Beyond Three-Neutrino Mixing

Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino
mailto://giunti@to.infn.it
Neutrino Unbound: http://www.nu.to.infn.it

νTURN 2012

Neutrino at the Turning Point

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Three-Neutrino Mixing Paradigm

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, BOREXino
Super-Kamiokande
GALLEX/GNO, SAGE
Homestake
Kamiokande)

Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$$\to \begin{cases} \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{SOL}} \simeq 0.31 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

(Super-Kamiokande
Kamiokande
IMB, MACRO
Soudan-2)

Accelerator
ν_μ disappearance

(K2K, MINOS, T2K)

$$\to \begin{cases} |\Delta m^2_{31}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

Accelerator
$\nu_\mu \to \nu_e$

(T2K, MINOS)

Reactor
$\bar{\nu}_e$ disappearance

(Daya Bay, RENO
Double Chooz)

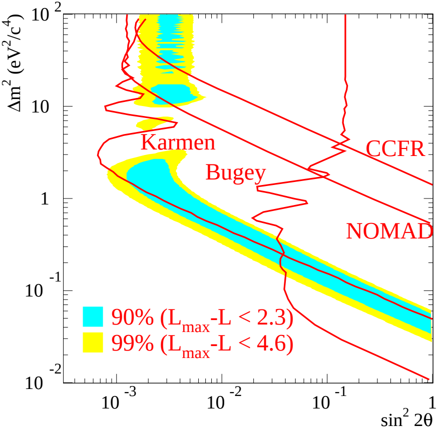
$$\to \sin^2\vartheta_{13} \simeq 0.025$$

LSND and MiniBooNE Antineutrinos

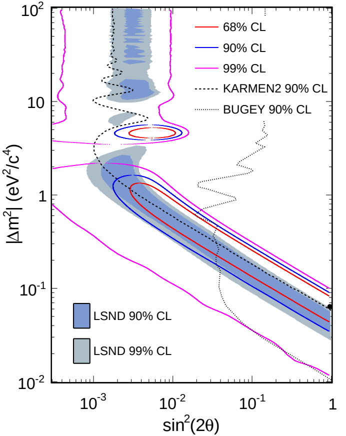
[LSND, PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

[MiniBooNE, PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$L \simeq 30\,\text{m}$ $\quad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



$L \simeq 541\,\text{m}$ $\quad 475\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$

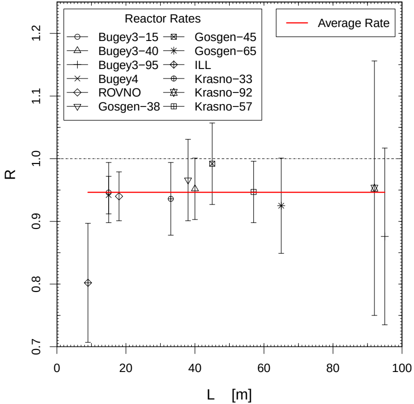


Similar L/E but different L and E $\Longrightarrow$ Oscillations!

$$\bar{\nu}_\mu \to \bar{\nu}_e \qquad \Delta m^2_{\text{SBL}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}})$$

Reactor and Gallium Anomalies

$\bar{\nu}_e$ Disappearance



$$\overline{R} = 0.946 \pm 0.024$$

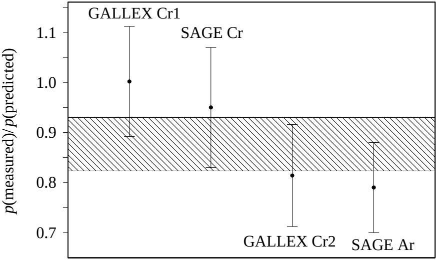
[Mention et al, PRD 83 (2011) 073006]

New Reactor $\bar{\nu}_e$ Fluxes

[Mueller et al, PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]

ν_e Disappearance



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

$$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$$

$$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$$

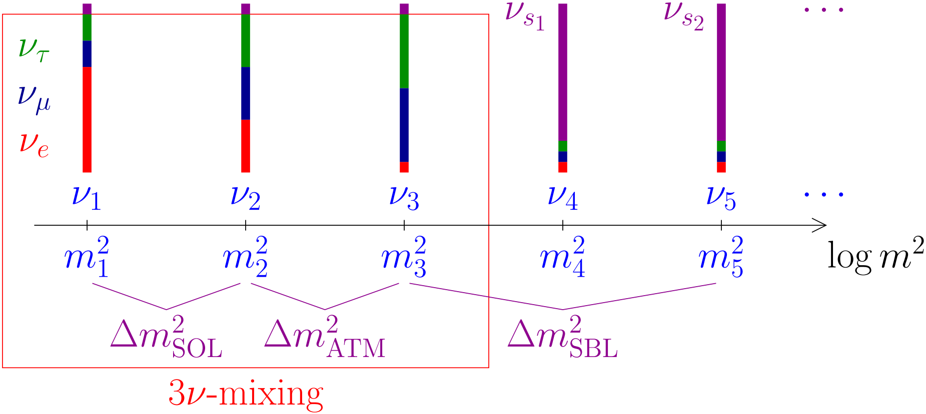
$$\overline{R} = 0.76^{+0.09}_{-0.08}$$

[Giunti, Laveder, PRC 83 (2011) 065504]

Haxton cross section with uncertainties

[Haxton, PLB 431 (1998) 110]

Beyond Three-Neutrino Mixing



Standard Model

- Neutrinos are the only massless fermions
- Neutrinos are the only fermions with only left-handed component ν_L

Extension of the SM: Massive Neutrinos

- Simplest extension: introduce right-handed component ν_R
- Dirac mass $m_{\text{D}}\overline{\nu_R}\nu_L$ + Majorana mass $m_{\text{M}}\overline{\nu_R^c}\nu_R$
- $\nu_{eL}\,,\ \nu_{\mu L}\,,\ \nu_{\tau L}\ +\ \nu_{eR}\,,\ \nu_{\mu R}\,,\ \nu_{\tau R}\quad\Longrightarrow$ 6 massive Majorana neutrinos

Sterile Neutrinos

- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \rightarrow \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

$$\begin{array}{ccccc} & \Delta m^2_{21} \ll & |\Delta m^2_{31}| \ll & |\Delta m^2_{41}| \leq & \ldots \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 & \ldots \\ \nu_e & \nu_\mu & \nu_\tau & \nu_{s_1} & \ldots \end{array}$$

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1\,, \quad |U_{\mu 4}|^2 \ll 1\,, \quad |U_{\tau 4}|^2 \ll 1\,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{ccc|c|} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{array}\right)$$

$\uparrow$

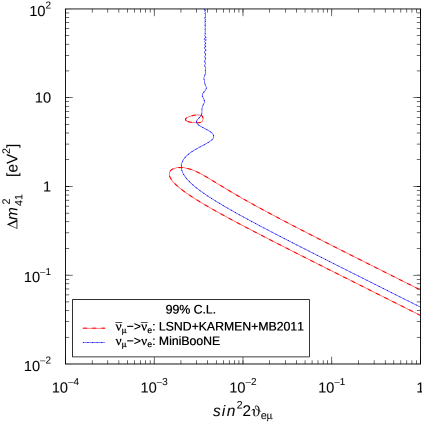
SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

CP Violation?



3+1

GoF = 36%

PGoF = 3.3%

- 3+1 $\Longrightarrow$ Tension between $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ data.
- 3+2 $\Longrightarrow$ CP Violation [Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004], [Maltoni, Schwetz, PRD 76 (2007) 093005], [Karagiorgi et al, PRD 80 (2009) 073001], [Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801], [Giunti, Laveder, PRD 84 (2011) 073008]
- 3+1+NSI $\Longrightarrow$ CP Violation [Akhmedov, Schwetz, JHEP 10 (2010) 115]

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L/4E$$

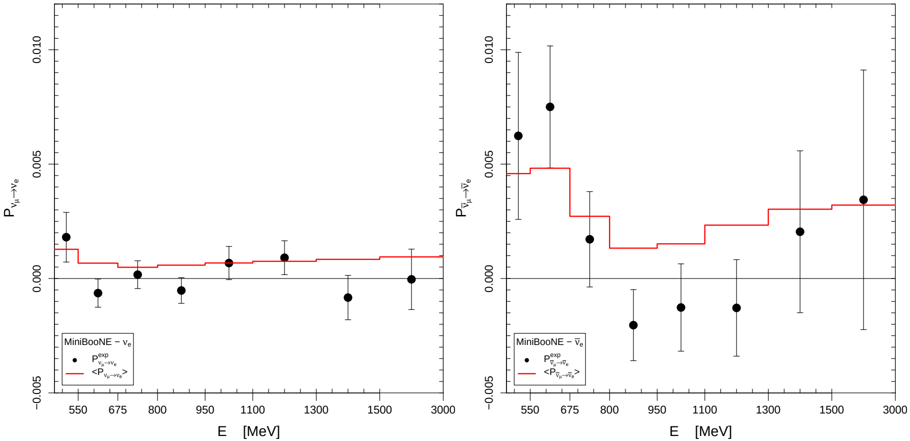
$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$\begin{aligned} P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\phi_{51} \\ & + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin\phi_{41} \sin\phi_{51} \cos(\phi_{54} \overset{(+)}{-} \eta) \end{aligned}$$

$$\begin{aligned} P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = {} & 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2\phi_{41} + |U_{\alpha 5}|^2 \sin^2\phi_{51}) \\ & - 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2\phi_{54} \end{aligned}$$

- Good: CP violation can solve the $\nu_\mu \to \nu_e$ vs $\bar{\nu}_\mu \to \bar{\nu}_e$ tension.
- Bad:
  - More parameters: 7 (vs 3 in 3+1)
  - 2 sterile neutrinos are disfavored by BBN.
  - A large sum of masses is disfavored by CMB+LSS.

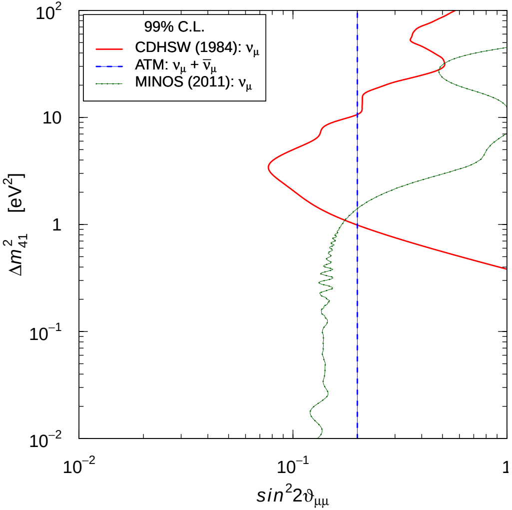
3+2: MiniBooNE $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$



$$
\begin{aligned}
P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2|U_{\mu4}|^2 \sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu5}|^2 \sin^2\phi_{51} \\
& + 4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}| \cos\eta \left[\sin^2\phi_{41} + \sin^2\phi_{51} - \sin^2\phi_{54}\right] \\
& \overset{(+)}{-} 2|U_{e4}U_{\mu4}U_{e5}U_{\mu5}| \sin\eta \left[\sin(2\phi_{41}) - \sin(2\phi_{51}) + \sin(2\phi_{54})\right]
\end{aligned}
$$

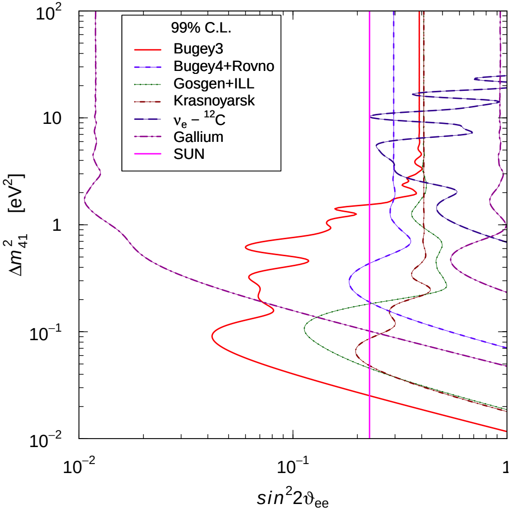
Disappearance Constraints

ν_μ and $\bar{\nu}_\mu$ Disappearance



- ATM constraint on $|U_{\mu 4}|^2$
  [Maltoni, Schwetz, PRD 76 (2007) 093005]
- MINOS constraint on $|U_{\mu 4}|^2$
  [Giunti, Laveder, PRD 84 (2011) 093006]

ν_e and $\bar{\nu}_e$ Disappearance



- New Reactor $\bar{\nu}_e$ Fluxes
  [Mueller et al., PRC 83 (2011) 054615]
  [Mention et al., PRD 83 (2011) 073006]
  [Huber, PRC 84 (2011) 024617]
- KARMEN + LSND
  $\nu_e + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + e^-$
  [Conrad, Shaevitz, PRD 85 (2012) 013017]
  [Giunti, Laveder, PLB 706 (2011) 200]
- SUN&KamLAND + ϑ_{13}
  [Giunti, Li, PRD 80 (2009) 113007]
  [Palazzo, PRD 83 (2011) 113013]
  [Palazzo, PRD 85 (2012) 077301]

SUN&KamLAND + ϑ_{13} bound on $|U_{e4}|^2$

[Giunti, Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301]

3+1 with simplifying assumptions: $U_{\mu 4} = U_{\tau 4} = 0$, no CP violation

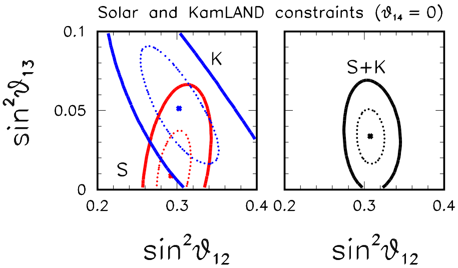
$$U_{e1} = c_{12} c_{13} c_{14} \quad U_{e2} = s_{12} c_{13} c_{14} \quad U_{e3} = s_{13} c_{14} \quad U_{e4} = s_{14}$$

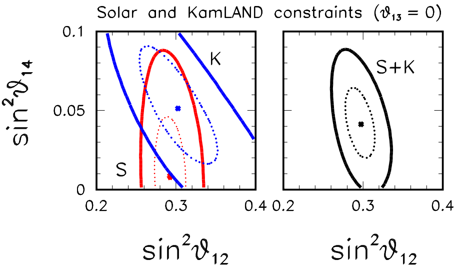
$$U_{s1} = -c_{12} c_{13} s_{14} \quad U_{s2} = -s_{12} c_{13} s_{14} \quad U_{s3} = -s_{13} s_{14} \quad U_{s4} = c_{14}$$

$$P_{\nu_e \to \nu_e} = c_{13}^4 c_{14}^4 P_{\nu_e \to \nu_e}^{2\nu} + s_{13}^4 c_{14}^4 + s_{14}^4$$

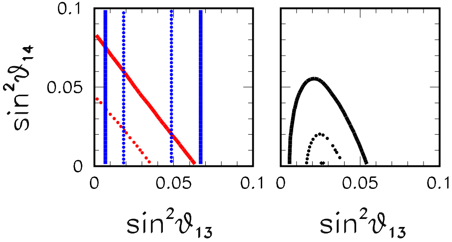
$$P_{\nu_e \to \nu_s} = c_{14}^2 s_{14}^2 \left(c_{13}^4 P_{\nu_e \to \nu_s}^{2\nu} + s_{13}^4 + 1 \right)$$

$$\begin{aligned} V &= c_{13}^2 c_{14}^2 V_{\text{CC}} - c_{13}^2 s_{14}^2 V_{\text{NC}} \\ &= (|U_{e1}|^2 + |U_{e2}|^2) V_{\text{CC}} - (|U_{s1}|^2 + |U_{s2}|^2) V_{\text{NC}} \end{aligned}$$





[Palazzo, PRD 83 (2011) 113013]



[Palazzo, PRD 85 (2012) 077301]

$$\sin^2\vartheta_{13} = 0.025 \pm 0.004$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \lesssim 0.02\,(1\sigma)$$

3+1

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

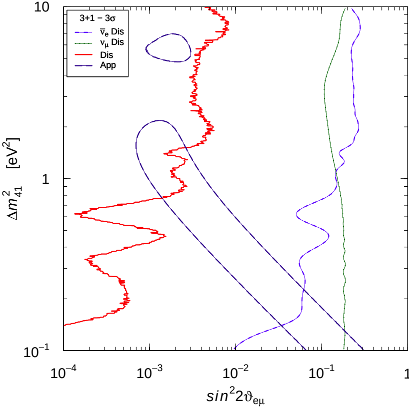
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu} \Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694]

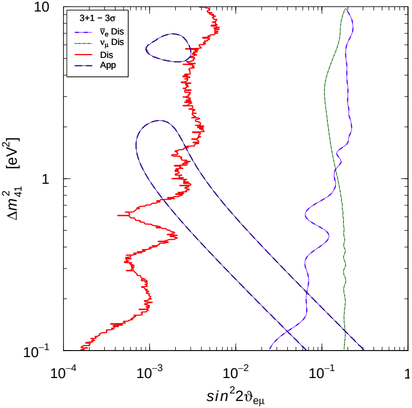
[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247]

$3{+}1$ without SUN&KL$+\vartheta_{13}$



GoF = 45%   PGoF = 0.4%

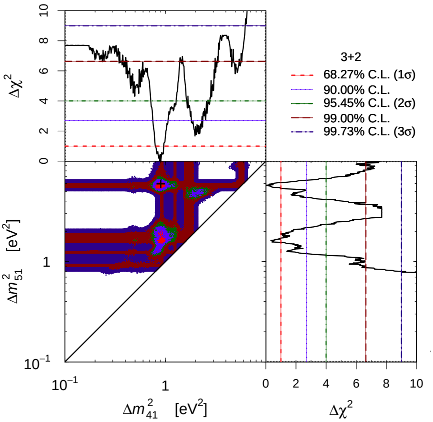
$3{+}1$ with SUN&KL$+\vartheta_{13}$

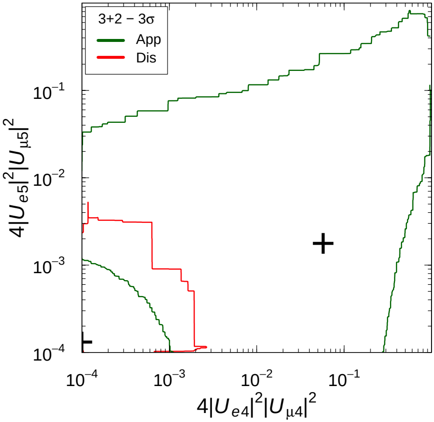


GoF = 38%   PGoF = 0.3%

- 3+1 & 3+2: Appearance-Disappearance tension
- Tension reduced in 3+1+NSI [Akhmedov, Schwetz, JHEP 10 (2010) 115]
- No tension in 3+1+CPTV [Barger, Marfatia, Whisnant, PLB 576 (2003) 303] [Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

3+2 with SUN&KamLAND+ϑ_{13}

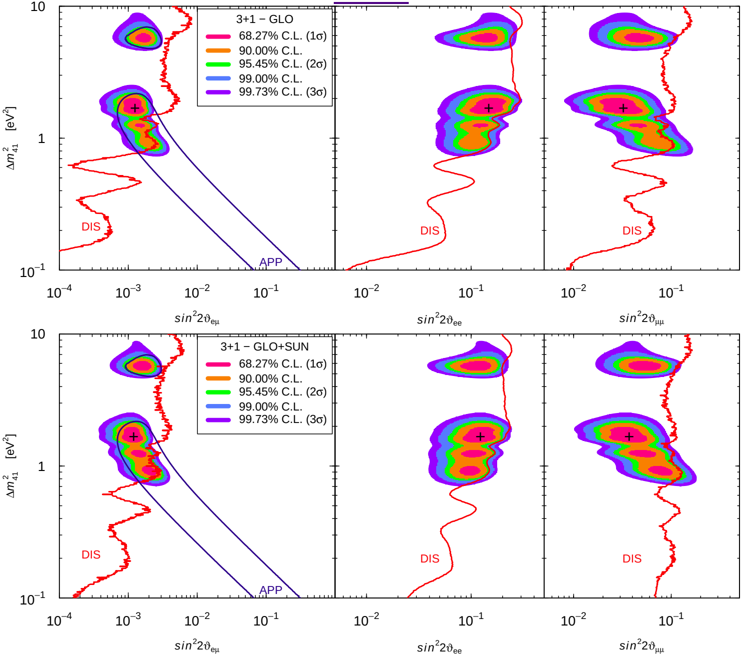




GoF = 46% PGoF = 0.29%

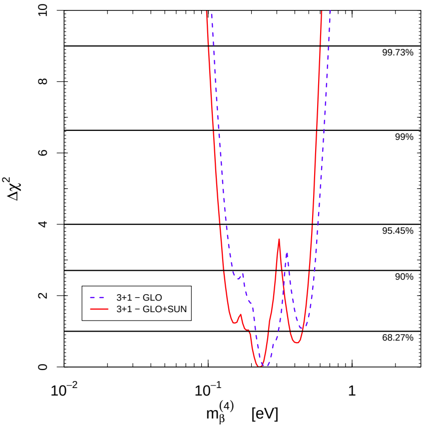
3+2 is preferred to 3+1 only for
CP-violating MiniBooNE neutrino antineutrino difference

3+1


3+1 − GLO
68.27% C.L. (1σ)
90.00% C.L.
95.45% C.L. (2σ)
99.00% C.L.
99.73% C.L. (3σ)
3+1 − GLO+SUN
DIS
APP
Δm^2_{41} [eV^2]
$\sin^2 2\vartheta_{e\mu}$
$\sin^2 2\vartheta_{ee}$
$\sin^2 2\vartheta_{\mu\mu}$
10
1
10^{-1}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

Testable Implications

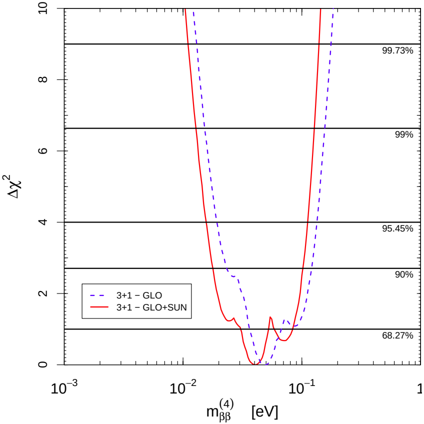
β Decay



$$m_\beta = \sqrt{\sum_k |U_{ek}|^2 m_k^2}$$

$$m_\beta^{(4)} = |U_{e4}| \sqrt{\Delta m_{41}^2}$$

$(\beta\beta)_{0\nu}$ Decay



$$m_{\beta\beta} = \left|\sum_k U_{ek}^2\, m_k\right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

Cosmology

- N_s = number of thermalized sterile neutrinos (not necessarily integer)
- CMB and LSS in ΛCDM: $N_s = 1.3 \pm 0.9 \quad m_s < 0.66\,\text{eV} \quad (95\%\ \text{C.L.})$

  [Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301]

  $$N_s = 1.61 \pm 0.92 \qquad m_s < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

  [Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023]

  $N_s = 1.12^{+0.86}_{-0.74} \quad (95\%\ \text{C.L.})$ [Archidiacono, Calabrese, Melchiorri, PRD 84 (2011) 123008]

- BBN: $\begin{cases} N_s = 0.22 \pm 0.59 \\ N_s = 0.64^{+0.40}_{-0.35} \\ N_s \leq 1 \text{ at } 95\%\ \text{C.L.} \end{cases}$

  [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313]

  [Izotov, Thuan, ApJL 710 (2010) L67]

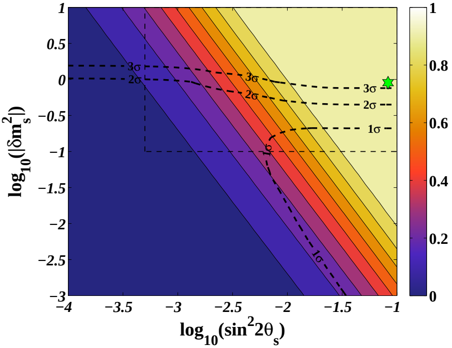
  [Mangano, Serpico, PLB 701 (2011) 296]

- CMB+LSS+BBN: $N_s = 0.85^{+0.39}_{-0.56} \quad (95\%\ \text{C.L.})$

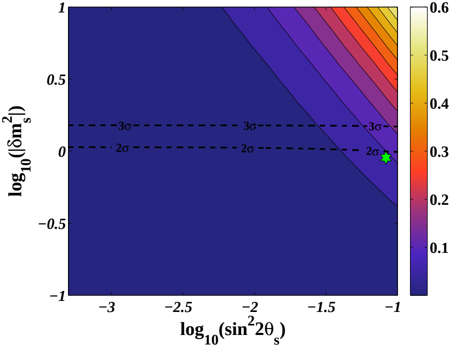
  [Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034]

Thermalisation of Sterile Neutrinos

[Hannestad, Tamborra, Tram, arXiv:1204.5861]



lepton asymmetry = 0



lepton asymmetry = 10^{-2}

Color: thermalized iso-N_s contours

Star: 3+1 best fit [Giunti, Laveder, PLB 706 (2011) 200]

Dashed lines: CMB+LSS constraints [Hamann et al, PRL 105 (2010) 181301]

Conclusions

- Suggestive LSND, MiniBooNE, Reactor and Gallium indications in favor of short-baseline neutrino oscillations $\Longrightarrow$ sterile neutrinos
- Recent measurement of $|U_{e3}|^2$ and SUN+KamLAND data restrict range of $|U_{e4}|^2$ allowed by reactor short-baseline data for $\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2$.
- 3+1 Neutrino Mixing:
  - No CP violation $\Longrightarrow$ Neutrinos vs Antineutrinos tension
  - Appearance vs Disappearance tension
  - Tension with standard cosmology (or large lepton asymmetry)
- 3+2 Neutrino Mixing:
  - CP violation $\Longrightarrow$ no Neutrinos vs Antineutrinos tension
  - Appearance vs Disappearance tension
  - More tension with standard cosmology (or large lepton asymmetry)
- Simpler 3+1 Neutrino Mixing may be enough (Occam's Razor).
- New short-baseline neutrino oscillation experiments are needed!