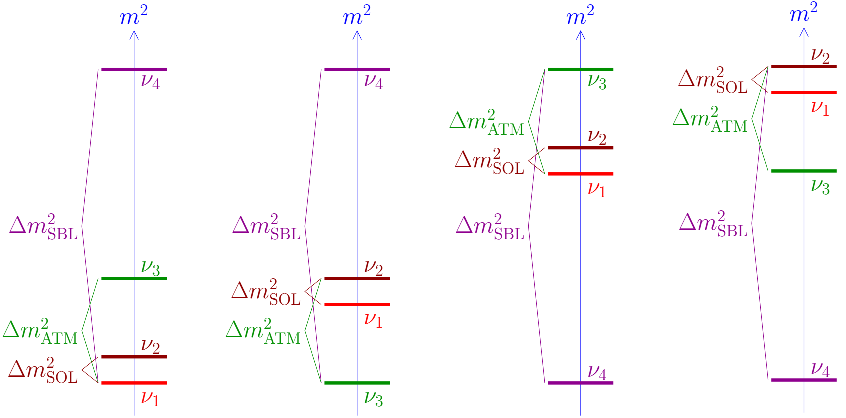
matter effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2$$
$$1 - \eta_s = |U_{s3}|^2 + |U_{s4}|^2$$

$$99\%\ \text{CL:} \quad \begin{cases} \eta_s < 0.25 & \text{(solar + KamLAND)} \\ \eta_s > 0.75 & \text{(atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

3+1 Four-Neutrino Schemes



"normal" "3ν-inverted" "4ν-inverted" "fully-inverted"

Perturbation of 3-ν Mixing

$|U_{e4}|^2 \ll 1$ $|U_{\mu 4}|^2 \ll 1$ $|U_{\tau 4}|^2 \ll 1$ $|U_{s4}|^2 \simeq 1$

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2$$

$$= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \to \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \qquad \Delta m_{41}^2 \to \Delta m^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left|\delta_{\alpha\beta} - U_{\alpha4}^{*} U_{\beta4}\left[1 - \exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha4}|^2 |U_{\beta4}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha4}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right)\left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right)\sin^2\frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha4}|^2|U_{\beta4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha4}|^2\left(1 - |U_{\alpha4}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1 \,, \quad |U_{\mu 4}|^2 \ll 1 \,, \quad |U_{\tau 4}|^2 \ll 1 \,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{ccc|c|} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{array}\right)$$

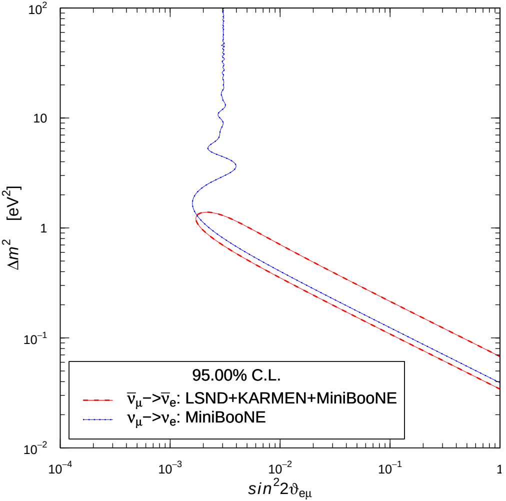
$\uparrow$

SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$



3+1 Schemes

GoF = 32%

PGoF = 0.89%

- Tension between LSND + KARMEN + MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ and MiniBooNE $\nu_\mu \to \nu_e$ $\Longrightarrow$ CP Violation?
- 3+2 $\Longrightarrow$ CP Violation OK [Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004, hep-ph/0305255; Maltoni, Schwetz, PRD 76, 093005 (2007), arXiv:0705.0107; Karagiorgi et al, PRD 80 (2009) 073001, arXiv:0906.1997]
- 3+1+NSI $\Longrightarrow$ CP Violation OK [Akhmedov, Schwetz, JHEP 10 (2010) 115, arXiv:1007.4171]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom
$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$
N_{D} = Number of Data $\qquad$ N_{P} = Number of Fitted Parameters
- $\langle \chi^2_{\text{min}} \rangle = \text{NDF} \qquad \text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz \qquad p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom
$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$
- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, dz$

SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L/4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

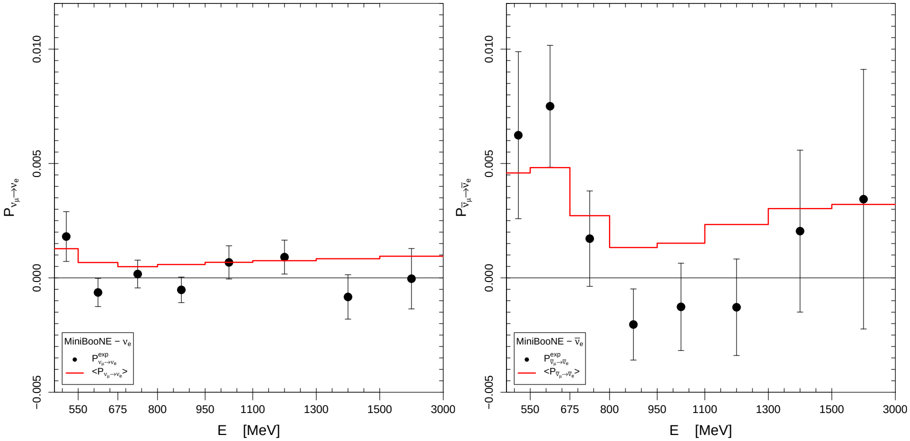
$$\begin{aligned} P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 \phi_{51} \\ & + 4|U_{e4} U_{\mu 4} U_{e5} U_{\mu 5}| \cos\eta \left[\sin^2 \phi_{41} + \sin^2 \phi_{51} - \sin^2 \phi_{54}\right] \\ & \overset{(+)}{-} 2|U_{e4} U_{\mu 4} U_{e5} U_{\mu 5}| \sin\eta \left[\sin(2\phi_{41}) - \sin(2\phi_{51}) + \sin(2\phi_{54})\right] \end{aligned}$$

$$\begin{aligned} P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = {} & 1 - 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2) \sin^2 \phi_{41} + 4|U_{\alpha 5}|^2 (1 - |U_{\alpha 5}|^2) \sin^2 \phi_{51} \\ & + 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \left[\sin^2 \phi_{41} + \sin^2 \phi_{51} - \sin^2 \phi_{54}\right] \end{aligned}$$

MiniBooNE $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

		MB$\bar{\nu}$ + MBν
No Osc.	χ^2	35.5
	NDF	32
	GoF	0.31
Osc. (3+2)	χ^2_{min}	21.1
	NDF	25
	GoF	0.69
	Δm^2_{41}	0.087
	Δm^2_{51}	4.57
	$4\|U_{e4}\|^2\|U_{\mu4}\|^2$	0.19
	$4\|U_{e5}\|^2\|U_{\mu5}\|^2$	0.002
	η/π	1.40
	$4\|U_{\mu4}\|^2(1-\|U_{\mu4}\|^2)$	0.93
	$4\|U_{\mu5}\|^2(1-\|U_{\mu5}\|^2)$	0.0025
PGoF	$\Delta\chi^2_{\text{min}}$	5.72
	NDF	7
	PGoF	0.57

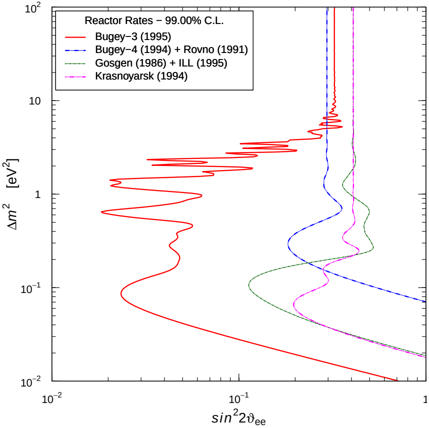
MiniBooNE $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$



$$
\begin{aligned}
P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2|U_{\mu4}|^2 \sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu5}|^2 \sin^2\phi_{51} \\
& + 4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}| \cos\eta \left[\sin^2\phi_{41} + \sin^2\phi_{51} - \sin^2\phi_{54}\right] \\
& \overset{(+)}{-} 2|U_{e4}U_{\mu4}U_{e5}U_{\mu5}| \sin\eta \left[\sin(2\phi_{41}) - \sin(2\phi_{51}) + \sin(2\phi_{54})\right]
\end{aligned}
$$

Disappearance Constraints

$\bar{\nu}_e$ Disappearance

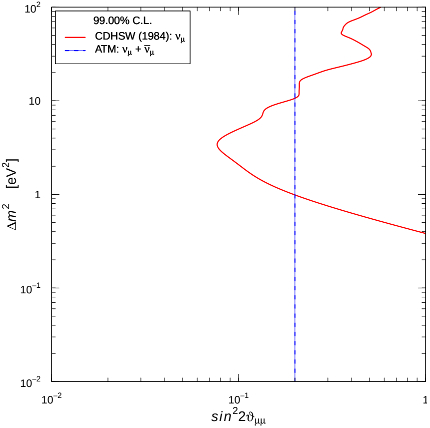


New Reactor $\bar{\nu}_e$ Fluxes

[Mueller et al., arXiv:1101.2663]

[Mention et al., arXiv:1101.2755]

ν_μ and $\bar{\nu}_\mu$ Disappearance



ATM constraint on $|U_{\mu 4}|^2$

[Maltoni, Schwetz, PRD 76 (2007) 093005, arXiv:0705.0107]

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

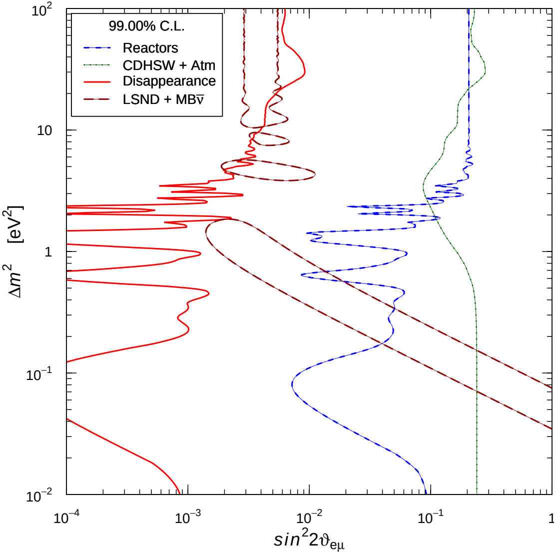
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

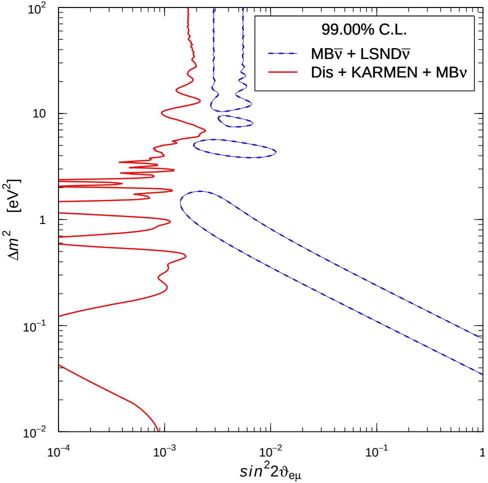
- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694, arXiv:hep-ph/9606411]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247, arXiv:hep-ph/9607372]

3+1 Schemes


99.00% C.L.
Reactors
CDHSW + Atm
Disappearance
LSND + MB$\bar{\nu}$
10^2
10
1
10^{-1}
10^{-2}
Δm^2 [eV^2]
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
$\sin^2 2\vartheta_{e\mu}$



PGoF $= 0.006\%$

- Strong tension between $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance and disappearance limits ($\bar{\nu}_e \to \bar{\nu}_e$ and mainly $\nu_\mu \to \nu_\mu$)
- Tension reduced in 3+2, 3+1+NSI
- CPT Violation?

[Barger, Marfatia, Whisnant, PLB 576 (2003) 303]
[Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

Global 3+2 Fit

[Kopp, Maltoni, Schwetz, arXiv:1103.4570]

Best fit: $$\begin{cases} \Delta m^2_{41} = 0.47\,\text{eV}^2 & \Delta m^2_{51} = 0.87\,\text{eV}^2 \\ |U_{e4}| = 0.128 & |U_{e5}| = 0.138 \\ |U_{\mu 4}| = 0.165 & |U_{\mu 5}| = 0.148 \qquad \eta = 1.64\pi \end{cases}$$

$$\chi^2_{\text{min}}/\text{NDF} = 110.1/130$$

LSND + MiniBooNE($\bar{\nu}$) vs rest

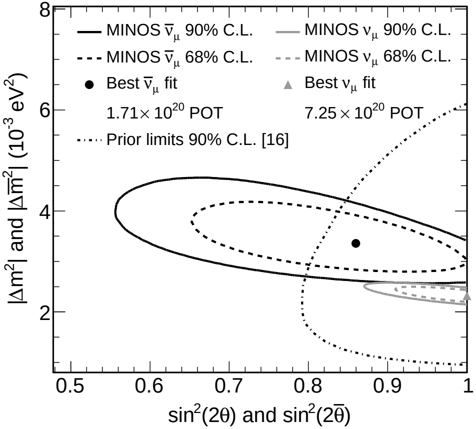
$\chi^2_{\text{PGof}}/\text{NDF}_{\text{PGof}} = 19.9/5$ PGof $= 0.13\%$

MINOS Hint of CPT Violation?

LBL ν_μ and $\bar{\nu}_\mu$ disappearance $E \sim 3\,\text{GeV}$

Near Detector at 1.04 km

Far Detector at 735 km



$$|\Delta m^2| = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta > 0.90 \quad (90\%\ \text{C.L.})$$

[arXiv:1103.0340v1]

$$|\Delta \overline{m}^2| = \left(3.36^{+0.46}_{-0.40}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\overline{\vartheta} = 0.86^{+0.11}_{-0.12}$$

[arXiv:1104.0344v2]

CDF Hint of CPT Violation?

Measurement of the mass difference between t and $\bar{t}$ quarks,

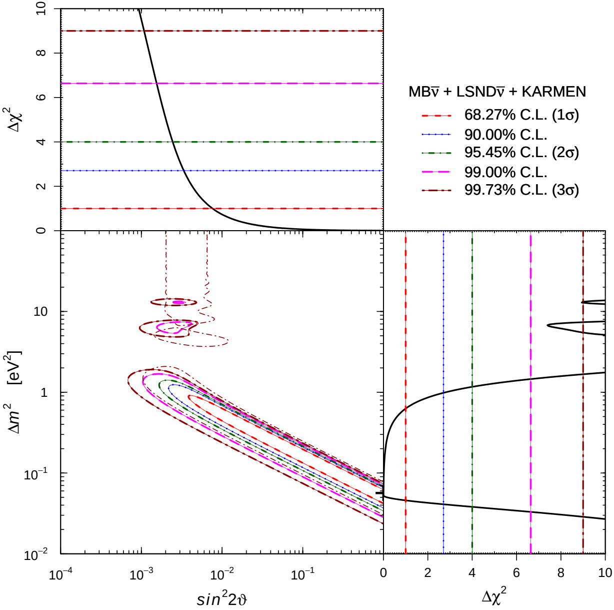
arXiv:1103.2782v1 [hep-ex]

$$m_t - m_{\bar{t}} = -3.3 \pm 1.4 \pm 1.0 \, \text{GeV}$$

“approximately two standard deviations away from the CPT hypothesis of zero mass difference”

Phenomenological Approach: Consider $\bar{\nu}$'s Only

$\bar{\nu}_{\mu} \to \bar{\nu}_{e}$



$\chi^2_{\text{min}} = 29.8$

$\text{NdF} = 26$

$\text{GoF} = 28\%$

$\sin^2 2\vartheta = 1.00$

$\Delta m^2 = 0.052\,\text{eV}^2$

Parameter Goodness-of-Fit

$\Delta\chi^2_{\text{min}} = 5.9$

$\text{NdF} = 4$

$\text{GoF} = 21\%$

[Giunti, Laveder, PRD 82 (2010) 093016, arXiv:1010.1395]

Conservation of Probability

$$\sum_{\alpha} P_{\bar{\nu}_\alpha \to \bar{\nu}_e} = 1$$

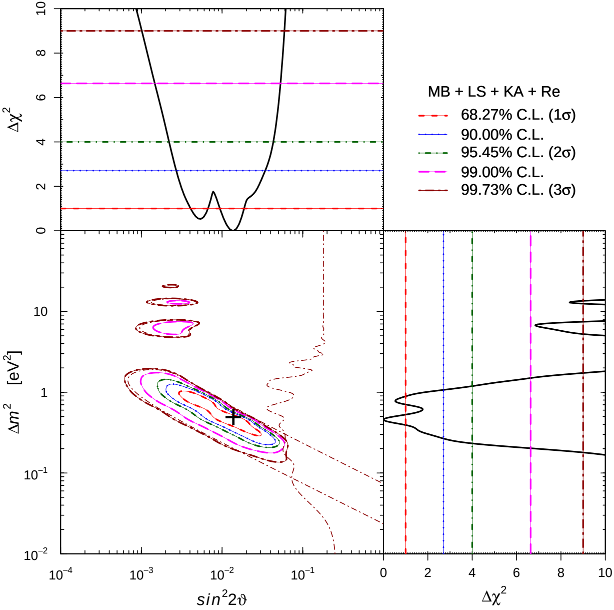
$$P_{\bar{\nu}_e \to \bar{\nu}_e} + P_{\bar{\nu}_\mu \to \bar{\nu}_e} + P_{\bar{\nu}_\tau \to \bar{\nu}_e} + P_{\bar{\nu}_s \to \bar{\nu}_e} = 1$$

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} = 1 - P_{\bar{\nu}_e \to \bar{\nu}_e} - P_{\bar{\nu}_\tau \to \bar{\nu}_e} - P_{\bar{\nu}_s \to \bar{\nu}_e}$$

$$\boxed{P_{\bar{\nu}_\mu \to \bar{\nu}_e} \leq 1 - P_{\bar{\nu}_e \to \bar{\nu}_e}}$$

Reactor $\bar{\nu}_e$ disappearance bound is unavoidable!

$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\bar{\nu}_e \to \bar{\nu}_e$



$\chi^2_{\text{min}} = 81.4$

$\text{NdF} = 82$

$\text{GoF} = 50\%$

$\sin^2 2\vartheta = 0.014$

$\Delta m^2 = 0.46\,\text{eV}^2$

Parameter Goodness-of-Fit

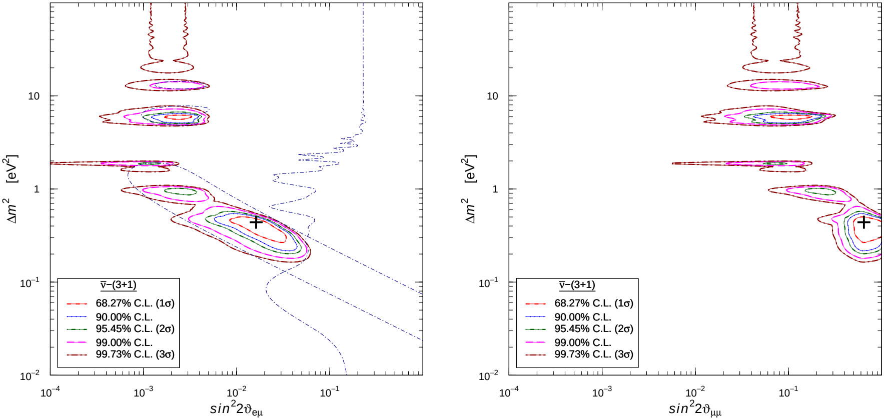
$\Delta\chi^2_{\text{min}} = 3.0$

$\text{NdF} = 2$

$\text{GoF} = 22\%$

[Giunti, Laveder, PRD 82 (2010) 093016, arXiv:1010.1395]

Antineutrino Oscillations in 3+1 Schemes



$$\chi^2_{\text{min}} = 87.0 \qquad \text{NdF} = 83 \qquad \text{GoF} = 36\%$$

$$\Delta m^2 = 0.42\,\text{eV}^2 \quad \sin^2 2\vartheta_{e\mu} = 0.016 \quad \sin^2 2\vartheta_{ee} = 0.020 \quad \sin^2 2\vartheta_{\mu\mu} = 0.65$$

Prediction: large SBL $\bar{\nu}_\mu$ disappearance at $\Delta m^2 \gtrsim 0.1\,\text{eV}^2$

[Giunti, Laveder, PRD 83 (2011) 053006, arXiv:1012.0267]

Gallium Anomaly

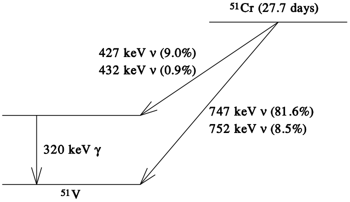
Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

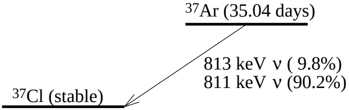
Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

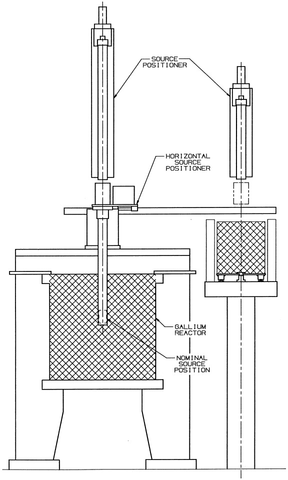
	^{51}Cr				^{37}Ar	
E [keV]	747	752	427	432	811	813
B.R.	0.8163	0.0849	0.0895	0.0093	0.902	0.098



[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]



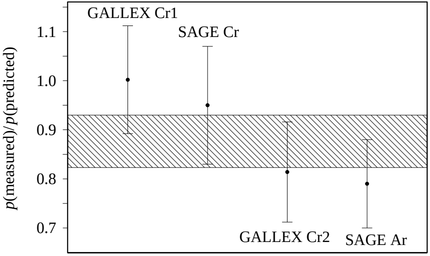
[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]



[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]

$$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$$

$$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$$



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

$$\begin{aligned}
R_{\text{B}}^{\text{Gallex-Cr1}} &= 0.953 \pm 0.11 \\
R_{\text{B}}^{\text{Gallex-Cr2}} &= 0.812^{+0.10}_{-0.11} \\
R_{\text{B}}^{\text{SAGE-Cr}} &= 0.95 \pm 0.12 \\
R_{\text{B}}^{\text{SAGE-Ar}} &= 0.791^{+0.084}_{-0.078}
\end{aligned}$$

$$\boxed{R_{\text{B}}^{\text{Ga}} = 0.86 \pm 0.05}$$

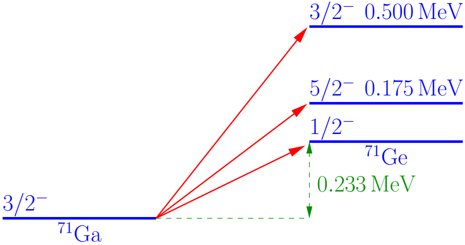
Bahcall cross section

Only exp uncertainties!

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391, hep-ph/9710491



- $\sigma_{\mathrm{G.S.}}$ related to measured $\sigma(e^- + {}^{71}\mathrm{Ge} \to {}^{71}\mathrm{Ga} + \nu_e)$:

$$\sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) = 55.3 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\mathrm{Cr}) = \sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) \left(1 + 0.669\,\frac{\mathrm{BGT}_{175\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}} + 0.220\,\frac{\mathrm{BGT}_{500\,\mathrm{keV}}}{\mathrm{BGT}_{\mathrm{G.S.}}}\right)$

- Contribution of Excited States only 5%!

- Bahcall: [Bahcall, PRC 56 (1997) 3391, hep-ph/9710491]

  from $p + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + n$ measurements [Krofcheck et al., PRL 55 (1985) 1051]

  $$\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} < 0.056 \Rightarrow \frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = \frac{0.056}{2} \qquad \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0.146$$

  3σ lower limit: $\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0$

  3σ upper limit: $\frac{\text{BGT}_{175\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} < 0.056 \times 2 \qquad \frac{\text{BGT}_{500\,\text{keV}}}{\text{BGT}_{\text{G.S.}}} = 0.146 \times 2$

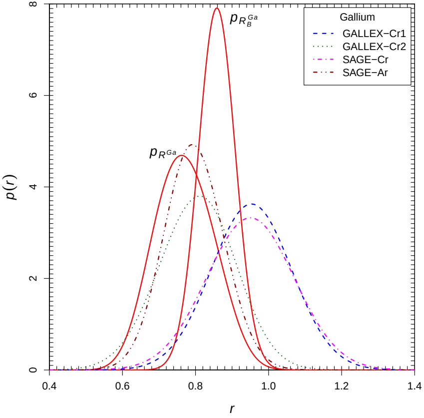
  $$\sigma({}^{51}\text{Cr}) = 58.1 \times 10^{-46}\,\text{cm}^2 \left(1^{+0.036}_{-0.028}\right)_{1\sigma} \quad \Longrightarrow \quad \boxed{R_{\text{B}}^{\text{Ga}} = 0.86 \pm 0.06}$$

- Haxton: [Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

  “a sophisticated shell model calculation is performed ... for the transition to the first excited state in ^{71}Ge. The calculation predicts destructive interference between the (p, n) spin and spin-tensor matrix elements.”

$$\sigma({}^{51}\text{Cr}) = 63.9 \times 10^{-46}\,\text{cm}^2\,(1 \pm 0.106)_{1\sigma} \quad \Longrightarrow \quad \boxed{R_{\text{H}}^{\text{Ga}} = 0.78 \pm 0.13}$$

- $R_{\mathrm{H}}^{\mathrm{Ga}} = 0.78 \pm 0.13$ exp and the uncertainties added in quadrature
- $R^{\mathrm{Ga}} = R_{\mathrm{exp}}^{\mathrm{Ga}} / R_{\mathrm{the}}^{\mathrm{Ga}}$ probability distribution of ratio is not Gaussian



$$p_{R^{\mathrm{Ga}}}(r) = \int_{R_{\mathrm{gs}}^{\mathrm{Ga}}}^{\infty} p_{R_{\mathrm{exp}}^{\mathrm{Ga}}}(rs)\, p_{R_{\mathrm{the}}^{\mathrm{Ga}}}(s)\, s \, \mathrm{d}s$$

$$\boxed{R^{\mathrm{Ga}} = 0.76^{+0.09}_{-0.08}}$$

$$\begin{aligned} R^{\text{Gallex-Cr1}} &= 0.84^{+0.13}_{-0.12} \\ R^{\text{Gallex-Cr2}} &= 0.71^{+0.12}_{-0.11} \\ R^{\text{SAGE-Cr}} &= 0.84^{+0.14}_{-0.13} \\ R^{\text{SAGE-Ar}} &= 0.70^{+0.10}_{-0.09} \end{aligned}$$

[Giunti, Laveder, arXiv:1006.3244]

Gallium Radioactive Source Experiments
are
Short-BaseLine Neutrino Oscillation Experiments

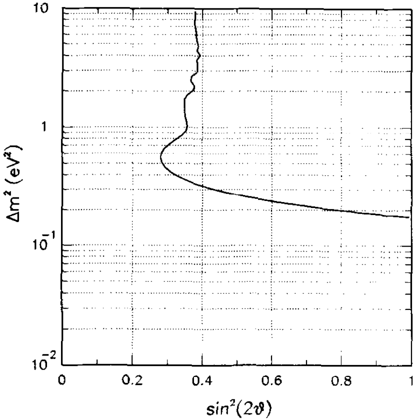


Fig. 1. Region of electron neutrino oscillation parameters ruled out at 90% C.L. by the GALLEX ^{51}Cr source experiment.

[Bahcall, Krastev, Lisi, PLB 348 (1995) 121]

$$P^{\text{SBL}}_{\nu_e\to\nu_e}(L,E) = 1 - \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$R^k(\sin^2 2\vartheta, \Delta m^2) = \frac{\int_k \mathrm{d}V \, L^{-2} \sum_i b_i^k \, \sigma_i^k \, P^{\text{SBL}}_{\nu_e\to\nu_e}(L,E_i)}{\sum_i b_i^k \, \sigma_i^k \int_k \mathrm{d}V \, L^{-2}}$$

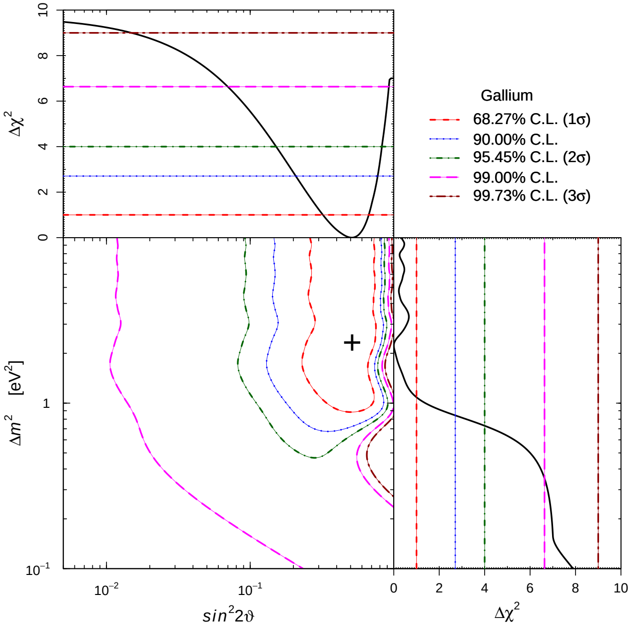
$$k = \text{GALLEX-Cr1}\,,\ \text{GALLEX-Cr2}\,,\ \text{SAGE-Cr}\,,\ \text{SAGE-Ar}$$

$R^k = R^k_{\text{exp}}/R^k_{\text{the}}$ fully correlated theoretical uncertainty!

$$p_{\vec{R}}(\vec{r}) = \int_{R^k_{\text{gs}}}^{\infty} \left[\prod_k p_{R^k_{\text{exp}}}(r^k s)\right] p_{R^k_{\text{the}}}(s) \, s^4 \, \mathrm{d}s$$

$$\mathcal{L}(\sin^2 2\vartheta, \Delta m^2) = p_{\vec{R}}(\vec{R}(\sin^2 2\vartheta, \Delta m^2))$$

$$\chi^2(\sin^2 2\vartheta, \Delta m^2) = -2 \ln \mathcal{L}(\sin^2 2\vartheta, \Delta m^2) + \text{constant}$$



$$\Delta\chi^2_{\text{No Osc.}} = 9.7$$

No Osc. disfavored at 99.23 % C.L (2.7σ)

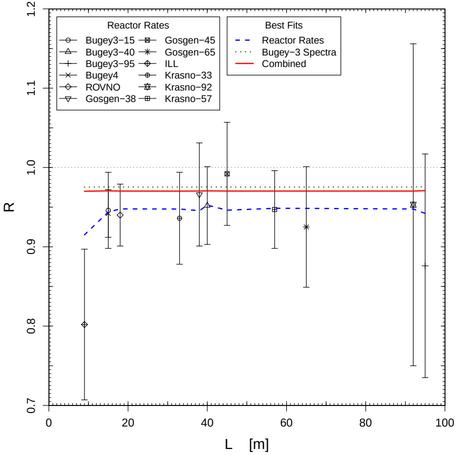
Osc.

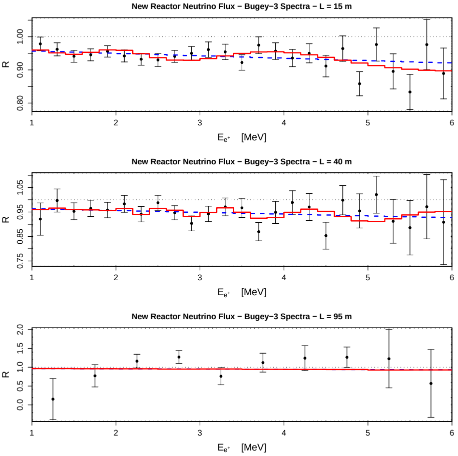
$$\sin^2 2\vartheta_{\text{bf}} = 0.51$$

$$\Delta m^2_{\text{bf}} = 2.24\,\text{eV}^2$$

[Giunti, Laveder, arXiv:1006.3244]

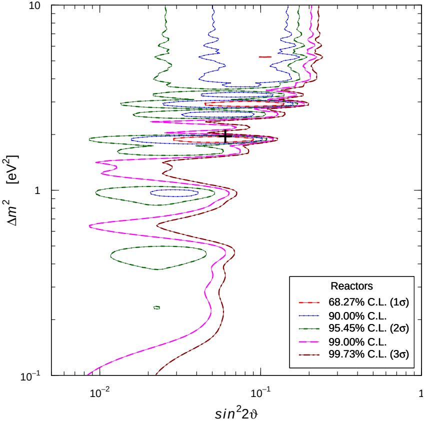
Reactor Antineutrino Anomaly





- $\overline{R}_{\text{rates}} = 0.946 \pm 0.024$
- Improved hint of oscillations given by Bugey energy spectrum with old reactor fluxes [Acero, Giunti, Laveder, PRD 78 (2008) 073009, arXiv:0711.4222]
- $\sin^2 2\vartheta_{\text{bf}} = 0.059 \qquad \Delta m^2_{\text{bf}} = 1.89\,\text{eV}^2$

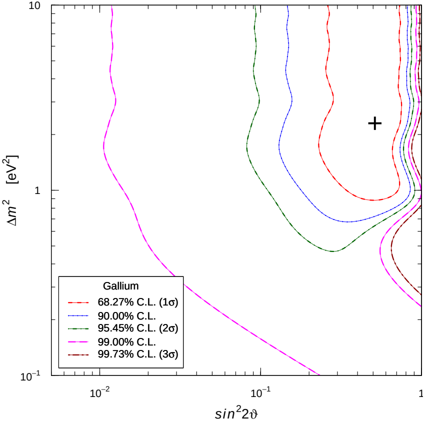
Reactor Antineutrino Anomaly



$$\sin^2 2\vartheta_{\rm bf} = 0.059$$

$$\Delta m^2_{\rm bf} = 1.89\,\text{eV}^2$$

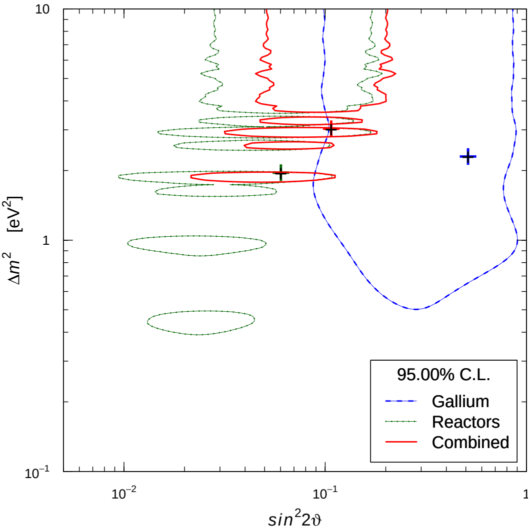
Gallium Neutrino Anomaly



$$\sin^2 2\vartheta_{\rm bf} = 0.51$$

$$\Delta m^2_{\rm bf} = 2.24\,\text{eV}^2$$

Gallium Anomaly + Reactor Anomaly



$\chi^2_{\text{min}} = 59.6$

NdF $= 71$

GoF $= 83\%$

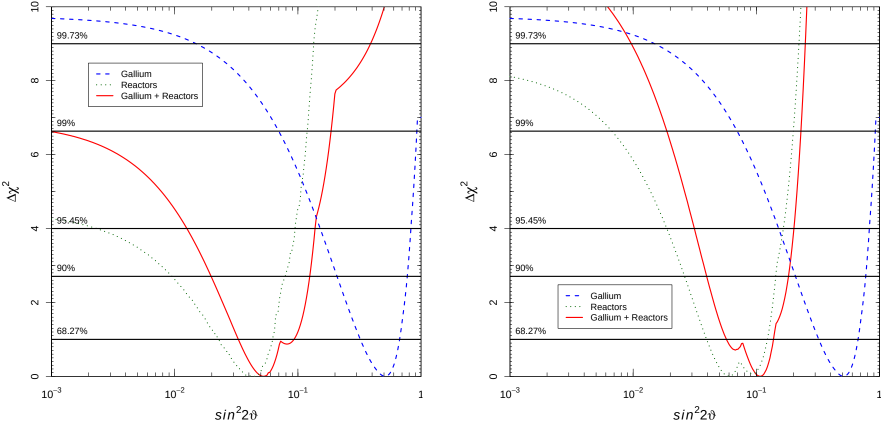
$\sin^2 2\vartheta = 0.11$

$\Delta m^2 = 2.95\,\text{eV}^2$

PGoF $= 4.6\%$

Old Reactor $\bar{\nu}_e$ Fluxes

New Reactor $\bar{\nu}_e$ Fluxes

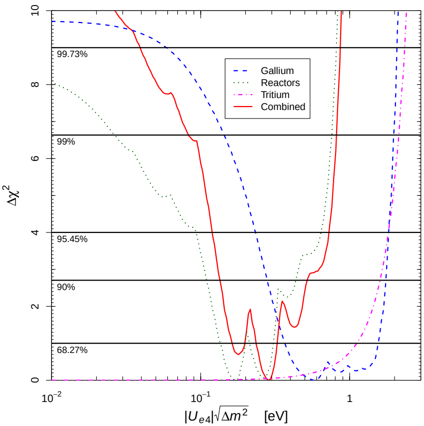


PGoF = 2.3%

PGoF = 4.6%

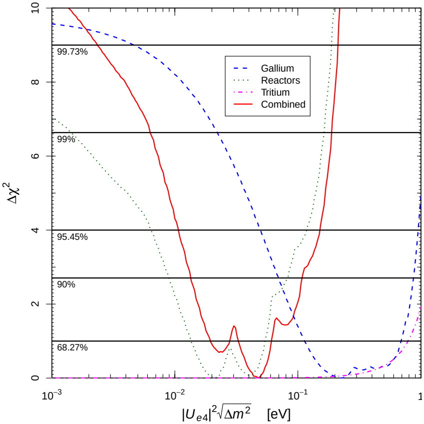
Implications of Gallium and Reactor Anomalies

β Decay



$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

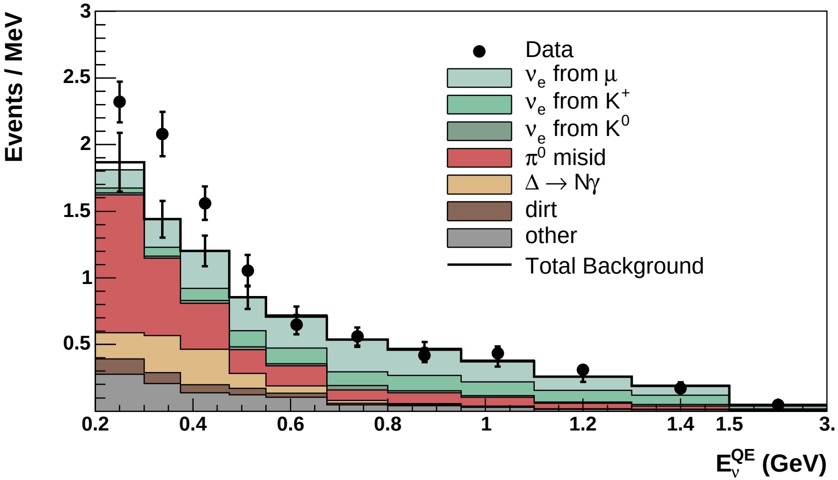
$(\beta\beta)_{0\nu}$ Decay



$$m_{\beta\beta} = \left|\sum_k U_{ek}^2\, m_k\right|$$

[Giunti, Laveder, In Preparation]

MiniBooNE Low-Energy Anomaly



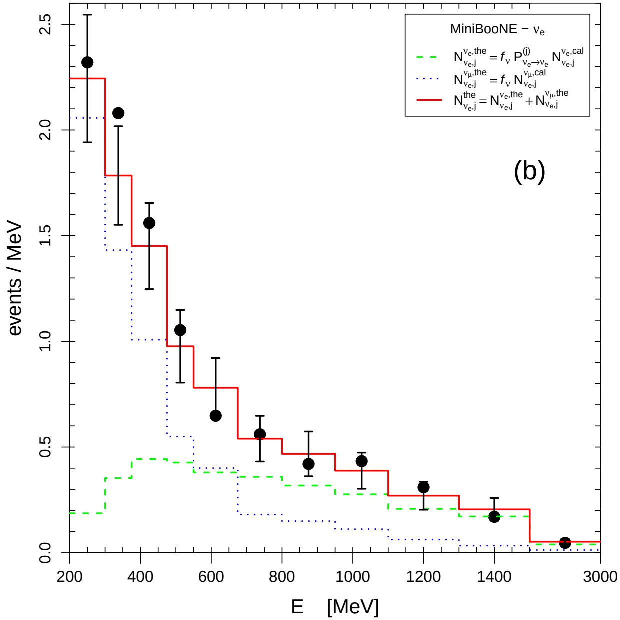
[PRL 102 (2009) 101802, arXiv:0812.2243]

Our Hypothesis: $N_{\nu,j}^{\text{the}} = f_\nu \left(P_{\nu_e \to \nu_e} N_{\nu_e,j}^{\text{cal}} + N_{\nu_\mu,j}^{\text{cal}} \right)$

[Giunti, Laveder, PRD 77 (2008) 093002, arXiv:0707.4593; PRD 80 (2009) 013005, arXiv:0902.1992]

$$\boxed{N_{\nu,j}^{\text{the}} = f_\nu \left(P_{\nu_e \to \nu_e} N_{\nu_e,j}^{\text{cal}} + N_{\nu_\mu,j}^{\text{cal}} \right)}$$

- Estimated 15% uncertainty of the calculated neutrino flux [MiniBooNE, PRD 79 (2009) 072002, arXiv:0806.1449] is consistent with measured ratio 1.21 ± 0.24 of detected and predicted charged-current quasi-elastic ν_μ events [MiniBooNE, PRL 100 (2008) 032301, arXiv:0706.0926]
- We fit MiniBooNE ν_e and ν_μ data using the info at `http://www-boone.fnal.gov/for_physicists/data_release/lowe/`



No Osc. & $f_\nu = 1$

$\chi^2_{\rm min} = 14.3 + 5.4$

NdF $= 3 + 16$

GoF $= 41\%$

Our Hypothesis

$\chi^2_{\rm min} = 2.0 + 7.6$

NdF $= 16$

GoF $= 89\%$

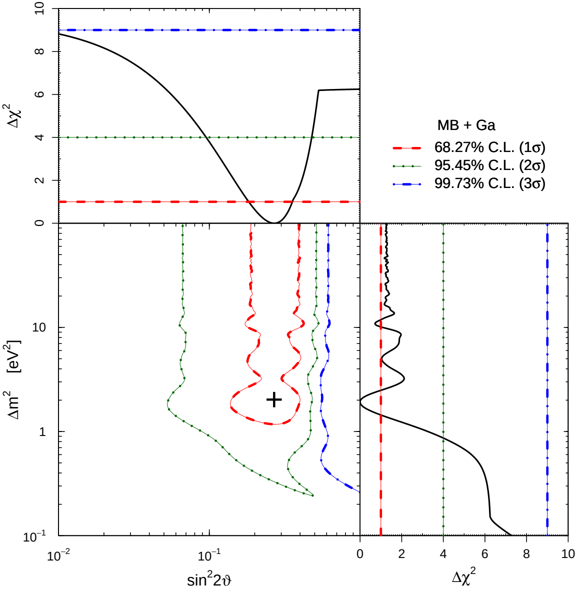
$f_\nu = 1.26$

$\sin^2 2\vartheta = 0.32$

$\Delta m^2 = 1.84\,\mathrm{eV}^2$

[Giunti, Laveder, PRD 82 (2010) 053005, arXiv:1005.4599]

MiniBooNE + Gallium



$\chi^2_{\text{min}} = 2.3 + 9.2$

$\text{NdF} = 20$

$\text{GoF} = 93\%$

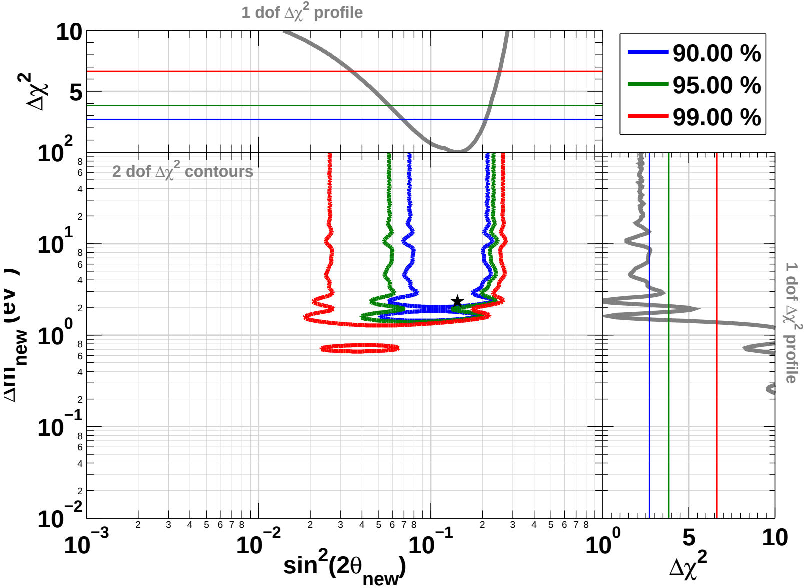
$\sin^2 2\vartheta = 0.27$

$\Delta m^2 = 1.92\,\text{eV}^2$

$\text{PGoF} = 93\%$

[Giunti, Laveder, PRD 82 (2010) 053005, arXiv:1005.4599]

MiniBooNE + Gallium + Reactors



[Mention et al., arXiv:1101.2755]

Future

- MiniBooNE is continuing to take antineutrino data.

$$\bar{\nu}_\mu \to \bar{\nu}_e + \bar{\nu}_\mu \to \bar{\nu}_\mu$$

- ICARUS@CERN-PS: $L \sim 1\text{km}$ $E \sim 1\text{GeV}$ [C. Rubbia et al, CERN-SPSC-2011-012]

$$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e + \overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu + \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$$

- MicroBooNE will test the MiniBooNE low-energy anomaly by measuring $\pi^0 \to 2\gamma$ background.

- ν_e disappearance: new SAGE Gallium source experiments with 2 spherical shells [Gavrin et al, arXiv:1006.2103]

- CPT test: ν_e and $\bar{\nu}_e$ disappearance

- Beta-Beam experiments: [Antusch, Fernandez-Martinez, PLB 665 (2008) 190, arXiv:0804.2820]

$$N(A,Z) \to N(A,Z+1) + e^- + \bar{\nu}_e \qquad (\beta^-)$$

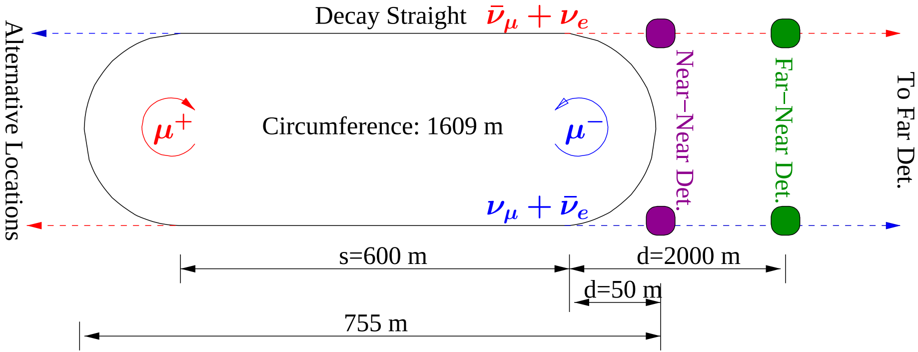
$$N(A,Z) \to N(A,Z-1) + e^+ + \nu_e \qquad (\beta^+)$$

- Neutrino Factory experiments: [Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

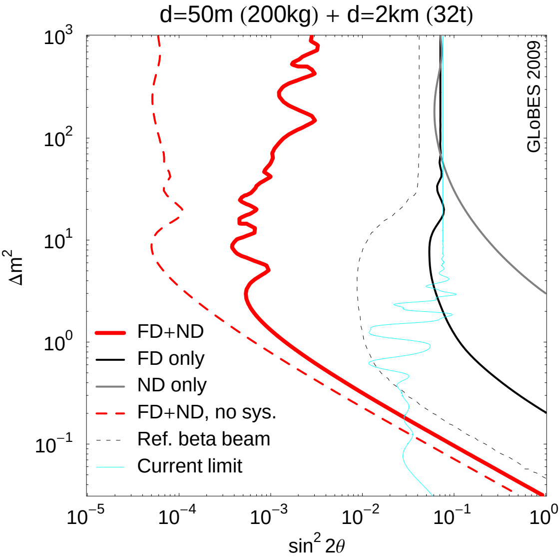
Neutrino Factory



[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

Near Detectors:	Scintillator or Iron Calorimeter with perfect flavor identification
Systematic Uncertainties:	Cross Section, Detector Normalization, Energy Resolution and Calibration, Backgrounds

ν_e Disappearance



[Giunti, Laveder, Winter, PRD 80 (2009) 073005, arXiv:0907.5487]

- New ν_e and $\bar{\nu}_e$ radioactive source experiments with low-threshold neutrino elastic scattering detectors.

- Borexino: $\overset{(-)}{\nu}_e + e^- \to \overset{(-)}{\nu}_e + e^-$ [Pallavicini, talk at BEYOND3NU]

  [Ianni, Montanino, Scioscia, EPJC 8 (1999) 609, arXiv:hep-ex/9901012]

- LENS (Low Energy Neutrino Spectroscopy): [Agarwalla, Raghavan, arXiv:1011.4509]

$$\nu_e + {}^{115}\mathrm{In} \to {}^{115}\mathrm{Sn} + e^- + 2\gamma \qquad E_{\mathrm{th}} = 0.1\,\mathrm{MeV}$$

$$\bar{\nu}_e + p \to n + e^+ \qquad E_{\mathrm{th}} = 1.8\,\mathrm{MeV}$$

- Spherical Gaseous TPC: [Vergados, Giomataris, Novikov, arXiv:1103.5307]
  - Targets: ^{131}Xe, ^{40}Ar, ^{20}Ne, ^{4}He.
  - Sources: ^{37}Ar, ^{51}Cr, ^{65}Zn, ^{32}P.

Conclusions - 1

- Suggestive LSND and MiniBooNE agreement on SBL $\bar{\nu}_\mu \to \bar{\nu}_e$
- Hint in favor of sterile neutrinos is compatible with cosmological data, but mass is limited
- Two experimental tensions:
    - LSND and MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ vs MiniBooNE $\nu_\mu \to \nu_e$ (CP violation?)
    - LSND and MiniBooNE $\bar{\nu}_\mu \to \bar{\nu}_e$ vs $\bar{\nu}_e$ and ν_μ disappearance limits
- CPT-invariant 3+1 Four-Neutrino Mixing is strongly disfavored (no CP violation and tension between appearance and disappearance)
- 3+2 can explain CP violation and reduce tension between appearance and disappearance with New Reactor $\bar{\nu}_e$ Fluxes
- 3+1+NSI has CP violation and reduced appearance-disappearance tension
- CPT-violating 3+1 Mixing $\Longrightarrow$ testable large SBL $\bar{\nu}_\mu$ disappearance

Conclusions - 2

- Interesting possible agreement of
  - Gallium Anomaly (SBL ν_e disappearance)
  - Reactor Anomaly (SBL $\bar{\nu}_e$ disappearance)
- Testable Predictions:
  - $m_\beta \sim 0.12 - 0.71\,\text{eV} \quad (2\sigma)$
  - $m_{\beta\beta} \sim 0.011 - 0.15\,\text{eV} \quad (2\sigma)$
- Exciting experimental results in favor of sterile neutrinos.
- More work to do because interpretation is not clear:
  - Explanation of all data needs at least two new physical effects.
  - Without CPT violation tensions do not disappear completely.
  - Possible that some experiments are giving misleading information.
- New short-baseline neutrino oscillation experiments are needed!