Neutrino Physics

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http://www.nu.to.infn.it/slides/2012/giunti-120611-phd-to-3.pdf

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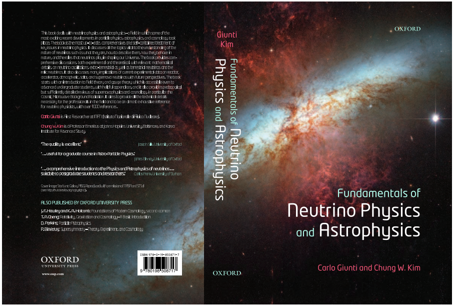

C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part I: Theory of Neutrino Masses and Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses

Part II: Neutrino Oscillations

Part III: Phenomenology

Part I

Phenomenology

Solar Neutrinos and KamLAND

- Solar Neutrinos and KamLAND
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  - Standard Solar Model (SSM)
  - Homestake
  - Gallium Experiments
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  - GALLEX: GALLium EXperiment
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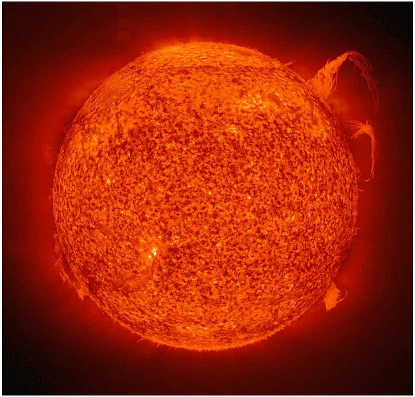
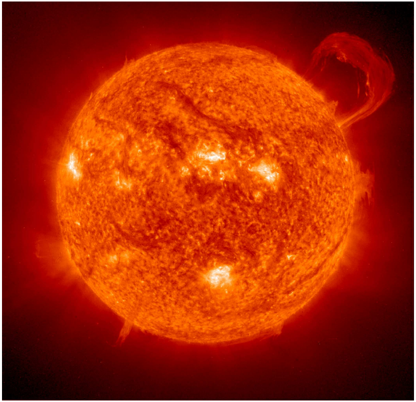
  - Sterile Neutrinos in Solar Neutrino Flux?
  - Determination of Solar Neutrino Fluxes
  - Details of Solar Neutrino Oscillations
  - BOREXino

The Sun





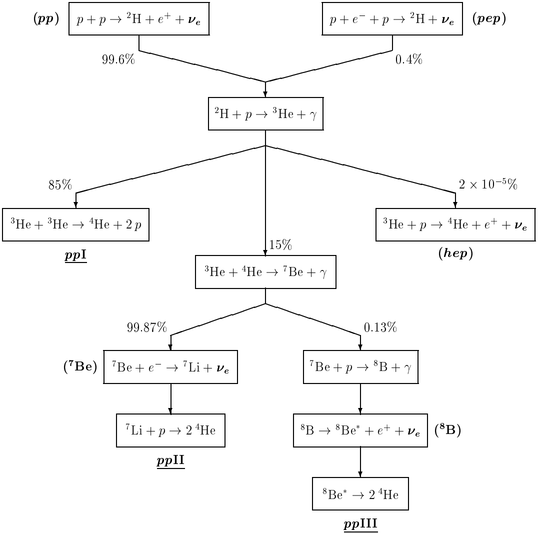
Extreme ultraviolet Imaging Telescope (EIT) 304 Å images of the Sun

emission in this spectral line (He II) shows the upper chromosphere

at a temperature of about 60,000 K

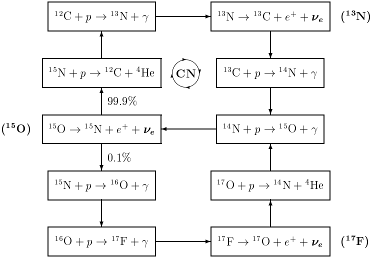
[The Solar and Heliospheric Observatory (SOHO), http://sohowww.nascom.nasa.gov/]

Standard Solar Model (SSM)



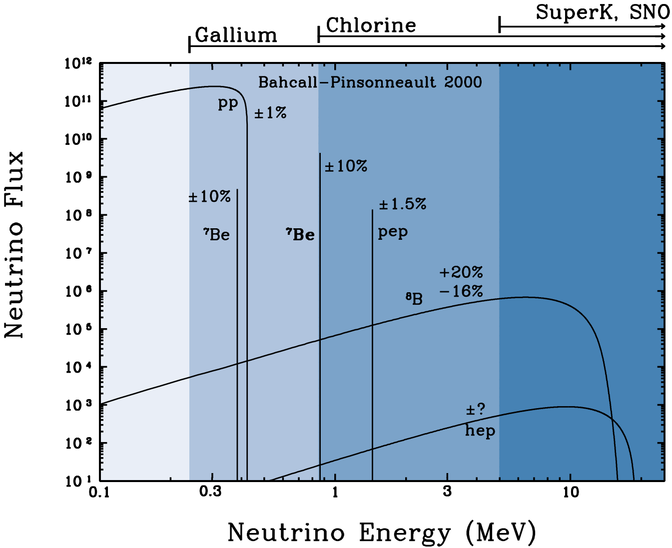
pp chain and CNO cycle

$$4\,p + 2\,e^- \to {}^4\mathrm{He} + 2\,\nu_e + 26.731\ \mathrm{MeV}$$

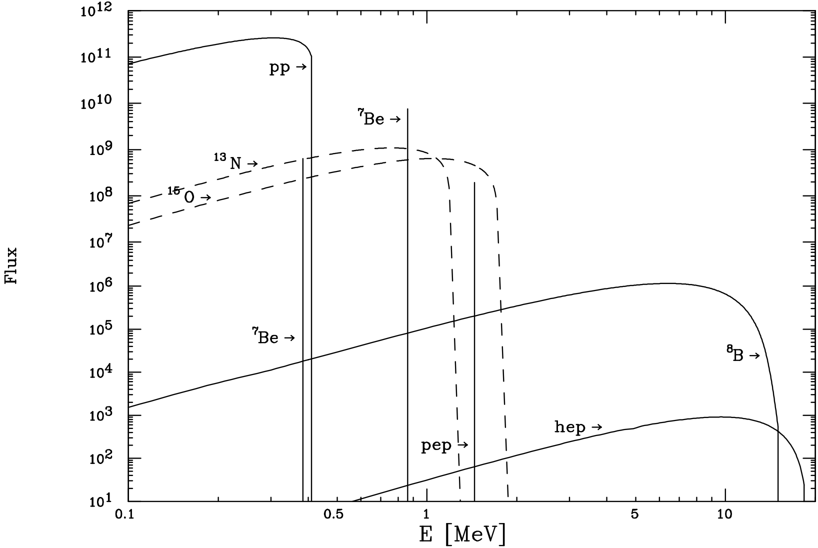


Bahcall SSMs

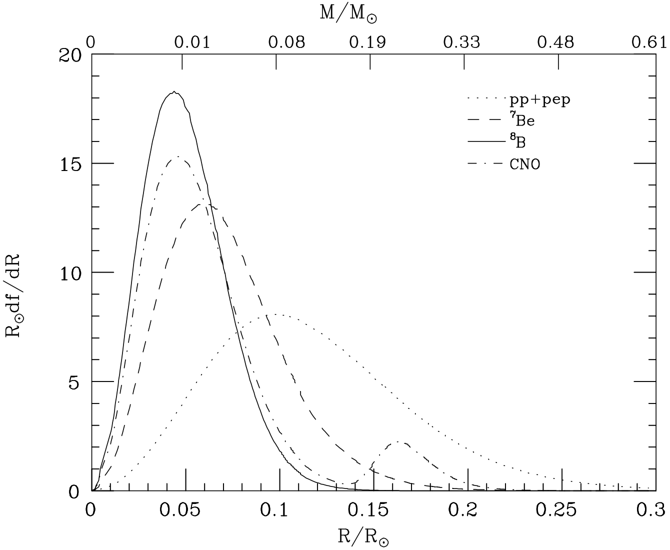
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



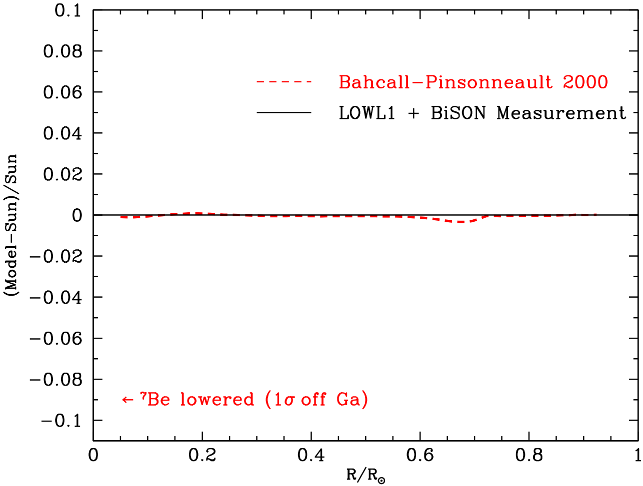
[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between $0.05\,R_\odot$ and $0.95\,R_\odot$ and is 0.08% for the deep interior region, $r < 0.25\,R_\odot$, in which neutrinos are produced

Homestake

$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$ [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e. $\Rightarrow \Phi_\mu \simeq 4\,\mathrm{m}^{-2}\,\mathrm{day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

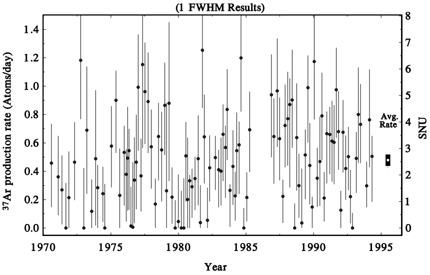
615 tons of tetrachloroethylene (C_2Cl_4), 2.16×10^{30} atoms of ^{37}Cl (133 tons)

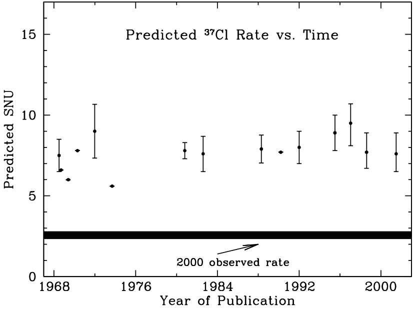
energy threshold: $E_{\mathrm{th}}^{\mathrm{Cl}} = 0.814\,\mathrm{MeV} \Longrightarrow {}^{8}\mathrm{B}\,,\ {}^{7}\mathrm{Be}\,,\ pep\,,\ hep\,,\ {}^{13}\mathrm{N}\,,\ {}^{15}\mathrm{O}\,,\ {}^{17}\mathrm{F}$

1970–1994, 108 extractions $\Longrightarrow$ $\dfrac{R_{\mathrm{Cl}}^{\mathrm{exp}}}{R_{\mathrm{Cl}}^{\mathrm{SSM}}} = 0.34 \pm 0.03$ [APJ 496 (1998) 505]

$R_{\mathrm{Cl}}^{\mathrm{exp}} = 2.56 \pm 0.23\,\mathrm{SNU}$ $R_{\mathrm{Cl}}^{\mathrm{SSM}} = 7.6^{+1.3}_{-1.1}\,\mathrm{SNU}$

1 SNU $= 10^{-36}$ events atom^{-1} s^{-1}





Gallium Experiments

SAGE, GALLEX, GNO

radiochemical experiments

$$\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$$ [Kuzmin (1965)]

threshold: $E_{\mathrm{th}}^{\mathrm{Ga}} = 0.233\,\mathrm{MeV} \Longrightarrow pp\,,\ {}^{7}\mathrm{Be}\,,\ {}^{8}\mathrm{B}\,,\ pep\,,\ hep\,,\ {}^{13}\mathrm{N}\,,\ {}^{15}\mathrm{O}\,,\ {}^{17}\mathrm{F}$

SAGE+GALLEX+GNO $\Longrightarrow$ $\dfrac{R_{\mathrm{Ga}}^{\mathrm{exp}}}{R_{\mathrm{Ga}}^{\mathrm{SSM}}} = 0.56 \pm 0.03$

$R_{\mathrm{Ga}}^{\mathrm{exp}} = 72.4 \pm 4.7\,\mathrm{SNU}$ $R_{\mathrm{Ga}}^{\mathrm{SSM}} = 128^{+9}_{-7}\,\mathrm{SNU}$

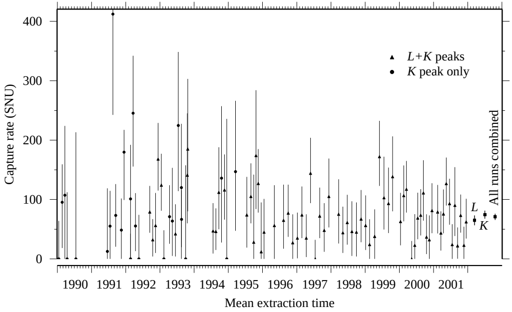
SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic ^{71}Ga, 2000 m deep, 4700 m.w.e. $\Rightarrow \Phi_{\mu} \simeq 2.6\,\text{m}^{-2}\,\text{day}^{-1}$

detector test: ^{51}Cr Source: $R = 0.95^{+0.11}_{-0.10}{}^{+0.06}_{-0.05}$ [PRC 59 (1999) 2246]

1990 – 2001 $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.54 \pm 0.05$ [astro-ph/0204245]



GALLEX: GALLium EXperiment

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

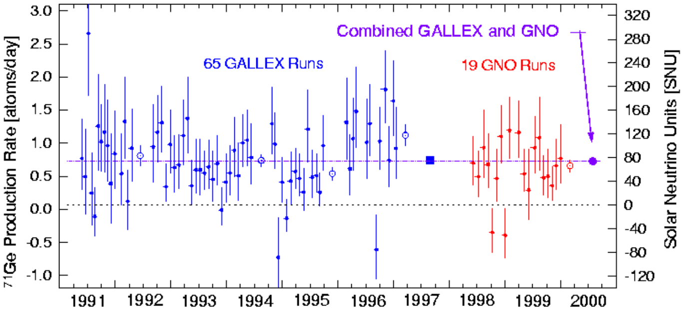
30.3 tons of gallium in 101 tons of gallium chloride ($GaCl_3$-HCl) solution

May 1991 – Jan 1997 $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{GALLEX}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.61 \pm 0.06$ [PLB 477 (1999) 127]

GNO: Gallium Neutrino Observatory

continuation of GALLEX: 30.3 tons of gallium

May 1998 – Jan 2000 $\Longrightarrow$ $$\frac{R_{\text{Ga}}^{\text{GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.51 \pm 0.08$$ [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

Kamiokande

water Cherenkov detector $\nu + e^- \to \nu + e^-$

Sensitive to ν_e, ν_μ, ν_τ, but $\sigma(\nu_e) \simeq 6\,\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75\,\text{MeV} \Longrightarrow {}^8\text{B}$, *hep*

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{SSM}}} = 0.55 \pm 0.08$$

[PRL 77 (1996) 1683]

Super-Kamiokande

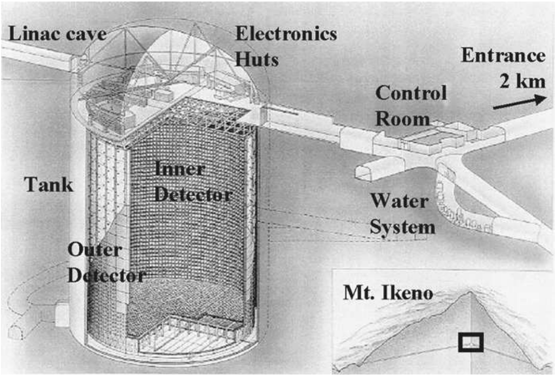
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}$, *hep*

1996 – 2001 (1496 days)

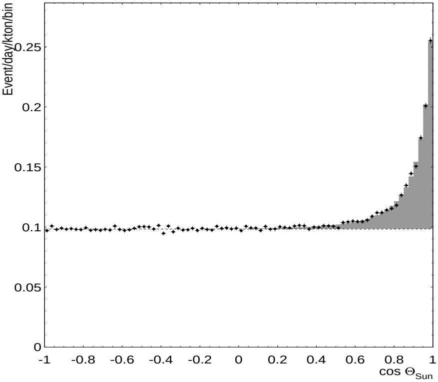
$$\frac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{SSM}}} = 0.465 \pm 0.015$$ [SK, PLB 539 (2002) 179]



the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

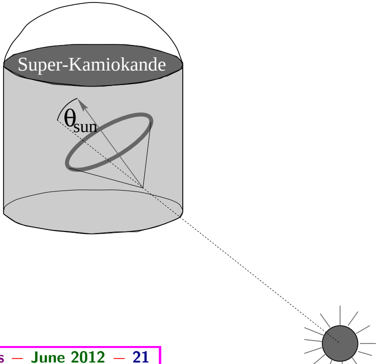
Super-Kamiokande $\cos\theta_{\text{sun}}$ distribution



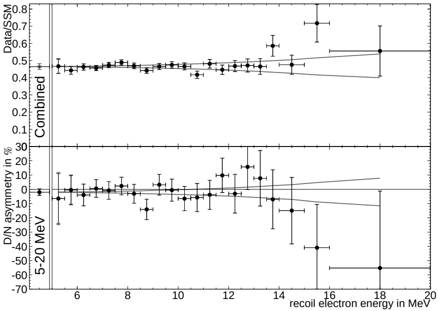
[Smy, hep-ex/0208004]

the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

the peak at $\cos\theta_{\text{sun}} = 1$ is due to solar neutrinos

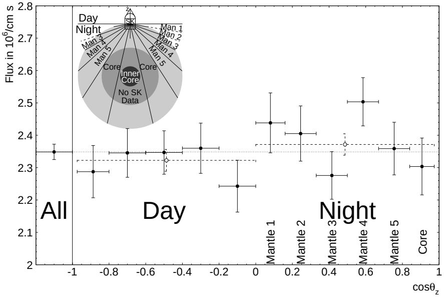


Super-Kamiokande energy spectrum normalized to BP2000 SSM



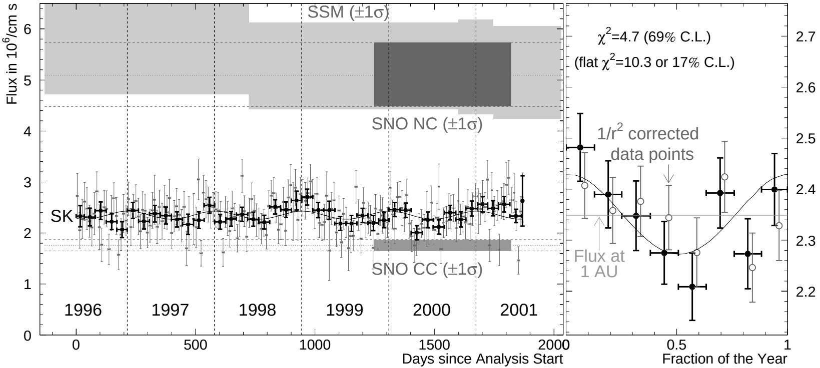
Day-Night asymmetry as a function of energy

solar zenith angle (θ_z) dependence of Super-Kamiokande data



[Smy, hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D_2O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

$$
\begin{array}{ll}
\text{CC:} & \nu_e + d \to p + p + e^- \\
\text{NC:} & \nu + d \to p + n + \nu \\
\text{ES:} & \nu + e^- \to \nu + e^-
\end{array}
$$

$$
\left.
\begin{array}{l}
\text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2\,\text{MeV} \\
\text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2\,\text{MeV} \\
\text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0\,\text{MeV}
\end{array}
\right\} \Longrightarrow {}^8\text{B}, \textit{hep}
$$

D_2O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.01 \pm 0.13$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.31 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.03 \pm 0.09$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

$$\Phi_{\nu_e}^{\text{SNO}} = 1.76 \pm 0.11 \times 10^6\,\text{cm}^{-2}\,\text{s}^{-1}$$

$$\Phi_{\nu_\mu,\nu_\tau}^{\text{SNO}} = 5.41 \pm 0.66 \times 10^6\,\text{cm}^{-2}\,\text{s}^{-1}$$

SNO solved

solar neutrino problem

$\Downarrow$

Neutrino Physics

(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

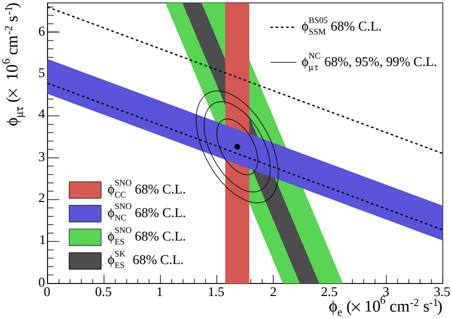
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

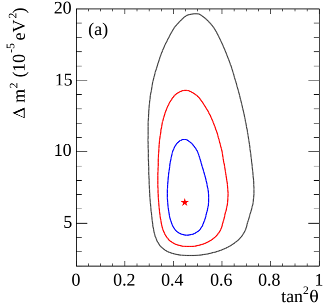
$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2$$

$$\tan^2\vartheta \simeq 0.45$$





[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km
- 6.7% of flux from one reactor at 88 km
- 79% of flux from 26 reactors at 138–214 km
- 14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$

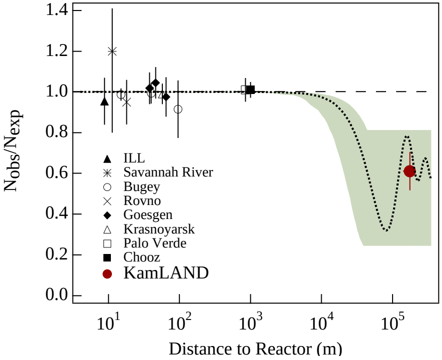
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$

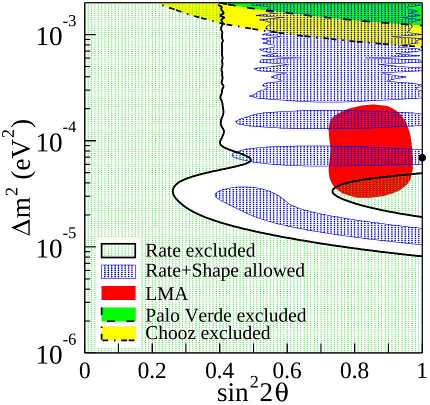
observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

99.95% C.L. evidence of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



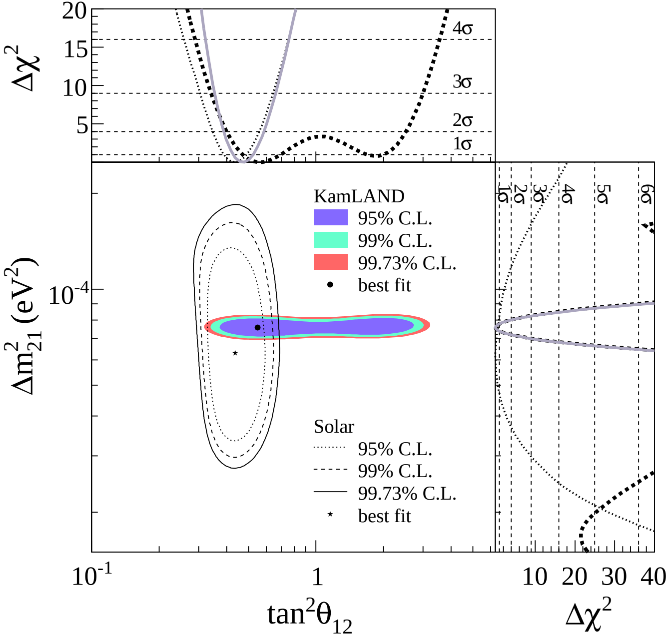


Shade: 95% C.L. LMA

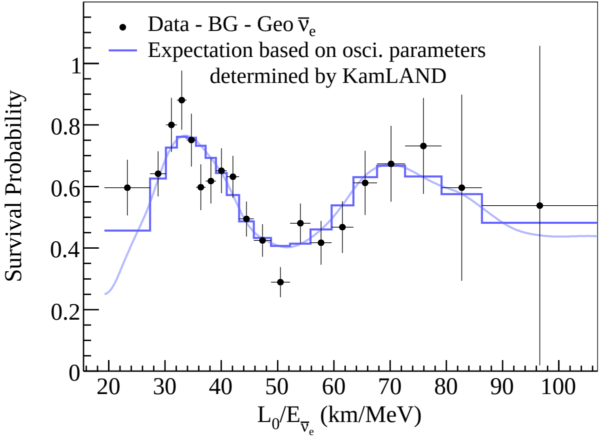
95% C.L.

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

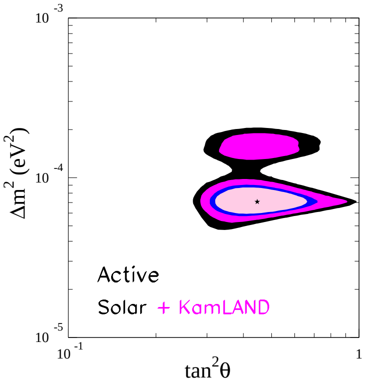


[KamLAND, PRL 100 (2008) 221803]



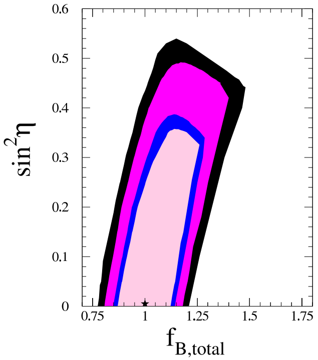
[KamLAND, PRL 100 (2008) 221803]

Sterile Neutrinos in Solar Neutrino Flux?



90%, 95%, 99%, 99.73% (3σ) C.L.

[Bahcall, Gonzalez-Garcia, Pena-Garay, JHEP 0302 (2003) 009]



$$\nu_e \to \cos\eta\,\nu_a + \sin\eta\,\nu_s$$

$$\sin^2\eta < 0.52\,(3\sigma)$$

$$f_{B,\text{total}} = \frac{\Phi_{^8B}}{\Phi_{^8B}^{SSM}} = 1.00 \pm 0.06$$

Determination of Solar Neutrino Fluxes

[Bahcall, Peña-Garay, hep-ph/0305159]

fit of solar and KamLAND neutrino data with fluxes as free parameters

+ luminosity constraint

$$\sum_r \alpha_r \Phi_r = K_\odot \quad (r = pp, pep, hep, {}^7\text{Be}, {}^8\text{B}, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F})$$

$$K_\odot \equiv \mathcal{L}_\odot / 4\pi (1\text{a.u.})^2 = 8.534 \times 10^{11}\, \text{MeV}\, \text{cm}^{-2}\, \text{s}^{-1}$$

solar constant

$$\Delta m^2 = 7.3^{+0.4}_{-0.6}\, \text{eV}^2 \qquad \tan^2\vartheta = 0.42^{+0.08}_{-0.06} \, \left({}^{+0.39}_{-0.19}\right)$$

$$\frac{\Phi_{^8\text{B}}}{\Phi^{\text{SSM}}_{^8\text{B}}} = 1.01^{+0.06}_{-0.06} \, \left({}^{+0.22}_{-0.17}\right)$$

moderate uncertainty

will improve with new SNO NC data (salt phase)

$$\frac{\Phi_{^7\text{Be}}}{\Phi^{\text{SSM}}_{^7\text{Be}}} = 0.97^{+0.28}_{-0.54} \, \left({}^{+0.85}_{-0.97}\right)$$

large uncertainty

needs ^{7}Be experiment (KamLAND, Borexino?)

$$\frac{\Phi_{pp}}{\Phi^{\text{SSM}}_{pp}} = 1.02^{+0.02}_{-0.02} \, \left({}^{+0.07}_{-0.07}\right)$$

small uncertainty

CNO luminosity: $\mathcal{L}_{\text{CNO}} / \mathcal{L}_\odot = 0.0^{+2.8}_{-0.0} \, \left({}^{+7.3}_{-0.0}\right)$

[Bahcall, Gonzalez-Garcia, Peña-Garay, PRL 90 (2003) 131301]

Details of Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{\text{d}\ln A}{\text{d}x}\right|_{\text{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2}EG_{\text{F}}N_e^{\text{c}}\exp\left(-\frac{x}{x_0}\right) \quad \Longrightarrow \quad \left|\frac{\text{d}\ln A}{\text{d}x}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15}\,\text{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad \Longrightarrow \quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad \Longrightarrow \quad P_{\text{c}} \ll 1 \quad \Longrightarrow \quad \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2}\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta_{\mathrm{M}}^{0} = \frac{\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, hep-ph/0305159]

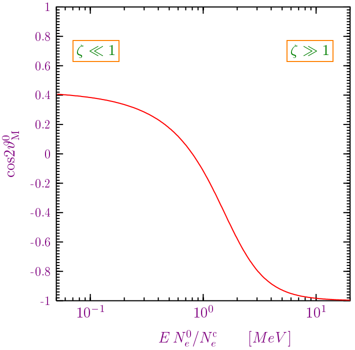
$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right)\left(\frac{N_e^0}{N_e^c}\right)$$

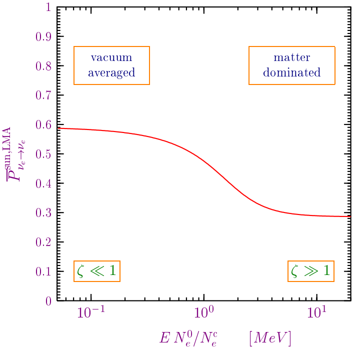
$$\zeta \ll 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \vartheta \implies \overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} \simeq 1 - \tfrac{1}{2}\sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \pi/2 \implies \overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} \simeq \sin^2\vartheta$$

matter dominated survival probability

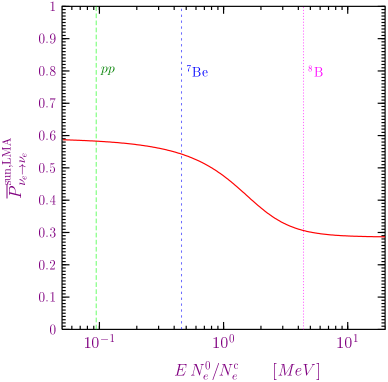




$\langle E \rangle_{pp} \simeq 0.27\,\mathrm{MeV}$, $\langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot$ $\Longrightarrow$ $\langle E\,N_e^0/N_e^c \rangle_{pp} \simeq 0.094\,\mathrm{MeV}$

$E_{^7\mathrm{Be}} \simeq 0.86\,\mathrm{MeV}$, $\langle r_0 \rangle_{^7\mathrm{Be}} \simeq 0.06\,R_\odot$ $\Longrightarrow$ $\langle E\,N_e^0/N_e^c \rangle_{^7\mathrm{Be}} \simeq 0.46\,\mathrm{MeV}$

$\langle E \rangle_{^8\mathrm{B}} \simeq 6.7\,\mathrm{MeV}$, $\langle r_0 \rangle_{^8\mathrm{B}} \simeq 0.04\,R_\odot$ $\Longrightarrow$ $\langle E\,N_e^0/N_e^c \rangle_{^8\mathrm{B}} \simeq 4.4\,\mathrm{MeV}$



each neutrino experiment is mainly sensitive to one flux

each neutrino experiment is mainly sensitive to ϑ

accurate *pp* experiment can improve determination of ϑ

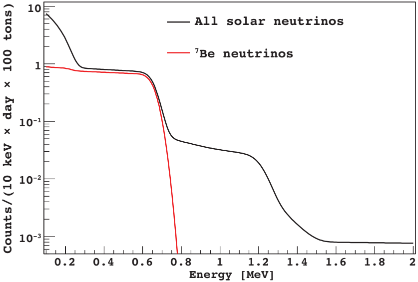
[Bahcall, Peña-Garay, hep-ph/0305159]

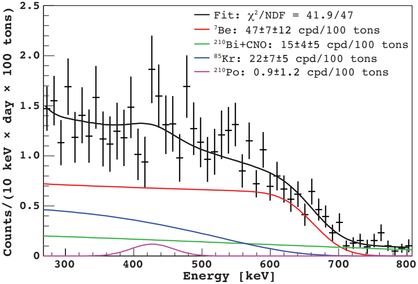
BOREXino

[BOREXino, arXiv:0708.2251]

Real-time measurement of ^{7}Be solar neutrinos (0.862 MeV)

$$\nu + e \to \nu + e \qquad E = 0.862\,\text{MeV} \quad \Longrightarrow \quad \sigma_{\nu_e} \simeq 5.5\,\sigma_{\nu_\mu,\nu_\tau}$$





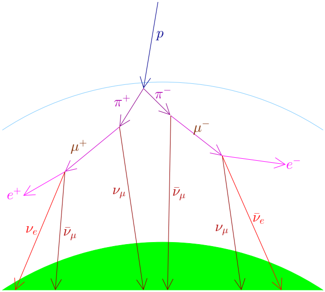
$$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad n_{\text{exp}} = 47 \pm 7 \pm 12\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$$

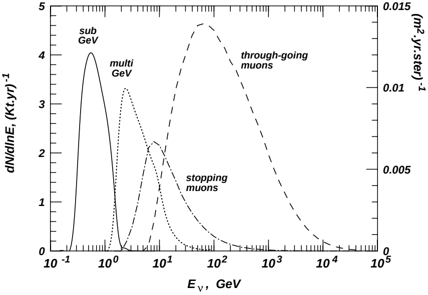
$$n_{\text{the}}^{\text{osc}} = 49 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad (n_{\text{the}}^{\text{no-osc}} - n_{\text{exp}})/\Delta n \simeq 1.9$$

Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
  - Atmospheric Neutrinos
  - Super-Kamiokande Up-Down Asymmetry
  - Fit of Super-Kamiokande Atmospheric Data
  - Kamiokande, Soudan-2, MACRO and MINOS
  - K2K
  - MINOS
  - Sterile Neutrinos in Atmospheric Neutrino Flux?
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses

Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

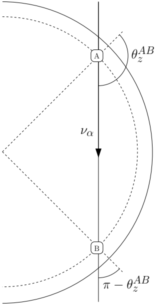
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB}) \qquad \phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z) = \phi^{(A)}_{\nu_\alpha}(\pi - \theta_z)$$

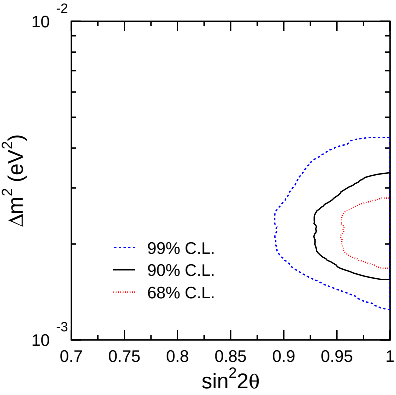
(December 1998)

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$\underline{\nu_\mu \to \nu_\tau}$

Best Fit: $\left\{ \begin{array}{l} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{array} \right.$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20\text{events/yr}$
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\text{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$$\boxed{N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}}$$

$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

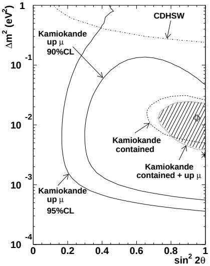
Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

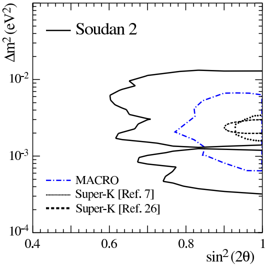
$L \simeq 732\,\text{km}$ $\langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

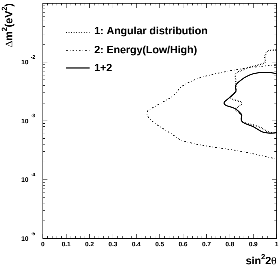
Kamiokande, Soudan-2, MACRO and MINOS



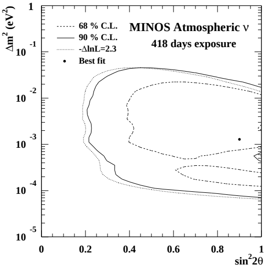
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



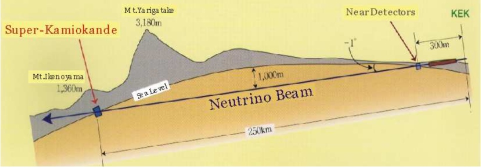
[MACRO, hep-ex/0304037]



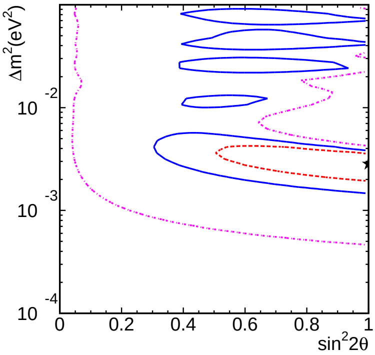
[MINOS, hep-ex/0512036]

K2K

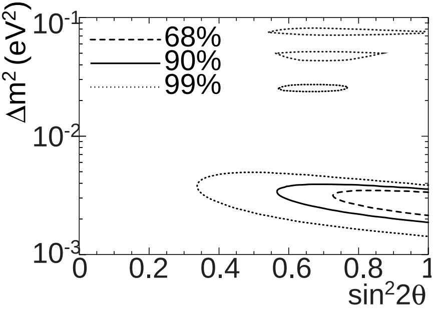
confirmation of atmospheric allowed region (June 2002)



KEK to Kamioka
(Super-Kamiokande)
250 km
$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]



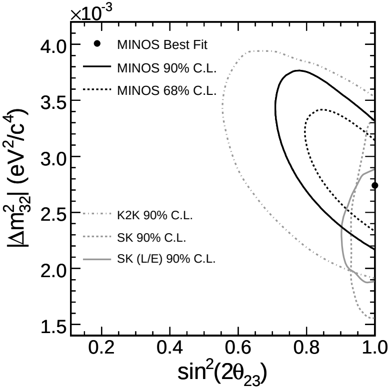
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

http://www-numi.fnal.gov/

Near Detector: 1 km

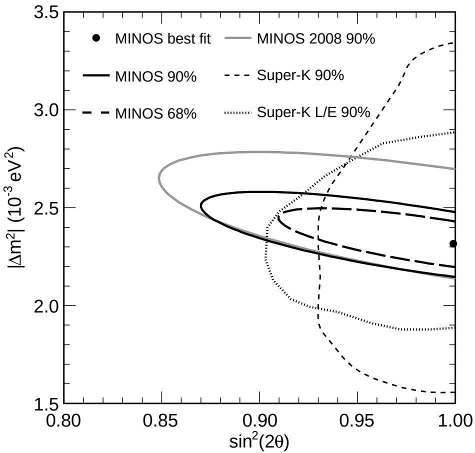


$$\nu_\mu \to \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \text{ @ } 68\%\,CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]



$$|\Delta m^2_{31}| = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta_{23} > 0.90 \quad (90\%\ \text{C.L.})$$

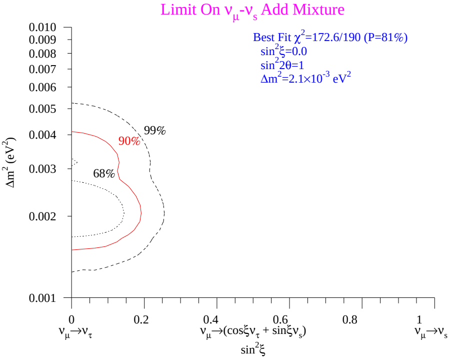
[arXiv:1103.0340v1]

Sterile Neutrinos in Atmospheric Neutrino Flux?

Nature of atmospheric Oscillation

Mode	Best fit	$\Delta\chi2$	σ
ν_μ-ν_τ	$\sin^2 2\theta=1.00$; $\Delta m^2=2.5\text{x}10^{-3}\text{eV}^2$	0.0	0.0
ν_μ-ν_e	$\sin^2 2\theta=0.97$; $\Delta m^2=5.0\text{x}10^{-3}\text{eV}^2$	79.3	8.9
ν_μ-ν_s	$\sin^2 2\theta=0.96$; $\Delta m^2=3.6\text{x}10^{-3}\text{eV}^2$	19.0	4.4
LxE	$\sin^2 2\theta=0.90$; $\alpha=5.3\text{x}10^{-4}$	67.1	8.2
ν_μ Decay	$\cos^2\theta=0.47$; $\alpha=3.0\text{x}10^{-3}\text{eV}^2$	81.1	9.0
ν_μ Decay to ν_s	$\cos^2\theta=0.33$; $\alpha=1.1\text{x}10^{-2}\text{eV}^2$	14.1	3.8

[Smy (SK), Moriond 2002]



[Nakaya (SK), hep-ex/0209036]

Phenomenology of Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
  - SOL and ATM Neutrino Oscillations
  - Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses

SOL and ATM Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Reactor
$\bar{\nu}_e$ disappearance

Homestake
Kamiokande
GALLEX/GNO & SAGE
Super-Kamiokande
SNO
BOREXino

(KamLAND)

$$\to \begin{cases} \Delta m^2_{\text{SOL}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{SOL}} \simeq 0.31 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Accelerator
ν_μ disappearance

Kamiokande
IMB
Super-Kamiokande
MACRO
Soudan-2

(K2K, MINOS, T2K)

$$\to \begin{cases} \Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{ATM}} \simeq 0.50 \end{cases}$$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\quad \vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

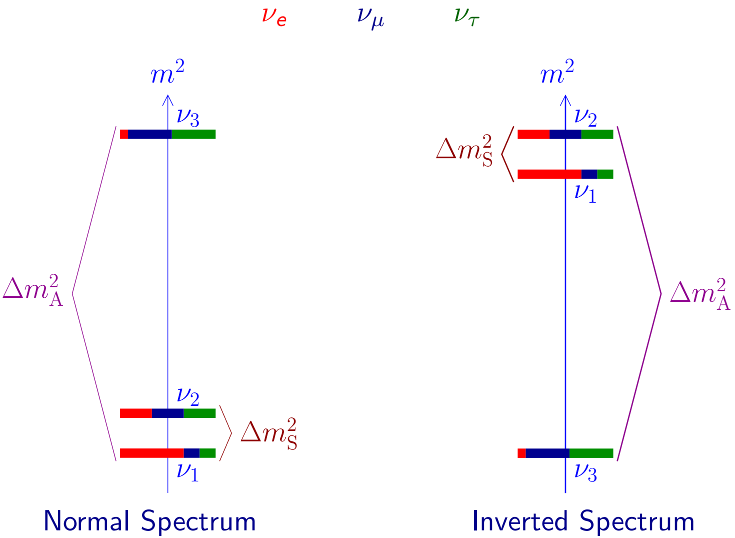
three flavor fields: ν_e, ν_μ, ν_τ

three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5} \, \text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \times 10^{-3} \, \text{eV}^2$$



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

$$\begin{array}{l} \text{SOL} \to \\ U = \end{array} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

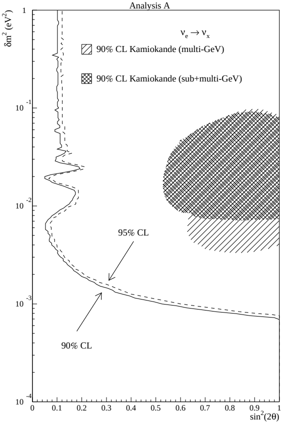
$\uparrow$ ATM & LBL

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1-|U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$

[Bilenky, Giunti, PLB 444 (1998) 379]



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau 3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

Effective ATM and LBL Oscillation Probability in Vacuum

$$P_{\nu_\alpha\to\nu_\beta} = \left|\sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t}\right|^2 * \left|e^{iE_1 t}\right|^2$$

$$= \left|\sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t}\right|^2 \to \left|\sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i\frac{\Delta m_{k1}^2 L}{2E}\right)\right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \Delta m_{31}^2 \to \Delta m^2$$

$$P_{\nu_\alpha\to\nu_\beta} = \left|U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left|\delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3}\left[1 - \exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha 3}|^2 \left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2\right)\left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2\right)\sin^2\frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & \boxed{U_{e3}} \\ U_{\mu 1} & U_{\mu 2} & \boxed{U_{\mu 3}} \\ U_{\tau 1} & U_{\tau 2} & \boxed{U_{\tau 3}} \end{pmatrix}$$

LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \simeq 4|U_{e3}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu3}|^2 \left(1 - |U_{\mu3}|^2\right)$$

$$|U_{\mu3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - \sin^2 2\vartheta_{\mu\mu}}\right)$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2|U_{\mu3}|^2$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\text{eV}^2 \qquad |\Delta m^2_{31}| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2\vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

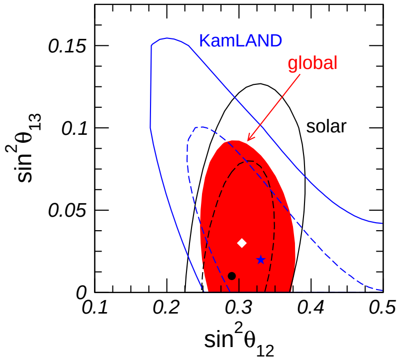
$$\sin^2\vartheta_{13} < 0.035 \quad (90\%\ \text{C.L.})$$

[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]

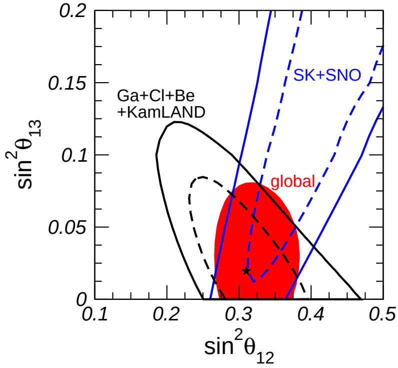
Hint of $\sin^2\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]



[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]



[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

$$P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \simeq \begin{cases} \left(1-\sin^2\vartheta_{13}\right)^2 \left(1-0.5\sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1-\sin^2\vartheta_{13}\right)^2 \sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

Measurements of ϑ_{13}

$0.03\,(0.04) < \sin^2 2\vartheta_{13} < 0.28\,(0.34)$	T2K, arXiv:1106.2822 (90% CL)
$\sin^2 2\vartheta_{13} = 0.041^{+0.047}_{-0.031}\,(0.079^{+0.071}_{-0.053})$	MINOS, arXiv:1108.0015
$\sin^2\vartheta_{13} = 0.022 \pm 0.013$	Double Chooz, arXiv:1112.6353
$\sin^2\vartheta_{13} = 0.024 \pm 0.004$	Daya Bay, arXiv:1203.1669
$\sin^2\vartheta_{13} = 0.029 \pm 0.006$	RENO, arXiv:1204.0626

$\sin^2\vartheta_{13} > 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

CP Violation

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to small ϑ_{13}
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

Mass Hierarchy

- $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $\cos 2\vartheta_{13} = \dfrac{2EV}{\Delta m^2_{13}}$

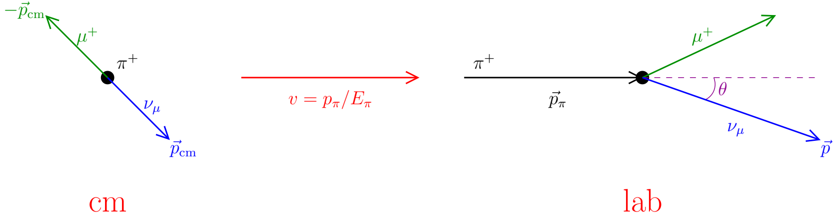
  Requires $\Delta m^2_{13} > 0$

- $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $\cos 2\vartheta_{13} = -\dfrac{2EV}{\Delta m^2_{13}}$

  Requires $\Delta m^2_{13} < 0$

Off-Axis Experiments

high-intensity WB beam
detector shifted by a small angle from axis of beam
almost monochromatic neutrino energy



$$E_{\rm cm} = p_{\rm cm} = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 29.79\,\text{MeV}$$

$$\gamma = \left(1 - v^2\right)^{-1/2} = E_\pi / m_\pi \gg 1 \qquad \begin{cases} E = \gamma\left(E_{\rm cm} + v\, p_{\rm cm}^z\right) \\ p^z = \gamma\left(v\, E_{\rm cm} + p_{\rm cm}^z\right) \end{cases}$$

$$p^z = p\,\cos\theta \qquad \Longrightarrow \qquad E = \frac{E_{\rm cm}}{\gamma\left(1 - v\,\cos\theta\right)}$$

$$\cos\theta \simeq 1 - \theta^2/2 \qquad \text{and} \qquad v \simeq 1$$

$$E = \frac{E_{\text{cm}}}{\gamma\,(1 - v\cos\theta)} \simeq \frac{\gamma\,(1+v)}{1 + \gamma^2\theta^2 v\,(1+v)\,/2}\,E_{\text{cm}} \simeq \frac{2\gamma}{1+\gamma^2\theta^2}\,E_{\text{cm}}$$

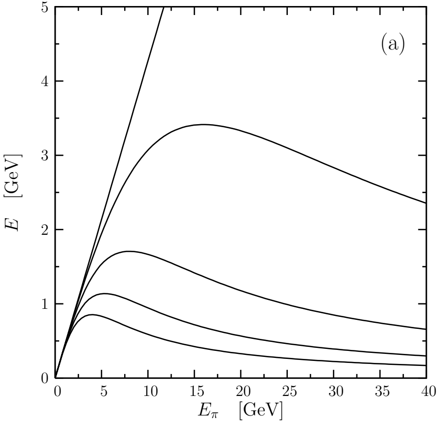
$$E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi}{1+\gamma^2\,\theta^2} = \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi\;m_\pi^2}{m_\pi^2 + E_\pi^2\,\theta^2}$$

- $\theta = 0 \quad \Longrightarrow \quad E \propto E_\pi$ WB beam
- $E_\pi\theta \gg m_\pi \quad \Longrightarrow \quad E \propto \dfrac{m_\pi^2}{E_\pi\,\theta^2}$ high-energy π^+ give low-energy ν_μ

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{1-\gamma^2\,\theta^2}{\left(1+\gamma^2\,\theta^2\right)^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq 0 \quad \text{for} \quad \theta = \gamma^{-1} = \frac{m_\pi}{E_\pi} \quad \Longrightarrow \quad E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{m_\pi}{2\theta} \simeq \frac{29.79\,\text{MeV}}{\theta}$$

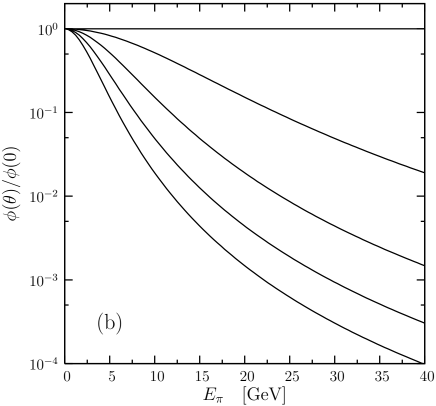
off-axis angle $\theta \simeq m_\pi / \langle E_\pi \rangle \implies E \simeq \dfrac{29.79\,\text{MeV}}{\theta}$



$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

- E can be tuned on oscillation peak $E_{\text{peak}} = \Delta m^2 L / 2\pi$
- small $E \implies$ short $L_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2} \implies$ sensitivity to small values of Δm^2

$$\frac{\phi(\theta)}{\phi(0)} = \frac{1}{4}\left(\frac{2}{1+\gamma^2\,\theta^2}\right)^2$$



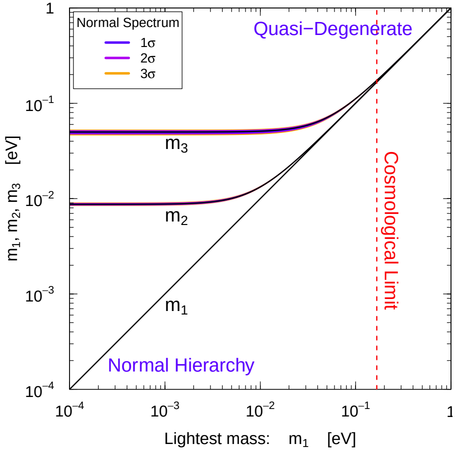
$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

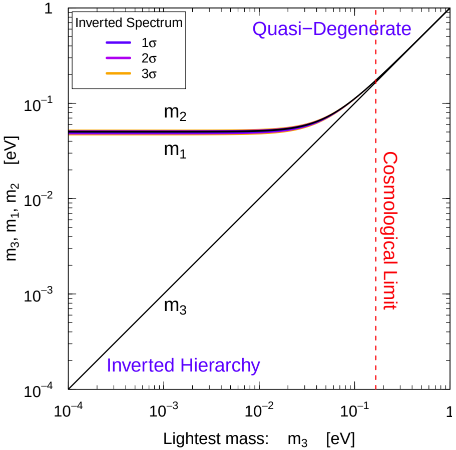
flux suppression requires superbeam

Absolute Scale of Neutrino Masses

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
  - Mass Hierarchy or Degeneracy?
  - Tritium Beta-Decay
  - Neutrinoless Double-Beta Decay
  - Cosmological Bound on Neutrino Masses

Mass Hierarchy or Degeneracy?





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{S}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{A}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{A}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{A}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\mathrm{A}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

Tritium Beta-Decay

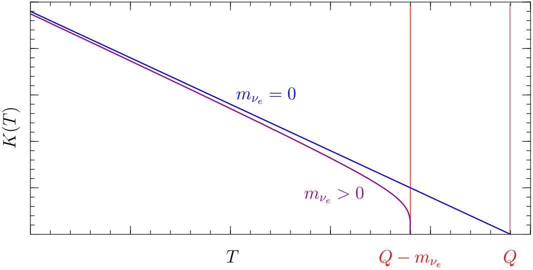
$^{3}\text{H} \to {}^{3}\text{He} + e^{-} + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

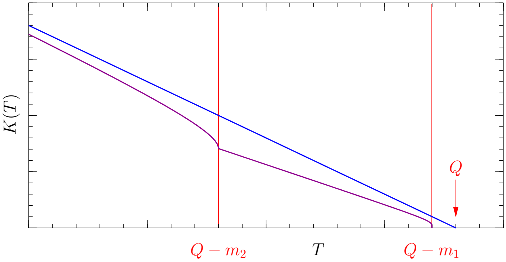
future: KATRIN

[www.katrin.kit.edu]

start data taking 2014

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses ($m_k \ll Q - T$)

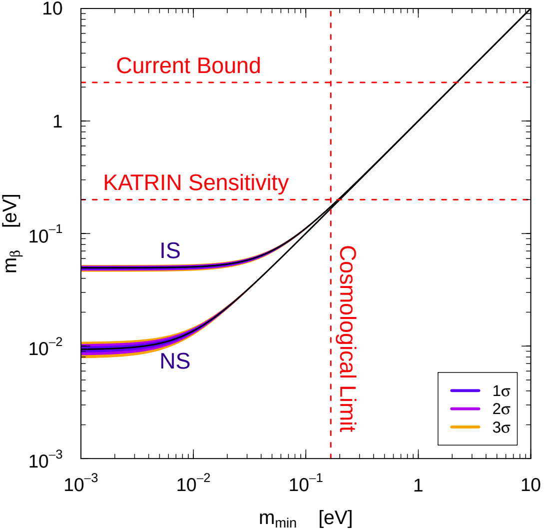
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$

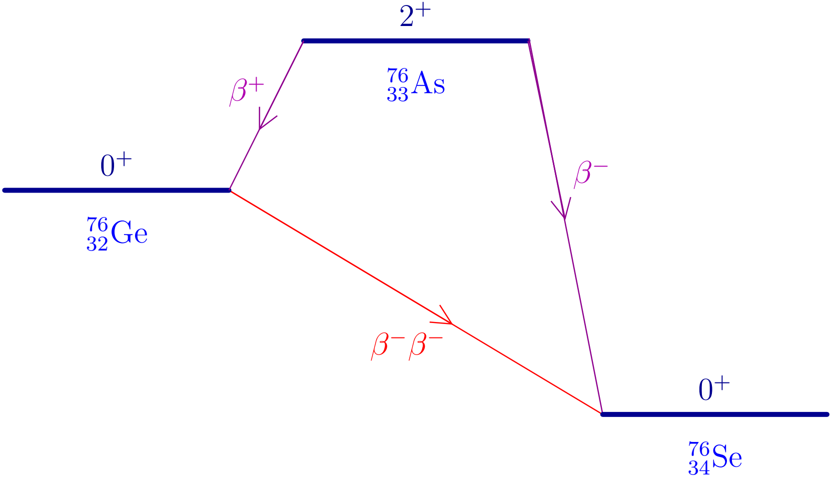


- Quasi-Degenerate:
  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$
- Inverted Hierarchy:
  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_{\rm A}^2 \simeq \Delta m_{\rm A}^2$$
- Normal Hierarchy:
  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_{\rm S}^2 + s_{13}^2 \Delta m_{\rm A}^2$$
  $$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\text{eV}^2$$
- If $\quad m_\beta \lesssim 4 \times 10^{-2}\,\text{eV}$

$$\Downarrow$$

Normal Spectrum

Neutrinoless Double-Beta Decay



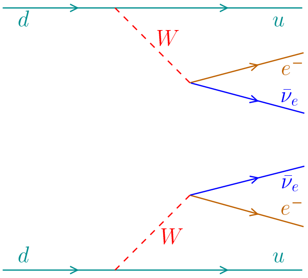
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

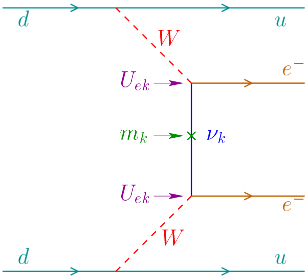


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$

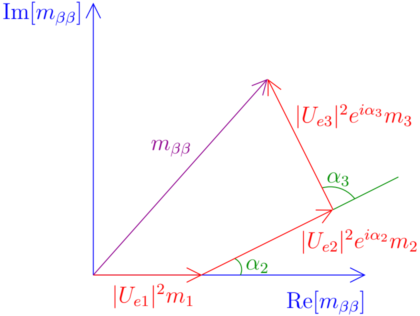


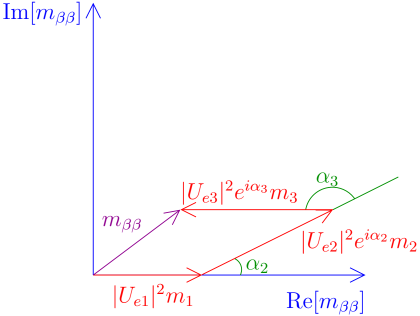
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$





Experimental Bounds

CUORICINO (^{130}Te) [AP 34 (2011) 822]

$$\boxed{T^{0\nu}_{1/2} > 2.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

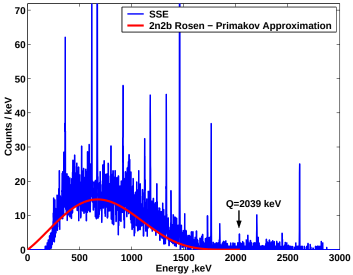
$$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

Experimental Positive Indication

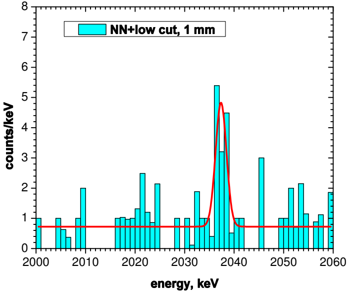
[Klapdor et al., MPLA 16 (2001) 2409]

$$T_{1/2}^{0\nu} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\,\mathrm{y}$$

6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

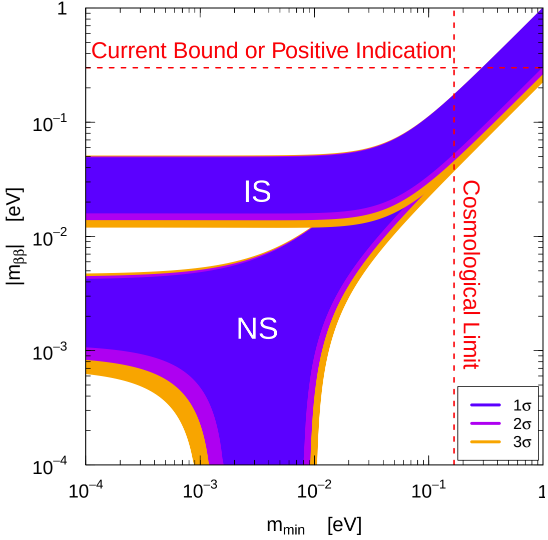
$$|m_{\beta\beta}| = 0.32 \pm 0.03\,\mathrm{eV}$$

[MPLA 21 (2006) 1547]

if confirmed, very exciting: Majorana ν and large mass scale

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Positive indication: tension with cosmology
- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\mathrm{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\mathrm{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\mathrm{A}}}|$$

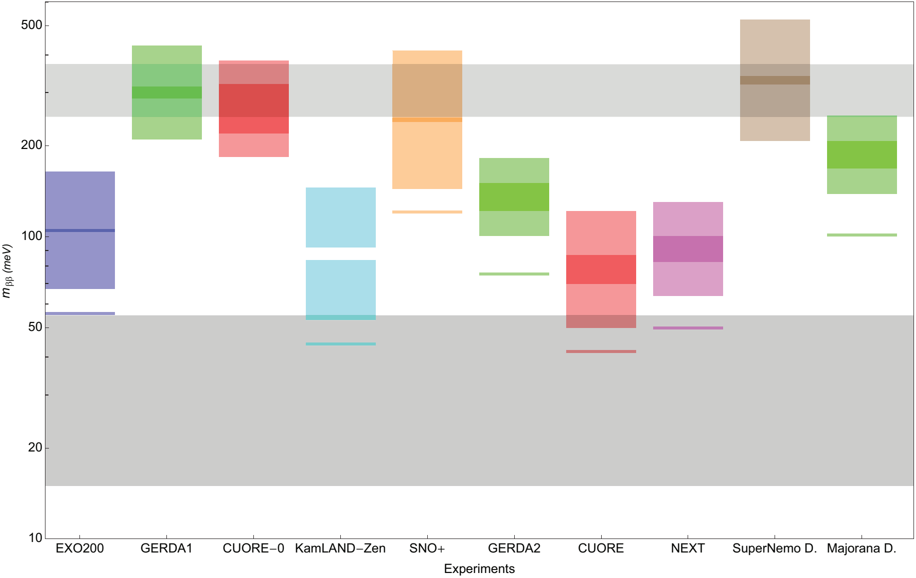
$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\mathrm{eV}$$

- If $|m_{\beta\beta}| \lesssim 10^{-2}\,\mathrm{eV}$

$\Downarrow$

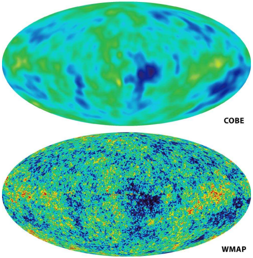
Normal Spectrum

New Experiments

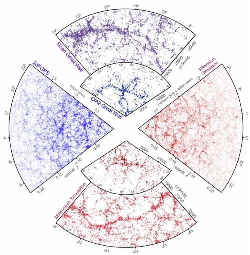


[Gomez-Cadenas, Martin-Albo, Mezzetto, Monrabal, Sorel, Riv. NC 35 (2012) 29]

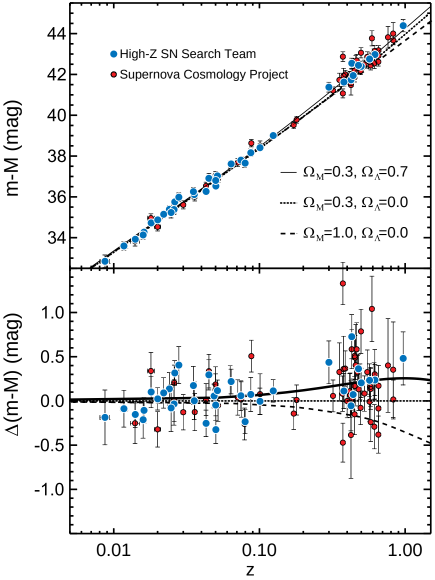
Cosmological Bound on Neutrino Masses



[WMAP, http://map.gsfc.nasa.gov]

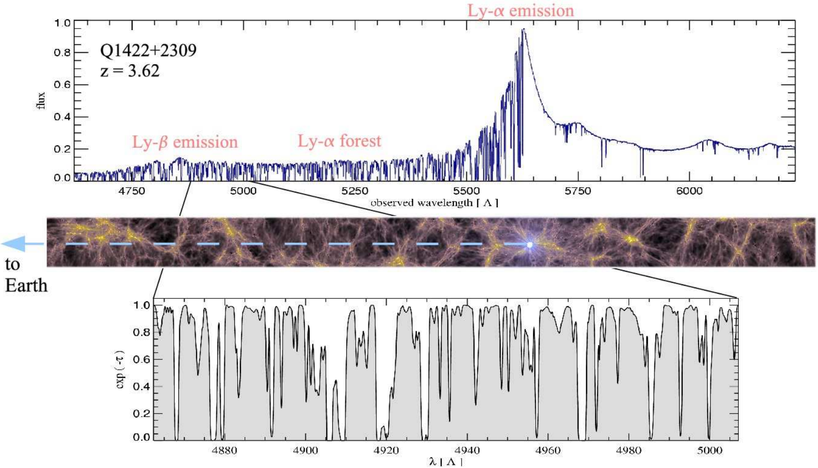


[Springel, Frenk, White, Nature 440 (2006) 1137]



[http://cfa-www.harvard.edu/supernova/]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_{\alpha}^{0} = 1215.67$ Å, $\lambda_{\beta}^{0} = 1025.72$ Å, $\lambda_{\gamma}^{0} = 972.54$ Å

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1 + z_q)\lambda_{\beta}^{0}, (1 + z_q)\lambda_{\alpha}^{0}]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

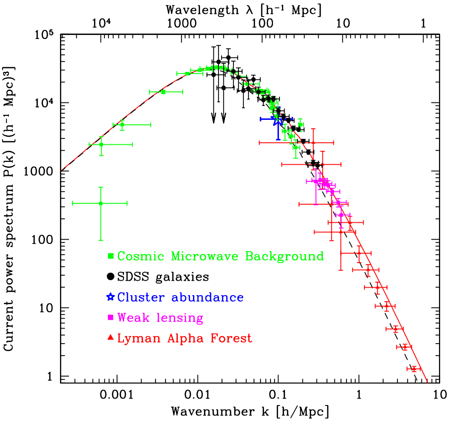
density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\, h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$

$\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_M^0 = 0.28\,,\ h = 0.72\,,\ \Omega_B^0/\Omega_M^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\,\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$

$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$

$$\Omega_\nu h^2 < 0.0076 \quad (95\%\ \text{conf.}) \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$

$-0.0179 < \Omega_0 - 1 < 0.0081 \quad (95\%\ \text{C.L.})$

$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad (95\%\ \text{C.L.}) \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

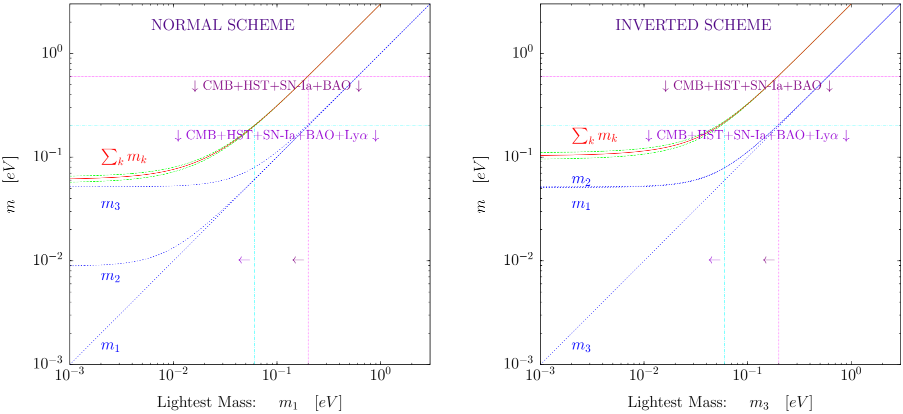
Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ.

$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO}$$

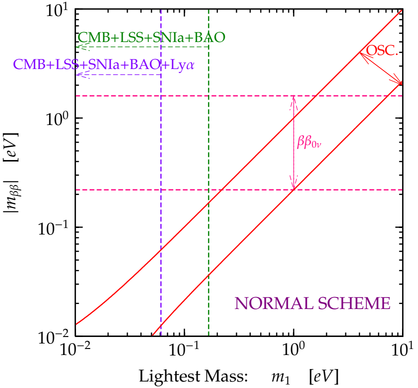
$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$

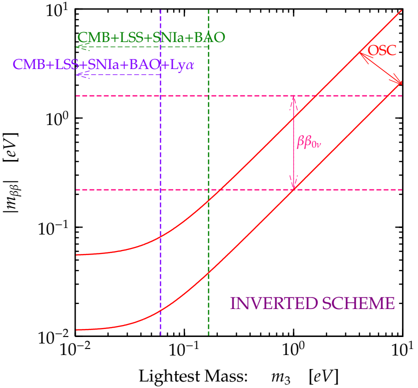


$$\text{FUTURE: IF} \quad \sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow \text{NORMAL HIERARCHY}$$

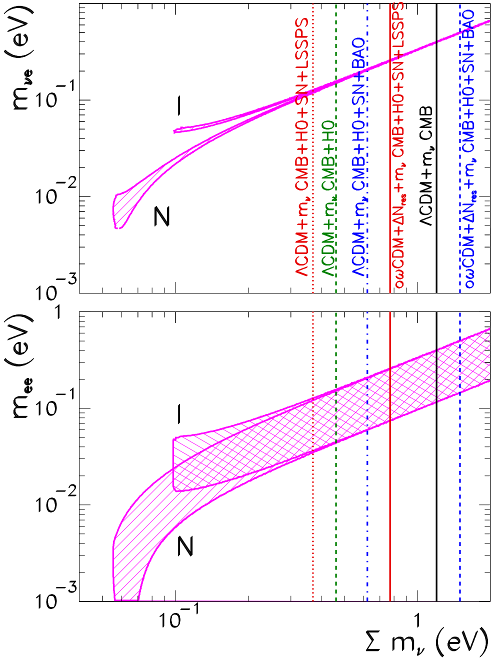
Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ ($\sim 3\sigma$ range)

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]





tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal



95% allowed regions (2 dof)

95% upper bounds on $\sum m_\nu$

[Gonzalez-Garcia, Maltoni, Salvado, JHEP08 (2010) 117, arXiv:1006.3795v2]

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$ [SOL, KamLAND]

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ [ATM, K2K, MINOS]

$\sin^2\vartheta_{12} \simeq 0.3$ $\sin^2\vartheta_{23} \simeq 0.5$ $\sin^2\vartheta_{13} \simeq 0.02$ [Daya Bay]

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

Exp.&Pheno.: Measure CP violation, matter effects, mass hierarchy.

Find absolute mass scale.

Find if sterile neutrinos exist.