# Neutrino Masses

**Carlo Giunti**

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

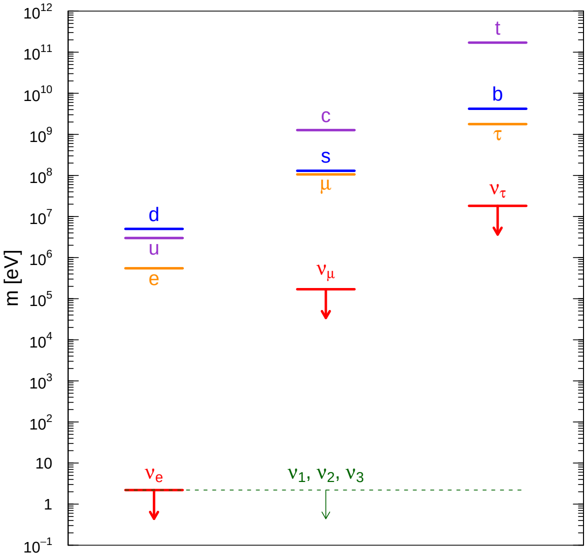
mailto://giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

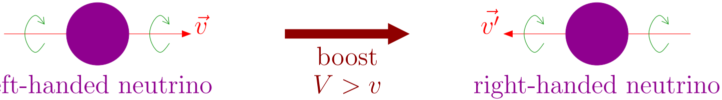
47th Rencontres de Moriond - Cosmology

10 - 17 March 2012, La Thuile (Val d'Aosta, Italy)

# Fermion Mass Spectrum



# Standard Model: Massless Neutrinos




Standard Model: $\nu_L \quad \Longrightarrow \quad$ no Dirac mass term

$$\mathcal{L}^{\mathrm{D}} = -m^{\mathrm{D}} \left( \overline{\nu_R} \nu_L + \overline{\nu_L} \nu_R \right) \qquad \text{(no } \nu_R\text{)}$$

Majorana Neutrino: $\nu = \nu^c \Longrightarrow \nu_R = \nu_L^c \Longrightarrow$ Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} m^{\mathrm{M}} \left( \overline{\nu_L^c} \nu_L + \overline{\nu_L} \nu_L^c \right)$$

Standard Model: Majorana mass term not allowed by $SU(2)_L \times U(1)_Y$

(no Higgs triplet)

# Extension of the SM: Massive Neutrinos

Standard Model can be extended with $\nu_R$

Dirac neutrino mass term $\mathcal{L}^{\mathrm{D}} = -m^{\mathrm{D}} \left( \overline{\nu_R} \nu_L + \overline{\nu_L} \nu_R \right) \Rightarrow m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$

surprise: Majorana neutrino mass for $\nu_R$ is allowed!

$$\mathcal{L}^{\mathrm{M}}_R = -\frac{1}{2} m^{\mathrm{M}}_R \left( \overline{\nu^c_R} \nu_R + \overline{\nu_R} \nu^c_R \right)$$

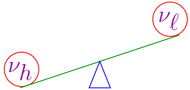
total neutrino mass term $\quad \mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu^c_L} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} + \mathrm{H.c.}$

$m^{\mathrm{M}}_R$ can be arbitrarily large (not protected by SM symmetries)

$m^{\mathrm{M}}_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m^{\mathrm{M}}_R \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m^{\mathrm{M}}_R \end{pmatrix} \quad \Rightarrow \quad m_\ell \simeq \frac{(m^{\mathrm{D}})^2}{m^{\mathrm{M}}_R}, \qquad m_h \simeq m^{\mathrm{M}}_R$

natural explanation of smallness of light neutrino masses




see-saw mechanism

massive neutrinos are Majorana!

3-GEN $\Rightarrow$ effective low-energy 3-$\nu$ mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

# Three-Neutrino Mixing

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \overline{\nu_e} & \overline{\nu_\mu} & \overline{\nu_\tau} \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

diagonalization of mass matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \qquad\qquad (\alpha = e, \mu, \tau)$$

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \overline{\nu_1} & \overline{\nu_2} & \overline{\nu_3} \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \sum_{k=1}^{3} m_k \overline{\nu_k} \nu_k$$

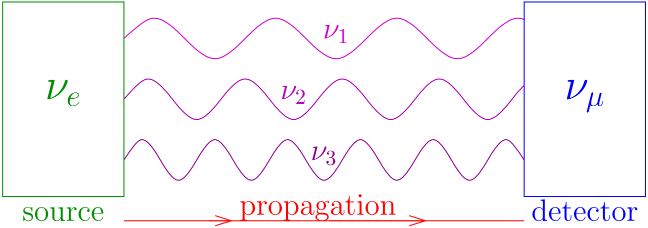
# Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955)
- Flavor Neutrinos: $\nu_e, \nu_\mu, \nu_\tau$ produced in Weak Interactions
- Massive Neutrinos: $\nu_1, \nu_2, \nu_3$ propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}
$$

- $U$ is the $3 \times 3$ Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$




$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

at the detector there is a probability $> 0$ to see the neutrino as a $\nu_\mu$

Neutrino Oscillations are Flavor Transitions

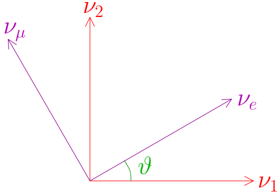
$$\nu_e \to \nu_\mu \qquad \nu_e \to \nu_\tau \qquad \nu_\mu \to \nu_e \qquad \nu_\mu \to \nu_\tau$$

$$\bar{\nu}_e \to \bar{\nu}_\mu \qquad \bar{\nu}_e \to \bar{\nu}_\tau \qquad \bar{\nu}_\mu \to \bar{\nu}_e \qquad \bar{\nu}_\mu \to \bar{\nu}_\tau$$

$$\Delta L_e, \Delta L_\mu, \Delta L_\tau = \pm 1 \qquad \Delta L = 0$$

# Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$




$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

# Experimental Evidences of Neutrino Oscillations

| | | |
|---|---|---|
| Solar<br>$\nu_e \to \nu_\mu, \nu_\tau$ | Homestake<br>Kamiokande<br>GALLEX/GNO & SAGE<br>Super-Kamiokande<br>SNO<br>BOREXino | |
| Reactor<br>$\bar{\nu}_e$ disappearance | (KamLAND) | |

$$\to \begin{cases} \Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2 \\ \tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06 \end{cases}$$

| | | |
|---|---|---|
| Atmospheric<br>$\nu_\mu \to \nu_\tau$ | Kamiokande<br>IMB<br>Super-Kamiokande<br>MACRO<br>Soudan-2 | |
| Accelerator<br>$\nu_\mu$ disappearance | (K2K & MINOS) | |

$$\to \begin{cases} \Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07 \end{cases}$$

Two scales of $\Delta m^2$: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ\,, \quad \vartheta_{\text{SOL}} \simeq 34^\circ$

# Three-Neutrino Mixing Paradigm

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: $\nu_e$, $\nu_\mu$, $\nu_\tau$

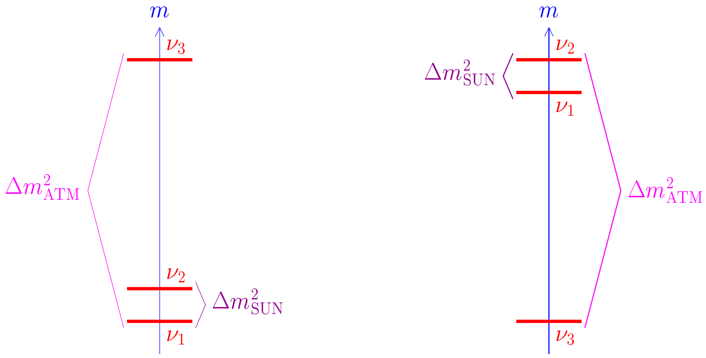
three massive fields: $\nu_1$, $\nu_2$, $\nu_3$

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{31} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$

# Allowed Three-Neutrino Schemes




"normal" "inverted"

different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

# Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

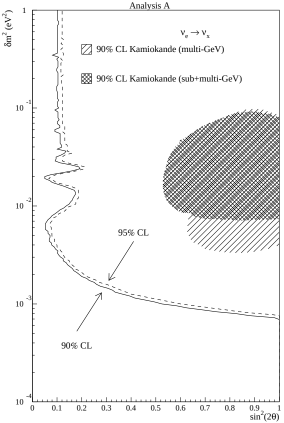
$$\begin{array}{rc} \text{SOL} \rightarrow & \\ & U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ & \text{ATM \& LBL} \end{array}$$

Chooz: $\begin{cases} \Delta m^2_{\text{Chooz}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{Chooz}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$

[Bilenky, Giunti, PLB 444 (1998) 379]




[Chooz, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC $\nu$ OSCILLATIONS
ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau 3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

$$U^{\mathrm{D}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\mathrm{ATM}}} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\mathrm{SOL}}}$$

$$\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\mathrm{eV}^2 \qquad |\Delta m^2_{31}| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2\vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2\vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

[Schwetz, Tortola, Valle, arXiv:1108.1376]

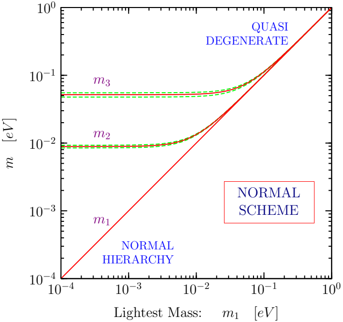
6 days ago: $\boxed{\sin^2\vartheta_{13} = 0.023 \pm 0.004}$ [Daya Bay, arXiv:1203.1669]

Previous indications of $\sin^2\vartheta_{13} > 0$: [T2K, arXiv:1106.2822], [MINOS, arXiv:1108.0015], [Double Chooz, arXiv:1112.6353]

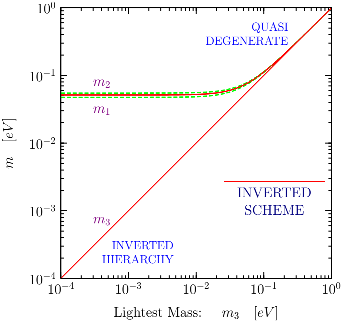
$\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

# Absolute Scale of Neutrino Masses

normal scheme




inverted scheme




$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

- Tritium Beta-Decay $\qquad m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$

$m_\beta < 2.2\,\text{eV}$ (95% C.L.) Mainz & Troitsk [hep-ex/0210050]

KATRIN sensitivity: $m_\beta \simeq 0.2\,\text{eV}$ [hep-ex/0109033, hep-ex/0309007]

- Neutrinoless Double-Beta Decay $\qquad m_{\beta\beta} = \sum_k U_{ek}^2 m_k$

$|m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}$ (90% C.L.) CUORICINO [arXiv:1012.3266]

- Cosmology

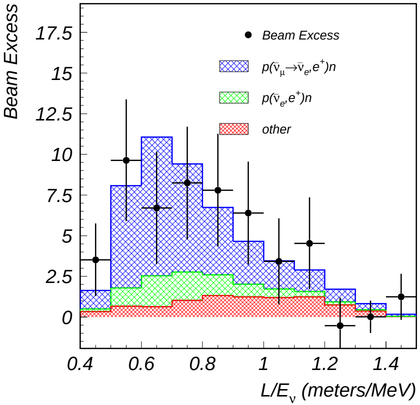
$$\sum_{k=1}^{3} m_k \lesssim 0.2 - 0.6\,\text{eV} \quad (95\%\ \text{C.L.})$$ [hep-ph/0805.2517, arXiv:1006.3795]

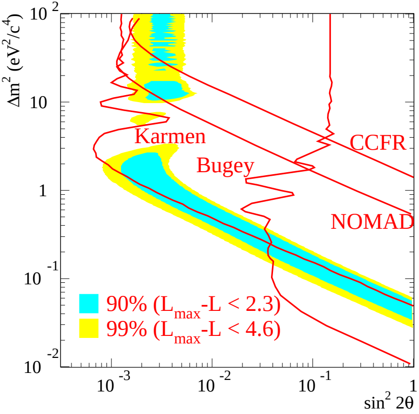
# Anomalies Beyond 3-$\nu$ Mixing

# LSND

[LSND, PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $L \simeq 30\,\mathrm{m}$ $20\,\mathrm{MeV} \leq E \leq 200\,\mathrm{MeV}$



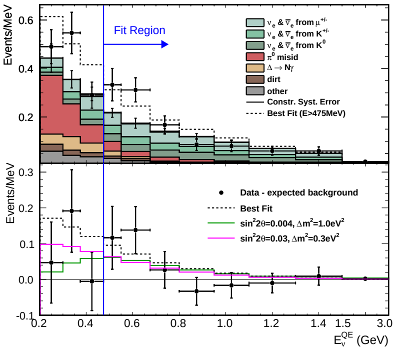




$$\Delta m^2_{\mathrm{LSND}} \gtrsim 0.2\,\mathrm{eV}^2 \qquad (\gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}})$$

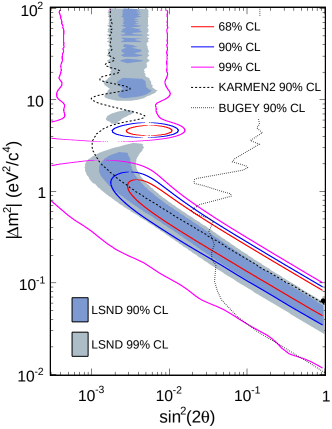
# MiniBooNE Antineutrinos

[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 541\,\mathrm{m}$ $\quad 475\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$




[MiniBooNE, PRL 105 (2010) 181801, arXiv:1007.1150]



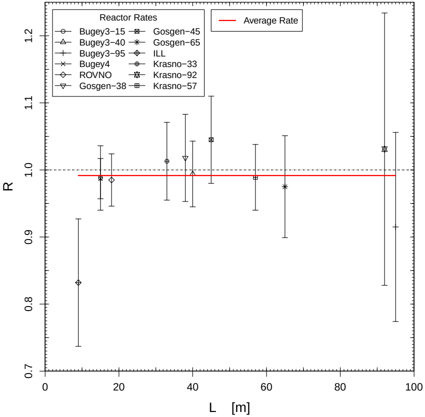

Agreement with LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal!

Similar $L/E$ but different $L$ and $E$ $\Longrightarrow$ Oscillations!

# Reactor Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006, arXiv:1101.2755]

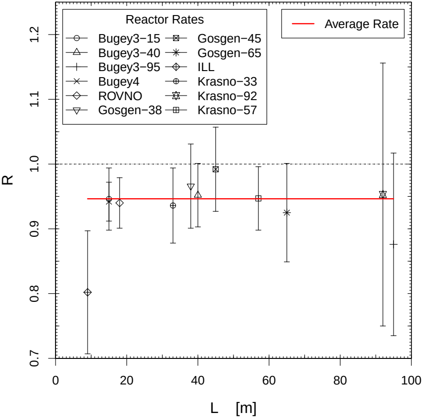
## Old Reactor $\bar{\nu}_e$ Fluxes




$$\overline{R} = 0.992 \pm 0.024$$

## New Reactor $\bar{\nu}_e$ Fluxes

[Mueller et al, PRC 83 (2011) 054615, arXiv:1101.2663]




$$\overline{R} = 0.946 \pm 0.024$$

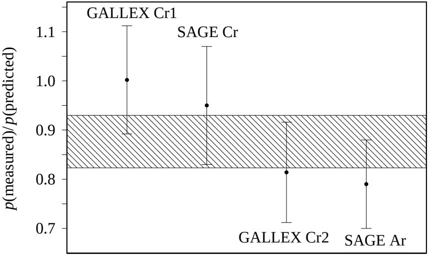
# Gallium Anomaly

Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

$\nu_e$ Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

Detection Process: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$




[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

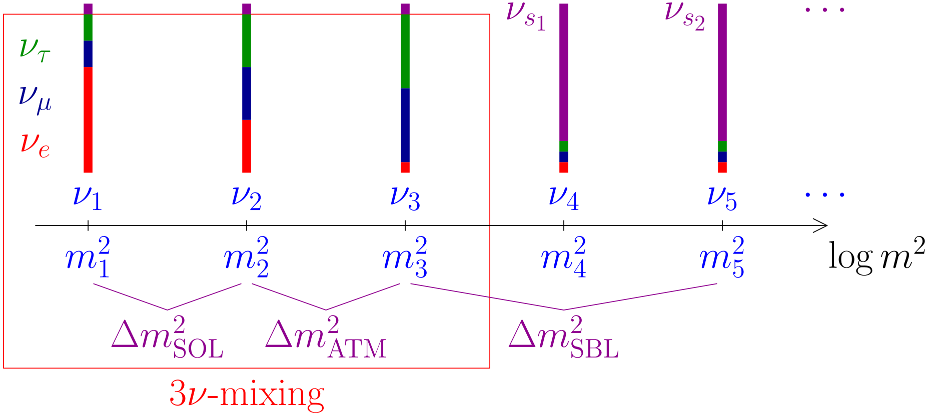
$$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$$

$$\langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$$

$$\boxed{R^{\mathrm{Ga}} = 0.76^{+0.09}_{-0.08}}$$

[Giunti, Laveder, PRC 83 (2011) 065504, arXiv:1006.3244]

# Beyond Three-Neutrino Mixing



# Sterile Neutrinos

- Light anti-$\nu_R$ are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos $(\nu_s)$
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Powerful window on new physics beyond the Standard Model
- Unfortunately such light sterile neutrinos are Hot Dark Matter

# Cosmology

- $N_s$ = number of thermalized sterile neutrinos (not necessarily integer)

- CMB and LSS in ΛCDM: $N_s = 1.3 \pm 0.9 \quad m_s < 0.66\,\text{eV} \quad (95\%\ \text{C.L.})$

[Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301, arXiv:1006.5276]

$$N_s = 1.61 \pm 0.92 \qquad m_s < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

[Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023, arXiv:1102.4774]

$N_s = 1.12^{+0.86}_{-0.74} \quad (95\%\ \text{C.L.})$ [Archidiacono, Calabrese, Melchiorri, PRD 84 (2011) 123008, arXiv:1109.2767]

- BBN: $\begin{cases} N_s = 0.22 \pm 0.59 \\ N_s = 0.64^{+0.40}_{-0.35} \\ N_s \leq 1 \text{ at } 95\%\ \text{C.L.} \end{cases}$
  - $N_s = 0.22 \pm 0.59$: [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]
  - $N_s = 0.64^{+0.40}_{-0.35}$: [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]
  - $N_s \leq 1$ at 95% C.L.: [Mangano, Serpico, PLB 701 (2011) 296, arXiv:1103.1261]

- CMB+LSS+BBN: $N_s = 0.85^{+0.39}_{-0.56} \quad (95\%\ \text{C.L.})$

[Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034, arXiv:1108.4136]

# Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of $3\nu$ Mixing

$$|U_{e4}|^2 \ll 1\,, \quad |U_{\mu 4}|^2 \ll 1\,, \quad |U_{\tau 4}|^2 \ll 1\,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

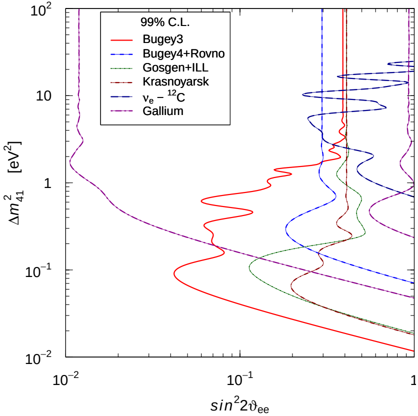
$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

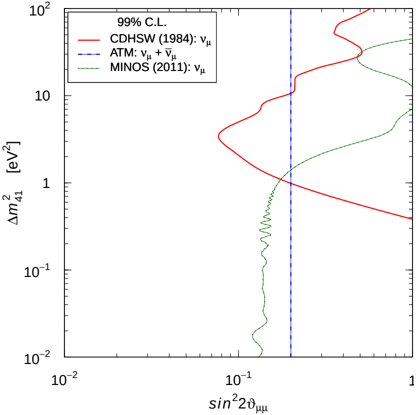
$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

# Disappearance Constraints

## $\bar{\nu}_e$ Disappearance




## $\nu_\mu$ and $\bar{\nu}_\mu$ Disappearance




- New Reactor $\bar{\nu}_e$ Fluxes
  [Mueller et al., arXiv:1101.2663]
  [Mention et al., arXiv:1101.2755]
- KARMEN+LSND $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$
  [Conrad, Shaevitz, arXiv:1106.5552]
  [Giunti, Laveder, arXiv:1111.1069]
- ATM constraint on $|U_{\mu 4}|^2$
  [Maltoni, Schwetz, arXiv:0705.0107]
- MINOS constraint on $|U_{\mu 4}|^2$
  [Giunti, Laveder, arXiv:1109.4033]

# 3+1

- $\nu_e$ disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- $\nu_\mu$ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

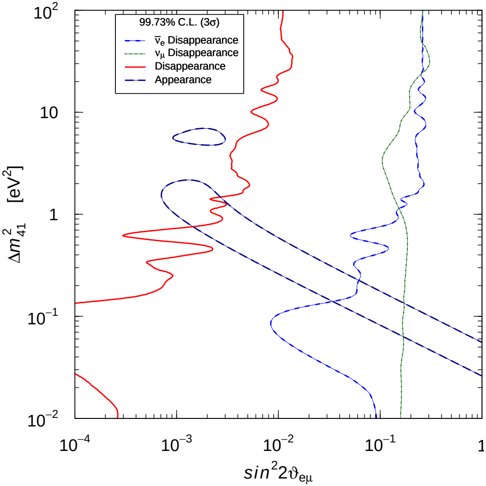
- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694, arXiv:hep-ph/9606411]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247, arXiv:hep-ph/9607372]




3+1

GoF = 50%

PGoF = 0.3%

[Giunti, Laveder, arXiv:1111.1069]

- 3+1: Appearance-Disappearance tension
- 3+2: same tension [Kopp, Maltoni, Schwetz, arXiv:1103.4570], [Giunti, Laveder, arXiv:1107.1452]
- Tension reduced in 3+1+NSI [Akhmedov, Schwetz, JHEP 10 (2010) 115, arXiv:1007.4171]
- No tension in 3+1+CPTV [Barger, Marfatia, Whisnant, PLB 576 (2003) 303] [Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

# Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$ [SOL, KamLAND]

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ [ATM, K2K, MINOS]

$\sin^2\vartheta_{12} \simeq 0.3$ $\sin^2\vartheta_{23} \simeq 0.5$ $\sin^2\vartheta_{13} \simeq 0.02$ [Daya Bay]

$\beta$ & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of $\nu$'s?)

Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

Exp.&Pheno.: Measure CP violation, matter effects, mass hierarchy.

Find absolute mass scale.

Understand anomalies and find if sterile neutrinos exist.